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Strategies to Fight Ad-sponsored Rivals

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Strategies to Fight Ad-sponsored Rivals*

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Abstract

We analyze the optimal strategy of a high-quality incumbent that faces a low-quality ad-sponsored competitor. In addition to competing through adjustments of tactical variables such as price or advertising intensity, we allow the incumbent to consider changes in its business model. We consider four alternative business models, two *pure* models (subscription-based and ad-sponsored) and two *mixed* models that are hybrids of the two pure models. We show that the optimal response to an ad-sponsored rival often entails business model reconfigurations, a phenomenon that we dub "competing through business models." We also find that when there is an ad-sponsored entrant, the incumbent is more likely to prefer to compete through a pure, rather than a mixed, business model because of cannibalization and endogenous vertical differentiation concerns. We discuss how our study helps improve our understanding of notions of strategy, business model, and tactics in the field of strategy.

1 Introduction

Ad-sponsored business models appear to be increasingly prevalent in today's economy. Many companies choose to finance themselves using ad revenues and offer their products or services free to consumers. These products and services range from newspapers to software applications and from television programs to online search.

The emergence of ad-sponsored entrants in various industries poses significant threats to the incumbents in these markets whose business models are often based on subscriptions or fees charged to their customers. For example, newly-launched music service providers such as *Imeem* give users free access to ad-supported, streaming music files, while industry leaders such as Apple's *iTunes* music services and RealNetwork's *Rhapsody* are fee or subscription-based. NetZero offered free ad-sponsored dial-up internet access and attracted many users away from AOL's subscription-based dial-up service.

Ad-sponsored business models are not limited to Internet-related industries. Free ad-sponsored broadcast television channels have been competing with subscription-based cable channels such as HBO for many years. And *Metro*, the world's largest newspaper measured by circulation, is free and ad-sponsored. It is published in more than 100 cities in 18 countries.[1] In each city it enters, it competes with local newspapers sold at positive prices.

Faced with the threat from ad-sponsored entrants, incumbents must choose strategies to respond. The New York Times Co., which owns *The Boston Globe*, bought a 49 percent stake in *Metro Boston* in 2006. In September 2007, the company also stopped charging readers to access certain articles on NYTimes.com and began to use ads exclusively to finance its online news services. *Recoletos*, one of the biggest Spanish media groups, launched *Qué!*, a free newspaper in 15 cities to compete against *Metro Spain*.[2] Apple, on the other hand, chose not to respond to ad-sponsored free music sites and continues its business as usual.

These empirical observations suggest that incumbents use a variety of measures to respond to ad-sponsored rivals. They not only use tactics such as adjusting their prices, but also consider the adoption of new business models by switching from subscription-based models to ad-sponsored models or by extending their product lines to include ad-sponsored versions of their offerings. Some of these strategic responses have not worked well. The New York Times Co., for example, is planning to charge for access to some of its online content once again.[3]

[1] Source: `http://www.metro.lu/about/metro_facts`, accessed April 2009.

[2] See Khanna et al. (2007).

[3] Source: `http://www.watblog.com/2009/02/05/nytimes-plans-paid-subscription-for-online-`

How should an incumbent react to an ad-sponsored entrant? The goal of this research is to develop an analytical framework to establish guidelines for incumbent firms facing these issues. We consider a competitive setting with an incumbent that faces an ad-sponsored entrant. Specifically, we set up a game where the the incumbent firm first chooses the business model through which it would like to compete. We consider four alternative business models:

- The *pure-subscription-based* model: the firm offers at positive price one product that comes without ads.
- The *pure-ad-sponsored* model: the firm's product comes with ads but it is given away for free. We assume that ads have a detrimental effect on the product's perceived quality. For example, most people would prefer to watch movies without interruptions for advertising.
- The *mixed-single-product* model: the firm's product has advertisements and it is sold at positive price.
- The *mixed-product-line-extension* model: In this case, the firm offers two products, a high-quality product that, just as in the mixed-single-product model, is sold at positive price and comes with a few ads, and a low-quality product that is ad-sponsored (has many ads but is given away for free).

Table 1 presents some examples under each business model. Building on Ghemawat (1991), we refer to the choice of business model as *strategy*: the business model is a set of committed choices that lays the ground for the competitive interaction that will occur between the incumbent and the ad-sponsored entrant down the line.

After the business model has been chosen, the incumbent and the entrant make *tactical* choices simultaneously: the entrant chooses advertising intensity as it is assumed to compete through a pure ad-sponsored business model, and the incumbent chooses price and/or advertising intensity, depending on the business model through which it has decided to compete. For example, if the incumbent competes through the pure-subscription-based model, then it chooses price as tactics, and if the incumbent competes through the mixed-product-line-extension model, it chooses price and advertising intensity for the high-quality product and advertising intensity for the low-quality product.

`content-again/`, accessed April 2009.

Business Models	Examples
Pure-subscription-based	iTunes or Rhapsody (audio and video distribution)
	eHarmony (dating sites)
	HBO (cable TV)
	Dell (PCs)
Pure-ad-sponsored	Metro (newspapers)
	Free-pc (PCs)
	Imeem (music distribution)
	Blyk (mobile service)
	Facebook (social networks)
	ABS notebooks (notebooks for college students)
	FreePaperCups.com (coffee cups)
	ABC, MSNBC, Fox... (TV channels)
	WARL, WAVM... (broadcasted radio stations)
Mixed-single-product	Time (magazines)
	Wall Street Journal (newspapers)
	TNT (cable channels)
	Ryanair (airlines)
Mixed-product-line-extension	Recoletos (newspapers)
	GMail (email)
	Match.com (dating sites)
	Pandora (music distribution)
	Flickr (photo storage and management)

Table 1: Examples of Different Business Models

We study how the incumbent's optimal strategy (choice of business model) and tactics (choice of price and/or advertising intensity) is affected by: (a) the presence of the entrant, (b) the prevailing advertising rates, and (c) the additional fixed costs of competing through a mixed, as opposed to a pure, business model.

Our analysis reveals that the optimal strategic and tactical choices change dramatically in the presence of an ad-sponsored rival. Compared to a situation where the incumbent is a monopolist, when there is an ad-sponsored competitor the incumbent is more likely to prefer to compete through a pure, rather than a mixed, business model. There are two reasons for this. First, the presence of the ad-sponsored rival puts an upper bound on the number of ads that an incumbent competing through a mixed-product-line-extension can set. Therefore, the low-quality, ad-sponsored product that the incumbent offers cannot be of very low quality (otherwise it obtains no sales) and, as a consequence, it cannibalizes sales of the high-quality product. Given this, the mixed-product-line-extension model is inferior to the pure-subscription-based model when the advertising rate is low.

Second, even if the incumbent can avoid cannibalization by using the mixed-*single-product* model, the incumbent may still prefer to use the pure-subscription-based model. The reason is that advertising intensities of the two firms are strategic substitutes. As the incumbent increases the number of ads, the entrant decreases its advertising intensity and vertical differentiation diminishes. This forces the incumbent to lower its price which ends up hurting its profits. When the quality difference between the incumbent and entrant's products is small and the incumbent competes through the mixed-single-product model, we find the counter-intuitive result that the incumbent's (equilibrium) profits are decreasing in the advertising rate!

We also find that there are situations where the best response to an ad-sponsored entrant is to do nothing. That is, it may be optimal for the incumbent not to change its business model and tactics. This happens only when the optimal business model under both monopoly and duopoly is the pure-subscription-based model and the quality difference between the incumbent and the entrant is large. In all other cases, even if the optimal business model does not change following entry, optimal tactics change.

The entrant is pushed out of the market only when the incumbent competes through a business model that has an ad-sponsored product.[4] Otherwise, both firms coexist with strictly positive profits. Eliminating the entrant is optimal only when the prevailing adver-

[4]The only two possibilities are: (a) the incumbent competes through the pure-ad-sponsored model or (b) the incumbent competes through the mixed-product-line-extension model.

tising rate is high. Therefore, the incumbent's reaction to the entry by an ad-sponsored rival is most aggressive when advertising rates are high. Ironically, this is the situation when, absent the incumbent's reaction, the entrant would have had the strongest incentives to enter.

Our study is the first to provide a comprehensive analysis of the competition between a free ad-sponsored entrant and an incumbent that has the option of choosing different business models. Our analysis shows the importance of considering modifications to a firm's business model when deciding how to face competition. We quantify the additional profit that the incumbent earns when, in addition to re-optimizing its tactics, it also takes into consideration possible changes in the business model and show that the profit implications of competing through business models could be substantial.

The paper is organized as follows. Section 2 discusses the relationship of our paper with the literature. Section 3 presents our model setup. Section 4 analyzes the monopoly benchmark. Section 5 examines the duopoly case in which an incumbent competes with an ad-sponsored entrant. Section 6 provides managerial implications. Section 7 discusses the contribution of our study in improving our understanding of strategy and tactics. Section 8 concludes after suggesting some extensions to the analysis.

2 Related literature

The paper is related to several strands of literature. First, it contributes to the literature on ad-sponsored business models. Prasad et al. (2003) and Gabszewicz et al. (2005) examine a monopolist's pricing decisions when it is ad-sponsored. Their results are akin to our mixed-single-product model for a monopoly in that the monopoly will lower the subscription price as the willingness to pay of the advertisers increases. Several studies (e.g., Steiner 1952; Beebe 1977; Spence and Owen 1977; Doyle 1998; Gal-Or and Dukes 2003; Bourreau 2003; Gabszewicz et al. 2006; Peitz and Valletti 2008) look at the product positioning of ad-sponsored firms. In general, they find that with advertising, firms tend to provide less differentiated products. These models often assume that products are horizontally differentiated and have the same quality level. In contrast, we do not consider horizontal differentiation, and instead focus on vertically differentiated products and allow firms to strategically decide the level of product quality. Hence, our study complements these extant studies. Choi (2006) and Crampes et al. (2009) examine entry of media firms that are financed both from ads and subscriptions and find that with free entry, there may be excessive levels of entry in

such markets. In their models, firms are symmetric and they adopt the same mixed-single-product business model. While we only look at the competition between one incumbent and one entrant, we allow them to use different business models. We discuss the case with more than one entrant in Section 7.

The economic model we work with is close in spirit to the literature on product-line extension. Mussa and Rosen (1978) consider the product line decisions of a price-discriminating monopolist able to offer a range of products of different qualities. They find that a monopolist may offer inefficiently low quality level to lower-value customers to reduce substitution possibilities of higher-value customers. Deneckere and McAfee (1996) show how a monopoly can degrade its product quality at no cost to create a low quality substitute to price discriminate. In our model, firms can use ads to degrade the quality of the product. Consistent with Mussa and Rosen (1978) and Deneckere and McAfee (1996), we show that when product-line extension is not costly, the monopolist indeed has an incentive to use ads to create a low-end product. In addition, the monopolist has an incentive to introduce the maximum possible number of ads to its low-end product to avoid cannibalizing its high-end product. Interestingly, we find that the monopolist also has an incentive to introduce ads to its high-end product, even at the cost of reducing the differentiation of its two products. In a duopoly setting, Shaked and Sutton (1982) show that when each firm is allowed to offer one quality, the two firms will want to maximize quality differentiation to soften price competition. In contrast, as we show after introducing the ads, in our mixed-single-product-line model the quality differentiation between the incumbent and the entrant is no longer maximized in equilibrium.

Champsaur and Rochet (1989) extend Shaked and Sutton (1982)'s analysis by allowing each firm to offer a whole range of products in a chosen interval of quality. They analyze equilibria in which both firms are active by assuming non-overlapping quality intervals. As a result, one firm does not have the option to push the other out of the market. In our model, the range of quality the incumbent can choose for its product(s) overlaps with the range of quality the entrant can choose. This setting allows us to analyze when the incumbent wants to push the entrant out of the market. A few other studies such as Johnson and Myatt (2003) and Gal-Or (1983) analyze settings in which vertically-differentiated firms set quantities of their output to compete, while we analyze a model in which firms set subscription rate and/or advertising intensities.

Our paper also contributes to an emerging literature in strategy that explores competitive interaction between organizations with different business models. While there are several

formal models of asymmetric competition that exist in strategy (differences in costs, resources endowments or information, mainly), the asymmetries that this literature wrestles with are of a different nature: firms with fundamentally different objective functions, opposed approaches to competing, or different governance structures. Casadesus-Masanell and Ghemawat (2006), Economides and Katsamakas (2006) and Lee and Mendelson (2008) for example, study mixed duopoly models in which a profit-maximizing competitor interacts with an open source competitor that is committed to zero price. Casadesus-Masanell and Yoffie (2007) study competitive interactions between two complementors, Microsoft and Intel, with asymmetries in their objectives functions stemming from technology—software vs. hardware. Seamans (2009) looks at strategies used by incumbent cable TV firms to deter entry by public and private entrants. Zhu (2008) examines networks' incentives to establish compatibility under subscription and ad-sponsored business models when the source of network differentiation comes from users of the networks.

Interest in the study of competitive interactions between organizations with different business models has increased in the last few years as new technologies, regulatory changes, and new customer demands have allowed firms to implement new approaches to competing in a wide range of industries spanning from airlines (e.g., Ryanair) to furniture (e.g., IKEA) and from the circus (e.g., Cirque du Soleil) to software (e.g., open source projects). In fact, many of the fastest-growing firms in the recent past appear to have taken advantage of opportunities sparked by globalization, deregulation, or technological change to "compete differently" and to innovate in their business models (see Kim and Mauborgne (2005) and Markides (2008) for additional examples). So far, the literature has studied interactions between exogenously given business models. This paper contributes to this literature by endogeneizing the choice of business model: We allow the incumbent to choose the business model with which it would like to fight a rival that competes with an ad-sponsored product.

Finally, this paper is related to the literature on two-sided markets (e.g., Rochet and Tirole 2003; Caillaud and Jullien 2003; Armstrong 2006; Hagiu 2009; Casadesus-Masanell and Ruiz-Aliseda 2009). A market is two-sided when it is intermediated by a platform which enables transactions between participants on both sides. In most applications in the literature (e.g., the video game industry), the two sides attract each other.[5] In contrast, when a platform is ad-sponsored, consumers are attracted by the product offered by the platform *per se*, rather than the ads, and they in general would prefer to watch fewer ads. Our paper contributes to this literature by explicitly addressing the question of when it is

[5]That is, *indirect* network effects are positive.

optimal for a firm to use a one-sided business model by excluding the side that produces the negative effect.

3 The model

We consider two alternative industry structures. In the benchmark model of Section 4, we study the case of a profit-maximizing monopolist that offers a high-quality product through one of four alternative business models. In Section 5, we study the effects on the incumbent's optimal choice of business model and tactics when a free ad-sponsored rival enters the market.

On the demand side, there is one unit mass of consumers. Consumers are differentiated by their type θ, which represents their marginal willingness to pay for product quality and is uniformly distributed on $[0, 1]$. If a consumer of type θ purchases a product of quality q at price p, her utility is $U(\theta) = \theta q - p$. We impose a non-negativity constraint on price and normalize consumers' utility from outside options to be zero. Each consumer adopts one product only. In addition, we adopt a tie-breaking rule that if a consumer receives zero utility from adopting a product, she will choose to adopt the product.

Let $q_i > 0$ denote the (exogenous) quality of product i. q_i is the maximum consumer with type $\theta = 1$ would ever be willing to pay for i if it contains no ads. Let A_i denote the (endogenous) number of ads that product i carries. When a product is sponsored by advertisers, the larger the number of consumers, the more attractive the product is for the advertisers. Following Gabszewicz et al. (2004), we assume that the advertising fee charged to each advertiser, r_i, is an increasing linear function of the demand for the product, D_i. Mathematically, $r_i = \alpha D_i$, where $\alpha > 0$ and is the (per consumer) advertising rate charged to each advertiser.

Consumers have to view ads that come with the product. Several recent work on media industries characterize advertising as a nuisance (e.g., Anderson and Coate 2005). Empirical studies in the television industry and the magazine industry (Wilbur 2008; Depken and Wilson 2004) find that ads indeed reduce viewers' utilities. Hence, we assume that the total nuisance cost of the ads is βA_i^2, where $\beta > 0$. The functional form implies that the marginal disutility of ads increases with advertising intensity. Moreover, the first few ads are tolerated well by consumers but as more and more ads are shown, consumers become increasingly irritated by them.

In summary, a consumer of type θ receives utility $U(\theta) = \theta(q_i - \beta A_i^2)$ from product i. We refer to $q_i - \beta A_i^2$ as the net quality of product i after taking the nuisance cost of ads into

consideration.

As mentioned above, we consider four possible business models: a pure-subscription-based model, a pure-ad-sponsored model, and two mixed models in which the firm makes money from both subscription fees and ad revenues. We normalize the fixed cost of competing through a pure business model to zero and denote by $f \geq 0$ the additional fixed cost incurred when a mixed business model is employed.[6] Finally, we assume, for simplicity, that the marginal cost of producing the product or introducing an ad is zero.

4 Monopoly benchmark

We begin by studying the optimal strategy of a monopolist with a product of quality q_h. The timing is as follows. The monopolist first chooses the business model. After the business model is in place, the monopolist makes its tactical decisions: price p_h and/or advertising intensity A_h (depending on the business model used).

4.1 Four business models

Before deriving the optimal strategy and tactics for the monopolist, we need to introduce our formalizations of the four business models.

Consistent with the common use of the expression, we think of a business model as "the logic of the firm, the way it operates and how it creates value for its stakeholders"(Baden-Fuller et al. 2008). To be able to work formally with business models, we represent them in the form of simple profit functions. Profit functions are extreme reduced form representations of *actual* business models. For example, while we represent the pure-ad-sponsored business model through profit function $\pi_h^A = \alpha A_h$, behind this simple expression there is an elaborate activity system that includes: product distribution activities, incentive systems, hiring policies, procurement contracts, information systems, and the like. We discuss this issue at length in Section 7.

4.1.1 Pure-subscription-based model

In the pure-subscription-based model, the monopolist prices its product at p_h. Let θ^* be the consumer who is indifferent from purchasing the product and not purchasing it. Thus, θ^* is

[6] It is reasonable to expect that dealing with both advertisers and consumers will be more costly: As the two groups are very different and do not overlap with each other, there will be little economy of scope. On the other hand, it is trivial to generalize the analysis to the case $f < 0$.

implicitly defined by $\theta^* q_h - p_h = 0$. Consumers with type $\theta \geq \theta^*$ will purchase the product and the rest will not purchase it. The profit is:

$$\pi_h^S = (1 - \theta^*) p_h. \tag{1}$$

4.1.2 Pure-ad-sponsored model

In this case, the monopolist offers the product for free to consumers, i.e., $p_h = 0$, and makes money exclusively from advertisers.[7] As $p_h = 0$, all consumers will adopt the product as long as $q_h - \beta A_h^2 \geq 0$ and none will adopt the product otherwise. Hence, the profit of the monopolist is:

$$\pi_h^A = \alpha A_h \tag{2}$$

$$s.t.\ q_h - \beta A_h^2 \geq 0.$$

4.1.3 Mixed-single-product model

The monopolist offers a single product at a positive price, $p_h > 0$, and at the same time introduces A_h ads with the product. The consumer of type θ receives a utility of $U(\theta) = \theta(q_h - \beta A_h^2) - p_h$. The type of the indifferent consumer between purchasing and not purchasing the product, θ^*, is thus defined by $U(\theta^*) = 0$. The profit of the monopolist is:

$$\pi_h^{MS} = (1 - \theta^*)(p_h + \alpha A_h) - f \tag{3}$$

$$s.t.\ 0 \leq \theta^* \leq 1, q_h - \beta A_h^2 \geq 0.$$

This business model is meaningful only when $A_h > 0$ and $p_h > 0$. Clearly, if $A_h = 0$ then the business model is pure-subscription-based (as opposed to the mixed-single-product model) and if $p_h = 0$ then the business model is pure-ad-sponsored (as opposed to the mixed-single-product model). As argued above, the activity system supporting the pure-subscription-based model or the pure-ad-sponsored model are different (and less costly) than that supporting the mixed-single-product model.

Notice that as $p_h > 0$, $\theta^* > 0$ and hence not all consumers will adopt the product. For

[7] Given our assumption of zero marginal cost of production, it happens that the pure-ad-sponsored model has $p_h = mc = 0$. We should stress, however, that the distinctive feature of the pure-ad-sponsored business model is that revenue accrues from ads only. Therefore, even if $mc > 0$, the correct specification would have $p_h = 0$. Assuming $mc > 0$ complicates the analysis because additional corners must be considered and it does not affect our results qualitatively.

each consumer adopting the product, the monopolist makes money from selling the product and also displaying ads.

4.1.4 Mixed-product-line-extension model

In the mixed-product-line-extension model, the monopolist offers two products, h and h'. The first product, h, is priced at p_h and comes with A_h ads. The second product, h', comes with A'_h ads and is free. The most profitable way for the monopolist to come up with an ad-sponsored product h' is by just adding ads to h. While the monopolist could create a different, brand-new h', it would prefer to use ads to lower the quality of h as ads will bring additional profit.

In this case, the indifferent consumer between h and h' is determined by: $\theta^*(q_h - \beta A_h^2) - p_h = \theta^*(q_h - \beta A_h'^2)$. The profit is:

$$\pi_h^{ME} = (1-\theta^*)(p_h + \alpha A_h) + \alpha A'_h \theta^* - f \tag{4}$$

$$s.t.\ 0 \leq \theta^* \leq 1,\ q_h - \beta A_h'^2 \geq 0$$

This business model is meaningful only when $A'_h > A_h \geq 0$, $p_h > 0$ and $q_h - \beta A_h'^2 \geq 0$. Note that if $A'_h \leq A_h$, then nobody will buy h at a positive price because h' is of at least as much quality and is offered for free, and the model effectively becomes the pure-ad-sponsored model. And if $p_h = 0$, then nobody will adopt h' (even if free) because h is of higher quality and it is also free, and the model effectively becomes the pure-ad-sponsored model. If $q_h - \beta A_h'^2 < 0$, then product h' is not active and we have the mixed-single-product model. Also, if $A'_h = A_h = 0$, then it is as if there was one product only offered through the pure-subscription-based model.

While it may seem as if the pure-ad-sponsored, the pure-subscription-based and mixed-single-product business models are special cases of the mixed-produce-line extension model (as if they were points on a continuum), once we zoom down to the concrete system of activities that those profit functions aim to capture it becomes clear that they are *not* points in a continuum. Put differently, real companies do not think of their profit functions as completely "plastic." For example, if one initially competes through the pure-subscription-based model and considers putting "a little bit of ads," or introducing an ad-sponsored product, he will likely need some important changes in his activity system (activities to negotiate with advertisers, collect ad revenues, access to different distribution channels...). In other words, he will need to conduct activities that he did not perform (or that he

configured differently) when competing through the pure-subscription-based model.

4.2 Optimal strategy, business model, and tactics

Having presented the four business models, we now proceed to solving for the optimal tactical choices implied by each model and for the monopolist's optimal business model. We solve the optimization problem backwards: We first find the optimal tactical choices and later study the optimal choice of business model.

4.2.1 Monopoly tactics

We use the term *tactics* to refer to the choices that the firm makes after the business model has been chosen. We note that the tactical options available to the firm depend on the business model under consideration.

The following proposition summarizes the optimal tactical choices for each business model that we consider.

Proposition 1 *The optimal price and number of ads under each business model are:*

- *Pure-subscription-based model:* $p_h = q_h/2$.
- *Pure-ad-sponsored model:* $A_h = (q_h/\beta)^{1/2}$.
- *Mixed-single-product model:*

  A_h *solves* $A_h^3\beta^2 + q_h(\alpha - A_h\beta) = 0$.[8]

  $p_h = (q_h - \beta A_h^2 - \alpha A_h)/2$.
- *Mixed-product-line-extension model:*

  $A'_h = (q_h/\beta)^{1/2}$.

  $A_h = \frac{1}{2}((A'_h(4\alpha + A'_h\beta)/\beta)^{1/2} - A'_h)$.

  $p_h = \frac{1}{2}(A'_h - A_h)(\alpha + (A_h + A'_h)\beta)$.

Proof. We provide all the proofs in the appendix. ■

The intuitions for these results are as follows:

[8]The solution to A_h is too lengthy to be shown here.

Pure-subscription-based model. The monopolist trades off demand against mark-up. It is well-known that when the demand function is linear and marginal cost is zero, the optimal solution has a price equal to $\frac{1}{2}$ the choke price and half of the market is served.

Pure-ad-sponsored model. The monopolist introduces the maximum number of ads possible making sure that the resulting quality is not so low that there is no willingness to pay. Because $p_h = 0$, as long as $q_h - \beta A_h^2 \geq 0$, every consumer buys the product regardless of the number of ads that it contains. Thus profit is maximized at $A_h = (q_h/\beta)^{1/2}$.

We note also that as β increases, the equilibrium A_h is smaller. Clearly, if consumers become easily irritated by ads, the number of ads that results in zero net quality is smaller.

Mixed-single-product model. Given A_h, the net quality of the product is $q_h - \beta A_h^2$. A monopolist earning profits from subscription only with a product of quality $q_h - \beta A_h^2$ would charge a price of $\hat{p}_h = (q_h - \beta A_h^2)/2$. However, the optimal price in the case of a monopolist that *also* earns profits from advertising is $p_h = (q_h - \beta A_h^2 - \alpha A_h)/2 < \hat{p}_h$. The reason is that with advertising the firm considers the profits accrued from both sides of the market. To earn more from advertising, the firm must decrease p_h to increase demand. In addition, as α increases, p_h decreases. This inverse relationship between advertising rate and subscription price is also found in other model setups (e.g., Armstrong 2006).

The equilibrium A_h is increasing in α. As α increases, the monopolist will typically decrease p_h to increase the number of adopters of the ad-sponsored product and earn even more from the advertisers. Less expected, perhaps, is the fact that there is a discontinuity in the equilibrium advertising intensity A_h when considered a function of α. When α grows, A_h increases and p_h decreases. Before p_h reaches zero, the monopolist will want to increase discretely the number of ads to the point that the net quality of h is zero. As discussed above, at this point the mixed-single-product model is not meaningful and the relevant business model has become the pure-ad-sponsored model.

Mixed-product-line-extension model. The ad-sponsored product has the lowest possible quality in equilibrium. In other words, A'_h is such that the willingness to pay for the low-quality product is zero. Therefore the individuals that consume the ad-sponsored product would not have bought the high-quality product had the ad-sponsored product not existed. As a consequence, there is no cannibalization between the two products.

When α is sufficiently large ($\alpha \geq 2(\beta q_h)^{1/2}$), the optimal p_h is zero and all the profit comes from the ads. Of course, in this case the monopolist is effectively offering one product

only (both products are identical). Thus, when α is large, the mixed-product-line extension model becomes the pure-ad-sponsored model.

When β grows, the monopolist is better off reducing both A_h and A'_h (for the same reason as in the pure-ad-sponsored model).

4.2.2 Monopoly strategy

We use the term *strategy* to refer to the choice of business model by the monopolist. An optimal strategy takes into account that each business model leads to different equilibrium tactical choices. We now characterize the monopolist's optimal strategy through a series of simple lemmas.

Lemma 1 *When $f = 0$, the two mixed models dominate the pure-subscription-based model.*

The reason is that the marginal effect of ads on consumer utility evaluated at $A_h = 0$ is zero. On the other hand, the marginal revenue of ads is constant and equal to α, which is assumed to be positive. Therefore, when the additional cost of using a mixed model f is zero, it is always optimal to have a few ads, even if α is very small. Of course, with a large α the relative advantage of the mixed models over the pure-subscription-based model is even larger.

Lemma 2 *The mixed-product-line-extension model dominates the mixed-single-product model.*

Intuitively, under the mixed-single-product model not all consumers adopt the product because the price is positive. The monopolist could improve its payoff by offering an ad-sponsored free product that gives zero utility. The ad-sponsored product does not cannibalize the sales of the high quality product. Those who choose the outside option would now adopt the ad-sponsored product and would bring ad revenue to the monopolist.

Lemma 3 *When α is sufficiently large, the pure-ad-sponsored model is always optimal; when α is sufficiently low, the pure-ad-sponsored model is never optimal.*

Clearly, the monopolist prefers to earn all profits from the advertising side of the market when α is large. The business model where profits from advertising are largest is the pure-ad-sponsored model because the optimal tactics implied maximize the volume of customers. To understand the second part of the lemma, note that the monopolist can always guarantee a profit of $q_h/4 > 0$ by competing through the pure-subscription-based model. However, profits under the pure-ad-sponsored model converge to zero when α approaches 0.

Lemma 4 *When $f > 0$ and α is low, the pure-subscription-based model is optimal.*

Given Lemmas 2 and 3, when α is low, the only business models that need to be considered are the pure-subscription-based model and the mixed-product-line-extension model. Noticing that the additional profit from advertising in the mixed-product-line-extension model converges to zero when α approaches 0, $f > 0$ renders that model inferior to the pure-subscription-based model when α is low.

Figure 1 summarizes the optimal strategy for the monopolist as a function of α and f.[9] Having established the benchmark monopoly case, we proceed to the analysis of the duopoly.

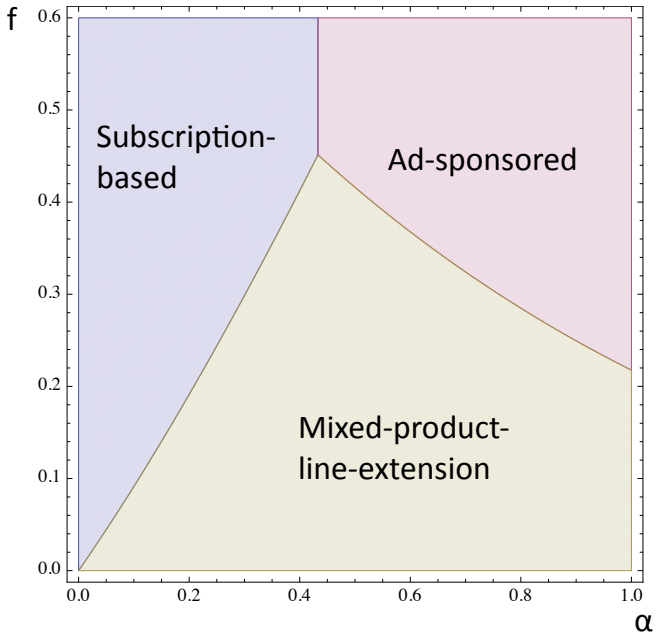

Figure 1: Optimal business model in the monopoly case.

5 Duopoly

We now examine the firm's optimal strategy and tactics in a competitive environment. Specifically, we examine the optimal strategy of an incumbent that faces an ad-sponsored entrant. We assume that the entrant faces no entry costs and, thus, that it enters as long as the profits that it expects to earn are greater than zero.

[9] In this figure, the parameter values are: $q_h = 3$ and $\beta = 1$.

Let the quality of the entrant's product be $q_l < q_h$. Because the entrant uses a pure-ad-sponsored business model, its product is given away for free. Let A_l be the amount of ads the entrant introduces. On the one hand, the entrant would like to have many ads; on the other hand, having many ads reduces the product's quality and, as a result, reduces the number of consumers adopting it. The entrant also needs to take the incumbent's responses into consideration when choosing the number of ads. Note that consumers will consider the entrant's product only if $q_l - \beta A_l^2 \geq 0$.

We again examine the four possible business models that the incumbent may want to adopt: the pure-subscription-based model, the pure-ad-sponsored model, the mixed-single-product model, and the mixed-product-line-extension model.

The timing of the game is as follows. First, the entrant decides whether to enter or not. Second, the incumbent chooses a business model. Third, tactical choices are made by both the entrant and the incumbent, and demand and profits are realized.

5.1 Four business models

We now present our formalizations of the four business models.

5.1.1 Pure-subscription-based model

The incumbent maximizes profits by setting p_h and the entrant maximizes profits by setting A_l subject to the constraint that $q_l - \beta A_l^2 \geq 0$. As the entrant product is free, consumers who do not adopt product h will adopt product l. The type of the indifferent consumer between the two products, θ^*, is defined by $\theta^* q_h - p_h = \theta^*(q_l - \beta A_l^2)$.

Profits of the incumbent and the entrant are:

$$\begin{aligned} \pi_h^S &= (1-\theta^*)p_h. \\ \pi_l^S &= \alpha\theta^* A_l. \\ s.t. \quad & 0 \leq \theta^* \leq 1,\ q_l - \beta A_l^2 \geq 0. \end{aligned}$$

5.1.2 Pure-ad-sponsored model

When both the incumbent and the entrant provide free products, all consumers will buy the product with higher net quality. This competitive situation is similar to Bertrand competition, except that now the two firms are setting the number of ads, not prices.

The profits are:

$$\begin{aligned}
\pi_h^A &= \begin{cases} \alpha A_h & \text{if } \; q_h - \beta A_h^2 \geq q_l - \beta A_l^2 \\ 0 & \text{otherwise.} \end{cases} \\
\pi_l^A &= \begin{cases} 0 & \text{if } \; q_h - \beta A_h^2 \geq q_l - \beta A_l^2 \\ \alpha A_l & \text{otherwise.} \end{cases} \\
s.t. & \quad q_h - \beta A_h^2 \geq 0 \;\text{ and }\; q_l - \beta A_l^2 \geq 0.
\end{aligned}$$

Note that we are assuming that when both products, h and l, are of the same quality, consumers prefer the incumbent's offering.[10]

5.1.3 Mixed-single-product model

The incumbent product now comes with ads, A_h, and is priced at $p_h > 0$. The indifferent consumer is defined by $\theta^*(q_h - \beta A_h^2) - p_h = \theta^*(q_l - \beta A_l^2)$. Hence, the profits are:

$$\begin{aligned}
\pi_h^{MS} &= (1-\theta^*)(p_h + \alpha A_h) - f. \\
\pi_l^{MS} &= \theta^* \alpha A_l. \\
s.t. & \quad 0 \leq \theta^* \leq 1,\; q_h - \beta A_h^2 \geq 0,\; q_l - \beta A_l^2 \geq 0.
\end{aligned}$$

Just as in the monopoly case, for this business model to be meaningful, we need that $p_h > 0$ and $A_h > 0$. Otherwise, one of the pure business models is the effective one.

5.1.4 Mixed-product-line-extension model

The incumbent introduces two products, product h that is both subscription and ad-based and product h' that is purely ad-sponsored. Suppose that the advertising intensities A_h' and A_l are such that the entrant is pushed out of the market. Then, consumers either buy the high quality product of the incumbent or consume the free ad-sponsored product of the incumbent. In this case, the indifferent consumer θ^* is determined by $\theta^*(q_h - \beta A_h^2) - p_h = \theta^*(q_h - \beta A_h'^2)$. Suppose, instead, that the advertising intensities A_h' and A_l are such that the entrant is *not* pushed out of the market. Then, consumers either buy the high quality product of the incumbent or consume the free ad-sponsored product of the entrant. In this case, the

[10] As $q_h > q_l$, the incumbent could always set its net quality, $q_h - \beta A_h^2$, at $q_l - \beta A_l^2 + \epsilon$ to attract all consumers.

indifferent consumer θ^{**} is determined by $\theta^{**}(q_h - \beta A_h^2) - p_h = \theta^{**}(q_l - \beta A_l^2)$.

As product h is not free, we must have that $A_h < A'_h$. That is, the net quality of h has to be greater than that of h'. Otherwise, product h will have no demand. The profits are:

$$\begin{aligned}
\pi_h^{ME} &= \begin{cases} (1-\theta^*)(p_h + \alpha A_h) + \theta^*(\alpha A'_h) - f & \text{if } \; q_h - \beta A_h'^2 \geq q_l - \beta A_l^2 \\ (1-\theta^{**})(p_h + \alpha A_h) - f & \text{otherwise.} \end{cases} \\
\pi_l^{ME} &= \begin{cases} 0 & \text{if } \; q_h - \beta A_h'^2 \geq q_l - \beta A_l^2 \\ \alpha\theta^{**} A_l & \text{otherwise.} \end{cases} \\
s.t. \qquad & 0 \leq \theta^* \leq 1, \; 0 \leq \theta^{**} \leq 1, \; q_h - \beta A_h'^2 \geq 0, \quad q_l - \beta A_l^2 \geq 0, \quad \text{and} \quad A_h < A'_h.
\end{aligned}$$

This business model is meaningful only when $p_h > 0$, $A_h > 0$, and $A'_h > 0$.

5.2 Optimal strategy, business model, and tactics

We proceed to finding the equilibrium tactical choices of the incumbent and the entrant and the incumbent's equilibrium choice of business model. We look for the sub-game perfect equilibria of the game and, thus, proceed to solving it by backwards induction.

5.2.1 Duopoly tactics

The following proposition shows the equilibrium tactics for each business model.

Proposition 2 *The optimal price and number of ads under each business model are:*

- *Pure-subscription-based model:*

  We may have a corner solution in which $q_l - \beta A_l^2 = 0$ or an interior solution in which $q_l - \beta A_l^2 > 0$.

  At the interior solution, $p_h = q_h - q_l$ and $A_l = (\frac{q_h - q_l}{\beta})^{1/2}$. This happens when $q_h < 2q_l$.

  At the corner solution, $p_h = q_h/2$ and $A_l = (q_l/\beta)^{1/2}$. This happens when $q_h \geq 2q_l$.

- *Pure-ad-sponsored model:*

  $A_h = ((q_h - q_l)/\beta)^{1/2}$.

  $A_l = 0$. *The entrant is pushed out the market.*

- *Mixed-single-product model:*

  We may have a corner solution in which $q_l - \beta A_l^2 = 0$ *or an interior solution in which* $q_l - \beta A_l^2 > 0$.

  At the interior solution, A_h *and* A_l *solve the following system:*[11]

$$\begin{cases} (q_h - q_l)/\beta + A_l^2 = \frac{A_h^3}{A_h - \alpha/\beta} \\ A_l = ((q_h - q_l - A_h^2\beta)/\beta)^{1/2} \end{cases},$$

  and $p_h = \frac{1}{2}\left(q_h - q_l - \alpha A_h - \beta(A_h^2 - A_l^2)\right)$.

  At the corner solution, A_h *solves:* $A_h^3\beta^2 + q_h(\alpha - A_h\beta) = 0$,

  $A_l = (q_l/\beta)^{1/2}$, *and*

  $p_h = (q_h - A_h(\alpha + A_h\beta))/2$.

- *Mixed-product-line-extension model:*

  $A_h' = ((q_h - q_l)/\beta)^{1/2}$.

  $A_h = \frac{1}{2}((A_h'(4\alpha + A_h'\beta)/\beta)^{1/2} - A_h')$.

  $p_h = \frac{1}{2}(A_h' - A_h)(\alpha + \beta(A_h + A_h'))$.

  $A_l = 0$. *The entrant is pushed out the market.*

We now present the intuitions behind these results:

Pure-subscription-based model. The optimal tactics of the incumbent depend on whether the entrant sets its number of ads at the corner or not ($A_l = (q_l/\beta)^{1/2}$ or $A_l > (q_l/\beta)^{1/2}$), which in turn depends on the exogenous vertical differentiation between the incumbent's and the entrant's products. Recall that the entrant's profits increase with its market share and the number of ads its product has. When the entrant's product is of very low quality ($q_h \geq 2q_l$), it is best for it to maximize the number of ads because its market share, $\theta^* = \frac{p_h}{q_h - q_l + \beta A_l{}^2}$, is insensitive to the amount of ads that it offers (the derivative of θ^* with respect to A_l approaches zero as the difference between q_h and q_l grows). On the other hand, if its quality is close to the high quality one ($q_h < 2q_l$), θ^* is sensitive to the number of ads and it makes sense for the entrant to reduce the number of ads to gain some market share.

[11] The solution to A_h is too lengthy to be shown here.

When $q_h \geq 2q_l$, there is no cannibalization between the two products: $q_l - \beta A_l^2 = 0$. The indifferent consumer obtains zero utility. When $q_h < 2q_l$, the net quality of the entrant in equilibrium is positive: $q_l - \beta A_l^2 = q_l - q_h/2 > 0$. The indifferent consumer has positive utility from both products. Note that the solution when $q_h \geq 2q_l$ is the same as in the monopoly case for the incumbent. This result suggests that the incumbent may not have to adjust its tactics when facing an ad-sponsored rival.

It is interesting to note that $\theta^* = 1/2$ in both cases. That is, the incumbent and the entrant always split the market equally, regardless of their quality difference. Given any A_l, the residual demand for product h is $D_h = 1 - \theta^* = 1 - \frac{p_h}{q_h - q_l + \beta A_l^2}$. The marginal revenue implied by this demand function equals marginal cost (which is zero) at $D_h = \frac{1}{2}$ regardless of the value of A_l. Of course, the equilibrium p_h changes with A_l, and so does the incumbent profit, but the equilibrium D_h does not change.

Pure-ad-sponsored model. The incumbent uses the free ad-sponsored product to "kill" the entrant. This means that the incumbent cannot introduce too many ads as it has to offer at least the same amount of utility as the entrant *without ads* because the entrant will respond by lowering the amount of ads to survive. Hence, the optimal amount of ads is constrained by $q_h - \beta A_h^2 \geq q_l$. Under this constraint, all consumers will adopt product h. Therefore, it is in the interest of the incumbent to maximize A_h, subject to the constraint that $q_h - \beta A_h^2 \geq q_l$.

Mixed-single-product model. As the incumbent product is not free, consumers with low θ will not buy from the incumbent. As long as the entrant's product offers positive utility, these consumers will adopt the entrant's product. As a result, both the incumbent and the entrant coexist in equilibrium.

Similarly to the pure-subscription-based-model, the solution may be at a corner where the entrant sets the maximum number of ads ($A_l = (q_l/\beta)^{1/2}$) such that the utility for its product is zero or it may be interior ($A_l < (q_l/\beta)^{1/2}$). When $q_h \leq 2q_l$, we are at the interior solution and the entrant's product offers strictly positive utility. The indifferent consumer thus gets positive utility. Surprisingly, in this case, the derivative of π_h^{MS} w.r.t. α is negative.

Lemma 5 *Under the mixed-single-product model, when we have interior solutions, the incumbent's profits, π_h^{MS}, decrease with the advertising rate, α; when we have corner solutions, the incumbent's profits, π_h^{MS}, increase with the advertising rate, α. In addition, when $q_h \leq 2q_l$, we will always have interior solutions; when $q_h > 2q_l$ and α is smaller than a given threshold, $\alpha^* > 0$, we will have corner solutions.*

To understand this result, recall that when we have interior solutions, the entrant's best-response function is $A_l = ((q_h - q_l - A_h^2\beta)/\beta)^{1/2}$. Note that A_l does not depend on α directly but indirectly through A_h. In particular, increases in A_h result in decreases in A_l. It is easy to see that A_h increases with α. As a result, A_l will decrease with α through its inverse relationship with A_h. Therefore, as α increases, the vertical differentiation between the two products diminishes and the increased intensity of competition lowers the profit for the incumbent. The entrant's profit, however, increases with α. The interior case happens when A_l is less than $(q_l/\beta)^{1/2}$, the maximum number of ads that the entrant can possibly have. A sufficient condition for the equilibrium to be interior is $q_h < 2q_l$.

The corner solution happens when the quality difference is large (i.e., $q_h > 2q_l$). When $q_h > 2q_l$, the unconstrained profit-maximizing A_l (i.e., $((q_h - q_l - A_h^2\beta)/\beta)^{1/2}$) would exceed the maximum number of ads that the entrant can possible have (i.e., $(q_l/\beta)^{1/2}$). In this case, the entrant chooses to set A_l at $(q_l/\beta)^{1/2}$, and the indifferent consumer receives zero utility. When at the corner solution, the number of ads and the price that the incumbent sets are the same as in the monopoly mixed-single-product model because there is no interaction between the entrant and the incumbent. Moreover, the derivative of π_h w.r.t. α is positive. The entrant's profits, $\pi_l = \alpha(q_l/\beta)^{1/2}$, also increase with α. But as α keeps increasing, the entrant eventually finds it optimal to have fewer ads to enlarge its market share and we move into the interior case. Once we are in the interior case, the incumbent's profits decrease with α while the entrant's profits still increase with α. (The upper bound to the green area in Figure 4 illustrates this situation.)

Mixed-product-line-extension model. In this case, the entrant will be pushed out of the market by the pure-ad-sponsored product of the incumbent for the same reason as in the pure-ad-sponsored business model. All consumers adopt the incumbent's products. In order to push the entrant out, the net quality of the pure-ad-sponsored product, $q_h - \beta A_h'^2$, has to be no less than q_l. On the other hand, in order to minimize cannibalization between incumbent's two products, the incumbent wants to set A_h' such that the net quality of the pure-ad-sponsored product to be as low as possible. Hence, A_h' is determined by $q_h - \beta A_h'^2 = q_l$. The utility of the indifferent consumer over the two products is thus $\theta^* q_l > 0$.

We also note that the equilibrium A_h is greater than zero. That is, it is always optimal for the incumbent to introduce some ads with product h. Hence, this business model dominates the one where the incumbent offers a pure-subscription-based product and a pure-ad-sponsored product.[12]

[12]This is the reason why we do not consider this business model in this paper.

The entrant and the incumbent co-exist in the equilibrium under the pure-subscription-based model and mixed-single-product model. The entrant is pushed out only when the the incumbent competes with an ad-sponsored product (pure-ad-sponsored model or mixed-product-line-extension model).

5.2.2 Duopoly strategy

We now characterize the incumbent's optimal strategy when there is an ad-sponsored entrant through a series of simple lemmas.

Lemma 6 *When α is small, either the pure-subscription-based or the mixed-single-product model is optimal; when α is large, the pure-ad-sponsored model is optimal.*

When α is small, the incumbent prefers to coexist with the entrant as the additional ad profit from its ad-sponsored product after killing the entrant would be small and there is also cannibalization in the case of the mixed-product-line-extension model; but when α is large, the incumbent has incentives to push the entrant out as it wants the market share from the entrant to earn ad profit even at the cost of cannibalization.

Lemma 7 *Compared to the monopoly case, neither the mixed-single-product model nor the mixed-product-line-extension model dominates the pure-subscription-based model for all α when $f = 0$.*

As Lemma 5 indicates, when $q_h \leq 2q_l$ the incumbent's profit decreases with α when competing through the mixed-single-product model. The incumbent profit is maximized when the advertising rate approaches zero in this case. But when $\alpha = 0$, the incumbent is effectively using the pure-subscription-based model. Hence, the pure-subscription-based model provides greater profit than the mixed-single-product model.

In the case of mixed-product-line-extension model, the incumbent uses the free ad-sponsored product to "kill" the entrant. This means that the incumbent cannot introduce too many ads to its ad-sponsored product. As a result, there is cannibalization between the incumbent's two products, which lowers the profit for the incumbent. Cannibalization becomes more intense when q_l approaches q_h. Thus, competing through a pure-subscription-based model may be better when the effect of cannibalization dominates the additional ad profit from the pure-ad-sponsored product of the incumbent.

When $f > 0$, the two mixed business models become even less desirable.

Lemma 7 implies that the pure-subscription-based model may be the superior business model when $\alpha > 0$ and $f = 0$. This was *never* the case in the monopoly setting.

Lemma 8 *Compared to the monopoly case, the mixed-product-line-extension model no longer dominates the mixed-single-product model.*

The intuition is the same as in Lemma 7. Compared to the monopoly case, we now have cannibalization between the two products offered by the incumbent in the mixed-product-line-extension model. When the cannibalization is intense (this happens when q_h and q_l are close), the mixed-single-product model may be better. This was *never* the case in the monopoly setting.

Lemma 9 *When α is sufficiently large, the pure-ad-sponsored model is the optimal business model. When f is sufficiently large, only one of the two pure business models can be optimal.*

When α is very large, the incumbent wants to give away the product for free to maximize its market share. The situation is similar to the monopoly but because the incumbent needs to make sure that the entrant is pushed out, there is a tighter constraint on the amount of ads that the incumbent can include with the product. The profit is lower than in the monopoly case. The second part of the lemma is straightforward as f is incurred for mixed models only.

Proposition 3 *When $q_h \leq 2q_l$, three possible business models might be optimal (the one dominated is the mixed-single-product model); When $q_h > 2q_l$, all four business models may be optimal.*

Figures 2, 3, and 4 illustrate our results.[13]

Comparing Figures 2 and 3, we see that depending on q_h and q_l, the set of possibly optimal business models changes. When $q_h \leq 2q_l$, the equilibrium tactics in the mixed-single-product model are such that $q_l - \beta A_l^2 > 0$ (interior solution). As argued above, in this case, the mixed-single-product model is dominated by the pure-subscription-based model and, thus, we are left with three possibly optimal business models.

The shape of the region over which the mixed-single-product model is optimal (Figures 3 and 4) is interesting. When $q_h > 2q_l$ and α is small, we are at the corner ($q_l - \beta A_l^2 = 0$). In

[13]The three figures have $q_h = 3$ and $\beta = 1$. Only q_l varies. For Figure 2, $q_l = 1.6$. For Figure 3, $q_l = 1.3$. And for Figure 4, $q_l = 1.49$.

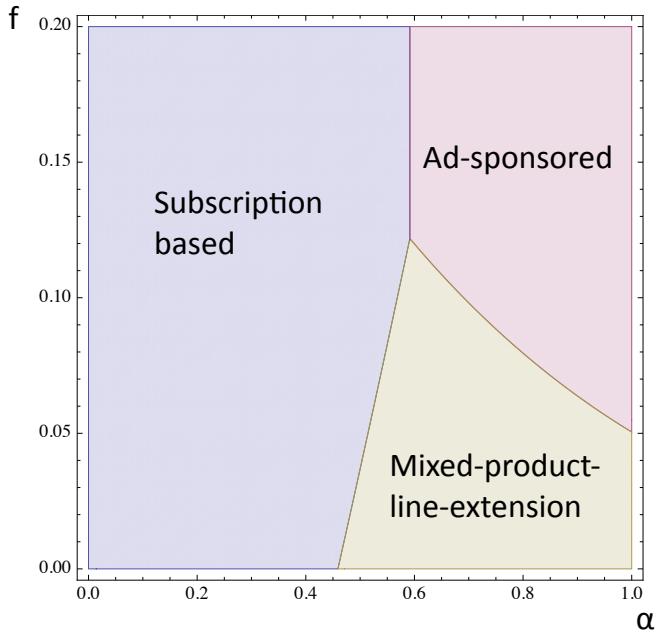

Figure 2: Optimal business models in duopoly when $q_h \leq 2q_l$.

this case, because there is no strategic reaction by the entrant, as α increases, the profit can only raise for the incumbent. As α grows, the incumbent increases its advertising intensity and the vertical differentiation between both products becomes smaller. Two cases may happen as α increases. When $q_h \gg 2q_l$, before we reach the point where the entrant has an incentive to lower A_l to gain share, the incumbent finds that α is large enough to introduce an ad-sponsored product to push the entrant out of the market. Thus, in this case, whenever the incumbent competes through the mixed-single-product model, we are at the corner were willingness to pay for the entrant's product is zero. Figure 3 illustrates this case.

The second case happens when the quality difference is not very large (i.e., $q_h > 2q_l$). In this case, as α grows, the point where the incumbent finds that α is large enough to introduce an ad-sponsored product to push the entrant out occurs *after* the point where the entrant has an incentive to lower A_l. From this point on, we are at an interior solution and the incumbent profit decreases with α. Figure 4 illustrates this situation.

We end our discussion by noting that, as illustrated in Figure 4, for fixed f, as α increases, a pure-subscription-based model may initially be optimal, then the mixed-single-product model becomes superior, but for even larger α the pure-subscription-based model is optimal once again. As α keeps growing, the mixed-product-line-extension model is preferred. And when α is very large, the pure-ad-sponsored model yields the largest profit.

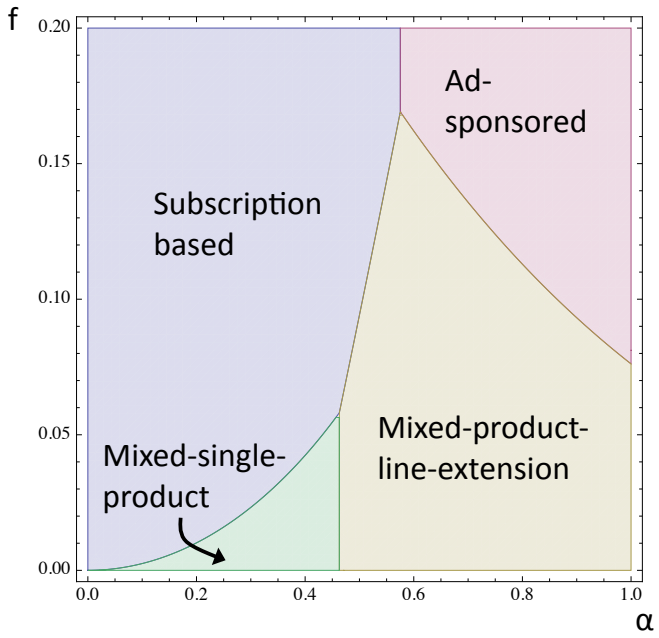

Figure 3: Optimal business models in duopoly when $q_h \gg 2q_l$.

6 Managerial implications

We now explore the implications of our work for management practice. In Section 6.1 we show that the optimal response to an ad-sponsored entrant often leads to increased strategic focus. In Section 6.2, we compute the value of reconfigurations in the firm's business model. In Section 6.3, we present a simple two-by-two matrix that summarizes our results. Finally, in Section 6.4 we revisit some of the examples that we have used to motivate our analysis.

6.1 Increased strategic focus

We have shown that when the incumbent competes through the mixed-product-line-extension model, it winds up serving the entire market. Therefore, the mixed-product-line-extension model results in a situation where the incumbent firm is "all things to all people." Our model suggests that while a monopoly will often find it optimal to adopt this model, a profit-maximizing duopolist incumbent will be less likely to compete through a mixed business model.

To see this, compare the regions in (α, f) space where each business model is optimal. Figure 1 shows the region for the monopolist and Figures 2, 3, and 4 show the regions for

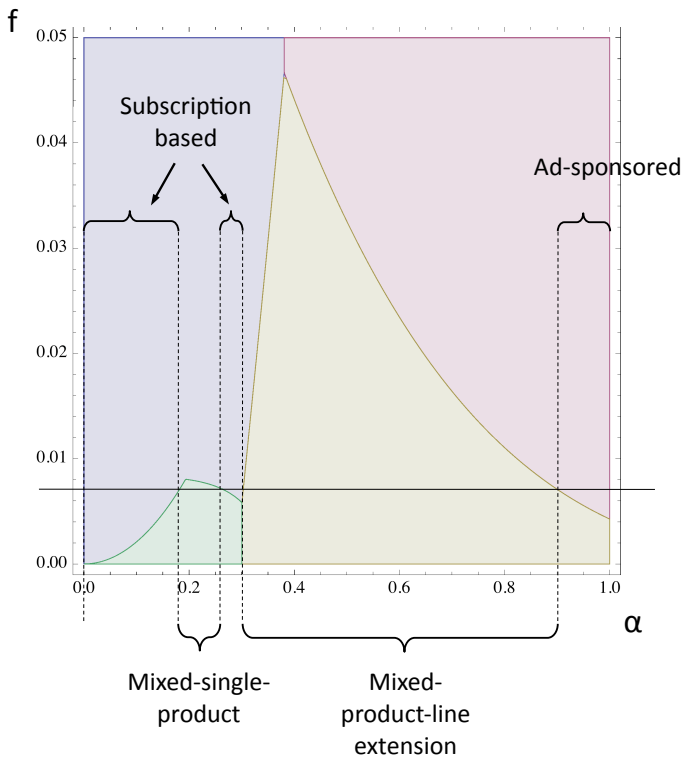

Figure 4: Optimal business models in duopoly when $q_h > 2q_l$.

the incumbent duopolist. Figure 5 superimposes Figures 1 on top of 3.[14] It reveals that the region of parameters such that it is optimal to compete through a mixed business model shrinks when there is a competitor that competes through a pure ad-sponsored business model. That is, a pure business model becomes more desirable when there is a potential entrant that is ad-sponsored. This effect becomes stronger as q_l approaches q_h. The reason is that, compared to the monopoly case, in the duopoly case the use of a mixed model implies either cannibalization (in the case of the mixed-product-line-extension model) or erosion of vertical differentiation (in the case of the mixed-single-product model). These two forces reduce incumbent's incentives to using mixed business models.

We conclude that *increased focus* and *narrower scope* (by competing through a pure business model or a mixed business model that does not serve the entire market) is more likely to be optimal when facing an ad-sponsored rival compared to the monopoly situation.

Remark 1 follows directly:

Remark 1 *If it is not optimal for the monopolist to compete through a mixed business model,*

[14]Of course, the same conclusions can be drawn by superimposing Figures 1 and 2, or Figures 1 and 4.

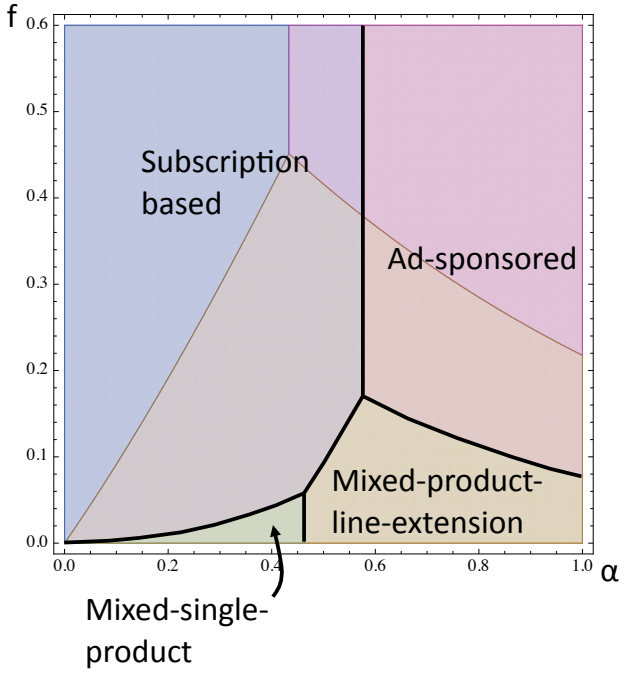

Figure 5: Monopoly and duopoly regions.

it is even less optimal to do so in the presence of an ad-sponsored rival.

Interestingly, that firms should not try to be "all things to all people" was identified first by Porter (1996). Porter's argument is that when a firm attempts to make everyone happy, its activity system will likely lead to internal inconsistencies resulting in a loss of competitive advantage.

Our reasoning is different. The suboptimality of mixed-product-line-extension business model is due to the nature of the competitive interactions that ensue when there is an ad-sponsored rival. While the mixed-product-line-extension model is often optimal when there is no competitor, duopolistic interactions result in damaging product cannibalization that hurt the attractiveness of the mixed business model. Thus, our analysis delivers an alternative/complementary theory as to why being "all things to all people" is often not wise.

6.2 Competing through business models

Our work suggests that the optimal response of an incumbent when an ad-sponsored entrant emerges often involves reconfiguration of its business model. We now illustrate quantitatively the cost of *not* competing through business models. Suppose that the incumbent

reacts by modifying its tactics but not by changing its business model. We ask: how much more profit could have been obtained if the incumbent had *also* considered business model reconfigurations?

Figure 6 shows the region (α, f) where the incumbent should have (optimally) reconfigured its business model after the entry of the ad-sponsored rival.[15]

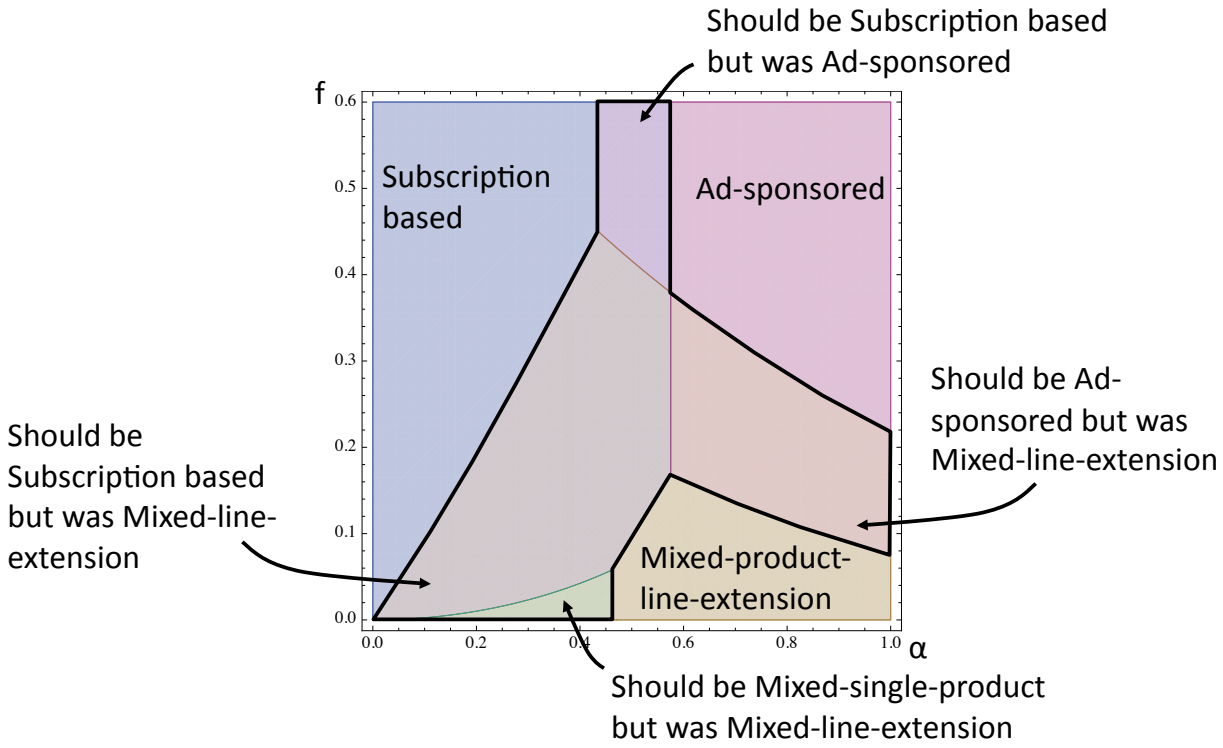

Figure 6: An incumbent who reacts to the entrant through tactics only.

Let $\pi_{\max}$ be the incumbent's profits when it competes through business models; that is, the profits when the incumbent makes use of strategy and tactics to fight the ad-sponsored rival. Let $\pi_{\text{constrained}}$ be the incumbent's profits when it competes through tactics only; that is, the profits when the incumbent does not consider changing its business model but reacts by optimally changing its tactical choices.

Figure 7 plots percent loss in profit, which is defined as

$$\% \text{ loss in profit} = \frac{\pi_{\max} - \pi_{\text{constrained}}}{\pi_{\max}}.$$

The plot shows that the profit implications of not considering business model reconfigurations may be substantial. In this simple example, the profit loss ranges from 0% to about

[15] In this particular example, $\beta = 1$, $q_h = 3$, and $q_l = 1.3$.

60%.

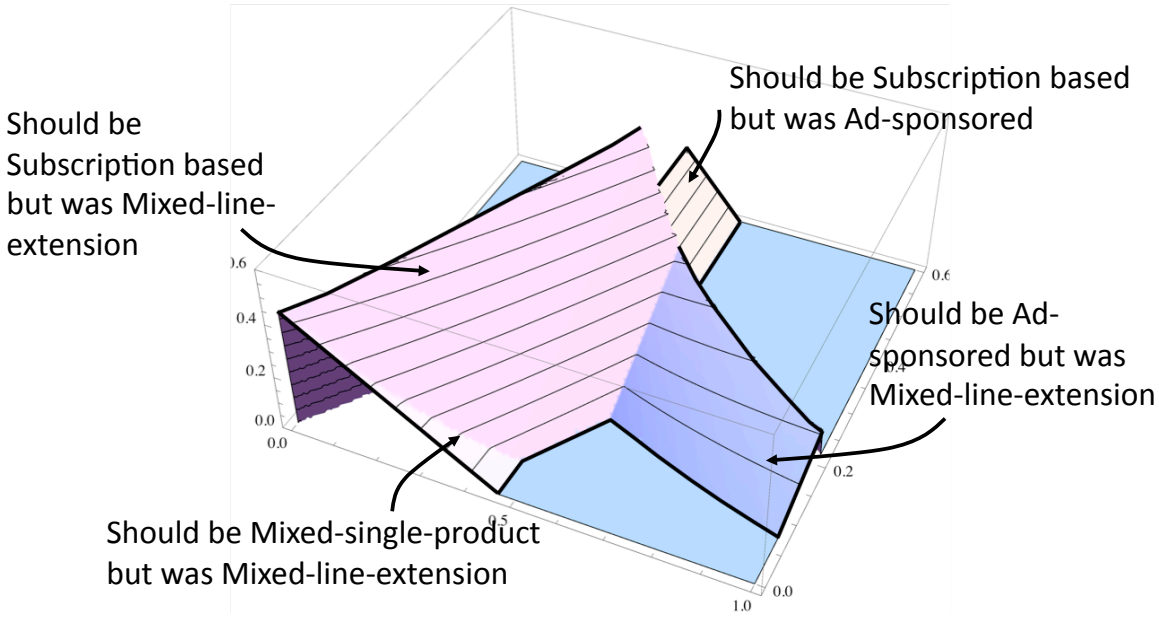

Figure 7: Profit implications of not competing through business models.

6.3 A two-by-two

Our theoretical results can be easily embedded into simple two-by-two matrices, a format that managers understand well. Figure 8 shows "two-by-two versions" of Figures 1, 2, and 3.[16] Specifically, the horizontal dimension α/β captures the attractiveness of bundling the product with ads. A large value of α/β means either that the advertising rates are large or that consumers are not too bothered by the ads (or both). The vertical dimension is f, the additional cost of running a mixed model. The matrix on the top row corresponds to the monopoly. The matrixes on the bottom row are for the duopoly. The matrix on the left is for the case $q_h < 2q_l$ and the one on the right for the case $q_h > 2q_l$.

Simple two-by-two matrices are helpful because practitioners often do not know the exact values of parameters (such as α, β, or f) but have a good idea of their orders of magnitude. In addition, it may be difficult for managers to foresee the full implications (including the equilibrium tactics) of competing through business models that they have not employed in the past. The matrices provide simple advice grounded on careful theory development that managers may consider when facing a new ad-sponsored entrant.

[16]The insight from Figure 4 is essentially the same as that of Figure 3.

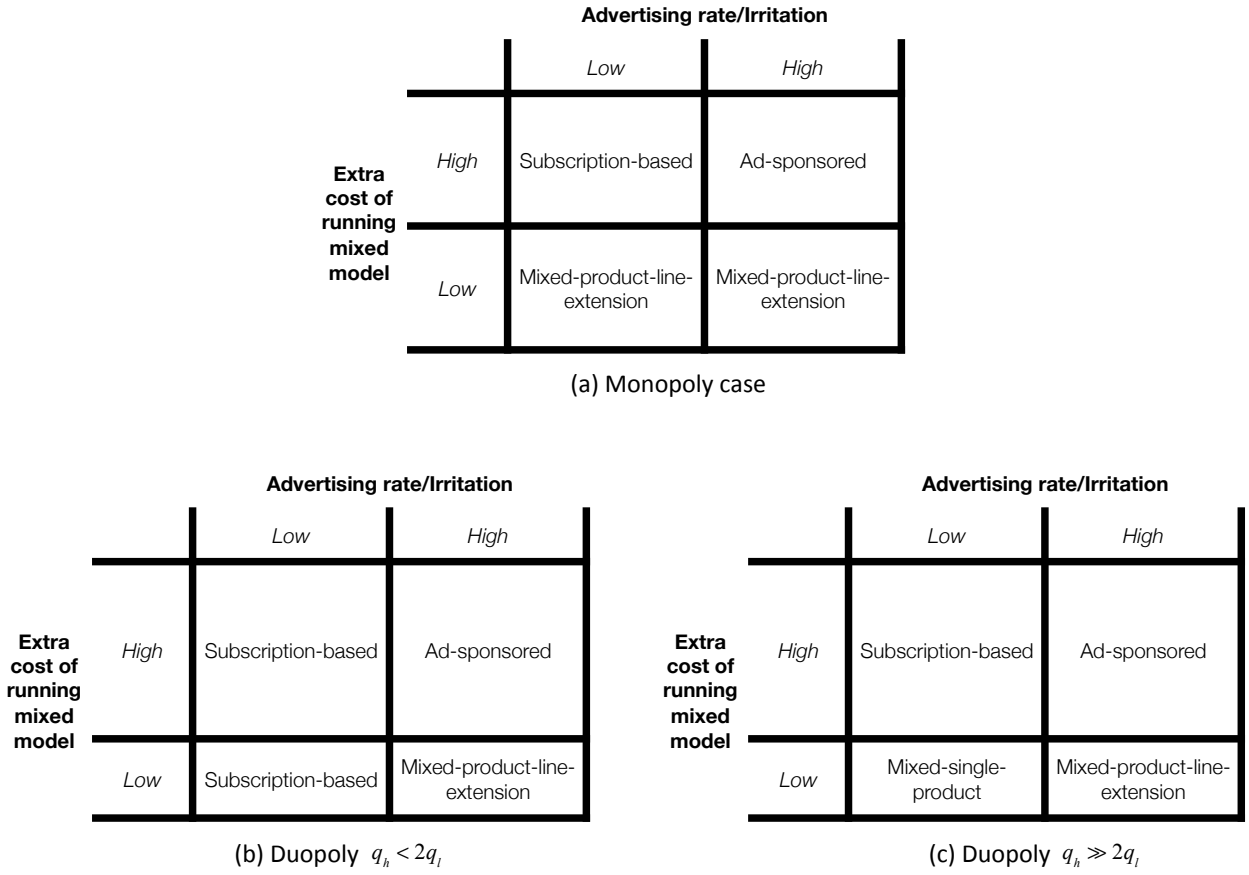

Figure 8: Optimal business models.

The most important implication of these matrices is that there is *one* cell where the recommended course of action changes drastically depending on whether there is an ad-sponsored rival or not and on the relative values of q_h and q_l.[17] We are in this interesting cell when the additional cost of running a mixed model f is low, the advertising rate α is low, and/or buyer irritability by ads β is high. It is when incumbent firms fall in that quadrant that reactions to ad-sponsored rivals are most critical. And it is in that quadrant that firms may be confused as to how to fight ad-sponsored rivals.

Our framework also delivers managerial implications for the ad-sponsored entrant. The entrant may have a product of high or low quality. A higher quality product increases the chances of successful entry. In addition, *absent incumbent reaction*, the profit potential for the entrant is larger when α/β is large. However, the analysis reveals that when α/β is large, the incumbent will compete through a business model with an ad-sponsored product which

[17] A second implication from the matrices is that when moving from monopoly to duopoly, mixed business models are less likely to be optimal. We have elaborated this point in Subsection 6.1.

will end up killing the entrant. When α/β is low, however, although the profit potential is not so great, the incumbent prefers to accommodate entry and both firms coexist.

To see this graphically, Figure 9 superimposes the "attractiveness of the market & likelihood of success" continuum onto a two-by-two matrix with dimensions "Quality of the entrant" and "Advertising rate/Irritation." We see that when the market is least attractive to the entrant, its likelihood of success is much larger than when the market is attractive if the incumbent did not react to the entry. The implication is that ad-sponsored entrants are condemned to either low profitability or to be pushed out of the market. This observation seems consistent with the poor profitability that ad-sponsored newspapers have had to date.

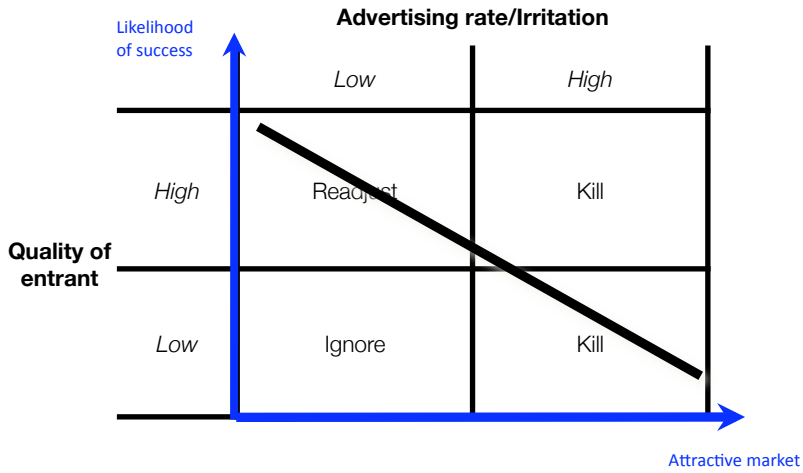

Figure 9: The entrant's viewpoint.

6.4 Examples revisited

We end this section by examining the responses of a few incumbents in real business situations in light of our theoretical results.

- *The Boston Globe.* By taking a significant stake in *Metro Boston* in 2006, *The Boston Globe* effectively switched from a mixed-single-product model to a mixed-product-line-extension model. This strategy helped *The Globe* push *BostonNow*, a free ad-sponsored entrant launched in 2007, out of the market in less than a year.

  The mixed-product-line-extension model appears to be the right business model for incumbents facing ad-sponsored entrants in the newspaper industry. It is reasonable to expect that the cost for newspapers to manage the advertiser side, f, is low as

established newspapers manage relationships with advertisers already. In addition, α seems to be relatively high. For example, *The Boston Globe* earns $577 per column inch on average with an average circulation of 386,415 per day in 2007. Hence, an advertiser pays about 18 cents per reader for a full-page ad while the newspaper is sold at 75 cents at the newsstands.[18] Moreover, β is likely to be relatively low, as readers have grown accustomed to having ads in newspapers over the years.

We also observe a similar pattern in Spain. While *Metro* in Spain was successfully launched in 2001 and by 2004 had achieved profitability, the competition with *Qué!*, an ad-sponsored paper launched in 2005 by *Recoletos* (a large media group), forced it to shut down its operation in January 2009.[19]

These examples are consistent with two predictions of our model. First, the mixed-product-line-extension model ends up killing the pure ad-sponsored entrant. Second, the best response to ad-sponsored rivals when α/β is high and f is low is the mixed, product-line-extension model.

- *Apple's iTunes.* Over the past few years a number of players have emerged that distribute music over the web for free and that earn revenues from advertising only. Examples include *Imeem*, *Grooveshark*, and *Spotify*. Apple's *iTunes* illustrates a situation where the incumbent's strategy has been *not* to respond to the ad-sponsored entrants; Apple has kept competing through a pure-subscription-based business model, just as it did before these players entered the market. Moreover, Apple has chosen not to lower prices.

  Our model suggests that one possible reason for inaction is that either α is low or β is high (or any combination of the two). Our best guess is that the issue for Apple is that β is too large for it to ever consider including ads in iTunes. Users of iTunes would likely become irritated if significant numbers of ads where shown on iTunes (when playing music or movies, for example). Ads would certainly detract from the slick and polished image that Apple's products and services have. Ads would not only have detrimental effects on iTunes' sales but also on Apple's other offerings such as the iPod, the iPhone, and the many models of personal computers that it offers. (We discuss this reputation effect in Section 8.6.)

[18] Authors' calculation based on data from TNS Media Intelligence.

[19] Source: `http://www.spanishnews.es/20090201-free-metro-newspaper-ceases-to-exist-in-spain/id=195/`, accessed April 2009.

7 Discussion: strategy vs. business model vs. tactics

We have been careful in our use of the expressions strategy, business model, and tactics. In this section we discuss the three notions in light of the formal model that we have just analyzed.[20]

In popular parlance, *business model* refers to "the logic of the firm, the way it operates and how it creates value for its stakeholders." We have operationalized this notion by calling business model the alternative, discrete profit functions that relate choices (such as prices or advertising intensities) with payoffs. For example, in the pure-subscription-based business model, the profit function is $\pi_h^S = (1-\theta^*)p_h$, and thus the firm earns profits from subscriptions only, while in the mixed-single-product business model, the profit function is $\pi_h^{MS} = (1-\theta^*)(p_h + \alpha A_h) - f$ and the firm earns profits from both subscriptions and advertising. Because in our formalization different business models correspond to different profit functions, every business model typically implies different optimal competitive choices (such as prices or advertising).

The description of a business model is not complete before the tactical choices that are enabled by the business model are spelled out. *Tactics* refers to the possible choices open to a firm by virtue of the business model that it employs. For example, in the pure-ad-sponsored business model, $\pi_h^A = \alpha A_h$, the only possible tactics are concerned with the choice of advertising intensity, A_h, while in the mixed-product-line extension business model

$$\pi_h^{ME} = \begin{cases} (1-\theta^*)(p_h + \alpha A_h) + \theta^* \alpha A'_h - f & \text{if } \; q_h - \beta A_h'^2 \geq q_l - \beta A_l^2 \\ (1-\theta^{**})(p_h + \alpha A_h) - f & \text{otherwise,} \end{cases}$$

and the tactics entail the choice of price and advertising intensity for the high-quality product, p_h and A_h, and advertising intensity for the low-quality product, A'_h. Therefore, business models set the stage for the tactical choices that firms make when interacting in the marketplace. The implication is that different business models give rise to different tactical choices and, thus, to tactical interactions of different nature. Specifically, we have seen that competitive outcomes are different depending on the business model used by the incumbent and that the optimality of a given business model depends on whether or not there is a competitor and on the values of the model's parameters (α, f, β, and $q_h - q_l$).

Profit functions are highly simplified, reduced form representations of business models.

[20] Casadesus-Masanell and Ricart (2009) offer a discussion of these and related issues based on a framework derived through inductive grounded theory.

These stylized representations allow detailed and tight mathematical analyses of optimal tactical choices for a small number of tactical variables such as price and advertising, which have been the focus of our paper. We think of profit functions as representations of business models as if looked at from the distance. We could "zoom down" closer to the actual business model used by the firm and generate more complex profit functions that explicitly accounted for additional elements in the business models that we have not considered. For example, the particular human resource management policies in place, the production technologies used, or the marketing policies (just to name a few) are all part of "the logic of the firm, the way it operates and how it creates value for its stakeholders" and, thus, are all part of a firm's business model, and could be included in the profit function to have a more detailed representation of the firm's business model. However, in most cases, these "closer," more complete representations of business models are too complex to be amenable to mathematical analyses. Depending on the question being studied, the simplifications are justified. For example, if a researcher is interested in tactical variables such as pricing that are not expected to interact much with tactical variables not considered (such as compensation policies or the pool from which talent is hired), then the use of simple profit functions would seem reasonable. This approach is used in most of the theoretical literature on industrial organization and its application to strategy. We should point out that this approach can be quite limiting because, as identified by Porter (1996, p. 68), the object of strategy is "the creation of a unique and valuable position, involving a different set of activities;" a unique business model, in our terminology.

Porter's (1996) activity systems are richer representations of business models, compared to the highly stylized profit functions that economic analyses use. The advantage of using activity systems is that they give a rich picture of "the logic of the firm, the way it operates and how it creates value for its stakeholders." Activity systems emphasize that a firm is more than the mere addition of activities; complementarities may result in important competitive advantages. On the negative side, activity systems, unlike profit functions, are not amenable to game theoretical analysis because, in most cases, they are too complex.

Having discussed the concepts of business model and tactics, we are now ready to define *strategy*. Loosely speaking, a *strategy* refers to the choice of business model through which (tactical) competition will take place. Note that strategy is a high-order choice that has profound implications on competitive outcomes. Choosing a particular business model means choosing a particular way to compete, a particular "logic of the firm:" a profit function and the associated set of possible tactics that will be used to maximize profits in the market

place. This concept of strategy agrees with Porter's (1996, p. 68) notion: "strategy is the creation of a unique and valuable position, involving a different set of activities." According to this definition, the activity system is a reflection of the firm's strategy. Strategy proper, however, is not the activity system itself but the *creation* of the activity system. Likewise, in our language strategy is concerned with the choice of a business model, and business models are represented formally through profit functions.

We have loosely defined *strategy* as a choice of business model. With this definition, it looks as if one could interchangeably use the notions *strategy* and *business model*. For example, if the firm's strategy is to choose the mixed-product-line-extension business model, then we could refer without loss of meaning to the firm's strategy as a mixed-product-line-extension strategy. So, what is the difference between strategy and business model? Why is there a need for the two expressions if they appear to refer to the same thing? We argue that *strategy* and *business model* are different but related concepts and that the strategy literature would benefit from clearly separating both notions. We will show that in many competitive situations, equating the two notions can lead to confusion.[21] The model that we have presented in this paper allows us to clearly discriminate between the two concepts.

A *strategy* is *a plan of action as to what business model is to be chosen for the different situations that might arise*. Note that in this definition, a strategy is *much more* than the mere selection of a business model; it is a *contingent* plan of action as to what business model will be selected, depending on different contingencies that might occur.

To illustrate the difference, we use the formal model of Sections 4 and 5. Suppose first that there is a monopolist just like the one we have modeled in Section 4, and that there is no firm that may challenge the monopolist's turf. Assume that the parameter values are such that the optimal business model is the mixed-product-line-extension model. In this simple situation, the notions of business model and strategy coincide: The monopolist competes through a mixed-product-line-extension business model or, equivalently, the monopolist has a mixed-product-line-extension strategy. The reason is that in this simple setting, there is no contingency that may make the monopolist change business models.

Consider now that the monopolist becomes aware that there is a potential entrant, a firm just as the one modeled in Section 5. A *strategy* for the monopolist is a complete plan of action as to what business model it will employ, contingent on the potential entrant's choice of whether to enter or not. More concretely, one possible strategy for the monopolist is: if the potential entrant does not enter, then remain with the mixed-product-line-extension

[21] We will also see that in the simplest competitive situations, both notions coincide.

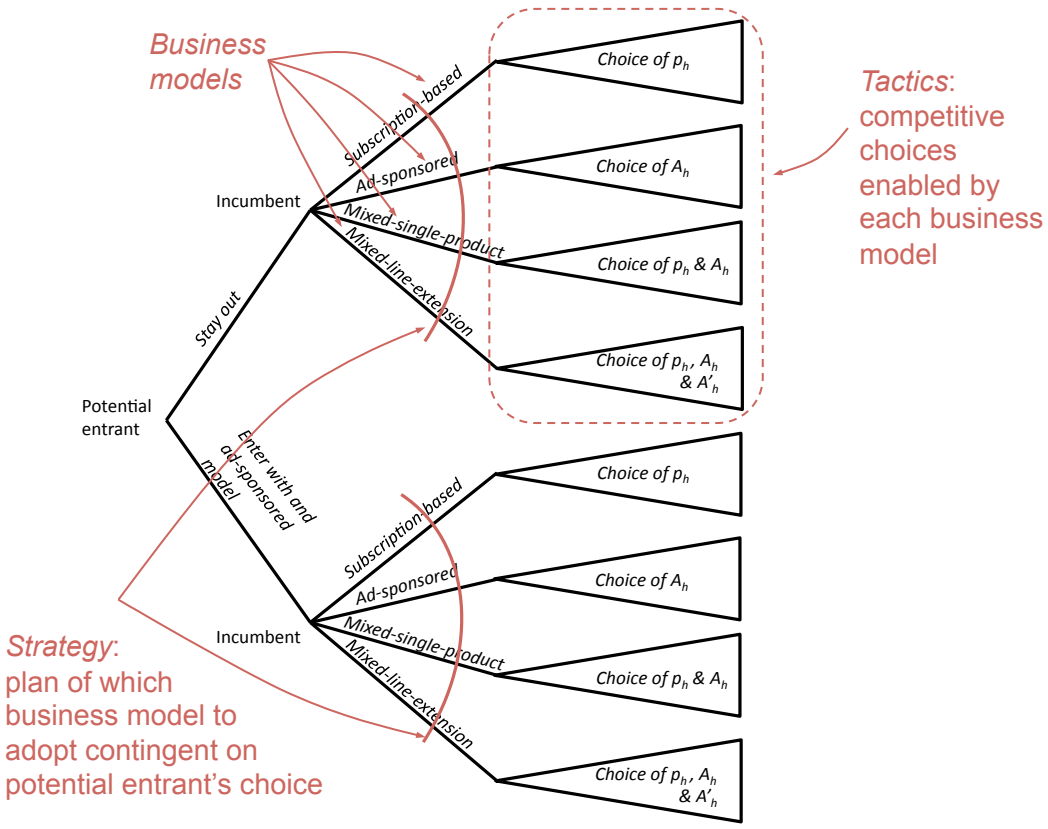

Figure 10: Strategy vs. Business Model vs. Tactics.

business model, but if the potential entrant does enter, then adopt the pure-subscription-based business model. Figure 10 summarizes the distinction between strategy, business model, and tactics.

A *crucial difference* between business models and strategies implied by our conceptualization is that while the business model used by a firm is observable, its strategy is typically not (fully) observed: The only responses (as prescribed by the strategy) that do take place are those to contingencies that have occurred. For example, the incumbent's strategy may be: if the potential entrant does not enter, then remain with the mixed-product-line-extension business model, but if the potential entrant does enter, then adopt the pure-subscription-based business model. Suppose that the potential entrant does not enter, then the incumbent stays put with the mixed-product-line-extension business model and its strategy (the complete plan as to what business model is to be chosen for whatever situation that might arise) is never fully observed. Put differently, all that we can observe are the equilibrium outcomes of strategies but *not* the strategies themselves. Therefore, business models are reflections of the *realized* strategy.

8 Limitations, extensions, and conclusions

We end the paper by discussing some of our main assumptions and by suggesting possible extensions to the model.

8.1 Higher quality ad-sponsored rivals

We have assumed throughout that the ad-free quality of the incumbent is at least as large as that of the ad-sponsored rival: $q_h \geq q_l$. An implication of this assumption is that if an advertising war arises, it is always the incumbent that wins. In addition, in equilibrium, the incumbent always has the higher quality product. The internet, however, allows the offering of free high-quality products sponsored by ads. For instance, Google set up GMail as a free email system supported by advertisements. GMail is a high-quality email system: It offers an unmatched search capability, over seven gigabytes of free storage space, online and offline access, and all the bells and whistles that users of paid email systems, such as Microsoft *Outlook* or Apple's *Mail*, have grown accustomed to.

If we allowed $q_l > q_h$, the incumbent could always be pushed out of the market by the entrant. The reason is that the entrant could always include a number of ads such that its product would be of higher net quality than that of the incumbent (i.e., set A_l such that $q_l - \beta A_l^2 \geq q_h$). Interestingly, even if $q_l > q_h$, it may be to the entrant's best interest to set a number of ads such that $q_l - \beta A_l^2 < q_h$, allowing the incumbent to remain a viable competitor. This will happen when q_l and q_h are close. The reason is that in this case, the only way to kill the incumbent is by having few ads, but given the ad-sponsored business model, the entrant's profits are low. The entrant may be better off setting a large number of ads and end up with a net quality lower than that of the incumbent, $q_l - \beta A_l^2 < q_h$. The entrant may earn larger profit because its high q_l allow it to introduce many ads and its revenue is proportional to the number of ads.

8.2 Reduced-form advertiser side

While in our model firms may earn profits from both sides of the market (consumers and advertisers), we have assumed that the advertising rate, α, is fixed and does not depend on the amount of ads, A. This assumption is motivated by the popularity of pay-per-click or pay-per-view business models used in the online advertising market today.[22] If we assume

[22]Under these business models, an advertiser pays every time its ad is clicked or viewed by a consumer.

that each advertiser generates a profit of α every time its ad is viewed, and also assume that the advertising market is perfectly competitive (i.e., advertisers are price-takers), then it is optimal for the firms to charge α to each advertiser for contacting a viewer. In this setting, the advertising rate charged to each advertiser is indeed independent of the amount of ads. Similar assumptions have been made before in the literature (e.g., Hansen and Kyhl 2001; Gabszewicz et al. 2004).

A more complicated model may allow the advertising rates to change with the number of ads that are being placed. Similar to the negative effect of ads on the users' utility ($U(\theta) = \theta q - \beta A^2$), the advertisers' willingness to pay for ads is likely to decrease with the number of ads already in place. An advertiser may be willing to pay a lot if its ad is the only one in the *Boston Globe*'s Friday edition. However, if most of the paper is made up of ads, its willingness to pay for advertising is likely to fall. One could also extend our model by allowing the incumbent to compete with the entrant for advertisers when the incumbent uses a mixed business model or a pure-ad-sponsored business model.

Our current model does not take these issues into consideration as our focus has been on the strategic, rather than the tactical, aspects of the model. To have a tractable model of strategic choice of business models, we have simplified the tactical interactions on the advertiser side.[23]

If we allow the advertising rate to decrease with the amount of ads or allow firm competition on the advertiser side, the incumbent will find it less profitable to introduce ads as now it is more difficult to earn profits from advertisers. Hence, the incumbent will find entry deterrence less attractive, while the entrant will still enter albeit its profit will be lower.

8.3 Multiple entrants

Our model can be extended to consider the possible entry of more than one low-quality ad-sponsored entrant. This was the case, for example, in Spain in the early and mid-2000s. Metro International launched *Metro Spain* in March 2001 in a competitive space where media group *Schibsted* competed with free newspaper *20 Minutos*. And in 2005, media group *Recoletos* launched the free newspaper *Qué!* and Grupo Planeta (a large publishing house) launched *ADN*. Therefore, in 2005 there were four(!) free, ad-sponsored newspapers

[23]As we allow the net quality of the product to decrease with the number of ads, if we assume that the advertising rate decreases with ads, the profit function will be a cubic function of the number of ads, making the analysis more complicated. Similarly, allowing the two firms to compete for advertisers will also significantly complicate the analysis. Indeed, theoretical models on ad-sponsored business models (with the exception of Reisinger (2004)) often only consider firm competition on the consumer side.

backed by large media corporations in Spain, making this perhaps the most competitively crowded market in Europe for ad-sponsored newspapers.

Although we have not dealt explicitly with multiple ad-sponsored entrants in our analysis, it is easy to examine the main effect. Assume for simplicity that there are two identical low-quality, ad-sponsored entrants with products l_1 and l_2 with equal quality (i.e., $q_{l_1} = q_{l_2}$). In this case, a Bertrand competition argument implies that both entrants are condemned to zero profits, no matter what the incumbent does. The effect of multiple entrants on the incumbent is clear also. The increased competition for share, leads to relatively high qualities for the entrants' products because when $q_{l_1} = q_{l_2}$, the only possible equilibrium advertising intensities for the entrants are $A_{l_1} = A_{l_2} = 0$. The competition between ad-sponsored rivals in the low-end market is bad news for the incumbent. The higher quality of the entrants' products make it harder for the incumbent to gain share and its profits could be lower than if there was one entrant only with a product of quality $q_l = q_{l_1} = q_{l_2}$. The incumbent has a greater incentive to introduce an ad-sponsored product to push the entrants out as cannibalization with its high-end product is unavoidable. The news for consumers is good, however, as the average quality of the products could be higher with competition.

8.4 Fondness for advertising

Recall our utility specification: $U(\theta) = q - \beta A^2$. We have assumed that $\beta > 0$ to capture the fact that consumers dislike advertisements: more ads lead to lower utility. If consumers *enjoy* ads, then we would have $\beta < 0$. To some extent, this may be the case for classifieds in the newspaper or ads in magazines such as *Vogue*. When $\beta < 0$, the equilibrium tactics involve $A \to \infty$ and consumer utility grows without bound.

While we understand that consumers may like to see some ads, as more and more ads are shown it gets to a point after which, for most consumers, ads become annoying, irritating, and exasperating. Clearly, all of our results go through if we interpret q_i not as product i's quality in the absence of ads but as its quality exactly at the point in which having one more ad begins to decrease utility (the point at which β becomes positive).

8.5 Vertical vs. horizontal differentiation

In our model, products are vertically differentiated only: All consumers rank the products equally but have different willingness to pay for them. We do not consider the possibility of horizontal differentiation, a situation where consumers rank the products differently. Our

assumption of vertical differentiation between the incumbent and the entrant's products is motivated by the phenomena that we study, as in many cases the products offered by ad-sponsored rivals are of lower quality than those of the incumbents. For example, most readers would agree that the ad-sponsored newspaper *Metro* is of lower quality than *the Boston Globe* or *the New York Times*, two newspapers sold at positive price against which *Metro* competes. Or the free personal computers that *Free-PC* gave away,[24] were bare-bones low-end computers of lower quality than the average computer sold by Dell, Compaq, HP, or IBM at the time.

While our model is motivated by the vertical differentiation observed between incumbent firms and ad-sponsored rivals, in reality there is always some degree of horizontal differentiation. For example, while most newspaper readers would agree that *the Boston Globe* is of higher quality than *Metro*, a few individuals may be unhappy about the format (page size and larger number of pages) of *the Boston Globe* and may still prefer *Metro* even if *the Boston Globe* were given away for free. While this may be the case, as long as the dominant differentiation between incumbents and ad-sponsored rivals is along the vertical dimension, our results on strategy should prevail even if both forms of differentiation are considered jointly in a formal model.

8.6 Reputation effects

One concern that firms may have when considering the use of the mixed-product-line-extension model is that the offering of a low-quality version of the company's flagship product may have adverse effects on the reputation for the high quality product. That is, perceived quality of the high-end product may be affected by the presence of the low-end, ad-sponsored version of the product.

Our model can be easily modified to take into account this effect. The most obvious way to incorporate such reputation concerns is by having $q_h^{ME} < q_h$ in the profit function for the mixed-product-line-extension model. Clearly, if the difference between q_h^{ME} and q_h is low, our qualitative results remain unchanged. The only difference is that the region of parameter values for which the mixed-product-line-extension business model is optimal shrinks. Of course, if the reputation effect is significant (i.e., the difference between q's is large), then the mixed-product-line-extension model should never be used in equilibrium.

[24] *Free-PC* was the company that kicked off the free PC craze in 1999 by giving away computers with complementary Internet service and subsidizing the cost through advertising.

8.7 Multi-homing

A final point that deserves discussion is the issue of multi-homing. In our setting, multi-homing refers to the possibility that consumers consider consuming one unit of each offering, the paid product and the free product. There are several ways to introduce multi-homing in our setting, but perhaps the easiest way is by having each consumer derive separate utilities for the paid high-quality product and for the free low-quality product and allowing consumers to consume one unit of each product if they so desire. Mathematically, and individual of type θ obtains the following utility when consuming the paid product (which is sold at $p \geq 0$) and the free product:

$$\begin{aligned} U(\theta)_{\text{paid}} &= \theta q_{\text{paid}} - \beta A_{\text{paid}}^2 - p \\ U(\theta)_{\text{free}} &= \theta q_{\text{free}} - \beta A_{\text{free}}^2. \end{aligned}$$

Consumers will adopt both products if $U(\theta)_{\text{paid}} \geq 0$ and $U(\theta)_{\text{free}} \geq 0$ and obtain a total utility of $U(\theta)_{\text{paid}} + U(\theta)_{\text{free}}$. In this case, the incumbent will have a strong incentive to compete through the mixed-product-line-extension model as with multi-homing, there is no competition, and thus no cannibalization, between its high-quality and low-quality products. The only reason why the incumbent may not compete through this business model is when f is too large. Thus, in most instances, the entrant is pushed out when consumers are adopting both paid and free products.

8.8 Conclusion

Competing through business model reconfiguration is everyday more relevant given the increasing number of opportunities for business model innovations enabled by technological progress, changes in customer preferences, and deregulation. IBM's 2006 and 2008 *Global CEO Study*,[25] for example, show that top management in a broad range of industries are actively seeking guidance on how to innovate in their business models to improve their ability to both create and capture value.

We hope that our analysis of strategies to fight ad-sponsored rivals is helpful to researchers and practitioners willing to consider competition beyond tactics in all sorts of competitive settings. From a conceptual point of view, the two-period game that we have presented with firms choosing business models that set the boundaries of the tactical game that follows,

[25]IBM Global Business Services, "The Global CEO Study 2006," IBM Corporation, 2006; IBM Global Business Services, "The Global CEO Study 2008," IBM Corporation, 2008.

are applicable to other competitive situations where firms choose strategies to fight low-cost entrants (Ryanair, Telmore...), open source projects (Linux, Apache...), platform players (shopping malls, video game systems...), mass customizers (Dell, Timbuk2...), or the like.

The most obvious aspect of our approach to modeling competition through business models that demands further development is allowing not only the focal firm (the incumbent in our setting) but also all other industry participants (the entrant in our setting) to choose business models. The analysis of endogenous business models for all players is technically challenging as it requires working with best response functions at the business model level, a construct that is difficult to handle. It is our hope to have provided a solid first step towards a more general framework for the study of competition through business models.

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Online Appendix

Proof of Proposition 1.

Pure-subscription-based model.

We have $\theta^* = \frac{p_h}{q_h}$. The demand is thus $1-\theta^* = 1-\frac{p_h}{q_h}$. The monopoly profit is: $\pi_h = (1-\frac{p_h}{q_h})p_h$. Solving the first-order condition (FOC) w.r.t. p_h gives the profit-maximizing price as $p_h = \frac{q_h}{2}$. The second order condition (SOC) w.r.t. p_h is negative. Therefore, the monopoly profit with under this business model is $\pi_h^S = \frac{q_h}{4}$.

Pure-ad-sponsored model.

In this case, the monopolist will introduce as many ads as possible subject to the constraint that $q_h - \beta A_h^2 \geq 0$. Therefore, $A_h = \sqrt{\frac{q_h}{\beta}}$ and the monopoly profit is $\pi_h^A = \alpha\sqrt{\frac{q_h}{\beta}}$.

Mixed-single-product-model.

In this case, the type of the consumer who is indifferent from purchasing and not purchasing the product, θ^*, is $\frac{p_h}{q_h-\beta A_h^2}$. Therefore,

$$\pi_h = (1 - \frac{p_h}{q_h - \beta A_h^2})(p_h + \alpha A_h).$$

The FOC w.r.t. p_h gives:

$$p_h = \frac{1}{2}(q_h - \alpha A_h - \beta A_h^2).$$

We can thus re-write the profit function as

$$\pi_h = \frac{(q_h + A_h(\alpha - \beta A_h))^2}{4(q_h - \beta A_h^2)}.$$

The FOC w.r.t. A_h implies that A_h solves the equation $A_h^3\beta^2 + q_h(\alpha - A_h\beta) = 0$.

Mixed-product-line-extension-model.

The indifferent consumer between h and h' is determined by: $\theta^*(q_h - \beta A_h^2) - p_h = \beta^*(q_h - \beta A_h'^2)$. Hence, $\theta^* = \frac{p_h}{\beta(A_h'^2 - A_h^2)}$. It is important to note that for both products to be active, we need $\sqrt{\frac{q_h}{\beta}} \geq A_h' > A_h$. The profit is:

$$\pi_h^{ME} = (1 - \frac{p_h}{\beta(A_h'^2 - A_h^2)})(p_h + \alpha A_h) + \frac{p_h}{\beta(A_h'^2 - A_h^2)}(\alpha A_h') - f \tag{1}$$

The monopolist maximizes π_h by setting p_h, A_h and A'_h. The FOC w.r.t. p_h gives:

$$p_h = \frac{1}{2}(A'_h - A_h)(\alpha + \beta(A_h + A'_h)).$$

Hence $\theta^* = \frac{1}{2}(1 + \frac{\alpha}{\beta(A_h + A'_h)})$. For $\theta^* \in [0,1]$, we need $A_h + A'_h \geq \frac{\alpha}{\beta}$. We then substitute p_h into the profit function and obtain:

$$\pi_h^{ME} = \frac{1}{4}\left(2\alpha(A_h + A'_h) + (1 - \frac{2A_h}{A_h + A'_h})\frac{\alpha^2}{\beta} + \beta(A'^2_h - A_h^2)\right) - f. \tag{2}$$

It is easy to see that π_h^{ME} increases in A'_h. We conclude that the monopolist will set A'_h to the maximum. Hence, $A'_h = \sqrt{\frac{q_h}{\beta}}$.

We then take FOC of equation (2) w.r.t. A_h and solve for optimal A_h. We have:

$$A_h = -\frac{A'_h}{2} + \frac{\sqrt{A'_h}\sqrt{4\alpha + \beta A'_h}}{2\sqrt{\beta}} = \frac{-\sqrt{q_h} + (\frac{q_h}{\beta})^{\frac{1}{4}}\sqrt{4\alpha + \sqrt{\beta q_h}}}{2\sqrt{\beta}}. \tag{3}$$

Hence,

$$\theta^* = \frac{1}{2} + \frac{\alpha}{\sqrt{\beta}\left(\sqrt{q_h} + (\frac{q_h}{\beta})^{\frac{1}{4}}\sqrt{4\alpha + \sqrt{\beta q_h}}\right)}.$$

For the solution to be interior, we need $\theta^* = \frac{1}{2}(1 + \frac{\alpha}{\beta(A_h + A'_h)}) \in [0,1]$. Hence, we need $\alpha \leq 2\sqrt{\beta q_h}$. Under this condition, the SOC of π_h^{ME} w.r.t. A_h is negative. Substituting the expressions of A_h and A'_h into equation (2), we obtain the profit:

$$\pi_h^{ME} = \frac{2\alpha^3 + \left(9\sqrt{\beta q_h} - 5\sqrt[4]{\beta q_h}\sqrt{4\alpha + \sqrt{\beta q_h}}\right)\alpha^2 + 2\alpha\beta q_h + (\beta q_h)^{5/4}\left(\sqrt{4\alpha + \sqrt{\beta q_h}} + \sqrt[4]{\beta q_h}\right)}{4\beta\left(-2\alpha + \sqrt{\beta q_h} + \sqrt[4]{\beta q_h}\sqrt{4\alpha + \sqrt{\beta q_h}}\right)} - f.$$

When $\alpha > 2\sqrt{\beta q_h}$, only the ad-sponsored product is active and the business model effectively becomes an ad-sponsored model. ■

Proof of Lemma 1. Under the mixed-single-product model, the optimal amount of A_h is given by $A_h^3\beta^2 + q_h(\alpha - A_h\beta) = 0$. It is easy to see that as long as $\alpha > 0$, the optimal A_h is greater than 0. Hence, the profit from the mixed-single-product model when $f = 0$ has to be greater than the one from the pure-subscription-based model as otherwise the monopolist could set $A_h = 0$ to earn larger profit. Hence, the mixed-single-product model dominates the pure-subscription-based model. From Lemma 2, we know that the mixed-product-line-

extension model dominates the mixed-single-product model. Hence, when $f = 0$, the pure-subscription-based model is dominated by the two mixed models. ■

Proof of Lemma 2. Under the mixed-single-product-model, a consumer with type θ receives utility $\theta(q_h - \beta A_h^2) - p_h$. Hence, for consumers with $\theta < \frac{p_h}{q_h - \beta A_h^2}$, they do not adopt the product. The monopolist could increase its profit by offering a second product with $A'_h = \sqrt{\frac{q_h}{\beta}}$ number of ads and these non-adopters will adopt this ad-sponsored product and ad revenue will accrue. The demand for the first product remains unchanged. Hence, the monopolist can always earn more profit with the mixed-product-line-extension strategy. ■

Proof of Lemma 3. As the mixed-single-product model is dominated, we only need to consider the other three models. We know from the proof of Proposition 1 that when $\alpha > 2\sqrt{\beta q_h}$, the mixed-product-line-extension model effectively becomes the pure-ad-sponsored model. We also know that $\pi_h^S = \frac{q_h}{4}$ and $\pi_h^A = \alpha\sqrt{\frac{q_h}{\beta}}$. It is easy to see that when $\alpha > \frac{\sqrt{\beta q_h}}{4}$, $\pi_h^A > \pi_h^S$. Hence, when $\alpha > 2\sqrt{\beta q_h}$, the pure-ad-sponsored model is the optimal business model. Similarly, when $\alpha < \frac{\sqrt{\beta q_h}}{4}$, the pure-subscription-based model dominates the pure-ad-sponsored model. ■

Proof of Lemma 4. When α is low, we only need to consider the pure-subscription-based model and the mixed-product-line-extension model as the pure-ad-sponsored model is dominated by the pure-subscription-based model as shown in Lemma 3. We know that π_h^{ME} increases with α, as the monopolist could at minimum keep p_h, A_h and A'_h at the same level and gain more profit. Hence, π_h^{ME} increases with α and decreases with f. We also know that when α is zero, the mixed-product-extension model effectively becomes a pure-subscription-based model. Therefore, given a level of f, when α is sufficiently low, the pure-subscription-based model is better than the mixed-product-line-extension model. ■

Proof of Proposition 2.

Pure-subscription-based model.

Under the subscription-based model, $\theta^* = \frac{p_h}{q_h - q_l + \beta A_l^2}$. The FOC condition of the profit function, $\pi_h^S = (1 - \theta^*)p_h$, gives the optimal price of product h, $p_h = \frac{1}{2}(q_h - q_l + \beta A_l^2)$ and the SOC is negative.

The FOC condition of the profit function, $\pi_l^S = \alpha\theta^* A_l$, gives the optimal amount of ads of product l, $A_l = \sqrt{\frac{q_h - q_l}{\beta}}$ and the SOC is negative.

The constraint that $q_l - \beta A_l^2 \geq 0$ gives $A_l \leq \sqrt{\frac{q_l}{\beta}}$.

Therefore, when $q_h < 2q_l$, we have an interior solution. In this case, $A_l = \sqrt{\frac{q_h - q_l}{\beta}}$. Substituting it to the expression of equilibrium p_h, we have $p_h = q_h - q_l$. Hence, $\pi_h^S = \frac{q_h - q_l}{2}$ and $\pi_l^S = \frac{\alpha}{2}\sqrt{\frac{q_h - q_l}{\beta}}$.

When $q_h \geq 2q_l$, we have a corner solution. In this case, $A_l = \sqrt{\frac{q_l}{\beta}}$. Thus, $p_h = \frac{q_h}{2}$, $\pi_h^S = \frac{q_h}{4}$ and $\pi_l^S = \frac{\alpha}{2}\sqrt{\frac{q_l}{\beta}}$.

Pure-ad-sponsored model.
Under the ad-sponsored model, there is always an incentive to maximize the number of ads because all consumers buy and more profit is generated from advertisers as the number of ads grow. If $q_h - \beta A_h^2 < q_l$, then the entrant will choose a small A_l such that $q_h - \beta A_h^2 < q_l - \beta A_l^2$ and get all the demand. The best response for the incumbent is to decrease A_h. Then the entrant will decrease A_l. This process ends when $q_h - \beta A_h^2 = q_l$.

Hence, the equilibrium amount of ads for the incumbent is $A_h = \sqrt{\frac{q_h - q_l}{\beta}}$. All consumers purchase product h. Hence, the profit of the incumbent is $\alpha\sqrt{\frac{q_h - q_l}{\beta}}$. The entrant receives no demand and thus no profit.

Mixed-single-product model.
We begin by finding the indifferent consumer between h and l. Its type is $\theta^* = \frac{p_h}{q_h - q_l - \beta A_h^2 + \beta A_l^2}$. Profit for the incumbent is

$$\pi_h^{MS} = (1 - \frac{p_h}{q_h - q_l - \beta A_h^2 + \beta A_l^2})(p_h + \alpha A_h) - f.$$

The FOC of π_h^{MS} w.r.t. p_h gives $p_h = \frac{1}{2}(q_h - q_l - \alpha A_h - \beta A_h^2 + \beta A_l^2)$. Substituting p_h into the profit function, we have:

$$\pi_h^{MS} = \frac{(q_h - q_l + \alpha A_h + \beta(A_l^2 - A_h^2))^2}{4(q_h - q_l + \beta(A_l^2 - A_h^2))} - f \tag{4}$$

We can then take FOC w.r.t. A_h and obtain

$$A_l^2 + \frac{q_h - q_l}{\beta} = \frac{A_h^3}{A_h - \alpha/\beta}. \tag{5}$$

The entrant profit is

$$\pi_l^{MS} = \frac{p_h}{q_h - q_l - \beta A_h^2 + \beta A_l^2}\alpha A_l.$$

Hence, its best response function is

$$A_l = \sqrt{\frac{q_h - q_l - \beta A_h^2}{\beta}}. \tag{6}$$

The SOC for A_l is always negative. We also need $q_l - \beta A_l^2 \geq 0$, i.e., $A_l < \sqrt{\frac{q_l}{\beta}}$. Hence, when $q_h \leq 2q_l$, $\sqrt{\frac{q_h - q_l - \beta A_h^2}{\beta}} \leq \sqrt{\frac{q_l}{\beta}}$ and we always have an interior solution. In this case, we could solve equations (5) and (6) for A_h and A_l, and obtain the expressions for equilibrium profits π_h^{MS} and π_l^{MS}.

When $q_h > 2q_l$, we may have a corner solution: this happens when A_l computed from equation (6) is greater than $\sqrt{\frac{q_l}{\beta}}$. When we are at a corner, $A_l = \sqrt{\frac{q_l}{\beta}}$ and A_h is solved by equation (5).

Mixed-product-line-extension model.
The incumbent maximizes π_h^{ME} by setting p_h, A_h and A'_h. The FOC w.r.t. p_h gives:

$$p_h = \frac{1}{2}(A'_h - A_h)(\alpha + \beta(A_h + A'_h)).$$

Hence $\theta^* = \frac{1}{2}(1 + \frac{\alpha}{\beta(A_h + A'_h)})$. We then substitute p_h into the profit function and obtain:

$$\pi_h^{ME} = \frac{1}{4}\left(2\alpha(A_h + A'_h) + (1 - \frac{2A_h}{A_h + A'_h})\frac{\alpha^2}{\beta} + \beta(A'^2_h - A_h^2)\right) - f. \tag{7}$$

It is easy to see that π_h increases in A'_h. We conclude that the incumbent will set A'_h to the maximum. The upper bound of A'_h is imposed by q_l. Hence, $A'_h = \sqrt{\frac{q_h - q_l}{\beta}}$.

We then take the FOC of equation (7) w.r.t. A_h and solve for optimal A_h. We have:

$$A_h = -\frac{A'_h}{2} + \frac{\sqrt{A'_h}\sqrt{4\alpha + \beta A'_h}}{2\sqrt{\beta}} = \frac{-\sqrt{q_h - q_l} + (\frac{q_h - q_l}{\beta})^{\frac{1}{4}}\sqrt{4\alpha + \sqrt{\beta(q_h - q_l)}}}{2\sqrt{\beta}}. \tag{8}$$

Hence,

$$\theta^* = \frac{1}{2} + \frac{\alpha}{\sqrt{\beta}\left(\sqrt{q_h - q_l} + (\frac{q_h - q_l}{\beta})^{\frac{1}{4}}\sqrt{4\alpha + \sqrt{\beta(q_h - q_l)}}\right)}.$$

For the solution to be interior, we need $\theta^* \in [0, 1]$. Hence, we need $\alpha \leq 2\sqrt{\beta(q_h - q_l)}$.

Substituting the expressions of A_h and A'_h into equation (7), we obtain the profit for firm h:

$$\begin{aligned}\pi_h^{ME} &= (2\alpha^3 + (9\sqrt{\beta(q_h - q_l)} - 5\sqrt[4]{\beta(q_h - q_l)}\sqrt{4\alpha + \sqrt{\beta(q_h - q_l)}})\alpha^2 + 2\alpha\beta(q_h - q_l) + \\ &\quad (\beta(q_h - q_l))^{5/4}(\sqrt{4\alpha + \sqrt{\beta(q_h - q_l)}} + \sqrt[4]{\beta(q_h - q_l)}))/ \\ &\quad \left(4\beta(-2\alpha + \sqrt{\beta(q_h - q_l)} + \sqrt[4]{\beta(q_h - q_l)}\sqrt{4\alpha + \sqrt{\beta(q_h - q_l)}})\right) - f\end{aligned}$$

The profit expression is the same as in the mixed-product-line-extension model of the monopolist except that we now have $q_h - q_l$ instead of q_h.

When $\alpha > 2\sqrt{\beta(q_h - q_l)}$, only the ad-sponsored product is active and it has demand of 1. Hence, the business model effectively becomes an ad-sponsored model.

In both cases, the entrant is pushed out of the market. ■

Proof of Lemma 5. When we have corner solutions, $A_l = \sqrt{\frac{q_l}{\beta}}$, the profit function (i.e., equation (4)) can be simplified to:

$$\pi_h^{MS} = \frac{(q_h + A_h(\alpha - \beta A_h))^2}{4\left(q_h - \beta A_h^2\right)} - f.$$

The FOC of π_h^{MS} w.r.t. A_h gives:

$$\alpha = \frac{A_h q_h \beta - A_h^3 \beta^2}{q_h}.$$

Thus,

$$\frac{dA_h}{d\alpha} = 1/\frac{d\alpha}{dA_h} = \frac{q_h}{q_h\beta - 3A_h^2\beta^2}.$$

We then differentiate π_h^{MS} w.r.t. α, taking into consideration that A_h, the equilibrium ad intensity, is a function of α. We have:

$$\frac{d\pi_h}{d\alpha} = \frac{\left(q_h + \alpha A_h - \beta A_h^2\right)\left(q_h\alpha\frac{dA_h}{d\alpha} + \beta A_h^3\left(-1 + \beta\frac{dA_h}{d\alpha}\right) + A_h\left(q_h - q_h\beta\frac{dA_h}{d\alpha}\right)\right)}{2\left(q_h - \beta A_h^2\right)^2}.$$

Substituting the expression for $\frac{dA_h}{d\alpha}$ into the above expression, we have:

$$\frac{d\pi_h}{d\alpha} = \frac{\left(q_h + \alpha A_h - \beta A_h^2\right)\left(q_h^2\alpha - 3q_h\beta^2 A_h^3 + 3\beta^3 A_h^5\right)}{2\beta\left(q_h - 3\beta A_h^2\right)\left(q_h - \beta A_h^2\right)^2}.$$

Using conditions such as $q_h > \beta A_h^2$ and $\alpha = \frac{A_h q_h \beta - A_h^3 \beta^2}{q_h}$, we could show that $\frac{d\pi_h}{d\alpha} > 0$. Therefore, when we have the corner solution, the incumbent profit increases with the advertising rate.

We now proceed to examining the interior case following a similar approach. In the interior case, A_h and A_l are the solutions of a system of two equations:

$$\begin{cases} (q_h - q_l)/\beta + A_l^2 = \frac{A_h^3}{A_h - \alpha/\beta} \\ A_l = ((q_h - q_l - A_h^2\beta)/\beta)^{1/2} \end{cases}.$$

Substituting the expression of A_l from the second equation to the first equation and solving for α, we have:

$$\alpha = \frac{2\beta A_h(q_h - q_l - \beta A_h^2)}{2(q_h - q_l) - \beta A_h^2}. \tag{9}$$

Thus,

$$\frac{dA_h}{d\alpha} = 1/\frac{d\alpha}{dA_h} = \frac{\left(2(q_h - q_l) - \beta A_h^2\right)^2}{2\beta\left(2(q_h - q_l)^2 - 5(q_h - q_l)\beta A_h^2 + \beta^2 A_h^4\right)}. \tag{10}$$

We can also substitute the expression for A_l into equation (4), the profit function of the incumbent:

$$\pi_h^{MS} = \frac{(2(q_h - q_l) + A_h(\alpha - 2A_h\beta))^2}{8\left(q_h - q_l - \beta A_h^2\right)} - f.$$

We note that A_h here is the equilibrium ad intensity and is a function of α. We now differentiate π_h^{MS} w.r.t. α and obtain:

$$\frac{d\pi_h^{MS}}{d\alpha} = \frac{(2(q_h - q_l) + A_h(\alpha - 2\beta A_h))\left((q_h - q_l)\alpha\frac{dA_h}{d\alpha} + \beta A_h^3\left(-1 + 2\beta\frac{dA_h}{d\alpha}\right) + (q_h - q_l)A_h\left(1 - 2\beta\frac{dA_h}{d\alpha}\right)\right)}{4\left((q_h - q_l) - \beta A_h^2\right)^2}$$

We then substitute $\frac{dA_h}{d\alpha}$ into the above equation and obtain:

$$\begin{aligned} \frac{d\pi_h^{MS}}{d\alpha} \quad = \quad & \frac{(q_h - q_l)(2(q_h - q_l) + A_h(\alpha - 2\beta A_h))}{8\beta(q_h - q_l - \beta A_h^2)^2(2(q_h - q_l)^2 - 5(q_h - q_l)\beta A_h^2 + \beta^2 A_h^4)} \times \\ & \left(4(q_h - q_l)^2\alpha + \beta A_h(-4(q_h - q_l)^2 + A_h(-4(q_h - q_l)\alpha + \beta A_h(2(q_h - q_l) + A_h(\alpha + 2\beta A_h))))\right). \end{aligned}$$

We check the sign for each component in the above expression. We find that except $(4(q_h - q_l)^2\alpha + \beta A_h(-4(q_h - q_l)^2 + A_h(-4(q_h - q_l)\alpha + \beta A_h(2(q_h - q_l) + A_h(\alpha + 2\beta A_h)))))$, the four other terms: $(q_h - q_l)$, $(2(q_h - q_l) + A_h(\alpha - 2\beta A_h))$, $8\beta(q_h - q_l - \beta A_h^2)^2$ and $(2(q_h - q_l)^2 - 5(q_h - q_l)\beta A_h^2 + \beta^2 A_h^4)$ are all positive. Hence, we conclude that $\frac{d\pi_h}{d\alpha}$ is negative.

Therefore, when we have interior solutions, the incumbent profit decreases with the advertising rate.

We now show that when $q_h < 2q_l$, we always have interior solutions. We have interior solutions if $A_l = ((q_h - q_l - \beta A_h^2)/\beta)^{1/2} < (q_l/\beta)^{1/2}$. That is:

$$q_h - \beta A_h^2 < 2q_l.$$

We know that in equilibrium $A_h > 0$. Hence, this condition is always satisfied. Hence, we always have interior solutions when $q_h < 2q_l$.

We now show that when $q_h > 2q_l$ and α is small, we always have corner solutions. We have corner solutions when

$$q_h - \beta A_h^2 > 2q_l.$$

Consider the case where $\alpha = 0$. In this case, the business model is equivalent to the pure-subscription-based model and we know when $q_h > 2q_l$, we have corner solutions. Now consider when $\alpha = \epsilon$, a very small positive number. From equations (9) and (10), we have when $\alpha \to 0$: $A_h \to 0$ and $\frac{dA_h}{d\alpha} = 1/\beta$. Hence, $A_h = \epsilon/\beta$. We can always find a small enough ϵ such that when $q_h > 2q_l$, $q_h - \beta A_h^2 = q_h - \beta(\epsilon/\beta)^2 = q_h - \epsilon^2/\beta > 2q_l$. Hence, we know when $\alpha < \alpha^* = \epsilon$, we are at the corner. ■

Proof of Lemma 6. It is easy to see that when α is small, the pure-subscription-based model is better than pure-ad-sponsored model. We now compare the profit from the pure-subscription-based model to the mixed-product-line-extension model. When $\alpha \to 0$, $\pi_h^{ME} \to \frac{q_h - q_l}{4} - f$. We also know that

$$\frac{d\pi_h^{ME}}{d\alpha} = \frac{3((q_h - q_l)\beta)^{1/4}\sqrt{4\alpha + \sqrt{(q_h - q_l)\beta}} - 2\alpha - \sqrt{(q_h - q_l)\beta}}{4\beta},$$

which is positive when $\alpha < 2\sqrt{\beta(q_h - q_l)}$. Hence, we know that π_h^{ME} increases from $\frac{q_h - q_l}{4} - f$ as α increases. The profit from the pure-subscription-based model, π_h^S, does not change with α and is either $\frac{q_h - q_l}{2}$ or $\frac{q_h}{4}$. In either case, $\pi_h^S > \frac{q_h - q_l}{4} - f$. Hence, when α is small, the pure-subscription-based model provides more profit than the mixed-product-line-extension model. We also know that when $q_h \leq 2q_l$, the pure-subscription-based model is better than the mixed-single-product model. Hence, in this case, when α is small, the pure-subscription-based model is the best model among the four. When $q_h > q_l$ and α is small, the mixed-

single-product model is better than the pure-subscription-based model. Hence, the mixed-single-product model is the best model among the four. Therefore, when α is small, either the pure-subscription-based or the mixed-single-product model is optimal.

The second part is straightforward. As α increases, in both the mixed-single-product model and the mixed-product-line-extension model, the optimal price of the high quality product decreases. At some point, the incumbent is willing to give the product away for free and make money exclusively from ads. It is also easy to see that with a big α, the profit from the pure-ad-sponsored model is greater than the one from pure-subscription-based model. Hence, when α is sufficiently large, the optimal business model is the pure-ad-sponsored model. ■

Proof of Lemma 7. The simplest way to show this is to provide an example in which the pure-subscription-based model is better than the mixed-single-product model and the mixed-product-line-extension model. Consider the case in which $q_h = 3$, $q_l = 1.6$, $\alpha = 0.4$, $\beta = 1$ and $f = 0$. We have $\pi_h^S = 0.7$, $\pi_h^A = 0.473$, $\pi_h^{MS} = 0.696$ and $\pi_h^{ME} = 0.648$. Hence, the pure-subscription-based model is the best among the four even in the case where $f = 0$. ■

Proof of Lemma 8. As the example in the proof of Lemma 7 shows, π_h^{MS} could be greater than π_h^{ME}. Hence, the mixed-product-line-extension model no longer dominates the mixed-single-product model. ■

Proof of Lemma 9. It is easy to see that when α is sufficiently large, $\pi_h^A = \alpha\sqrt{\frac{q_h - q_l}{\beta}}$ will be greater than π_h^S, which is $\frac{q_h - q_l}{2}$ or $\frac{q_h}{4}$ depending on the relative size of q_h and q_l. Similarly, in the mixed-single-product model, as α increases, A_h will increase and A_l will eventually decrease (it could be at the corner initially). Hence, p_h will decrease. When α is sufficiently large, p_h becomes zero and effectively we have a pure-ad-sponsored model. In the mixed-product-line-extension model, we know when $\alpha > \sqrt{\beta(q_h - q_l)}$, the model becomes a pure-ad-sponsored model. Hence, when α is sufficiently large, the optimal business model is the pure-ad-sponsored model.

As the additional cost f is only introduced in the mixed models, we know that when f is sufficiently large, only the pure business models can be optimal. ■

Proof of Proposition 3. The proposition follows straightly from Lemma 5. Consider the mixed-single-product model. When $q_h \leq 2q_l$, we are always at the interior and π_h^{MS} decreases

with α. Hence, the incumbent will earn more profit if $\alpha = 0$. In other words, the profit from the pure-subscription-based model will be higher. Thus, the mixed-single-product model is always dominated by the pure-subscription-based model (even if $f = 0$). As a result, we may only have three optimal business models, as shown in Figure 2.

When $q_h > 2q_l$, we know that when α is small, we are at the corner and π_h^{MS} increases with α. Hence, in this case, when f is small, π_h^{MS} will be greater than π_h^S, the profit from the pure-subscription-based model. Therefore, there is always a region in which the mixed-single-product model is better than the pure-subscription-based model and is thus not dominated. As illustrated in Figure 3, indeed all four business models may be optimal. ■