The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium*

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Abstract

We study a two-sector general equilibrium model of housing and non-housing production where heterogenous households face limited opportunities to insure against aggregate and idiosyncratic risks. The model generates large variability in the national house price-rent ratio, both because it fluctuates endogenously with the state of the economy and because it rises in response to a relaxation of credit constraints and decline in housing transaction costs (*financial market liberalization*). These factors, together with a rise in foreign ownership of U.S. debt calibrated to match the actual increase over the period 2000-2006, generate an increase in the model price-rent ratio comparable to that observed in U.S. data over this period. The model also predicts a sharp decline in home prices starting in 2007, driven by the economic contraction and by a presumed reversal of the financial market liberalization. Fluctuations in the model's price-rent ratio are driven by changing risk premia, which fluctuate endogenously in response to cyclical shocks, the financial market liberalization, and its subsequent reversal. By contrast, we show that the inflow of foreign money into domestic bond markets plays a small role in driving home prices, despite its large depressing influence on interest rates. Finally, the model implies that procyclical increases in equilibrium price-rent ratios reflect rational expectations of lower future housing returns, not higher future rents.

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1 Introduction

Residential real estate is a large and volatile component of household wealth. Moreover, volatility in housing wealth is often accompanied by large swings in house prices relative to housing fundamentals. For example, Figure 1 shows that national house price-rent ratios climbed to unusual heights by the end of 2006, but have since exhibited sharp declines.

This paper studies the macroeconomic consequences of fluctuations in housing wealth and housing finance. To what extent can episodes of national house price appreciation be attributed to a liberalization in housing finance, such as declines in collateral constraints or reductions in the costs of borrowing and conducting transactions? How do movements in house prices affect expectations about future housing fundamentals and future home price appreciation? To what extent do changes in housing wealth and housing finance affect output and investment, risk premia in housing and equity markets, measures of cross-sectional risk-sharing, and life-cycle patterns in wealth accumulation and savings?

In this paper we address these questions by studying a two-sector general equilibrium model of housing and non-housing production where heterogenous households face limited risk-sharing opportunities as a result of incomplete financial markets. The goal of this research is to provide theoretical answers to the questions posed above using a model that is sufficiently general as to account for the endogenous interactions among financial and housing wealth, output and investment, rates of return and risk premia in both housing and equity assets, and consumption and wealth inequality.

A house in our model is a residential durable asset that provides utility to the household, is illiquid (expensive to trade), and can be used as collateral in debt obligations. The model economy is populated by a large number of overlapping generations of households who receive utility from both housing and nonhousing consumption and who face a stochastic life-cycle earnings profile. We introduce market incompleteness by modeling heterogeneous agents who face idiosyncratic and aggregate risks against which they cannot perfectly insure, and by imposing collateralized borrowing constraints on households.

Within the context of this model, we focus our theoretical investigation on the macroeconomic consequences of three systemic changes in housing finance. First, we investigate the impact of changes in housing collateral requirements. Second, we investigate the impact of changes in housing transactions costs. Third, we investigate the impact of an influx of foreign capital into the domestic bond market. We argue below that all three factors fluctuate over time and changed markedly during or preceding the period of rapid home price appreciation from 2000-2006. In particular, this period was marked by a widespread relaxation of collateralized borrowing constraints and declining housing transactions costs, a combination we refer to hereafter as *financial market liberalization.* The period was also marked by a sustained depression of long-term interest rates that coincided with a vast inflow of capital from foreign governmental holders into U.S. bond markets. In the aftermath of the credit crisis that began in 2007, the erosion in credit standards and transactions costs has been reversed.[1] We use our framework as a laboratory for studying the impact of fluctuations in either direction of these features of housing finance.

We summarize the model's main implications as follows.

House prices relative to measures of fundamental value are volatile. The model generates substantial variability in the national house price-rent ratio, both because it fluctuates procyclically with the state of the economy, and because it rises in response to a relaxation of credit constraints and decline in housing transaction costs. When we combine a financial market liberalization with an inflow of foreign capital into the domestic bond market calibrated to match the rise in foreign ownership of U.S. Treasury and agency debt over the period 2000-2006, the model generates an increase in the price-rent ratio comparable to that observed in U.S. data from 2000-2006. The model also predicts a sharp decline in home prices starting in 2007, driven by the economic contraction and by a presumed reversal of the financial market liberalization (but not the foreign capital inflow).

A financial market liberalization drives price-rent ratios up because it drives risk premia down. The main impetus for rising price-rent ratios in the model is the simultaneous occurrence of positive economic shocks and a financial market liberalization, phenomena that generate an endogenous decline in risk premia on housing and equity assets. A financial market liberalization reduces risk premia for two reasons, both of which are related to the ability of heterogeneous households to insure against aggregate and idiosyncratic risks. First, lower collateral requirements directly increase access to credit, which acts as a buffer against unexpected income declines. Second, lower transactions costs reduce the expense of obtaining the collateral required to increase borrowing capacity and provide insurance. These factors lead to an increase in risk-sharing, or a decrease in the cross-sectional variance of marginal utility.

It is important to note that the rise in price-rent ratios caused by a financial market liberalization must be attributed to a decline in risk premia and not to a fall in interest rates.

[1] Some analysts have argued that, since the credit crisis, borrowing restrictions and credit constraints have become even more stringent than historical norms in the pre-boom period (e.g., Streitfeld (2009)).

Indeed, the very changes in housing finance that accompany a financial market liberalization drive the endogenous interest rate up, rather than down. It follows that price-rent ratios rise after a financial market liberalization because the decline in risk premia more than offsets the rise in equilibrium interest rates. These findings underscore the crucial role of foreign capital in maintaining low interest rates during a financial market liberalization. Without an infusion of foreign capital, any period of looser collateral requirements and lower housing transactions costs (such as that which characterized the period of rapid home price appreciation from 2000-2006) would be accompanied by an increase in equilibrium interest rates, as households endogenously respond to the improved risk-sharing opportunities afforded by a financial market liberalization by reducing precautionary saving.

Foreign purchases of U.S. bonds play a central role in lower interest rates but a small role in housing booms. The model implies that a rise in foreign purchases of domestic bonds, equal in magnitude to those observed in the data from 2000-2006, leads to a quantitatively large decline in the equilibrium real interest rate. In partial equilibrium analyses where risk premia are held fixed, a decline in the interest rate of this magnitude would be sufficient–by itself–to explain the rise in price-rent ratios observed from 2000-2006. But we show that, in general equilibrium, borrowed sums from the rest of the world can play at most a limited role in asset booms, despite their large depressing influence on interest rates. Foreign purchases of U.S. bonds crowd domestic savers out of the safe bond market, exposing them to greater systematic risk in equity and housing markets. In response, risk premia on housing and equity assets rise, substantially offsetting the lower interest rates and limiting the impact of foreign capital inflows on home prices.

Procyclical increases in equilibrium price-rent ratios reflect rational expectations of lower future returns, not higher future rents. It is commonly assumed that increases in national house-price rent ratios reflect an expected increase in future housing fundamentals, such as rental growth. In partial equilibrium analyses where discount rates are held constant, this is the only outcome possible (e.g., Sinai and Souleles (2005), Campbell and Cocco (2007)). This reasoning, however, ignores the general equilibrium response of both residential investment and discount rates to economic growth. In the model here, positive economic shocks stimulate greater housing demand and greater residential investment. Under plausible parameterizations, the latter can lead to an equilibrium *decline* in future rental growth as the housing stock rises. It follows that high price-rent ratios in expansions must entirely reflect expectations of future house price depreciation (lower discount rates), driven in the model by falling risk premia as collateral values and risk-sharing opportunities rise with the economy.

Financial market liberalization plus foreign capital leads to a shift in the composition of wealth towards housing, increases financial wealth inequality, but has ambiguous effects on consumption inequality. A financial market liberalization plus an inflow of foreign capital into the domestic bond market leads households of all ages and incomes to shift the composition of their wealth towards housing, consistent with observed changes in household-level data from 2000 to 2007. These factors also have implications for inequality. Although a financial market liberalization improves risk sharing and drives risk premia down, an inflow of foreign governmental capital into the safe bond market reduces risk sharing because it increases the exposure of domestic savers to risky asset markets. We show that a financial market liberalization and foreign capital infusion have offsetting effects on consumption inequality but reinforcing upward effects on financial wealth inequality.

The paper is organized as follows. The next subsection briefly discusses related literature. Section 2 describes recent changes in the three key aspects of housing finance discussed above: collateral constraints, housing transactions costs, and foreign capital in U.S. debt markets. Section 3 presents the theoretical model. Section 4 presents our main findings, including benchmark business cycle and financial market statistics. Here we show that the model generates forecastable variation in equity and housing returns, and a sizable equity premium and Sharpe ratio simultaneously with a plausible degree of variability in aggregate consumption. Section 5 concludes.

1.1 Related Literature

Our paper is related to a growing body of literature in finance that studies the asset pricing implications of incomplete markets models. The focus of this literature, however, is typically on the equity market implications of pure exchange economies with exogenous endowments, with no role for housing or the production side of the economy.[2] Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008), and Favilukis (2008) explicitly model the production side of the economy, but focus on single-sector economies without housing.

Within the incomplete markets environment, our work is related to several papers that study questions related to housing and/or consumer durables more generally. These papers typically either do not model production (instead studying a pure exchange economy), and/or

[2]See for example Aiyagari and Gertler (1991), Telmer (1993), Lucas (1994), Heaton and Lucas (1996), Basak and Cuoco (1998), Luttmer (1999)), for a study of single sector exchange economies, or Lustig and Van Nieuwerburgh (2005) for a two-sector exchange economy model.

the portfolio choice problem underlying asset allocation between a risky and a risk-free asset, or are analyses of partial equilibrium environments. See for example, the general equilibrium exchange-economy analyses that embed bond, stock and housing markets of Ríos-Rull and Sánchez-Marcos (2006), Lustig and Van Nieuwerburgh (2007, 2008), Piazzesi and Schneider (2008), and the partial equilibrium analyses of Peterson (2006), Ortalo-Magné and Rady (2006), and Corbae and Quintin (2009).

Other researchers have studied the role of incomplete markets in housing decisions in models without aggregate risk. Fernández-Villaverde and Krueger (2005) study how consumption over the life-cycle is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibria in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2008) study a life-cycle model with housing and non-housing production, but focus their analysis on the perfect foresight equilibria of an economy without aggregate risk and an exogenous interest rate. One recent analysis that does combine aggregate risk, production, and incomplete markets is Iacoviello and Pavan (2009). These authors study the role of housing and debt for the volatility of the aggregate economy in a model with a single production and single saving technology. Because there is no risk-free asset, however, their model is silent about the role of risk premia in the economy, a central focus of our paper.

Outside of the incomplete markets environment, a strand of the macroeconomic literature studies housing behavior in a two-sector, general equilibrium business cycle framework either with production (e.g., Davis and Heathcote (2005), Kahn (2008)) or without production (e.g., Piazzesi, Schneider, and Tuzel (2007)). The focus in these papers is on environments with complete markets for idiosyncratic risks and a representative agent representation. These models are silent on questions involving risk-sharing, inequality, and age and income heterogeneity.

It is important to note that our paper does not address the question of *why* credit market conditions changed so markedly in recent decades (we discuss this in the conclusion). It is widely understood that the financial market liberalization we study was preceded by a number of revolutionary changes in housing finance, notably by the rise in securitization. These changes initially decreased the risk of individual home mortgages and home equity loans, allowing for a more efficient allocation of risk and, some have argued, making it optimal for lending contracts to feature lower collateral requirements and housing transactions fees (e.g. Green and Wachter (2008); Piskorski and Tchistyi (2008); Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). As these researchers note, however, these initially risk-reducing changes in housing finance were accompanied by government deregulation of financial institutions that ultimately increased risk, by permitting such institutions to alter the composition of their assets towards more high-risk securities, by permitting higher leverage ratios, and by presiding over the spread of complex financial holding companies that replaced the long-standing separation between investment bank, commercial bank and insurance company. Industry analysis suggests that the market's subsequent revised expectation upward of the riskiness of the underlying mortgage assets since 2007 has led to a reversal in collateral requirements and transactions fees. It is precisely these changes in credit conditions that are a focus of this study.

2 Changes in Housing Finance

This section documents the empirical relevance of changes in three features of housing finance. First are changes in collateralized borrowing requirements, broadly defined. Collateralized borrowing constraints can take the form of an explicit down payment requirement for new home purchases, but they also apply to home equity borrowing. Recent data suggests that down payment requirements for a range of mortgage categories declined during or preceding the period of rapid home price appreciation from 2000 to 2006. Loan-to-value (LTV) ratios on subprime loans rose from 79% to 86% over the period 2001-2005, while debt-income ratios rose (Demyanyk and Hemert (2008)). Other reports suggest that the increase LTV ratios for prime mortgages was even greater, with one industry analysis finding that LTV ratios for conforming first and second mortgages rose from 60.4% in 2002 to 75.2% in 2006.[3] These changes coincided with a surge in borrowing against existing home equity between 2002 and 2006 (Mian and Sufi (2009b)).

More generally, there was a widespread relaxation of underwriting standards in the U.S. mortgage market during the period leading up to the credit crisis of 2007, which provide a back-door means of reducing collateral requirements for home purchases. The loosening of standards can be observed in the marked rise in simultaneous second-lien mortgages and in no-documentation or low-documentation loans.[4] By the end of 2006 households routinely bought homes with 100% financing using a piggyback second mortgage or home equity loan.

[3]Source: UBS, April 16, 2007 Lunch and Learn, "How Did We Get Here and What Lies Ahead," Thomas Zimmerman, page 5.

[4]FDIC Outlook: Breaking New Ground in U.S. Mortgage Lending, December 18, 2006. <http://www.fdic.gov/bank/analytical/regional/ro20062q/na/2006_summer04.html#10A>.

See also Mian and Sufi (2009a). Loans for 125% of the home value were even available if the borrower used the top 25% to pay off existing debt. Industry analysts indicate that LTV ratios for combined (first and second) mortgages have since returned to more normal levels of no greater than 75-80% of the appraised value of the home. We assess the impact of these changes collectively by modeling them as a reduction in collateralized borrowing constraints and subsequent rise.

Second in our study of housing finance are transactions costs. The period of rapid home price appreciation was marked by a decline in the cost of conducting housing transactions; houses, in effect, became more liquid. Closing costs for mortgages, mortgage refinancing, and home equity extraction all fell sharply in the years during and preceding the housing boom that ended in 2006. The Federal Housing Financing Board reports monthly data on mortgage and mortgage refinancing closing costs (based on a survey of the largest lenders). Figure 2 shows closing costs on first mortgages and mortgage refinancings combined. These costs declined from 2.70% of the loan balance in January 1985 to 0.46% in April 2008. Expressed as an interest rate, these costs decline 90%, from 50 basis points to 5 basis points over the period 1985-2007. For Freddie Mac 30-year conforming mortgages, the same closing costs declined 83% over this period, 40% from the end of 2000 to end of 2006. These costs began moving back up in the aftermath of the credit crisis of 2007/2008. From 2007 to 2009, closing costs on Freddie Mac 30-year conforming mortgages surged back up 56%.

Researchers focusing on subprime borrowing have documented sharp declines in housing transactions costs during or proceeding the housing boom. Berndt, Hollifield, and Sandas (2010) use data from New Century Financial Corporation, a large subprime mortgage lender from 1996 until 2006. Their Table 2 shows that the broker fees paid by subprime mortgage borrowers declined 52% from 1997-2006 and 48% from 2000-2006. Moreover, these declines are uniform across loan types (fixed rate mortgages with and without full documentation, hybrid loans with and without documentation), suggesting that comparable reductions in transactions costs were present for other mortgage categories.

Finally, transactions costs associated with home equity extraction declined significantly and coincided with a surge of 350% in mortgage equity withdrawal rates from 2000-2006.[5] Kennedy and Greenspan (2007) compiled data on closing costs for home equity loans (HEL) and home equity lines of credit (HELOC) from periodic releases of the Home Equity Survey Report, published by the American Bankers Association. The data indicate that these costs

[5]Figures based on updated estimates provided by James Kennedy of the mortgage analysis in Kennedy and Greenspan (2005).

trended down significantly: for HELOCs, they were 76% lower in 2004 than they were in 1988. For closed-end HELs, the costs declined 41% from 1998 to 2004. The surveys indicate that non-pecuniary costs, in the form of required documentation, time lapsed from loan application to loan closing, and familiarity with available opportunities for refinancing and home-equity extraction, also declined substantially.

Third in our study of housing finance are foreign purchases of U.S. assets. A key development in the housing market in recent years is the secular decline in interest rates, which coincided with a surge in foreign ownership of U.S. bonds. Figure 3 shows that both 30-year FRMs and the 10-year Treasury bond yield have trended downward, with mortgage rates declining from around 18 percent in the early 1980s to near 6 percent by the end of 2007. This was not merely attributable to a decline in inflation: the real annual interest rate on the ten-year Treasury bond fell from 3.6% in 2000 to 0.93% in 2006 using the consumer price index as a measure of inflation. At the same time, foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from $118 billion in 1984, or 13.5% of marketable Treasuries outstanding, to $2.2 trillion in 2008, or 61% of marketable Treasuries (Figure 4, Panel A). Foreign holdings of U.S. agency and Government Sponsored Enterprise-backed agency securities quintupled between 2000 and 2007, rising from $261 billion to $1.3 trillion, or from 7% to 21% of total agency debt. Foreign holdings of U.S. Treasury and Agency debt as a fraction of GDP more than doubled from 14% to 30% over the period 2000-2006 (Figure 4, Panel B). By pushing real interest rates lower, the rise in foreign capital has been directly linked to the surge in mortgage originations over this period (e.g., Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). Economic policymakers, such as Federal Reserve Chairman Ben Bernanke, have also emphasized the role of foreign capital in driving interest rates lower and in fueling house price inflation.[6]

In the model, interest rates are determined in equilibrium by a market clearing condition for bondholders. We consider one specification of the model in which we introduce an exogenous foreign demand for domestic bonds into the market clearing condition, referred to hereafter as *foreign capital*. This foreign capital is modeled as owned by governmental holders who place all of their funds in domestic riskless bonds. We do this for two reasons. First, by the end of 2008, Foreign Official Institutions held 70% of all foreign holdings of U.S. Treasuries. Moreover, as explained in Kohn (2002), government entities have specific

[6] For example, see remarks by then Governor Ben S. Bernanke at the Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, March 10, 2005, and by Chairman Bernanke, at the International Monetary Conference, Barcelona, Spain (via satellite), June 3, 2008.

regulatory and reserve currency motives for holding U.S. Treasuries and face both legal and political restrictions on the type of assets that can be held, forcing them into safe securities. Second, Krishnamurthy and Vissing-Jorgensen (2010) find that demand for U.S. Treasury securities by governmental holders is extremely inelastic, implying that when these holders receive funds to invest they buy U.S. Treasuries, regardless of their price relative to other U.S. assets. This motivates our modeling of foreign capital as both exogenous and as restricted to investments in the safe asset. In the model, we assume domestic borrowers may obtain credit at a fixed interest rate spread with the governmental rate. Because our model abstracts from default, we set this spread to zero in our calibration.

3 The Model

3.1 Firms

The production side of the economy consists of two sectors. One sector produces the non-housing consumption good, and the other sector produces the housing good. We refer to the first as the "consumption sector" and the second as the "housing sector." Time is discrete and each period corresponds to a year. In each period, a representative firm in each sector chooses labor (which it rents) and investment in capital (which it owns) to maximize the value of the firm to its owners.

3.1.1 Consumption Sector

Denote output in the consumption sector as

$$Y_{C,t} \equiv Z_{C,t} K_{C,t}^{\alpha} N_{C,t}^{1-\alpha}$$

where $Z_{C,t}$ is the stochastic productivity level at time t, K_C is the capital stock in the consumption sector, α is the share of capital, and N_C is the quantity of labor input in the consumption sector. Let I_C denote investment in the consumption sector. The firm's capital stock $K_{C,t}$ accumulates over time subject to proportional adjustment costs, $\phi_C\left(\frac{I_{C,t}}{K_{C,t}}\right) K_{C,t}$, modeled as a deduction from the earnings of the firm. The firm maximizes the present discounted value $V_{C,t}$ of a stream of earnings:

$$V_{C,t} = \max_{N_{C,t}, I_{C,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left(Y_{C,t+k} - w_{t+k} N_{C,t+k} - I_{C,t+k} - \phi_C \left(\frac{I_{C,t+k}}{K_{C,t+k}} \right) K_{C,t+k} \right), \quad (1)$$

where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is a stochastic discount factor discussed below, and w_t is the wage rate (equal across sectors in equilibrium). The evolution equation for the firm's capital stock is

$$K_{C,t+1} = (1 - \delta) K_{C,t} + I_{C,t},$$

where δ is the depreciation rate of the capital stock.

The firm does not issue new shares and finances its capital stock entirely through retained earnings. The dividends to shareholders are equal to

$$D_{C,t} = Y_{C,t} - w_t N_{C,t} - I_{C,t} - \phi_C \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t}.$$

3.1.2 Housing Sector

The housing firm's problem is analogous to the problem solved by the representative firm in the consumption sector, except that, in our most general specification, housing production utilizes an additional fixed factor of production, $\mathcal{L}_t$, representing a combination of land and government permits for residential construction.[7] Denote output in the residential housing sector as

$$Y_{H,t} = Z_{H,t} (\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu} \right)^{\phi},$$

$Y_{H,t}$ represents construction of new housing (residential investment), $1 - \phi$ is the share of land/permits in housing production, and ν is the share of capital in the construction component $\left(K_{H,t}^{\nu} N_{H,t}^{1-\nu} \right)$ of housing production. Variables denoted with an "H" subscript are defined exactly as above for the consumption sector but now pertain to the housing sector, e.g., $Z_{H,t}$ denotes the stochastic productivity level in the housing sector.

Below we consider two specifications for the role of land/permits, including a baseline specification that sets their share in housing production to zero, or $\phi = 1$. For the specification in which land/permits have a non-zero share ($\phi < 1$), we assume that, each period, the government makes available a fixed supply L of land/permits for residential construction by renting them at the competitive rental rate equal to the marginal product of $\mathcal{L}_t$. The proceeds from land rentals are used by the government to finance (wasteful) government spending G_t. When a house is sold, the government issues a transferable lease for the land/permits in perpetuity at no charge to the homeowner. This implies that, for all practical purposes, the buyer of the home operates as owner even though, by eminent domain, the government retains the legal right to the land/permits.

[7] Glaeser, Gyourko, and Saks (2005) argue that the increasing value of land for residential development is tied to government-issued construction permits, rather than to the acreage itself.

The housing firm maximizes

$$V_{H,t} = \max_{N_{H,t}, I_{H,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left(\begin{array}{c} p_{t+k}^H Y_{H,t+k} - p_{t+k}^L \mathcal{L}_{t+k} - w_{t+k} N_{H,t+k} \\ -I_{H,t+k} - \phi_H \left(\frac{I_{H,t+k}}{K_{H,t+k}} \right) K_{H,t+k} \end{array} \right), \tag{2}$$

where p_{t+k}^H is the relative price of one unit of housing in units of the non-housing consumption good and p_{t+k}^L is the price of land/permits. Note that p_t^H is the time t price of a unit of housing of fixed quality and quantity. The dividends to shareholders in the housing sector are denoted

$$D_{H,t} = p_t^H Y_{H,t} - p_{t+k}^L \mathcal{L}_t - w_t N_{H,t} - I_{H,t} - \phi_H \left(\frac{I_{H,t}}{K_{H,t}} \right) K_{H,t}.$$

Capital in the housing sector evolves:

$$K_{H,t+1} = (1 - \delta) K_{H,t} + I_{H,t}.$$

Note that $Y_{H,t}$ represents residential construction; thus the law of motion for the aggregate residential housing stock H_t is

$$H_{t+1} = (1 - \delta_H) H_t + Y_{H,t},$$

where δ_H denotes the depreciation rate of the housing stock.

3.2 Risky Asset Returns

The firms' values $V_{H,t}$ and $V_{C,t}$ are the *cum* dividend values, measured before the dividend is paid out. Thus the *cum* dividend returns to shareholders in the housing sector and the consumption sector are defined, respectively, as

$$R_{Y_H,t+1} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \qquad R_{Y_C,t+1} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.$$

We define $V_{j,t}^e = V_{j,t} - D_{j,t}$ for $j = H, C$ to be the *ex* dividend value of the firm.[8]

3.3 Individuals

The economy is populated by A overlapping generations of individuals, indexed by $a = 1, ..., A$, with a continuum of individuals born each period. Individuals live through two

[8] Using the *ex* dividend value of the firm the return reduces to the more familiar *ex* dividend definition: $R_{j,t+1}^e = \frac{V_{j,t+1}^e + D_{j,t+1}}{V_{j,t}^e}$.

stages of life, a working stage and a retirement stage. Adult age begins at age 21, so a equals this effective age minus 20. Agents live for a maximum of $A = 80$ (100 years). Workers live from age 21 ($a = 1$) to 65 ($a = 45$) and then retire. Retired workers die with an age-dependent probability calibrated from life expectancy data. The probability that an agent is alive at age $a+1$ conditional on being alive at age a is denoted $\pi_{a+1|a}$. Upon death, any remaining net worth of the individual in that period is counted as terminal "consumption," e.g., funeral and medical expenses.

Individuals have an intraperiod utility function given by

$$U(C_{a,t}, H_{a,t}) = \frac{\widetilde{C}_{a,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \widetilde{C}_{a,t} = \left[\chi C_{a,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\chi) H_{a,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_{a,t}$ is non-housing consumption of an individual of age a, and $H_{a,t}$ is the stock of housing, σ is the coefficient of relative risk aversion, χ is the relative weight on non-housing consumption in utility, and ε is the constant elasticity of substitution between C and H. Implicit in this specification is the assumption that the service flow from houses is proportional to the stock $H_{a,t}$.

Financial market trade is limited to a one-period riskless bond and to risky capital, where the latter is restricted to be a mutual fund of equity in the housing and consumption sectors. The mutual fund is a value-weighted portfolio with return

$$R_{K,t+1} = \frac{V_{H,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_H,t+1} + \frac{V_{C,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_C,t+1}. \tag{3}$$

The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where q_{t-1} is the bond price known at time $t-1$. Individuals are born with no initial endowment of risky capital or bonds.

Individuals are heterogeneous in their labor productivity. To denote this heterogeneity, we index individuals i. Before retirement households supply labor inelastically. The stochastic process for individual income for workers is

$$Y_{a,t}^i = w_t L_{a,t}^i,$$

where $L_{a,t}^i$ is the individual's labor endowment (hours times an individual-specific productivity factor), and w_t is the aggregate wage per unit of productivity. Labor productivity is specified by a deterministic age-specific profile, G_a, and an individual shock $Z_{a,t}^i$:

$$\begin{aligned} L_{a,t}^i &= G_a Z_t^i \\ \log\left(Z_t^i\right) &= \log\left(Z_{t-1}^i\right) + \epsilon_t^i, \qquad \epsilon_t^i \sim i.i.d.\left(0, \sigma_t^2\right), \end{aligned}$$

where G_a is a deterministic function of age capturing a hump-shaped profile in life-cycle earnings and $\epsilon^i_{a,t}$ is a stochastic i.i.d. shock to individual earnings. To capture countercyclical variation in idiosyncratic risk of the type documented by Storesletten, Telmer, and Yaron (2004), we use a two-state specification for the variance of idiosyncratic earnings shocks:

$$\sigma_t^2 = \begin{cases} \sigma_E^2 & \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ \sigma_R^2 & \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \end{cases}, \qquad \sigma_R^2 > \sigma_E^2 \tag{4}$$

This specification implies that the variance of idiosyncratic labor earnings is higher in "recessions" ($Z_{C,t} \leq E\left(Z_{C,t}\right)$) than in "expansions" ($Z_{C,t} \geq E\left(Z_{C,t}\right)$). The former is denoted with an "R" subscript, the latter with an "E" subscript. Finally, labor earnings are taxed at rate τ in order to finance social security retirement income.

At age a, agents enter the period with wealth invested in bonds, B^i_a, and shares θ^i_a of risky capital. The total number of shares outstanding of the risky asset is normalized to unity. We rule out short-sales in the risky asset,

$$\theta^i_{a,t} \geq 0. \tag{5}$$

An individual who chooses to invest in the mutual fund pays a fixed, per-period participation cost, $F_{K,t}$.

We assume that the housing owned by each individual depreciates at rate δ_H, the rate of depreciation of the aggregate housing stock. Households may choose to increase the quantity of housing consumed at time $t+1$ by making a net investment $H^i_{a,t+1} - \left(1-\delta_H\right) H^i_{a,t} > 0$. Because houses are illiquid, it is expensive to change housing consumption. An individual who chooses to change housing consumption pays a transaction cost $F^i_{H,t}$. Denote the sum of the per period equity participation cost and housing transaction cost for individual i as

$$F^i_t \equiv F^i_{H,t} + F_{K,t}.$$

Define the individual's gross financial wealth at time t as

$$W^i_{a,t} \equiv \theta^i_{a,t}\left(V^e_{C,t} + V^e_{H,t} + D_{C,t} + D_{H,t}\right) + B^i_{a,t}.$$

The budget constraint for an agent of age a who is not retired is

$$\begin{aligned} C^i_{a,t} + B^i_{a+1,t+1} q_t + \theta^i_{a+1,t+1}\left(V^e_{C,t} + V^e_{H,t}\right) \leq\ & W^i_{a,t} + \left(1-\tau\right) w_t L^i_{a,t} \\ & + p^H_t \left((1-\delta_H) H^i_{a,t} - H^i_{a+1,t+1}\right) - F^i_t \end{aligned} \tag{6}$$

$$W^i_{a+1,t+1} \geq -\left(1-\varpi\right) p^H_t H^i_{a,t+1}, \qquad \forall a, t \tag{7}$$

where τ is a social security tax rate and where

$$F^i_{H,t} = \begin{cases} 0, & H^i_{a+1,t+1} = (1-\delta_H) H^i_{a,t} \\ \psi_0 + \psi_1 p^H_t H^i_{a,t}, & H^i_{a+1,t+1} \neq (1-\delta_H) H^i_{a,t} \end{cases}.$$

$$F_{K,t} = \begin{cases} 0 & \text{if } \theta^i_{a+1,t+1} = 0 \\ \overline{F} & \text{if } \theta^i_{a+1,t+1} > 0 \end{cases}.$$

$F^i_{H,t}$ is the housing transactions cost which contains both a fixed and variable component and depends on age only through $H^i_{a,t}$. Equation (7) is the collateral constraint, where $0 \leq \varpi \leq 1$. It says that households may borrow no more than a fraction $(1-\varpi)$ of the value of housing, implying that they must post collateral equal to a fraction ϖ of the value of the house. This constraint can be thought of as a down-payment constraint for new home purchases, but it also encompasses collateral requirements for home equity borrowing against existing homes. The constraint gives the maximum combined LTV ratio for first and second mortgages and home equity withdrawal. Notice that if the price p^H_t of the house rises and nothing else changes, the individual can finance a greater level of consumption of both housing and nonhousing goods and services.

Two points about the collateral constraint above are worth noting. First, it applies to any borrowing against home equity, not just to mortgages. Second, borrowing takes place using one-period debt. Thus, an individual's borrowing capacity fluctuates period-by-period with the value of the house.

We also prevent individuals from buying stock on margin. If the individual is a net borrower, this means we restrict holdings of the risky asset to be zero, $\theta^i_{a+1,t+1} = 0$. This restriction is stated mathematically as follows:

$$\begin{gathered} \text{if } W^i_{a,t} + (1-\tau) w_t L^i_{a,t} - \left(C^i_{a,t} + p^H_t \left(H^i_{a+1,t+1} - (1-\delta_H) H^i_{a,t}\right) - F^i_t\right) < 0 \\ \text{then} \qquad B^i_{a+1,t+1} < 0, \qquad \theta^i_{a+1,t+1} = 0. \end{gathered} \tag{8}$$

Net lenders may take a positive position in the risky asset but may not short the bond to do so:

$$\begin{gathered} \text{if } W^i_{a,t} + (1-\tau) w_t L^i_{a,t} - \left(C^i_{a,t} + p^H_t \left(H^i_{a+1,t+1} - (1-\delta_H) H^i_{a,t}\right) - F^i_t\right) \geq 0 \\ \text{then} \qquad B^i_{a+1,t+1} \geq 0, \qquad \theta^i_{a+1,t+1} \geq 0. \end{gathered} \tag{9}$$

Let Z^i_{ar} denote the value of the stochastic component of individual labor productivity, $Z^i_{a,t}$, during the last year of working life. Each period, retired workers receive a government pension $PE^i_{a,t} = Z^i_{ar} X_t$ where $X_t = \tau \frac{N^W}{N^R}$ is the pension determined by a pay as you go system, and N^W and N^R are the numbers of working age and retired households.[9] For agents who have reached retirement age, the budget constraint is identical to that for workers (6) except that wage income $(1-\tau)\,w_t L^i_{a,t}$ is replaced by pension income $PE^i_{a,t}$.

Let $Z_t \equiv (Z_{C,t}, Z_{H,t})'$ denote the aggregate shocks. The state of the economy is a pair, (Z,μ), where μ is a measure defined over $\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H})$, where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ is a distribution of agents across ages, idiosyncratic shocks, financial and housing wealth. The presence of aggregate shocks implies that μ evolves stochastically over time. We specify a law of motion, Γ, for μ,

$$\mu_{t+1} = \Gamma\left(\mu_t, Z_t, Z_{t+1}\right).$$

3.4 Stochastic Discount Factor

The stochastic discount factor (SDF), $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, appears in the dynamic value maximization problem (1) and (2) undertaken by each representative firm. As a consequence of our incomplete markets setting, a question arises about how to model $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$. For example, the intertemporal marginal rates of substitution (MRS) of any shareholder in this setting is a valid stochastic discount factor. Much of the existing literature has avoided this ambiguity about the SDF by assuming that firms rent capital from households on a period-by-period basis, thereby solving a series of static optimization problems. Since the problem is static, the question of discounting is then mute. In this static case, however, one needs to impose some other form of exogenous shock, for example stochastic depreciation in the rented capital stocks (e.g., Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008)), in order to make the volatility of the equity return realistic. Here we instead keep depreciation deterministic and model dynamic firms that own capital and face adjustment costs when changing their capital stocks, requiring us to take a stand on the SDF. We do this for several reasons. First, in our own experimentation we found that the amount of stochastic depreciation required to achieve reasonable levels of stock market volatility produced

[9] The decomposition of the population into workers and retirees is determined from life-expectancy tables as follows. Let X denote the total number of people born each period. (In practice this is calibrated to be a large number in order to approximate a continuum.) Then $N^W = 45 \cdot X$ is the total number of workers. Next, from life expectancy tables, if the probability of dying at age $a > 45$ is denoted p_a then $N^R = \sum_{a=46}^{80} (1-p_a)\,X$ is the total number of retired persons.

excessive volatility in investment. Second, it is difficult to know what amount of stochastic depreciation, if any, is reasonable. Third, an economy populated entirely of static firms is unrealistic. In the real world, firms own their own capital stocks and must think dynamically about shareholder value.

For these reasons, we assume that the representative firm in each sector solves the dynamic problem presented above and discount future profits using a weighted average of the individual shareholders' MRS in non-housing consumption, $\frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}}$, where the weights, $\theta^i_{a,t}$, correspond to the shareholder's proportional ownership in the firm. Let $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$ denote this weighted average. Recalling that the total number of shares in the risky portfolio is normalized to unity, we have

$$\frac{\beta\Lambda_{t+1}}{\Lambda_t} \equiv \int_{\mathcal{S}} \theta^i_{a+1,t+1} \frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} d\mu \tag{10}$$

$$\frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} = \beta \left[\left(\frac{C^i_{a+1,t+1}}{C^i_{a,t}} \right)^{-\frac{1}{\sigma}} \left[\frac{\chi + (1-\chi)\left(\frac{H^i_{a+1,t+1}}{C^i_{a+1,t+1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{\chi + (1-\chi)\left(\frac{H^i_{a,t}}{C^i_{a,t}}\right)^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\frac{\sigma-\varepsilon}{\sigma(\varepsilon-1)}} \right]. \tag{11}$$

Since we weight each individual's MRS by its proportional ownership (and since short-sales in the risky asset are prohibited), only those households who have taken a positive position in the risky asset (shareholders) will receive non-zero weight in the SDF.

Although this specification of the stochastic discount factor leads to an equilibrium that depends on the control of the firm being fixed according to the proportional ownership structure described above, it is not necessarily quantitatively sensitive to this assumption on ownership control. For example, Carceles Poveda and Coen-Pirani (2009) show that, given the firm's objective of value maximization, the equilibrium allocations in their incomplete markets models are invariant to the choice of stochastic discount factor within the set that includes the MRS of any household (or any weighted average of these) for whom the Euler equation for the risky asset return is satisfied. They show in addition that the equilibrium allocations of such economies are the same as the allocations obtained in otherwise identical economies with "static" firms that rent capital from households on a period-by-period basis. Although these results have been formally proved only in an environment without adjustment costs, we note that our calibration of adjustment costs (discussed below) makes them quantitatively small, amounting to less than one percent of investment per year. We have checked that our results are not affected by the following variants of the SDF above: (i) equally weighting the MRS of shareholders (gives proportionally more weight to small stakeholders), (ii) weighting the MRS of shareholders by the squares of their ownership stakes, $\left(\theta^i_{a+1,t+1}\right)^2$, (gives proportionally more weight to big stakeholders), (iii) using the MRS of the largest shareholder.

3.5 Equilibrium

An equilibrium is defined as a set of prices (bond prices, wages, risky asset returns) given by time-invariant functions $q_t = q(\mu_t, Z_t)$, $p^H_t = p^H(\mu_t, Z_t)$, $w_t = w(\mu_t, Z_t)$, and $R_{K,t} = R_K(\mu_t, Z_t)$, respectively, a set of cohort-specific value functions and decision rules for each individual i, $\left\{V_a, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}\right\}_{a=1}^{A}$ and a law of motion for μ, $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ such that:

1. Households optimize:

$$\begin{aligned} V_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t}) &= \max_{H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}} \{U(C^i_{a,t}, H^i_{a,t}) \\ &\quad + \beta \pi_{a+1|a} E_t[V_{a+1}(\mu_{t+1}, Z_{t+1}, Z^i_{a,t+1}, W^i_{a+1,t+1}, H^i_{a+1,t+1})]\} \end{aligned} \tag{12}$$

   subject to (6), (7), (8), and (9) if the individual of working age, and subject to (7) and the analogous versions of (6), (8), and (9) (using pension income in place of wage income), if the individual is retired.

2. Firm's maximize value: $V_{C,t}$ solves (1), $V_{H,t}$ solves (2).

3. The price of land/permits p^L_t satisfies $p^L_t = (1-\phi)\, p^H_t Z_{H,t} \mathcal{L}_t^{-\phi} \left(K^{\nu}_{H,t} N^{-\nu}_{H,t}\right)^{\phi}$.

4. Land/permits supply equals land/permits demand: $L = \mathcal{L}_t$.

5. Wages $w_t = w(\mu_t, Z_t)$ satisfy

$$w_t = (1-\alpha)\, Z_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} \tag{13}$$

$$w_t = (1-\nu)(1-\phi)\, p^H_t Z_{H,t} \mathcal{L}^{\phi}_t K^{\nu(1-\phi)}_{H,t} N^{-\phi(1-\nu)-\nu}_{H,t}. \tag{14}$$

6. The housing market clears: $p^H_t = p^H(\mu_t, Z_t)$ is such that

$$Y_{H,t} = \int_{\mathcal{S}} \left(H^i_{a,t+1} - H^i_{a,t}(1-\delta_H)\right) d\mu. \tag{15}$$

7. The bond market clears: $q_t = q(\mu_t, Z_t)$ is such that

$$\int_{\mathcal{S}} B^i_{a,t} d\mu + B^F_t = 0, \tag{16}$$

   where $B^F_t \geq 0$ is an exogenous supply of foreign capital discussed below.

8. The risky asset market clears:

$$1 = \int_{\mathcal{S}} \theta^i_{a,t} d\mu. \tag{17}$$

9. The labor market clears:

$$N_t \equiv N_{C,t} + N_{H,t} = \int_{\mathcal{S}} L^i_{a,t} d\mu. \tag{18}$$

10. The social security tax rate is set so that total taxes equal total retirement benefits:

$$\tau N_t w_t = \int_{\mathcal{S}} PE^i_{a,t} d\mu, \tag{19}$$

11. Government revenue from land/permit rentals equals total government spending, G_t:

$$p^L_t \mathcal{L}_t = G_t$$

12. The presumed law of motion for the state space $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ is consistent with individual behavior.

Equations (13), (14) and (18) determine the $N_{C,t}$ and therefore determine the allocation of labor across sectors:

$$(1-\alpha) Z_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} = (1-\nu) p^H_t Z_{H,t} K^{\nu}_{H,t} (N_t - N_{C,t})^{-\nu}. \tag{20}$$

Also, the aggregate resource constraint for the economy must take into account the housing and risky capital market transactions/participation costs and the wasteful government spending, which reduce consumption, the adjustment costs in productive capital, which reduce firm profits, and the change in net foreign capital in the bond market, which finances domestic consumption and investment. Thus, non-housing output equals non-housing consumption (inclusive of costs F_t) plus government spending plus aggregate investment (gross of adjustment costs) less the net change in the value of foreign capital:

$$\begin{aligned} Y_{C,t} = C_t + F_t + G_t + \left(I_{C,t} + \phi_C \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t} \right) + \left(I_{H,t} + \phi_H \left(\frac{I_{H,t}}{K_{H,t}} \right) K_{H,t} \right) \\ - \left(B^F_{t+1} q(\mu_t, Z_t) - B^F_t \right) \end{aligned} \tag{21}$$

where C_t and F_t are aggregate quantities defined as[10]

$$C_t \equiv \int_{\mathcal{S}} C^i_{a,t} d\mu \qquad F_t \equiv \int_{\mathcal{S}} F^i_t d\mu. \tag{22}$$

To solve the model, it is necessary to approximate the infinite dimensional object μ with a finite dimensional object. The appendix explains the solution procedure and how we specify a finite dimensional vector to represent the law of motion for μ.

[10]Note that (21) simply results from aggregating the budget constraints across all households, imposing all market clearing conditions, and using the definitions of dividends as equal to firm revenue minus costs.

3.6 Model Calibration

This section discusses our calibration of the model's primitive parameters under three alternative set of parameterizations. *Model 1* is our benchmark calibration, with "normal" collateral requirements and housing transactions costs calibrated to roughly match the data prior to the housing boom of 2000-2006. *Model 2* is identical to Model 1 except that it has undergone a financial market liberalization (decline in collateral requirements and housing transactions costs). In both Model 1 and Model 2, trade in the risk-free asset is entirely conducted between domestic residents: $B_t^F = 0$. The *Model 3* calibration is identical to that of Model 2 except that we add an exogenous foreign demand for the risk-free bond: $B_t^F > 0$. For convenience, the model's parameters and their numerical calibration are summarized in Table 1. We describe this calibration next.

3.6.1 Calibration of Parameters

The technology shocks Z_C and Z_H are assumed to follow two-state independent Markov chains; the calibration is described in the Appendix. The Appendix also describes our calibration of the individual productivity shocks.

Parameters pertaining to the firms' decisions are set as follows. The capital depreciation rate, δ, is set to 0.12, which corresponds to the average Bureau of Economic Analysis (BEA) depreciation rates for equipment and structures. The housing depreciation rate δ_H, is set to 0.025 following Tuzel (2009). Following Kydland and Prescott (1982) and Hansen (1985), the capital share for the non-housing sector is set to $\alpha = 0.36$. For the residential investment sector, the value of the capital share in production is taken from a BEA study of gross product originating, by industry. The study finds that the capital share in the construction sector ranges from 29.4% and 31.0% over the period 1992-1996. We therefore set the capital share in the housing sector to $\nu = 0.30$.[11] The adjustment costs for capital in both sectors are assumed to be the same quadratic function of the investment to capital-ratio, $\varphi \left(\frac{I}{K} - \delta\right)^2$, where the constant φ is chosen to represent a tradeoff between the desire to match aggregate investment volatility simultaneously with the volatility of asset returns. Under this calibration, firms

[11] From the November 1997 SURVEY OF CURRENT BUSINESS, "Gross Product by Industry, 1947–96, "by Sherlene K.S. Lum and Robert E. Yuskavage. http://www.bea.gov/scb/account_articles/national/1197gpo/maintext.htm

Gross Product Originating is equal to gross domestic income, whose components can be grouped into categories that approximate shares of labor and capital. Under a Cobb-Douglas production function, these equal shares of capital and labor in output.

pay a cost only for net new investment; there is no cost to replace depreciated capital. This implies that the total adjustment cost $\varphi\left(\frac{I}{K}-\delta\right)^2 K_t$ under our calibration is quite small: on average less than one percent of investment, I_t. The fixed quantity of land/permits available each period, L, is set to a level that permits the model to approximately match the housing investment-GDP ratio. In post-war data this ratio is 6%; under our calibration of L, the ratio ranges from 5% to 6.2% across Model, 1, 2 and 3. The share of land/permits in the housing production function is set either to zero (as a baseline), requiring $\phi = 1$, or to 10%, to match estimates in Davis and Heathcote (2005), requiring $\phi = 0.9$. A baseline of $\phi = 1$ allows us to illustrate the sensitivity of the results to the presence of the fixed factor while keeping our benchmark specification parsimonious.

Parameters of the individual's problem are set as follows. The subjective time discount factor is set to $\beta = 0.923$ at annual frequency, to allow the model to match the mean of a short-term Treasury rate in the data. The survival probability $\pi_{a+1|a} = 1$ for $a+1 \leq 65$. For $a+1 > 65$, we set $\pi_{a+1|a}$ equal to the fraction of households over 65 born in a particular year alive at age $a+1$, as measured by the U.S. Census Bureau. From these numbers, we obtain the stationary age distribution in the model, and use it to match the average earnings over the life-cycle, G_a, to that observed from the Survey of Consumer Finances. Risk aversion is set to $\sigma = 8$, to help the models match the high Sharpe ratio for equity observed in the data. The static elasticity of substitution between C and H is set to $\varepsilon = 1$ (Cobb-Douglas utility), following evidence in Davis and Ortalo-Magne (2010) that expenditure shares on housing are approximately constant over time and across U.S. metropolitan statistical areas. The weight, χ on C in the utility function is set to 0.70, corresponding to a housing expenditure share of 0.30. The regime-switching conditional variance in the unit root process in idiosyncratic earnings is calibrated following Storesletten, Telmer, and Yaron (2007) to match their estimates from the Panel Study of Income Dynamics. These are $\sigma_E = 0.0768$, and $\sigma_R = 0.1296$.

The other parameters of the individual's problem are less precisely pinned down from empirical observation. The costs of stock market participation could include non-pecuniary costs as well as explicit transactions fees. Vissing-Jorgensen (2002) finds support for the presence of a fixed, per period participation cost, but not for the hypothesis of variable costs. She estimates the size of these costs and finds that they are small, less than 50 dollars per year in year 2000 dollars. These findings motivate our calibration of these costs so that they are no greater than 1% of per capita, average consumption, denoted $\overline{C}^i$ in Table 1.

We are aware of no publicly available time series on collateral requirements for mortgages and home equity loans. However, our own conversations with government economists and industry analysts who follow the housing sector indicated that, prior to the housing boom that ended in 2006, the combined LTV for first and second conventional mortgages (mortgages without mortgage insurance) was rarely if ever allowed to exceed 75 to 80% of the appraised value of the home. In addition, home equity lines of credit were not widely available until relatively recently (McCarthy and Steindel (2007)). By contrast, during the boom years households routinely bought homes with 100% financing using a piggyback second or home equity loan. Our Model 1 sets the maximum combined LTV (first and second mortgages) to be 75%, corresponding to $\varpi = 25\%$. In Model 2, we lower this to $\varpi = 1\%$. It should be emphasized that $1 - \varpi$ gives the *maximum* combined (first and second mortgage) LTV ratio. This will differ from the *average* LTV ratio because not everyone borrows up to the credit limit.

The fixed and variable housing transactions costs for housing consumption are governed by the parameters ψ_0 and ψ_1. These costs are more comprehensive than the costs of buying and selling existing homes. They include costs of any change in housing consumption, such as home improvements and additions, that may be associated with mortgage refinancing and home equity extraction, as well as non-pecuniary psychological costs. As discussed in Section 2, mortgage closing costs for first and second (home equity) mortgages, home equity lines of credit, and refinancing eroded considerably in the period during or preceding the housing boom, by 90% in some cases. Although some of these costs began to decline in the late 1980s and early 1990s, industry analysts report that there was a delay in public recognition. Mortgage servicers only gradually implemented marketing tools designed to inform customers of lower costs for refinancing and home equity withdrawal. Likewise, news that borrowers could expect a reduction in financial documentation and shortened time periods from application to approval and from approval to closing also spread slowly (Peristiani, Bennett, Monsen, Peach, and Raiff (1997)). To anchor the baseline level of these costs, in Model 1 we set fixed costs ψ_0 and variable costs ψ_1 to match the average number of years individuals in the model go without changing housing consumption equal to the average length of residency (in years) for home owners in the Survey of Consumer Finances across the 1989-2001 waves of the survey. In the equilibrium of our model, this amount corresponds to a value for ψ_0 that is approximately 3.2% of annual per capita consumption, and a value for ψ_1 that is approximately 5.5% of the value of the house $p_t^H H_{a,t}^i$. In Models 2 and 3 we decrease ψ_0 by 31%, setting it to approximately 2.2% of per capita aggregate consumption, and we decrease ψ_1 by 36%, setting it to 3.5% of home value $p_t^H H_{a,t}^i$. Given the comprehensive (and therefore unobservable) nature of transactions costs in the model, the calibration of the Model 2 and 3 decline in costs is admittedly arbitrary, but is intended to be conservative compared to the larger percentage decline in observable costs associated with mortgage contracts, mortgage refinancing, and home equity extraction.

Finally, we calibrate foreign ownership of U.S. debt, B_t^F, by targeting a value for foreign bond holdings relative to GDP. Specifically, when we add foreign capital to the economy in Model 3, we experiment with several constant values for $B_t^F \equiv B^F$ until the model solution implies a value equal to 18% of average total output, $\overline{Y}$, an amount that is approximately equal to the rise in foreign ownership of U.S. Treasuries and agency debt over the period 2000-2008. Figure 4, Panel B shows that, as of the middle of 2008, foreign holdings of long-term Treasuries alone represent 15% of GDP. Higher values are obtained if one includes foreign holdings of U.S. agency debt and/or short-term Treasuries. Depending on how many of these categories are included, the fraction of foreign holdings in 2008 ranges from 15-30%.

3.6.2 Model Returns

Housing Return Abstracting from transactions costs and borrowing constraints, the first-order condition for optimal housing choice is

$$\frac{\partial U}{\partial C_{a,t}^i} = \frac{1}{p_t^H} \beta E_t \left[\frac{\partial U}{\partial C_{a+1,t+1}^i} \left(\frac{\frac{\partial U}{\partial H_{a+1,t+1}^i}}{\frac{\partial U}{\partial C_{a+1,t+1}^i}} + p_{t+1}^H (1 - \delta_H) \right) \right], \tag{23}$$

implying that each individual's housing return is given by $\frac{\partial U / \partial H_{a+1,t+1}^i}{\partial U_{t+1} / \partial C_{a+1,t+1}^i} + p_{t+1}^H (1 - \delta_H)$ where $\frac{\partial U / \partial H_{a+1,t+1}^i}{\partial U_{t+1} / \partial C_{a+1,t+1}^i}$ is the implicit rental price for housing services, referred to hereafter as "rent." For the national housing return, we define national rent, $\mathcal{R}_{t+1}$, as the average of $\frac{\partial U / \partial H_{a+1,t+1}^i}{\partial U_{t+1} / \partial C_{a+1,t+1}^i}$ across individuals. Given this definition of national rent, we define the corresponding national housing return as

$$R_{H,t+1} \equiv \frac{p_{t+1}^H (1 - \delta_H) + \mathcal{R}_{t+1}}{p_t^H}, \tag{24}$$

$$\mathcal{R}_{t+1} \equiv \int_{\mathcal{S}} \frac{\partial U / \partial H_{a+1,t+1}^i}{\partial U_{t+1} / \partial C_{a+1,t+1}^i} d\mu. \tag{25}$$

In the model, p_t^H is the price of a unit of housing stock, which holds fixed the composition of housing (quality, square footage, etc.) over time.

We compare our model results with three different measures of single-family residential price-rent ratios and associated housing returns. These are (i) a measure based on housing wealth for the household sector from the Flow of Funds, hereafter *FoF*, (ii) a measure based on the Freddie Mac Conventional Mortgage House Price index, hereafter *Freddie Mac*, (iii) a measure based on the Case-Shiller national house price index, hereafter *CS*. The FoF data are combined with a measure of housing services from the national income and product accounts (NIPA) to measure rent, or housing services, and compute a national price-rent ratio and housing return. The Freddie Mac and CS price indexes are combined with the Bureau of Labor Statistics (BLS) rental index for shelter to do the same. The Appendix details our construction of these variables.

It is important to note that the *level* of the average price-rent ratio in the data is not identified. For Freddie Mac and CS, the price-rent ratio cannot be inferred at all, since both price in the numerator and rent in the denominator are given by indexes. For FoF, we observe the stock of housing wealth and the flow of housing services from NIPA, where the latter is a measure of housing expenses for renters aggregated with an imputed rent measure for owner-occupiers. Although both the wealth and housing services are in dollar units, the difficulty here is that it is notoriously difficult to impute rents for owner-occupiers from the rental data of non-homeowners, a potentially serious problem since owners represent two-thirds of the population. Moreover, because owners are on average wealthier than non-homeowners, the NIPA imputed rent measure for owner-occupiers is likely to be biased down, implying that the level of the price-rent ratio is likely to be biased up and the average housing return biased down. For this reason, we do not attempt to match our model to the levels of the price-rent ratios and housing returns in the data, instead focusing on the changes in these ratios over time.

Equity Return The risky capital return $R_{K,t}$ in (3) is the return on a value-weighted portfolio of assets. This is not the same as the return on equity, which is a levered claim on the assets. To obtain an equity return, $R_{E,t}$, the return on assets, $R_{K,t}$, must be adjusted for leverage:

$$R_{E,t} \equiv R_{f,t} + (1 + B/E)\left(R_{K,t} - R_{f,t}\right),$$

where B/E is the fixed debt-equity ratio and where $R_{K,t}$ is the portfolio return for risky capital given in (3).[12] Note that this calculation explicitly assumes that corporate debt in the model is exogenous, and held in fixed proportion to the value of the firm. (There is no financing decision.) For the results reported below, we set $B/E = 2/3$ to match aggregate

[12]The cost of capital R_K is a portfolio weighted average of the return on debt R_f and the return on equity R_e: $R_K = aR_f + (1-a)\,R_e$, where $a \equiv \frac{B}{B+E}$.

debt-equity ratios computed in Benninga and Protopapadakis (1990).

4 Results

This section presents some of the model's main implications. Much of our analysis consists of a comparison of stochastic steady states across Models, 1, 2 and 3.[13] We also study a dynamic transition path for house prices and national price-rent ratios designed to mimic the state of the economy and housing market conditions over the period 2000-2009. For many results, the findings for the case with land/permits ($\phi = .9$) are quite close to those of the baseline $\phi = 1$ case. We therefore present here the complete set of results for the baseline case and present results for the specification with land/permits only if they differ significantly from the baseline case, or when they pertain to housing and financial wealth statistics, the main focus of the paper.[14] We start by presenting a set of benchmark business cycle and life-cycle results.

4.1 Benchmark Results

4.1.1 Business Cycle Variables

Table 2 presents benchmark results for Hodrick-Prescott (Hodrick and Prescott (1997)) detrended aggregate quantities. Panel A of Table 2 presents business cycle moments from U.S. annual data over the period 1953 to 2008. Panel B of Table 2 presents simulated data to summarize the implications for these same moments for the benchmark Model 1, baseline case ($\phi = 1$). Panel C presents the same results for Model 2. We report statistics for total output, or $GDP \equiv Y_C + p^H Y_H + C_H$, for non-housing consumption (inclusive of expenditures on financial services), equal to $C + F$, for housing consumption $C_{H,t}$, defined as price per unit of housing services times quantity of housing or $C_{H,t} \equiv \mathcal{R}_t H_t$, for total (housing and non-housing) consumption $C_T = C + F + C_H$, for non-housing investment (inclusive of adjustment costs) $I = (I_{C,t} + \phi_C(\cdot) K_{C,t}) + (I_{H,t} + \phi_H(\cdot) K_{H,t})$, for residential investment $p_t^H Y_{H,t}$ and for total investment $I_T = I + p^H Y_H$.

The standard deviation of total aggregate consumption divided by the standard deviation of GDP is 0.77 in Model 1 and 0.69 in Model 2, close to the 0.70 value found in the data.

[13] With all shocks in the model set to zero, the portfolio choice problem is indeterminant since all assets earn the risk-free return. Thus, there is no deterministic steady state in this model. We define stochastic steady state as the average equilibrium allocation over a large number of simulated sample paths.

[14] The complete set of results for $\phi = 0.9$ are available upon request.

In addition, the level of GDP volatility in the model is close to that in the data. Thus the model produces a plausible amount of aggregate consumption volatility. Total investment is more volatile than output, both in the model and in the data, but the model produces too little relative volatility: the ratio of the standard deviation of investment to that of output is 1.7 in Model 1 but is 2.9 in the data.[15] The model does a good job of matching the relative volatility of residential investment to output: in the data the ratio of these volatilities is 4.6, while it is 5.4 in Model 1 and 5.1 in Model 2. Finally, both in the model and the data, residential investment is less correlated with output than is consumption and total investment.

Table 3 shows the model's implications for the cyclical properties of national house prices. The housing price indexes in the data are all procyclical, but not as strongly so as in the model. As in the data, the model implies that both the level of house prices and price-rent ratios are procyclical, in Models 1, 2, and 3. Note that price-rent ratios are less procyclical than is the level of house prices because rents, in the denominator, are also procyclical. For the cases without land/permits ($\phi = 1$, panel A), the correlation between GDP and the national price-rent ratio ranges from 0.17 to 0.62 across the three models. The results for $\phi = 0.9$ are similar and are reported in panel B. In the data, these correlations vary substantially by data source and sample, ranging from 0.29 to 0.10.

4.1.2 Life Cycle Age-Income Profiles

Turning to individual-level implications, Figure 5 presents the age and income distribution of wealth, both in the model and in the historical data as given by the Survey of Consumer Finance (SCF). The figure shows wealth, by age, divided by average wealth across all households, for three income groups (low, medium and high earners). In both the model and the data, financial wealth is hump-shaped over the life-cycle, and is slightly negative or close to zero early in life when households borrow to finance home purchases. As agents age, wealth accumulates. In the data, financial (nonhousing) wealth peaks between 60 and 70 years old (depending on the income level). In the model, the peak for all three income groups is 65 years. After retirement, financial wealth is drawn down until death. Households in the model continue to hold some net worth in the final years of life to insure against the possibility of living long into old age. A similar observation holds in the data. For low and

[15] Volatility of investment could be increased by adding stochastic depreciation in capital as in Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008), or by adding investment-specific technology shocks.

medium earners, the model gets the average amount of wealth about right, but it some what under-predicts the wealth of high earners early in the life-cycle.

The right-hand panels in Figure 5 plot the age distribution of housing wealth. Up to age 65, the model produces about the right level of housing wealth for each income group, as compared to the data. In the data, however, housing wealth peaks around age 60 for high earners and age 67 for low and medium earners. By contrast, in the model housing wealth remains high until death. In the absence of an explicit rental market, owning a home is the only way to generate housing consumption, an argument in the utility function. For this reason, agents in the model continue to maintain a high level of housing wealth later in life even as they drawn down financial wealth.

What is the effect of a financial market liberalization and foreign capital influx on the optimal portfolio decisions of individuals? Table 4 exhibits the age and income distribution of housing wealth relative to total net worth, both over time in the SCF data and in Models, 1, 2 and 3 for $\phi = 1$ (panel A) and $\phi = 0.9$ (panel B). The benchmark model captures an empirical stylized fact emphasized by Fernández-Villaverde and Krueger (2005), namely that young households hold most of their wealth in consumer durables (primarily housing) and very little in financial assets. Indeed, our calibrations imply that young households (age 35 and under), hold slightly more of their wealth as durables than do households in the data.

By comparing the steady states of Model 1 and Model 3, we see that the model also predicts that a financial market liberalization plus an inflow of foreign capital leads households of all ages and income groups to shift the composition of their wealth towards housing. This occurs because the combination of lower interest rates, lower collateral constraints, and lower housing transactions costs in Model 3 makes possible greater housing investment by the young, whose incomes are growing and who rely on borrowing to expand their housing consumption. But the decline in housing transactions costs also has important effects on the asset allocation of net savers (primarily older, higher income individuals), consistent with the findings of Stokey (2009) who shows that such costs can have large effects on portfolio decisions. Here, a decline in housing transactions costs makes housing relatively less risky as compared to equity, which causes even unconstrained individuals to shift the composition of their wealth towards housing. Because of the simultaneous relaxation in credit constraints, the increased importance of housing is still largest for the young. Table 4, Panel A, shows that the housing wealth-total wealth ratio rises by 19% for the young between Model 1 and Model 3. But, the housing wealth-total wealth ratio also rises for other groups: it rises by 13% for households above age 35 and by 14% for high income individuals. Table 4 shows that these changes are in line with those in individual-level data from 2001 to 2007.

4.2 Asset Pricing

4.2.1 Return Moments

Table 5 presents asset pricing implications of the model, for the calibrations represented by Models 1, 2 and 3 and for $\phi = 1$ (panel A) and $\phi = 0.9$ (panel B). The statistics reported are averages over 1000 periods. We first discuss the implications of the benchmark Model 1 and then move on to discuss how the statistics change with a financial market liberalization and inflow of foreign money.

The benchmark model matches the historical mean return for the risk-free rate and only slightly overstates the volatility of the risk-free rate. The model produces a sizable equity return of 5.62% per annum (and annual equity premium of 4%) and an annual Sharpe ratio of 0.31 (panel A), compared to 0.34 in the data. Two factors related to the cyclicality of the cross-sectional distribution of consumption contribute to the model's high average Sharpe ratio. First, idiosyncratic income risk is countercyclical. Second, house prices and therefore collateral values are procyclical, making borrowing constraints countercyclical. These factors mean that insurance/risk-sharing opportunities are eliminated when households need them most–in recessions–resulting in a high risk premium and Sharpe ratio.

Turning to the implications for housing assets, the average housing return in the benchmark Model 1 with $\phi = 1$ is 13% per annum; the standard deviation of the housing return in the model is 6.2% per annum. The housing return Sharpe ratio for Model 1 is 1.52. The findings for the $\phi = 0.9$ case are similar.

Financial Market Liberalization and the Housing Boom How are these statistics affected by financial market liberalization? Table 5 shows that both the equity premium and the equity Sharpe ratio fall in an economy that has undergone a financial market liberalization. The equity premium falls from 4% to 3.6% from Model 1 to Model 2, while the Sharpe ratio falls from 0.31 to 0.23, a 26% decline (Panel B). A financial market liberalization lowers the risk premium on housing assets even more. The housing risk premium is cut by 40 percent from Model 1 to Model 2, from 11.39% per annum to 6.86%, while the housing Sharpe ratio declines by 47% from 1.52 to 0.8. The results in panel B of Table 7 for $\phi = 0.9$ are similar. This decline in the riskiness of both housing and equity assets reflects the greater amount of risk-sharing possible after a financial market liberalization, discussed further below. The housing Sharpe ratio declines more than the equity Sharpe ratio because there is an additional factor pushing down the housing risk premium that is inoperative for the equity market: a financial market liberalization is accompanied by a decline in transactions costs for housing but not for equity (or the risk-free asset).

The last column of Table 5 shows that, when $\phi = 1$, the national price-rent ratio $p^H/\mathcal{R}$ is 23.4% higher in Model 2 than it is in the benchmark Model 1 (Panel B, Table 5). These results isolate the effect of a financial market liberalization, since they are a comparison of steady states only. (Below we study a dynamic transition that includes economic shocks.) But since the price-rent ratio is procyclical, this finding implies that a financial market liberalization adds substantial fuel to the fire in an already booming housing market during an economic expansion. At the same time, Table 5 shows that a financial market liberalization by itself leads to a sharp *increase* in equilibrium interest rates. Indeed, the endogenous risk-free interest rate more than doubles in Model 2 to 3.56% per annum, from 1.63% in Model 1. This occurs because the relaxation of borrowing constraints and housing transactions costs reduces precautionary savings, as households endogenously respond to the improved risk-sharing/insurance opportunities afforded by financial market liberalization. It follows that the increase in price-rent ratios following a financial market liberalization is entirely attributable to the decline in the housing risk premium, which more than offsets the rise in equilibrium interest rates.[16] These results are similar for the $\phi = 0.9$ case in Panel C: here the price-rent ratio is 27.5% higher in Model 2 than in Model 1.

The Role of Foreign Capital in the Housing Boom Model 3 adds to Model 2 an inflow of foreign capital calibrated to match the increase in foreign ownership of U.S. Treasuries and U.S. agency debt over the period 2000-2006. Table 5 shows that such an increase has a large downward impact on the equilibrium interest rate, which falls from 3.56% in Model 2 to 0.0 in Model 3 ($\phi = 1$), or from 3.45% to 0.39% ($\phi = .9$). The magnitude of these declines are close to the reduction in real interest rates observed in U.S. data over the period 2000-2006.[17] The last column of Table 5 shows that the average price-rent ratio is 31% ($\phi = 1$) or 34% percent higher ($\phi = 0.9$) in the steady state of Model 3 than in it

[16] Note also that there are no differences in average annual rental growth rates across Models 1, and 2 and Model 3. Because the statistics for each model are computed from averages across 1000 periods, they give the long-run annualized values of rental growth. This is the same across all three models because it is pinned down by the steady state growth of technology, which is the same in each model, assumed to be two percent.

[17] Figure 3 shows the decline in nominal rates; subtracting off inflation to compute a real rate, we observe that the 10-year real Treasury bond rate fell from 3.6% to 0.93% from December 1999 to June 2006.

is in the benchmark Model 1. (As a comparison, the last column of Table 5, Panel A shows that these values represents all of the increase in the FoF and Freddie Mac house price-rent ratios over the 2000-2006 period, which each rose 31%, and 72 percent of the increase in the Case-Shiller index, which rose 43%.) The majority of the rise in the price-rent ratio over the benchmark Model 1, however, comes not from the foreign-capital-driven lower interest rates, but rather from the financial market liberalization. For both $\phi = 1$ and $\phi = 0.9$, the price-rent ratio is 6% higher in Model 3 than it is in Model 2. This represents less than a quarter of the total change from Model 1 to Model 3.

The reason lies with the endogenous response of the housing risk premium to an increase in foreign demand for the safe asset. By itself, foreign purchases of the safe asset make both equity and housing assets more *risky*. Both the risk premium and Sharpe ratio for equity and housing rise substantially from Model 2 to Model 3, for two reasons. First, the increase in foreign money forces domestic residents as a whole to take a leveraged position in the risky assets. This by itself increases the volatility of asset and housing returns, translating into higher risk premia. Second, domestic savers are crowded out of the bond market by foreign governmental holders who are willing to hold the safe asset at any price. As a result, they become more exposed to systematic risk in the equity and housing markets. This means that the equity and housing Sharpe ratios must rise from Model 2 to Model 3, as domestic savers shift the composition of their financial wealth towards risky securities. In addition, the volatility of the stochastic discount factor, $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, rises along with (as discussed below) a decrease in measured risk-sharing from Model 2 to Model 3.

Despite the increase in risk premia resulting from the foreign capital inflow, the housing risk premium is still lower in Model 3 than in the baseline Model 1 because the decline from Model 1 to Model 2 more than offsets the rise from Model 2 to Model 3. Still, the rise from Model 2 to Model 3 means that the endogenous response of risk premia to foreign purchases of U.S. government bonds substantially limits the extent to which foreign capital inflows can influence home prices. These findings underscore the importance of general equilibrium effects on risk premia for understanding the role of foreign capital inflows in a housing boom. In partial equilibrium models of the housing market (e.g., Titman (1982)), or in small open-economy models without aggregate risk (e.g., Kiyotaki, Michaelides, and Nikolov (2008)), the risk premium is held exogenously fixed. As a consequence, a decline in the interest rate equal in magnitude to that generated by the large influx of foreign money considered here, would be sufficient–by itself–to explain the rise in price-rent ratios observed from 2000-2006. In general equilibrium this is not possible because a foreign capital inflow causes the endogenous risk premium to rise at the same time that it causes interest rates to fall, substantially limiting the positive effect of lower interest rates on home prices.

4.2.2 Transition Dynamics: Housing Boom to Bust

Above we studied the effects of housing finance by comparing stochastic steady states. The steady state differences between models show long-run changes only and do not account for business cycle fluctuations. In this section we study a dynamic transition path for house prices and price-rent ratios, in response to a series of shocks designed to mimic both the state of the economy and housing market conditions over the period 2000-2009.[18] We assume that, at time 0 (taken to be the year 2000), the economy begins in the stochastic steady state of Model 1. In 2001, the economy undergoes an unanticipated shift to Model 3 (financial market liberalization and foreign holdings of U.S. bonds equal to 18% of GDP), at which time the policy functions and beliefs of Model 3 are applied.[19] The adjustment to the new stochastic steady state of model 3 is then traced out over the seven year period from 2001 to 2006, as the state variables evolve. Starting in 2007 and continuing through 2009, the economy is presumed to undergo a surprise reversal of the financial market liberalization but not the foreign capital inflow, and as such unexpectedly shifts to a new state in which all the parameters of Model 1 again apply except those governing the foreign capital inflow, which we assume remains equal to 18% of GDP annually, as in Model 3. This hybrid of Models 1 and 3 is referred to as *Model 4*.

In addition, we feed in a specific sequence of aggregate shocks designed to mimic the business cycle over this period. The aggregate technology shock processes Z_C and Z_H follow Markov chains, with two possible values for each shock, "low" and "high" (see the Appendix). Denote these possibilities with the subscripts "l" and "h":

$$Z_C = \{Z_{Cl}, Z_{Ch}\}, \ Z_H = \{Z_{Hl}, Z_{Hh}\}.$$

As the general economy began to decline in 2000, construction relative to GDP in U.S. data continued to expand, and did so in every quarter until the end of 2005. Thus, the recession of 2001 was a nonhousing recession. Starting in 2006, construction relative to GDP fell and has

[18] Ideally, we would study such a path after solving a larger framework that specified a probability law over parameters corresponding to the different models (1 through 3) defined above. Unfortunately, solving such a specification in the existing model would be computationally infeasible. We therefore pursue the simpler strategy described above.

[19] Along the transition path, foreign holdings of bonds are increased linearly from 0% to 18% of GDP from 2000 to 2006 and held constant at 18% from 2006 to 2009.

done so in every quarter through the most recent data at the time of this writing (2009:Q2). Thus, in contrast to the 2001 recession, housing led the recession of 2007-2009. To capture these cyclical dynamics, we feed in the following sequence of shocks for the period 2000-2009: $\{Z_{Cl}, Z_{Hh}\}_{t=2000}$, $\{Z_{Cl}, Z_{Hh}\}_{t=2001}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2002}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2003}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2004}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2005}$, $\{Z_{Ch}, Z_{Hl}\}_{t=2006}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2007}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2008}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2009}$.

Figure 6 shows the transition dynamics of the price-rent ratio, $p_t^H/\mathcal{R}_t$, (right scale) are such that it rises by 41% over the period 2000-2006 for the case with $\phi = 1$, boosted by economic growth, the financial market liberalization, and lower interest rates. House prices themselves (left scale) rise 18% from 2000-2006 when $\phi = 1$, and 26% for the model with land/permits ($\phi = 0.9$). The increase in home prices for the case $\phi = 0.9$ is significantly greater than that for the $\phi = 1$ case because the presence of land/permits makes housing supply more inelastic. The increase in $p_t^H/\mathcal{R}_t$ from 2000-2006 is larger than the increase in p_t^H because, in the model, rents fall modestly over this period as the housing stock expands in response to positive economic shocks. The model generates a decline of greater than 19% ($\phi = 0.9$) or 16% ($\phi = 1$) in the price-rent ratio and a decline of more than 14% ($\phi = 0.9$) or 12% ($\phi = 1$) in home prices p_t^H in just the two year period 2007 to 2009, driven by the economic contraction and by a presumed reversal of the financial market liberalization.

Finally, Figure 7 shows that the price of land/permits p_t^L for the model with $\phi = 0.9$ rises and falls over the transition with the price of housing. Thus, the expansion not only drives a construction and housing boom; it also raises the price of the fixed factor of housing production by 18% from 2000-2006. Land/permits prices subsequently fall along with house prices from 2007 to 2009, as the economy contracts and collateral constraints and transactions costs revert to previously higher levels.

4.2.3 Cyclical Dynamics of Housing: What Do Changes in House Price-Rent Ratios Forecast?

In this section we ask how cyclical increases in price-rent ratios affect expectations of future rental growth rates and future home price appreciation. In the model, 100% of the variability in the log price-rent ratio is attributable to variation in the rationally expected present discounted value of future rental growth rates. This variability can itself be divided into two parts: that attributable to variation in expected future rents and that attributable to variation in expected future housing returns (discount rates). Thus, to address this question, we look within each model at the relation between purely cyclical changes in price-rent ratios and subsequent movements in housing return and rents. The left panels of Table 6 show regression results (coefficient, t-stat, R^2) for predicting long-horizon future housing returns and rental growth rates using today's price-rent ratio. (The results reported are for the $\phi = 1$ case; results for $\phi = 0.9$ are very similar and are omitted.)

High price-rent ratios forecast *lower* future housing returns, or future home price depreciation. This aspect of the model is consistent with empirical evidence in the bottom left panels of Table 6 (see also Campbell, Davis, Gallin, and Martin (2010)). In the model this occurs in part because high price-rent ratios in an expansion forecast lower future *excess* returns to housing assets, driven by a lower housing risk premium. The housing risk premium falls as the economy grows for two reasons. First, economic growth reduces (but does not eliminate) uninsurable idiosyncratic income risk via (4). Second, the endogenous increase in house prices raises collateral values and relaxes borrowing restrictions, affording households more insurance against remaining income risk.

Table 6 also shows that high price-rent ratios forecast *lower* future rental growth. It is often suggested that increases in price-rent ratios reflect an expected increase in rental growth. For example, in a partial equilibrium setting where discount rates are constant, higher house prices relative to fundamentals can only be generated by higher implicit rental growth rates in the future (Sinai and Souleles (2005), Campbell and Cocco (2007)). The partial equilibrium setting, however, ignores the endogenous response of both discount rates and residential investment to economic growth. In general equilibrium, positive economic shocks can simultaneously drive discount rates down and residential investment up, leading high price-rent ratios to reflect an expected decline in rental growth. As the housing supply expands, the cost of future housing services (rent) is forecast to be lower. It follows that high price-rent ratios in expansions must entirely reflect expectations of future home price depreciation (lower future returns). Although future rental growth is expected to be lower, price-rent ratios still rise in response to positive economic shocks because the expected decline in future housing returns more than offsets the expected fall in future rental growth.[20]

For completeness, Table 6 also reports predictability results for equity returns. In model generated data, both the raw equity return and the excess return are forecastable over long horizons, consistent with evidence from U.S. stock market returns.[21] High price-dividend ratios forecast low future equity returns and low excess returns (low equity risk premia) over

[20] Predictable variation in housing returns must therefore account for more than 100 percent of the variability in price-rent ratios.

[21] See, for example, the summary evidence in Cochrane (2005), Chapter 20, Lettau and Ludvigson (2010), and Lettau and Van Nieuwerburgh (2008).

horizons ranging from 1 to 30 years. Compared to the data, the model produces about the right amount of forecastability in excess equity returns, but produces too much forecastability of dividend growth. This is not surprising since, unlike an endowment/exchange economy where dividends are set exogenously, in the model here both profits and the value of the firm respond endogenously to aggregate shocks.[22]

4.3 Risk Sharing and Inequality

In the limited risk-sharing environment here, risk premia are driven by amount of risk-sharing/insurance possible in the economy. Table 7 presents four measures of inequality or risk-sharing: (i) the cross-sectional standard deviation in the individual consumption share in aggregate consumption, (ii) the variance of log consumption, (iii) the Gini coefficient of consumption, and (iv) the cross-sectional standard deviation of the individual marginal rates of substitution, $\frac{\beta \partial U / \partial C^i_{a+1,t+1}}{\partial U / \partial C^i_{a,t}}$. The first three are measures of inequality in the numeraire consumption good. The last is a measure of risk-sharing. Under perfect risk-sharing (complete markets) individuals equate their marginal rates of substitution state by state. Thus the cross-sectional standard deviation of the marginal rates of substitution is a quantitative measure of market incompleteness, with higher values indicating less risk-sharing.

Table 7 shows that the decline in risk premia from Model 1 to Model 2 (documented above) coincides with an increase in risk-sharing and a decline in consumption inequality, according to all measures. Risk-sharing improves both because a financial liberalization directly increases access to credit, and because lower transactions costs reduce the expense of acquiring additional collateral, which increases borrowing capacity. Both factors allow heterogeneous households to insure more of their risks.

By contrast, Table 7 shows that these same measures of risk-sharing and consumption inequality rise from Model 2 to Model 3, isolating the effect of the foreign capital inflow. The rise in foreign capital, in effect, makes existing financial markets more incomplete because the foreign governmental holders' perfectly inelastic demand for the risk-free asset reduces the availability of this asset to domestic savers for insurance. Thus, the increase in risk-sharing and fall in consumption inequality resulting from a financial market liberalization is offset by a fall in risk-sharing and a rise in inequality resulting from foreign purchases of the risk-free asset. In the calibration here, the latter slightly more than offsets the former

[22] For this same reason, the model also produces too much predictability in raw returns driven by too much predictability in interest rates. Positive economic shocks increase consumption but not as much as income, thus saving and the capital stock rise, pushing down expected rates of return to saving, or interest rates.

so that the net change in consumption inequality is small but positive moving from Model 1 to Model 3.

What about wealth inequality? Unlike consumption inequality, a financial market liberalization and foreign demand for the risk-free asset have *reinforcing* effects on financial wealth inequality. Figure 8 shows the Gini index for inequality in total net worth, decomposed into financial wealth and housing wealth, for Models 1, 2, and 3 (right scale), as well as the Gini index based on the SCF data for the years 2001, 2004 and 2007 (left scale). The figure compares the change in the wealth Gini index from 2001 to 2007 with the change in the model Gini index between Models 1, 2 and 3.

While the present model does not explain the degree of wealth inequality in the data, it captures recent trends in wealth inequality.[23] In the data, the Gini index for financial wealth rises by almost 20 percent between 2001 and 2007. In the model, the Gini for financial wealth increases by about 10 percent as a result of financial market liberalization (Model 1 to Model 2), and by another 5.4 percent as a result of foreign governmental demand for the safe asset (Model 2 to Model 3). In addition, both in the model and in the data, housing wealth inequality increases far less than financial wealth inequality: the Gini index for housing wealth in the SCF data is flat from 2001 to 2007, while in the model it falls slightly from Model 1 to Model 3.

Why do a financial market liberalization and a foreign capital infusion have reinforcing upward affects on financial wealth inequality but offsetting affects on consumption inequality? A financial market liberalization relaxes financial frictions, making it easier to borrow against home equity and making it less costly to transact. This improves risk-sharing and reduces consumption inequality and housing inequality. But financial wealth inequality rises because, as domestic borrowers (mostly young individuals) take advantage of the market liberalization to increase current consumption, their net worth position deteriorates. At the same time, domestic savers as a whole (older households primarily concerned about retirement) are forced to shift the composition of their wealth toward risky securities as a result of the inelastic foreign demand for the safe asset. They therefore earn a higher rate of return on the risky asset and on their savings, as compared to Model 2, which drives their wealth more positive. These findings may help explain why wealth inequality has risen more than consumption inequality in recent decades.[24]

[23] The level of the Gini index in the model is lower than that in the data. Following Krusell and Smith (1997, 1998), the wealth distribution could be better approximated by introducing heterogeneity in the subjective time discount factor.

[24] Krueger and Perri (2006) and Heathcote, Perri, and Violante (2009) study income and consumption

5 Conclusion

In this paper we have studied the macroeconomic and household-level consequences of fluctuations in housing wealth and housing finance. The framework studied here endogenizes the interaction among financial and housing wealth, output and investment, rates of return and risk premia in both housing and equity assets, and consumption and wealth inequality. We have focused much of our investigation on studying the macroeconomic impact of systemic changes in housing finance that were a key characteristic of housing markets during the housing boom period from 2000-2006 and its aftermath.

The model implies that national house price-rent ratios may fluctuate considerably in response to a financial market liberalization, as well as in response to movements in the aggregate economy. A fundamental result of the paper is that these factors influence households' opportunities for risk-sharing, and it is through this mechanism that they influence home prices. In a simulated transition for the period 2000-2009, the model captures all of the run-up observed in U.S. national house price-rent ratios from 2000-2006 and predicts a sharp decline in housing markets starting in 2007. We found the general equilibrium environment to be particularly important for understanding some features of these results. For example, the model implies that procyclical increases in national house price-rent ratios must reflect lower future housing returns rather than higher future rents, a finding that is difficult to comprehend without taking into account the endogenous response of residential investment and discount rates to positive economic shocks.

A financial market liberalization increases house prices because it drives risk premia in both the housing and equity market down and shifts the composition of wealth for all age and income groups towards housing. These changes, along with economic shocks, are the largest drivers of volatility in the model price-rent ratio. By contrast, borrowed funds from the rest of the world–while having a large depressing effect on interest rates–were found to play a limited role in generating asset booms. This latter result runs contrary to the perception that, by driving interest rates lower, the vast inflow of foreign money into U.S. bond markets from 2000 to 2006 was a major factor in the housing boom.[25] We show that the general equilibrium effects of foreign capital on risk premia substantially offset the effects of

inequality directly, and show that consumption inequality has risen less than income inequality. Their results for saving and income inequality suggest that wealth inequality has risen more than consumption inequality over time.

[25] This perception has been voiced by policymakers, academics, and industry analysts. See for example, Bernanke (2005, 2008), and Stiglitz (2010).

lower interest rates, thereby limiting the role of foreign money in driving home prices. These results suggest that large fluctuations in borrowing from the rest of the world may not be the most important determinants of asset price fluctuations, and they lend interpretation to the observation that house prices have declined sharply in aftermath of the credit crisis even as foreign capital inflows have remained high and interest rates low.

Although the theoretical framework studied here generates a large boom-bust pattern in home prices, it has no role for a bubble: all of the variability in the model's price-rent ratio is attributable to variability in the rationally expected present discounted value of future rents. An important part of this variability is attributable to the changes in housing finance we have studied. The model takes no stand on whether these changes in housing finance can be characterized as a rational response to economic conditions and/or regulatory changes. Focusing on features of the recent housing boom, Piskorski and Tchistyi (2008) study the mortgage contracting problem in a partial equilibrium setting with stochastic (exogenous) home price appreciation. They find that many elements of the housing boom, such as the relaxation of credit limits, the subsidization of risky (subprime) borrowers, and the clustering of defaults among riskier borrowers, can be explained as the outcome of an optimal dynamic mortgage contracting problem in which both borrowers and lenders are fully rational. Combining the partial-equilibrium mortgage contracting problem with the general equilibrium model of limited risk-sharing is an important challenge for future research.

Appendix

This appendix describes how we calibrate the stochastic shock processes in the model, describes the historical data we use to measure house price-rent ratios and returns, and describes our numerical solution strategy.

Calibration of Shocks

The aggregate technology shock processes Z_C and Z_H are calibrated following a two-state Markov chain, with two possible values for each shock, $\{Z_C = Z_{Cl}, Z_C = Z_{Ch}\}$, $\{Z_H = Z_{Hl}, Z_H = Z_{Hh}\}$, implying four possible combinations:

$$\begin{aligned} Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hl} \\ Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hl} \\ Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hh} \\ Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hh.} \end{aligned}$$

Each shock is modeled as,

$$\begin{aligned} Z_{Cl} &= 1 - e_C, \qquad Z_{Ch} = 1 + e_C \\ Z_{Hl} &= 1 - e_H, \qquad Z_{Ch} = 1 + e_H, \end{aligned}$$

where e_C and e_H are calibrated to match the volatilities of GDP and residential investment in the data.

We assume that Z_C and Z_H are independent of one another. Let $\mathbf{P}^C$ be the transition matrix for Z_C and $\mathbf{P}^H$ be the transition matrix for Z_H. The full transition matrix equals

$$\mathbf{P} = \begin{bmatrix} p_{ll}^H \mathbf{P}^C & p_{lh}^H \mathbf{P}^C \\ p_{hl}^H \mathbf{P}^C & p_{hh}^H \mathbf{P}^C \end{bmatrix},$$

where

$$\mathbf{P}^H = \begin{bmatrix} p_{ll}^H & p_{lh}^H \\ p_{hl}^H & p_{hh}^H \end{bmatrix} = \begin{bmatrix} p_{ll}^H & 1 - p_{ll}^H \\ 1 - p_{hh}^H & p_{hh}^H \end{bmatrix},$$

and where we assume $\mathbf{P}^C$, defined analogously, equals $\mathbf{P}^H$. We calibrate values for the matrices as

$$\mathbf{P}^C = \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix}$$

$$\mathbf{P}^H = \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} =>$$

$$\mathbf{P} = \begin{bmatrix} .36 & .24 & .24 & .16 \\ .15 & .45 & .10 & .30 \\ .15 & .10 & .45 & .30 \\ .0625 & .1875 & .1875 & .5625 \end{bmatrix}.$$

With these parameter values, we match the average length of expansions divided by the average length of recessions (equal to 5.7 in NBER data from over the period 1945-2001). We define a recession as the event $\{Z_{Cl,}, Z_{Hl,}\}$, so that the probability of staying in a recession is $p_{ll}^H p_{ll}^C = 0.36$, implying that a recession persists on average for $1/(1-.36) = 1.56$ years. We define an expansion as either the event $\{Z_{Ch,} Z_{Hl,}\}$ or $\{Z_{Cl}, Z_{Hh}\}$ or $\{Z_{Ch,} Z_{Hh,}\}$. Thus, there are four possible states (one recession, three expansion). The average amount of time spent in each state is given by the stationary distribution (4×1) vector $\boldsymbol{\pi}$, where

$$\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}.$$

That is, $\boldsymbol{\pi}$ is the eigenvector for $\mathbf{P}$ with corresponding eigenvalue equal to 1. The first element of $\boldsymbol{\pi}$, denoted π_1, multiplies the probabilities in $\mathbf{P}$ for transitioning to any of the four states tomorrow conditional on being in a recession state today. π_1 therefore gives the average amount of time spent in the recession state, while π_2, π_3, and π_4 give the average amount of time spent in the other three (expansion) states. Given the matrix $\mathbf{P}$ above, the solution for $\boldsymbol{\pi}$ is

$$\boldsymbol{\pi} = \begin{pmatrix} 0.1479 \\ 0.2367 \\ 0.2367 \\ 0.3787 \end{pmatrix}.$$

This implies the chain spends 14.79% of the time in a recession state and 85.21% of the time in expansion states, so the average length of expansions relative to that of recessions is then $85.21/(14.79) = 5.76$ years.

Idiosyncratic income shocks follow the first order Markov process $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) +$

$\epsilon^i_{a,t}$, where $\epsilon^i_{a,t}$ takes on one of two values in each aggregate state:

$$\begin{aligned}
\epsilon^i_{a,t} &= \begin{cases} \sigma_E & \text{with } \Pr = 0.5 \\ -\sigma_E & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\
\epsilon^i_{a,t} &= \begin{cases} \sigma_R & \text{with } \Pr = 0.5 \\ -\sigma_R & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \\
\sigma_R &> \sigma_E.
\end{aligned}$$

Housing Price and Return Data

Our first measure of house prices uses aggregate housing wealth for the household sector from the Flow of Funds (FoF) (which includes the part of private business wealth which is residential real estate wealth) and housing consumption from the National Income and Products Accounts. The price-rent ratio is the ratio of housing wealth in the fourth quarter of the year divided by housing consumption summed over the year. The return is constructed as housing wealth in the fourth quarter plus housing consumption over the year divided by housing wealth in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities that is attributable solely to population growth. (Since the return is based on a price times quantity, it grows mechanically with the population. In the model, population growth is zero.) The advantage of this housing return series is that it is for residential real estate and for the entire population. The disadvantages are that it is not a per-share return (it has the growth in the housing stock in it, which we only partially control for by subtracting population growth), it is not an investable asset return, and it does not control for quality changes in the housing stock. There is also substantial measurement error in how the Flow of Funds imputes market prices to value the housing stock as well as in how the BEA imputes housing services consumption for owners. These errors, however, may be more likely to affect the level of the price-rent ratio more than the change in the ratio.

Our second series combines the Freddie Mac Conventional Mortgage House Price index for home purchases (Freddie Mac) and the rental price index for shelter from the Bureau of Labor Statistics (BLS). The price-rent ratio is the ratio of the price index in the last quarter of the year, divided by the rent index averaged over the quarters in the year. Since the level of the price-rent ratio is indeterminate (given by the ratio of two indexes), we normalize the level of the series by assuming that the 1970 Freddie Mac price-rent ratio is the same as that of the FoF price-rent ratio in 1970. The return is the price index plus the rent divided by the price index at the end of the previous year. We subtract CPI inflation to express the return in real terms. The FoF return has a correlation of 82% with the Freddie Mac return over 1973-2008. Since the Freddie Mac price index is a repeat-sales price index, it controls for quality changes in the housing stock (price changes are computed on the same house). It also is a per-share return (no quantities). Alternative repeat-sale price indices such as the Freddie Mac CMHPI which includes refinancing and purchases, or the OFHEO house price index, deliver similar results. The same is true if we use the BLS rental index for housing instead of shelter. (The rental index for housing includes utilities while the rental price index for shelter excludes them).

The third series is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics's price index of shelter (CS). The Case-Shiller price index is also a repeat-sales price index, which receives a lot of attention in the literature. It is available from 1987 on a quarterly basis. Since both the FoF and CS price-rent ratios are ratios of two indexes, we normalize the first observations of the Freddie Mac and CS price-rent ratio to be the same as the FoF ratio for that year.

Numerical Solution Procedure

The numerical solution strategy consists of solving the individual's problem taking as given her beliefs about the evolution of the aggregate state variables. With this solution in hand, the economy is simulated for many individuals and the simulation is used to compute the equilibrium evolution of the aggregate state variables, given the assumed beliefs. If the equilibrium evolution differs from the beliefs individuals had about that evolution, a new set of beliefs are assumed and the process is repeated. Individuals' expectations are rational once this process converges and individual beliefs coincide with the resulting equilibrium evolution.

The state of the economy is a pair, (Z_t, μ_t), where μ_t is a measure defined over

$$\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H}),$$

where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ_t is a distribution of agents across ages, idiosyncratic shocks, financial, and housing wealth. Given a finite dimensional vector to approximate μ_t, and a vector of individual state variables

$$\mu_t^i = (Z_t^i, W_t^i, H_t^i),$$

the individual's problem is solved using dynamic programming.

An important step in the numerical strategy is approximating the joint distribution of individuals, μ_t, with a finite dimensional object. The resulting approximation, or "bounded rationality" equilibrium has been used elsewhere to solve overlapping generations models with heterogenous agents and aggregate risk, including Krusell and Smith (1998); Ríos-Rull and Sánchez-Marcos (2006); Storesletten, Telmer, and Yaron (2007); Gomes and Michaelides (2008); Favilukis (2008), among others. For our application, we approximate this space with a vector of aggregate state variables given by

$$\mu_t^{AG} = (Z_t, K_t, S_t, H_t, p_t^H, q_t),$$

where

$$K_t = K_{C,t} + K_{H,t}$$

and

$$S_t = \frac{K_{C,t}}{K_{C,t} + K_{H,t}}.$$

The state variables are the observable aggregate technology shocks, the first moment of the aggregate capital stock, the share of aggregate capital used in production of the consumption good, the aggregate stock of housing, and the relative house price and bond price, respectively. The bond and the house price are natural state variables because the joint distribution of all individuals only matters for the individual's problem in so far as it affects asset prices. Note that knowledge of K_t and S_t is tantamount to knowledge of $K_{C,t}$ and $K_{H,t}$ separately, and vice versa ($K_{C,t} = K_t S_t$; $K_{H,t} = K_t(1 - S_t)$).

Because of the large number of state variables and because the problem requires that prices in two asset markets (housing and bond) must be determined by clearing markets every period, the proposed problem is highly numerically intensive. To make the problem tractable, we obviate the need to solve the dynamic programming problem of firms numerically by instead solving analytically for a recursive solution to value function taking the form $V(K_t) = Q_t K_t$, where Q_t (Tobin's q) is a recursive function. We discuss this below.

In order to solve the individual's dynamic programming problem, the individual must know μ_{t+1}^{AG} and μ_{t+1}^{i} as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Here we show that this can be achieved by specifying individuals' beliefs for the laws of motion of four quantities:

A1 K_{t+1},

A2 p^H_{t+1},

A3 q_{t+1}, and

A4 $[\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})]$, where $Q_{C,t+1} \equiv V_{C,t+1}/K_{C,t+1}$ and analogously for $Q_{H,t+1}$.

Let $\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t} \equiv M_{t+1}$. The beliefs are approximated by a linear function of the aggregate state variables as follows:

$$\varkappa_{t+1} = A^{(n)}\left(Z_t, Z_{t+1}\right) \times \widetilde{\varkappa}_t, \tag{26}$$

where $A^{(n)}\left(Z_t, Z_{t+1}\right)$ is a 4×5 matrix that depends on the aggregate shocks Z_t, and $Z_{t+1}$and where

$$\begin{aligned} \varkappa_{t+1} &\equiv \left[K_{t+1}, p^H_{t+1}, q_{t+1}, [M_{t+1}(Q_{C,t+1} - Q_{H,t+1})]\right]', \\ \widetilde{\varkappa}_t &\equiv \left[K_t, p^H_t, q_t, S_t, H_t\right]'. \end{aligned}$$

We initialize the law of motion (26) with a guess for the matrix $A^{(n)}\left(Z_t, Z_{t+1}\right)$, given by $A^{(0)}\left(Z_t, Z_{t+1}\right)$. The initial guess is updated in an iterative procedure (described below) to insure that individuals' beliefs are consistent with the resulting equilibrium.

Given (26), individuals can form expectations of μ^{AG}_{t+1} and μ^i_{t+1} as a function of μ^{AG}_t and μ^i_t and aggregate shocks Z_{t+1}. To see this, we employ the following equilibrium relation (as shown below) linking the investment-capital ratios of the two production sectors:

$$\frac{I_{H,t}}{K_{H,t}} = \frac{I_{C,t}}{K_{C,t}} + \frac{1}{2\varphi} E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]. \tag{27}$$

Moreover, note that $E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]$ can be computed from (26) by integrating the 4th equation over the possible values of Z_{t+1} given $\widetilde{\varkappa}_t$ and Z_t.

Equation (27) is derived by noting that the consumption firm solves a problem taking the form

$$V\left(K_{C,t}\right) = \max_{I_{C,t}, N_{C,t}} Z_{C,t} K^{\alpha}_{Ct} N^{1-\alpha}_{C,t} - w_t N_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 + E_t\left[M_{t+1} V\left(K_{C,t+1}\right)\right].$$

The first-order condition for optimal labor choice implies $N_{C,t} = \left(\frac{Z_{C,t}(1-\alpha)}{w_t}\right)^{1/\alpha} K_{C,t}$. Substituting this expression into $V\left(K_{C,t}\right)$, the optimization problem may be written

$$\begin{aligned} V\left(K_{C,t}\right) &= \max_{I_t} X_{C,t} K_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 K_{C,t} + E_t\left[M_{t+1} V\left(K_{C,t+1}\right)\right] \quad (28) \\ s.t. \quad K_{C,t+1} &= \left(1-\delta\right) K_{C,t} + I_{C,t} \end{aligned}$$

where

$$X_{C,t} \equiv \alpha \left(\frac{Z_{C,t}}{w_t} (1-\alpha) \right)^{(1-\alpha)/\alpha} Z_{C,t}$$

is a function of aggregate variables over which the firm has no control.

The housing firms solves

$$\begin{aligned} V(K_{H,t}) &= \max_{I_{H,t},N_{H,t}} p_t^H Z_{H,t} (\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu} \right)^{\phi} - w_t N_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t \\ & \quad -\varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 + E_t \left[M_{t+1} V(K_{H,t+1}) \right]. \end{aligned} \tag{29}$$

The first-order conditions for optimal labor and land/permits choice for the housing firm imply that $N_{H,t} = k_N K_{H,t}$, $\mathcal{L}_t = k_L K_{H,t}$, where

$$\begin{aligned} k_N &= \left(k_1^{\phi} k_2^{1-\phi} \right)^{1/\nu\phi} \\ k_L &= \left(k_1^{\phi(1-\nu)} k_2^{1-\phi(1-\nu)} \right)^{1/\phi\nu} \\ k_1 &= p_t^H Z_{H,t} \phi (1-\nu) / w_t \\ k_2 &= p_t^H Z_{H,t} (1-\phi) / p_t^L. \end{aligned}$$

Substituting this expression into $V(K_{H,t})$, the optimization problem may be written

$$\begin{aligned} V(K_{H,t}) &= \max_{I_t} X_{Ht} K_{H,t} - I_{H,t} - \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 K_{H,t} + E_t \left[M_{t+1} V(K_{H,t+1}) \right] \\ s.t. \quad K_{H,t+1} &= (1-\delta) K_{H,t} + I_{H,t} \end{aligned} \tag{30}$$

where

$$X_{H,t} = p_t^H Z_{H,t} \phi \nu k_N^{(1-\nu)\phi} k_L^{1-\phi}.$$

Let s index the sector as either consumption, C, or housing, H. We now guess and verify that for each firm, $V(K_{s,t+1})$, for $s = C, H$ takes the form

$$V(K_{s,t+1}) = Q_{s,t+1} K_{s,t+1}, \qquad s = C, H \tag{31}$$

where $Q_{s,t+1}$ depends on aggregate state variables but is not a function of the firm's capital stock $K_{s,t+1}$ or investment $I_{s,t}$. Plugging (31) into (28) we obtain

$$V(K_{s,t}) = \max_{I_t} X_{s,t} K_{s,t} - I_t - \varphi \left(\frac{I_{s,t}}{K_{s,t}} - \delta \right)^2 K_{s,t} + E_t \left[M_{t+1} Q_{s,t+1} \right] \left[(1-\delta) K_{s,t} + I_{s,t} \right]. \tag{32}$$

The first-order conditions for the maximization (32) imply

$$\frac{I_{s,t}}{K_{s,t}} = \delta + \frac{E_t \left[M_{t+1} Q_{s,t+1} \right] - 1}{2\varphi}. \tag{33}$$

Substituting (33) into (32) we verify that $V(K_{s,t})$ takes the form $Q_{s,t}K_{s,t}$:

$$\begin{aligned} V(K_{s,t}) \equiv\; & Q_{s,t}K_{s,t} = X_{s,t}K_{s,t} - \left(\delta + \frac{E_t[M_{t+1}Q_{s,t+1}] - 1}{2\varphi}\right)K_{s,t} - \varphi\left(\frac{E_t[M_{t+1}Q_{s,t+1}] - 1}{2\varphi}\right)^2 K_{s,t} \\ & + (1-\delta)\left(E_t[M_{t+1}Q_{s,t+1}]\right)K_{s,t} + E_t[M_{t+1}Q_{s,t+1}]\left(\delta + \frac{E_t[M_{t+1}Q_{s,t+1}] - 1}{2\varphi}\right)K_{s,t}. \end{aligned}$$

Rearranging terms, it can be shown that $Q_{s,t}$ is a recursion:

$$Q_{s,t} = X_{s,t} + (1-\delta) + 2\varphi\left(\frac{E_t[M_{t+1}Q_{s,t+1}] - 1}{2\varphi}\right) + \varphi\left(\frac{E_t[M_{t+1}Q_{s,t+1}] - 1}{2\varphi}\right)^2. \qquad (34)$$

Since $Q_{s,t}$ is a function only of $X_{s,t}$ and the expected discounted value of $Q_{s,t+1}$, it does not depend on the firm's own $K_{s,t+1}$ or $I_{s,t}$. Hence we verify that $V(K_{s,t}) = Q_{s,t}K_{s,t}$. Although $Q_{s,t}$ does not depend on the firm's individual $K_{s,t+1}$ or $I_{s,t}$, in equilibrium it will be related to the firm's investment-capital ratio via:

$$Q_{s,t} = X_{s,t} + (1-\delta)\left[1 + 2\varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)\right] + \varphi\left(\frac{I_{s,t}}{K_{s,t}}\right)^2 - 2\varphi\delta\left(\frac{I_{s,t}}{K_{s,t}}\right), \qquad (35)$$

as can be verified by plugging (33) into (34). Note that (33) holds for the two representative firms of each sector, i.e., $Q_{C,t}$ and $Q_{H,t}$, thus we obtain (27) above.

With (35), it is straightforward to show how individuals can form expectations of μ_{t+1}^{AG} and μ_{t+1}^{i} as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Given a grid of values for K_t and S_t individuals can solve for $K_{C,t}$ and $K_{H,t}$ from $K_{C,t} = K_tS_t$ and $K_{H,t} = K_t(1-S_t)$. Combining this with beliefs about K_{t+1} from (26), individuals can solve for $I_t \equiv I_{C,t} + I_{H,t}$ from $K_{t+1} = (1-\delta)K_t + I_t$. Given I_t and beliefs about $\left[\frac{\beta^k\Lambda_{t+k}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})\right]$ from (26), individuals can solve for $I_{C,t}$ and $I_{H,t}$ from (27). Given $I_{H,t}$ and the accumulation equation $K_{H,t+1} = (1-\delta)K_{H,t} + I_{H,t}$, individuals can solve for $K_{H,t+1}$. Given $I_{C,t}$ individuals can solve for $K_{C,t+1}$ using the accumulation equation $K_{C,t+1} = (1-\delta)K_{C,t} + I_{C,t}$. Using $K_{H,t+1}$ and $K_{C,t+1}$, individuals can solve for S_{t+1}. Given a grid of values for H_t, H_{t+1} can be computed from $H_{t+1} = (1-\delta_H)H_t + Y_{H,t}$, where $Y_{H,t} = Z_{H,t}(\mathcal{L}_t)^{1-\phi}\left(K_{H,t}^{\nu}N_{H,t}^{1-\nu}\right)^{\phi}$ is obtained from knowledge of $Z_{H,t}$, $K_{H,t}$ (observable today), from the equilibrium condition $\mathcal{L}_t = L$, and by combining (18) and (20) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Equation (26) can be used directly to obtain beliefs about q_{t+1} and p_{t+1}^H.

To solve the dynamic programming problem individuals also need to know the equity values $V_{C,t}$ and $V_{H,t}$. But these come from knowledge of $Q_{s,t}$ (using (35)) and $K_{s,t}$ via $V_{s,t} =$

$Q_{s,t}K_{s,t}$ for $s = C, H$. Values for dividends in each sector are computed from

$$\begin{aligned} D_{C,t} &= Y_{C,t} - I_{C,t} - w_t N_{C,t} - \phi_C \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t}, \\ D_{H,t} &= p_t^H Y_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t - w_t N_{H,t} - \phi_H \left(\frac{I_{H,t}}{K_{H,t}} \right) K_{H,t} \end{aligned}$$

and from

$$w_t = (1-\alpha) Z_{j,t} K_{j,t}^{\alpha} N_{j,t}^{-\alpha} = (1-\nu)(1-\phi) p_t^H Z_{H,t} \mathcal{L}_t^{\phi} K_{H,t}^{\nu(1-\phi)} N_{H,t}^{-\phi(1-\nu)-\nu}$$

and by again combining (18) and (20) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Finally, the evolution of the aggregate technology shocks Z_{t+1} is given by the first-order Markov chain described above; hence agents can compute the possible values of Z_{t+1} as a function of Z_t.

Values for $\mu_{t+1}^i = (Z_{t+1}^i, W_{t+1}^i, H_{t+1}^i)$ are given from all of the above in combination with the first order Markov process for idiosyncratic income $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$. Note that H_{t+1}^i is a choice variable, while $W_{t+1}^i = \theta_t^i(V_{C,t+1} + V_{H,t+1} + D_{C,t+1} + D_{H,t+1}) + B_{t+1}^i$ requires knowing $V_{s,t+1} = Q_{s,t+1}K_{s,t+1}$ and $D_{s,t+1}$, $s = C, H$ conditional on Z_{t+1}.These in turn depend on $I_{s,t+1}$, $s = C, H$ and may be computed in the manner described above by rolling forward one period both the equation for beliefs (26) and accumulation equations for $K_{C,t+1}$, and $K_{H,t+1}$.

The individual's problem, as approximated above, may be summarized as follows (where we drop age subscripts when no confusion arises):

$$V_{a,t}\left(\mu_t^{AG}, \mu_t^i\right) = \max_{H_{t+1}^i, \theta_{t+1}^i, B_{t+1}^i} U(C_t^i, H_t^i) + \beta \pi_i E_t[V_{a+1,t+1}\left(\mu_{t+1}^{AG}, \mu_{t+1}^i\right)] \quad s.t. \tag{36}$$

The above problem is solved subject to (6), (7), (8), and (9) if the individual of working age, and subject to the analogous versions of (6), (7), (8), and (9) (using pension income in place of wage income), if the individual is retired. The problem is also solved subject to an evolution equation for the state space:

$$\mu_{t+1}^{AG} = \Gamma^{(n)}(\mu_t^{AG}, Z_{t+1}).$$

$\Gamma^{(n)}$ is the system of forecasting equations that is obtained by stacking all the beliefs from (26) and accumulation equations into a single system. This dynamic programming problem is quite complex numerically because of a large number of state variables but is otherwise straightforward. Its implementation is described below.

Next we simulate the economy for a large number of individuals using the policy functions from the dynamic programming problem. The continuum of individuals born each period is approximated by a number large enough to insure that the mean and volatility of aggregate variables is not affected by idiosyncratic shocks. We check this by simulating the model for successively larger numbers of individuals in each age cohort and checking whether the mean and volatility of aggregate variables changes. We have solved the model for several different numbers of agents. For numbers ranging from a total of 2,400 to 40,000 agents in the population we found no significant differences in the aggregate allocations.

An additional numerical complication is that two markets (the housing and bond market) must clear each period. This makes p_t^H and q_t convenient state variables: the individual's policy functions are a response to a menu of prices p_t^H and q_t, Given values for $Y_{H,t}$, $H^i_{a+1,t+1}$, $H^i_{a,t}$, $B^i_{a,t}$ and B^F_t form the simulation, and given the menu of prices p_t^H and q_t and the beliefs (26), we then choose values for p^H_{t+1} and q_{t+1} that clear markets in $t+1$. The initial allocations of wealth and housing are set arbitrarily to insure that prices in the initial period of the simulation, p_1^H and q_1, clear markets. However, these values are not used since each simulation includes an initial burn-in period of 150 years that we discard for the final results.

Using data from the simulation, we calculate (A1)-(A4) as linear functions of $\widetilde{\varkappa}_t$ and an initial guess $A^{(0)}$. In particular, for every Z_t and Z_{t+1} combination we regress (A1)-(A4) on K_t, S_t, H_t, p_t^H, and q_t. This is used to calculate a new $A^{(n)} = A^{(1)}$ which is used to re-solve for the entire equilibrium. We continue repeating this procedure, updating the sequence $\left\{A^{(n)}\right\}$, $n = 0, 1, 2, ...$ until (1) the coefficients in $A^{(n)}$ between successive iterations is arbitrarily small, (2) the regressions have high R^2 statistics, and (3) the equilibrium is invariant to the inclusion of additional state variables such as additional lags and/or higher order moments of the cross-sectional wealth and housing distribution.

The R^2 statistics for the four equations (A1)-(A4) are (.999, .999, .989, .998), respectively. The lowest R^2 is for the bond price equation. We found that successively increasing the number of agents (beyond 2400) successively increases the R^2 in the bond price equation, without affecting the equilibrium allocations or prices. However, we could not readily increase the number of agents beyond 40,000 because attempts to do so exceeded the available memory on a workstation computer. Our interpretation of this finding is that the equilibrium is unlikely to be affected by an approximation using more agents, even though doing so could result in an improvement in the R^2 of the bond equation. For this reason, and because of the already high computational burden required to solve the model, we stopped at the slightly lower level of accuracy for the bond forecasting regression as compared to the other forecasting regressions.

Numerical Solution to Individual's Dynamic Programming Problem

We now describe how the individual's dynamic programming problem is solved.

First we choose grids for the continuous variables in the state space. That is we pick a set of values for W^i, H^i, K, H, S, p^H, and q. Because of the large number of state variables, it is necessary to limit the number of grid points for some of the state variables given memory/storage limitations. We found that having a larger number of grid points for the individual state variables was far more important than for the aggregate state variables, in terms of the effect it had on the resulting allocations. Thus we use a small number of grid points for the aggregate state variables but compensate by judiciously choosing the grid point locations after an extensive trial and error experimentation designed to use only those points that lie in the immediate region where the state variables ultimately reside in the computed equilibria. As such, a larger number of grid points for the aggregate state variables was found to produce very similar results to those reported using only a small number of points. We pick 25 points for W^i, 12 points for H^i, three points for K, H, S, p^H, and four points for q. The grid for W^i starts at the borrowing constraint and ends far above the maximum wealth reached in simulation. This grid is very dense around typical values of financial wealth and is sparser for high values. The housing grid is constructed in the same way.

Given the grids for the state variables, we solve the individual's problem by value function iteration, starting for the oldest (age A) individual and solving backwards. The oldest individual's value function for the period after death is zero for all levels of wealth and housing (alternately it could correspond to an exogenously specified bequest motive). Hence the value function in the final period of life is given by $V_A = \max_{H^i_{t+1}, \theta^i_{t+1}, B^i_{t+1}} U(C^i_A, H^i_A)$ subject to the constraints above for (36). Given V_A (calculated for every point on the state space), we then use this function to solve the problem for a younger individual (aged $A-1$). We continue iterating backwards until we have solved the youngest individual's (age 1) problem. We use piecewise cubic splines (Fortran methods PCHIM and CHFEV) to interpolate points on the value function. Any points that violate a constraint are assigned a large negative value.

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Figure 1: Price-Rent Ratios in the Data

The figure compares three measures of the price-rent ratio. The first measure ("Flow of Funds") is the ratio of residential real estate wealth of the household sector from the Flow of Funds to aggregate housing services consumption from NIPA. The second measure ("Freddie") is the ratio of the Freddie Mac Conventional Mortgage Home Price Index for purchases to the Bureau of Labor Statistics's price index of shelter (which measures rent of renters and imputed rent of owners). The third series ("Case-Shiller") is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics's price index of shelter. All indices are normalized to a value of 100 in 2000.Q4. The data are quarterly from 1970.Q1 until 2008.Q4. The REITs series starts in 1972.Q4 and the Case-Shiller series in 1987.Q1.

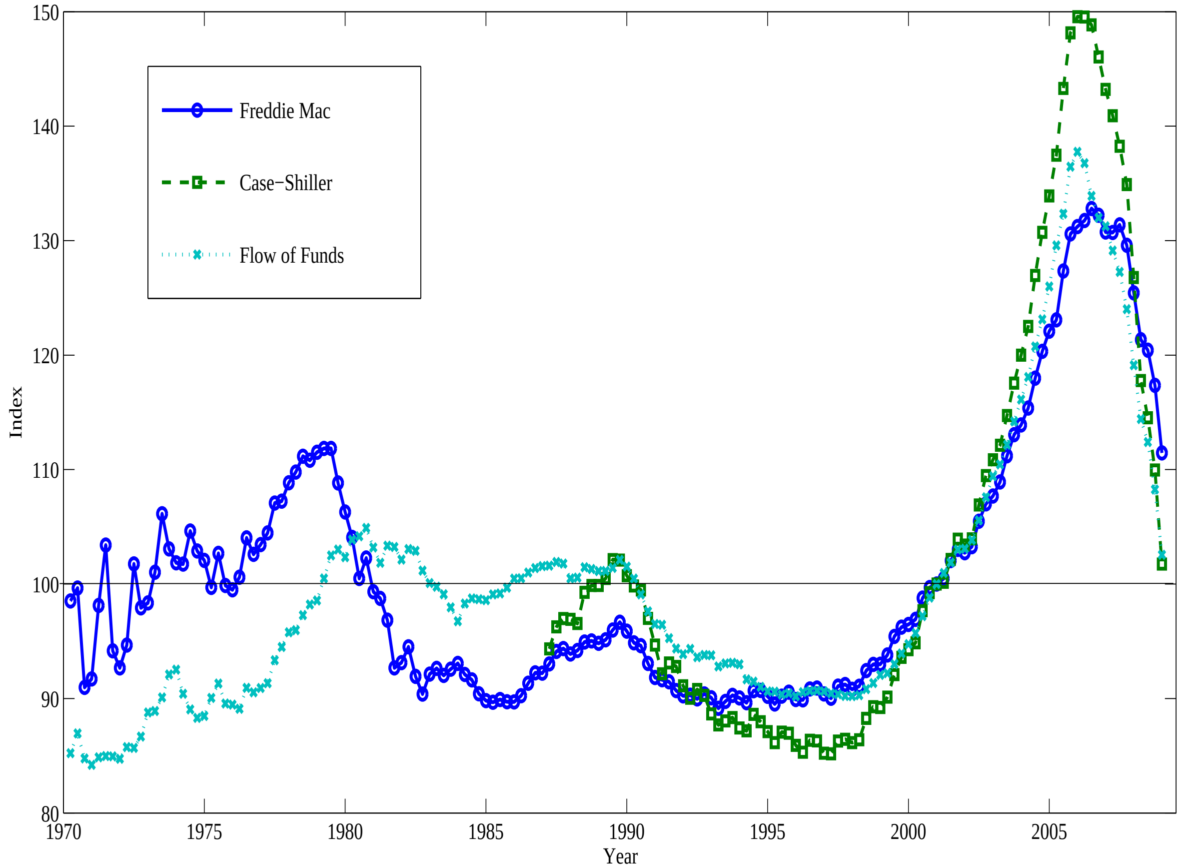


Figure 2: Mortgage Closing Costs

The solid line shows the closing costs (initial fees an charges when closing on a mortgage) from the Federal Housing Financing Board's Monthly Interest Rate Survey. The costs are expressed as a percentage of the value of the loan balance, and averaged across mortgage contracts. The data are monthly from January 1990 until December 2009.

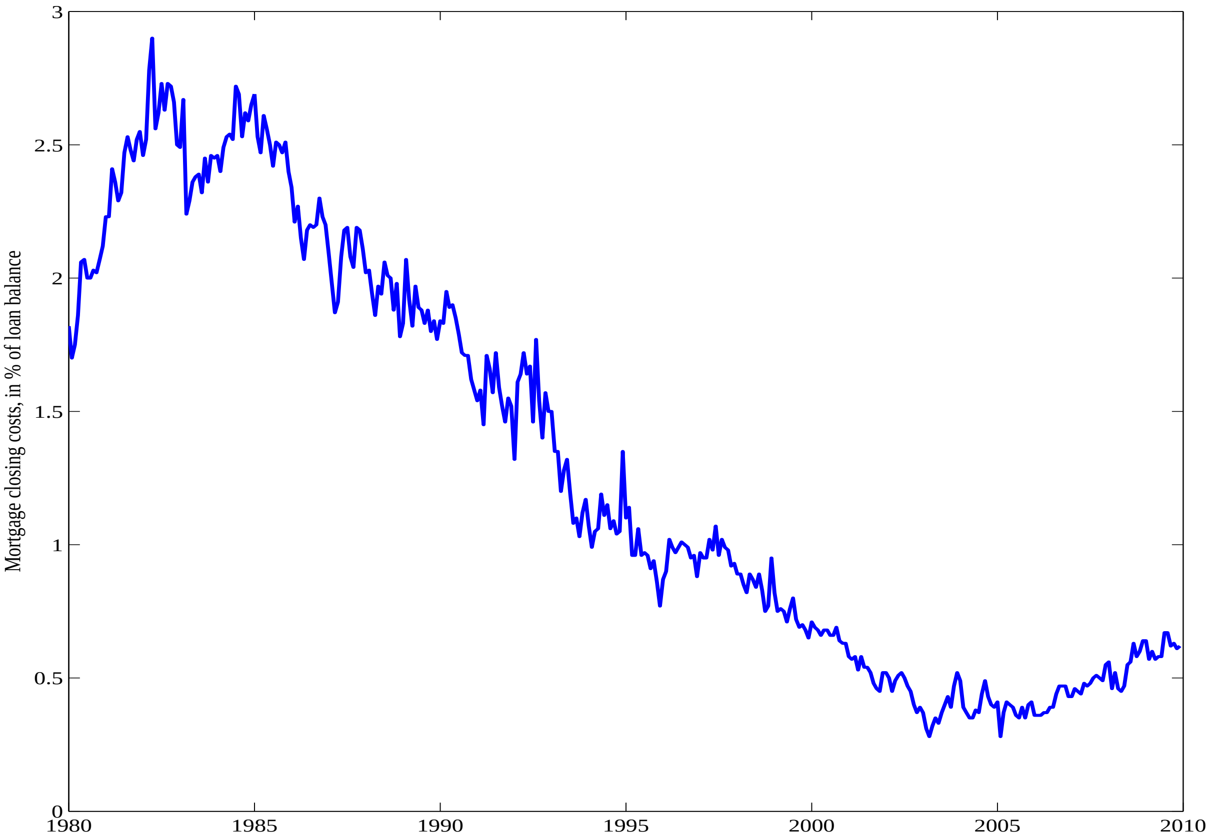


Figure 3: Fixed-rate Mortgage Rate and Ten-Year Constant Maturity Treasury Rate

The solid line plots the 30-year Fixed-Rate Mortgage rate (FRM); the dashed line plots the ten-year Constant Maturity Treasury Yield (CMT). The FRM data are from Freddie Mac's Primary Mortgage Market Survey. They are average contract rates on conventional conforming 30-year fixed-rate mortgages. The CMT yield data are from the St.-Louis Federal reserve Bank (FRED). The data are monthly from April 1971.4 until February 2009.

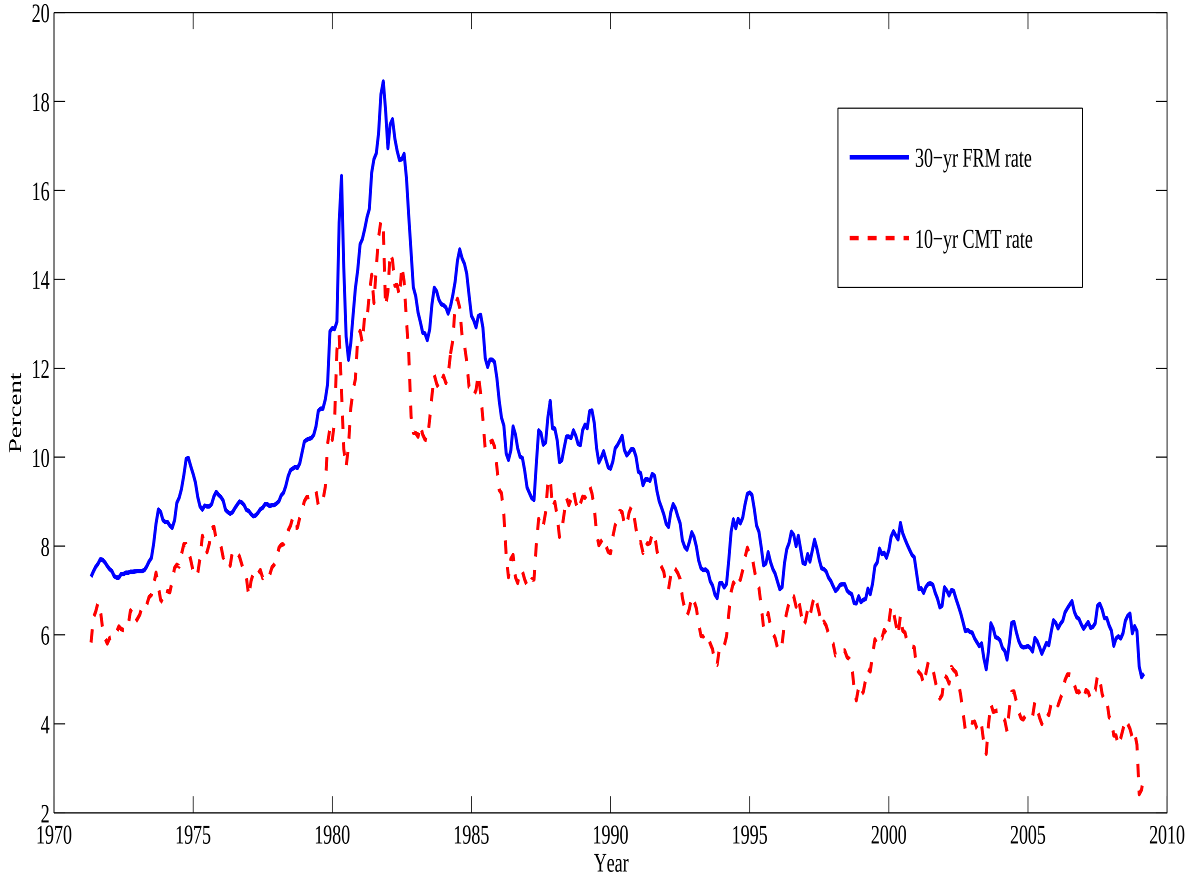
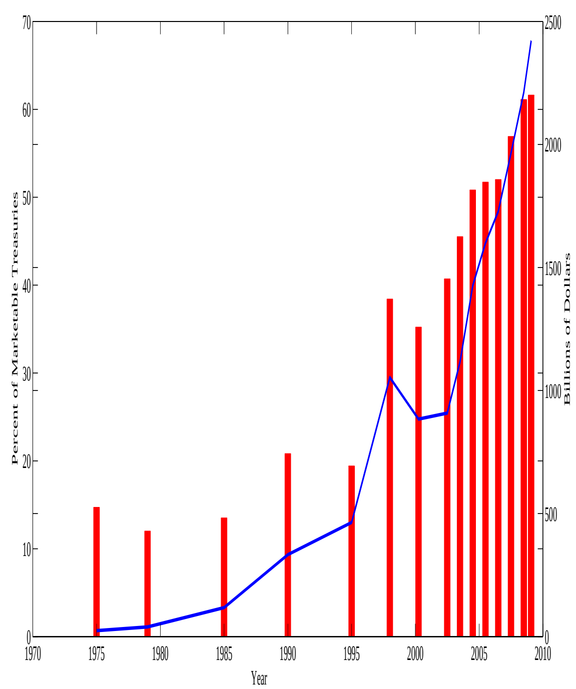


Figure 4: Foreign Holdings of US Treasuries

Panel A plots foreign holdings of U.S. Treasuries. The solid line, measured against the right axis, plots foreign holdings of long-term U.S. Treasury securities (T-notes, and T-bonds). It excludes (short-term) T-bills. The bars, measured against the left axis, plot those same holdings as a percent of total marketable U.S. Treasuries. Marketable U.S. Treasuries are available from the Office of Public Debt, and are measured as total marketable held by the public less T-bills. Panel B plots foreign holdings of U.S. Treasury securities (T-bills, T-notes, and T-bonds) and the sum of U.S. treasuries and U.S. Agency debt (e.g., debt issued by Freddie Mac and Fannie Mae), relative to GDP. The first two series report only long-term debt holdings, while the other two series add in short-term debt holdings. Since no short-term debt holdings are available before 2002, we assume that total holdings grow at the same rate as long-term holdings before 2002. The foreign holdings data from the Treasury International Capital System of the U.S. Department of the Treasury. The foreign holdings data are available for December 1974, 1978, 1984, 1989, 1994, 1997, March 2000, and annually for June 2002 through June 2008. Panel A also includes an observation for January 2009. Nominal GDP is from the National Income and Product Accounts, Table 1.1.5, line 1.

Panel A: Foreign Holdings of U.S. Treasuries



Panel B: Foreign Holdings Relative to GDP

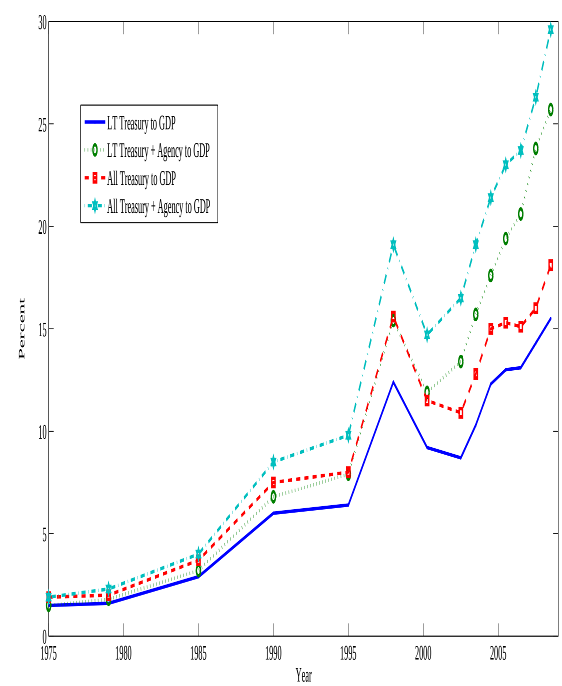


Figure 5: Wealth by Age and Income in Model and Data

The figure plots net financial wealth ("Wealth") by age in the left columns and housing wealth ("Housing") by age in the right columns. The top panels are for the Data, the middle panels for Model 1 ($\phi = 1$), and the bottom panels for Model 2 ($\phi = 1$). We use all 9 waves of the Survey of Consumer Finance (1983-2007, every 3 years). We construct housing wealth as the sum of primary housing and other property. We construct net financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. For each age between 22 and 81, we construct average net financial wealth and housing wealth using the SCF weights. To make information in the different waves comparable to each other and to the model, we divide housing wealth and net financial wealth in a given wave by average net worth (the sum of housing wealth and net financial wealth) across all respondents for that wave. We do the same in the model. The Low Earner label refers to those in the bottom 25% of the income distribution, where income is wage plus private business income. The Medium Earner group refers to the 25-75 percentile of the income distribution, and the High Earner is the top 25%. The model computations are obtained from a 1,000 year simulation. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%.

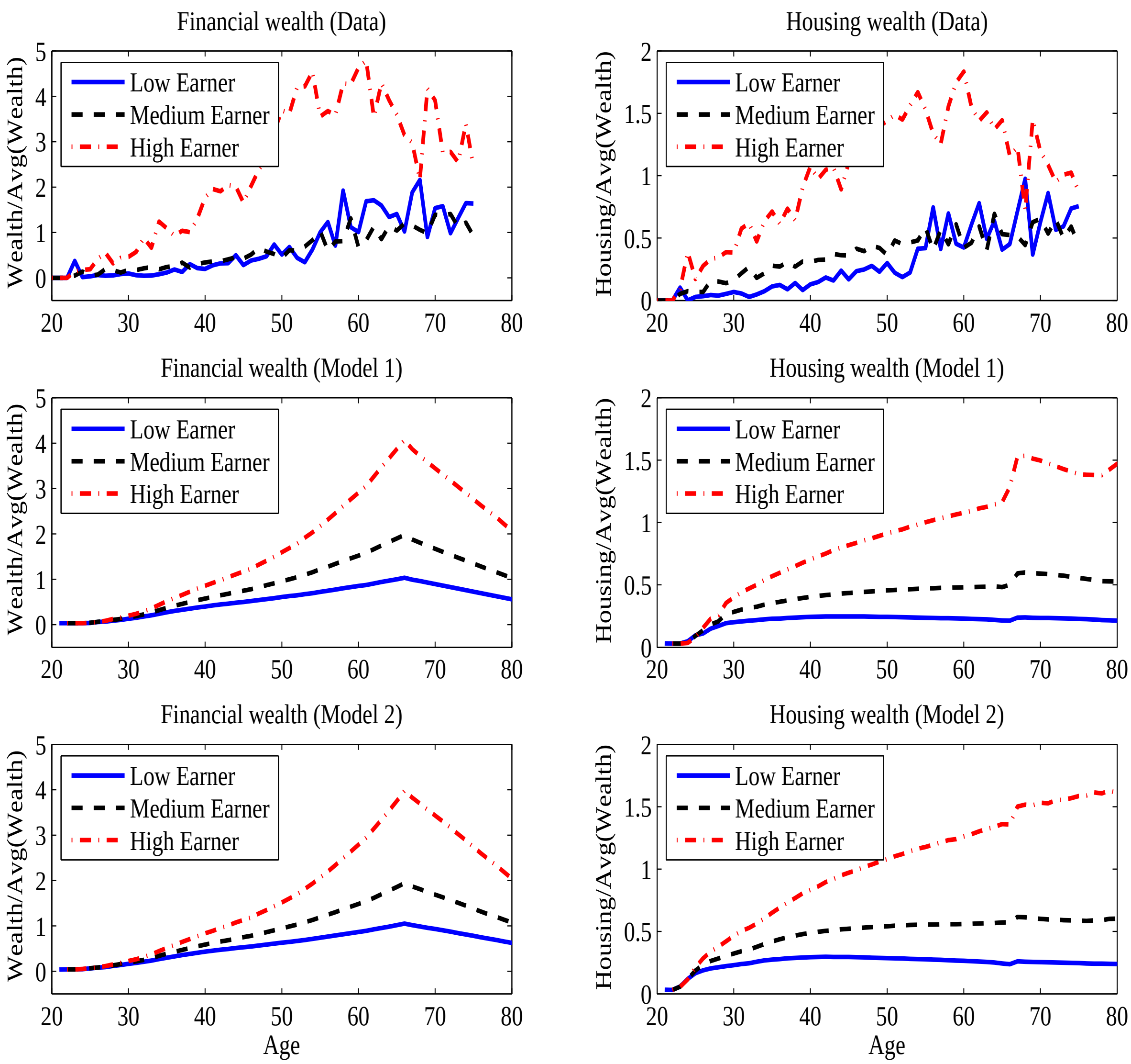


Figure 6: Transition Dynamics in Model: Price-Rent Ratio and Price

The figure plots the house price p^H, plotted against the left axis, and the price-rent ratio $p^H/\mathcal{R}$, plotted against the right axis for a transition generated from the model. The path begins in the year 2000 in the stochastic steady state of Model 1, the model with tight borrowing constraints and high transaction costs. In 2001, the world undergoes an unanticipated change to Model 3, the model with looser borrowing constraints, lower transaction costs, and foreign holdings of U.S. bonds equal to 18% of GDP. The figure traces the first 6 years of the transition from the stochastic steady state of Model 1 to the stochastic steady state of Model 3. Along the transition path, agents use the policy functions from Model 3 evaluated at state variables that begin at the stochastic steady state values of Model 1, and gradually adjust to their stochastic steady state values of Model 3. Along the transition path, foreign holdings of U.S. bonds increase linearly from 0% in 2000 to 18% of GDP by 2006, and remain constant thereafter. In 2007, the world unexpectedly changes to Model 4. Model 4 is the same as Model 1 but with foreign holdings of U.S. bonds equal to 18% of GDP, as in Model 3 ("Reversal of FML in 2007"). The transition path is drawn for a particular sequence of aggregate productivity shocks in the housing and non-housing sectors, as explained in the text. Panel A is for the model without land/permits ($\phi = 1$), while Panel B is for the model with land/permits ($\phi = .9$).

Panel A: $\phi = 1$ Panel B: $\phi = .9$

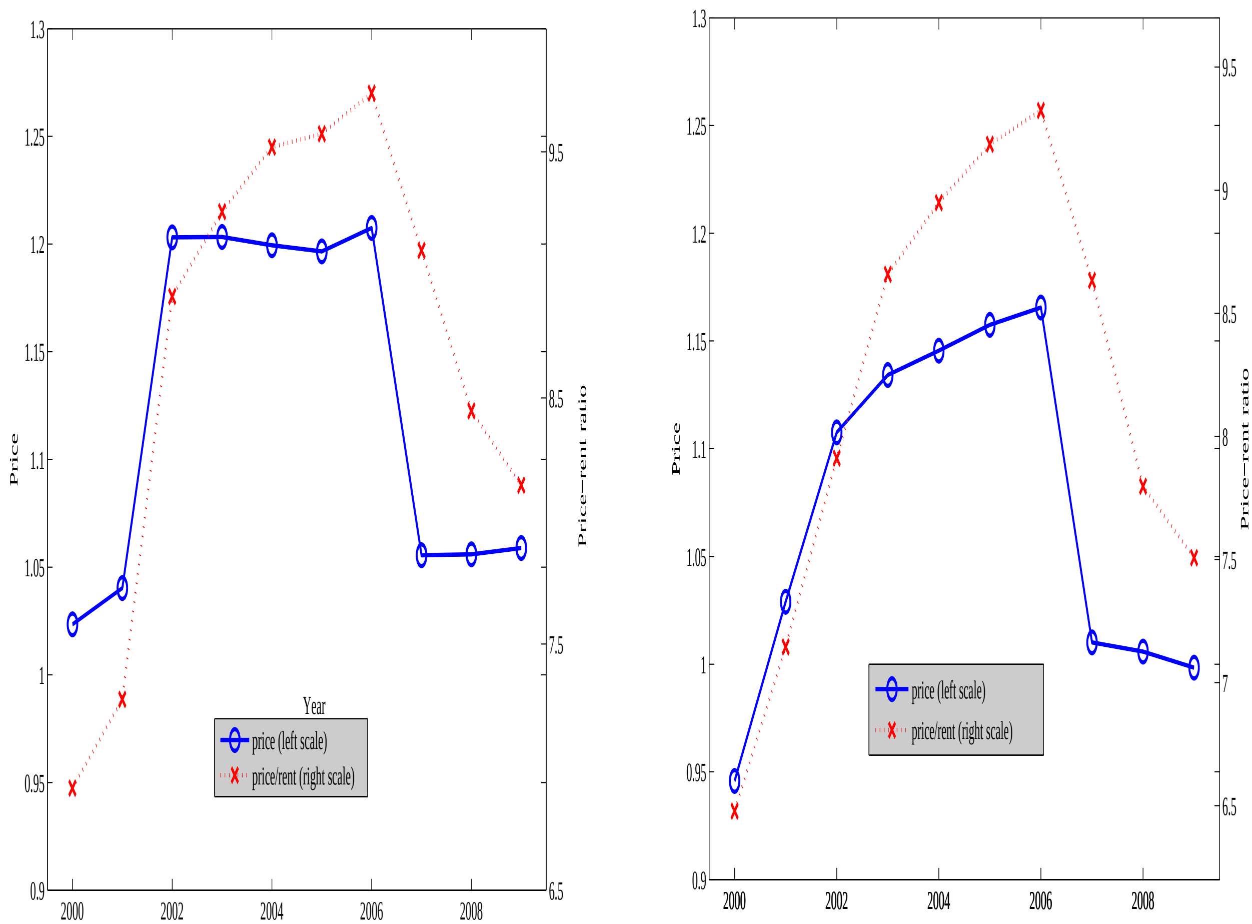


Figure 7: Transition Dynamics in Model: Land/Permits Price ($\phi = .9$)

The figure plots the price of land for a transition generated from the model. The path begins in the year 2000 in the stochastic steady state of Model 1, the model with tight borrowing constraints and high transaction costs. In 2001, the world undergoes an unanticipated change to Model 3, the model with looser borrowing constraints, lower transaction costs, and foreign holdings of U.S. bonds equal to 18% of GDP. The figure traces the first 6 years of the transition from the stochastic steady state of Model 1 to the stochastic steady state of Model 3. Along the transition path, agents use the policy functions from Model 3 evaluated at state variables that begin at the stochastic steady state values of Model 1, and gradually adjust to their stochastic steady state values of Model 3. Along the transition path, foreign holdings of U.S. bonds increase linearly from 0% in 2000 to 18% of GDP by 2006, and remain constant thereafter. In 2007, the world unexpectedly changes to Model 4. Model 4 is the same as Model 1 but with foreign holdings of U.S. bonds equal to 18% of GDP, as in Model 3 ("Reversal of FML in 2007"). The transition path is drawn for a particular sequence of aggregate productivity shocks in the housing and non-housing sectors, as explained in the text.

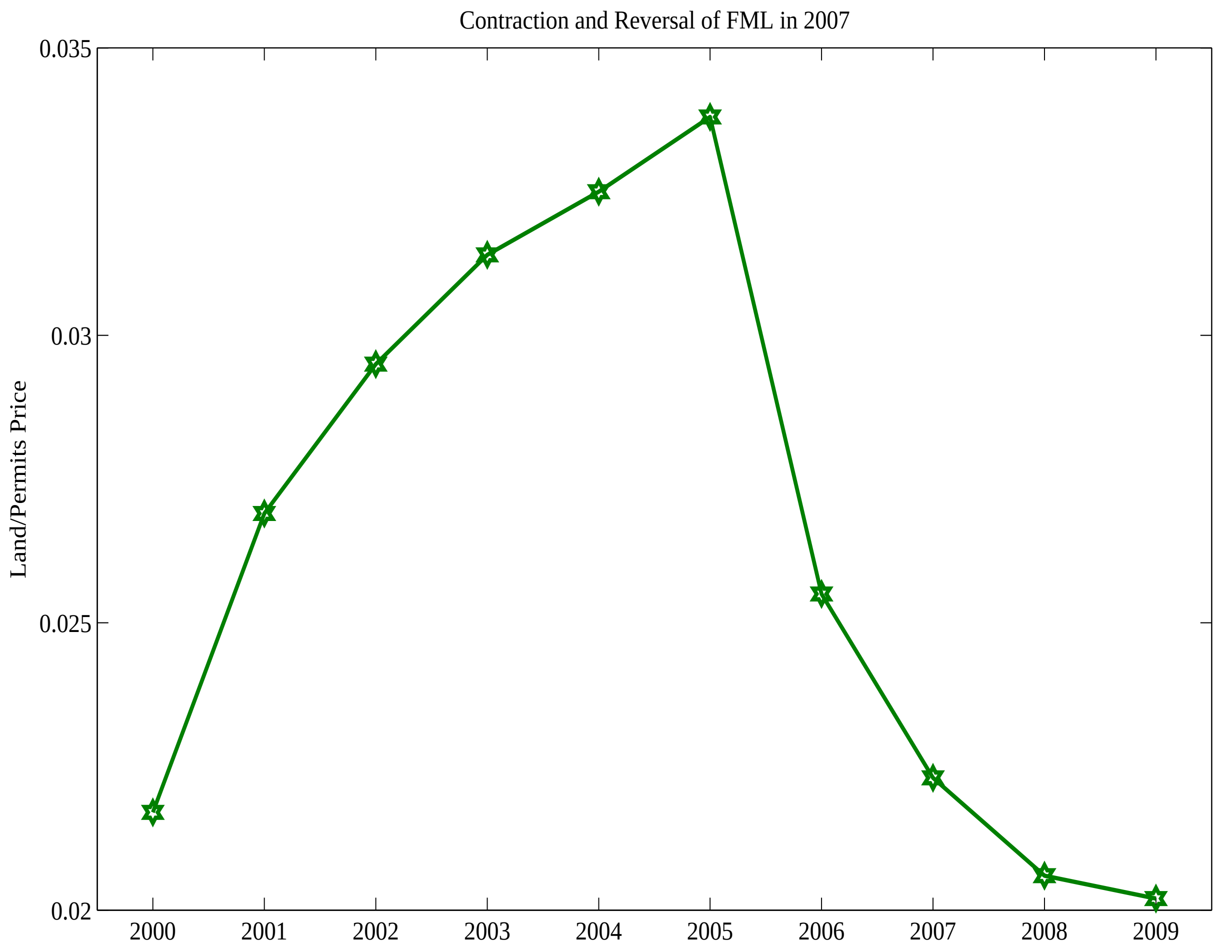


Figure 8: Wealth Inequality in Model and Data

The figure plots the Gini coefficient of total wealth (left panel), financial wealth (middle panel), and housing wealth (right panel). In each panel, the Gini in the data is measured against the left axis, while the Gini in the model is measured against the right axis. The data are shown for the years 2001, 2004, and 2007, indicated by the solid line with dots. For the model, we report the steady state Gini values in Models 1, 2 (star), and 3 (square). The model is the model without land ($\phi = 1$). The right axes are chosen so that the Model 1 Gini coincides with the value in Model 1. The data are from three waves of the Survey of Consumer Finance. We construct housing wealth as the sum of primary housing and other property. We construct financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. We use the SCF weights to calculate the Gini coefficients. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP.

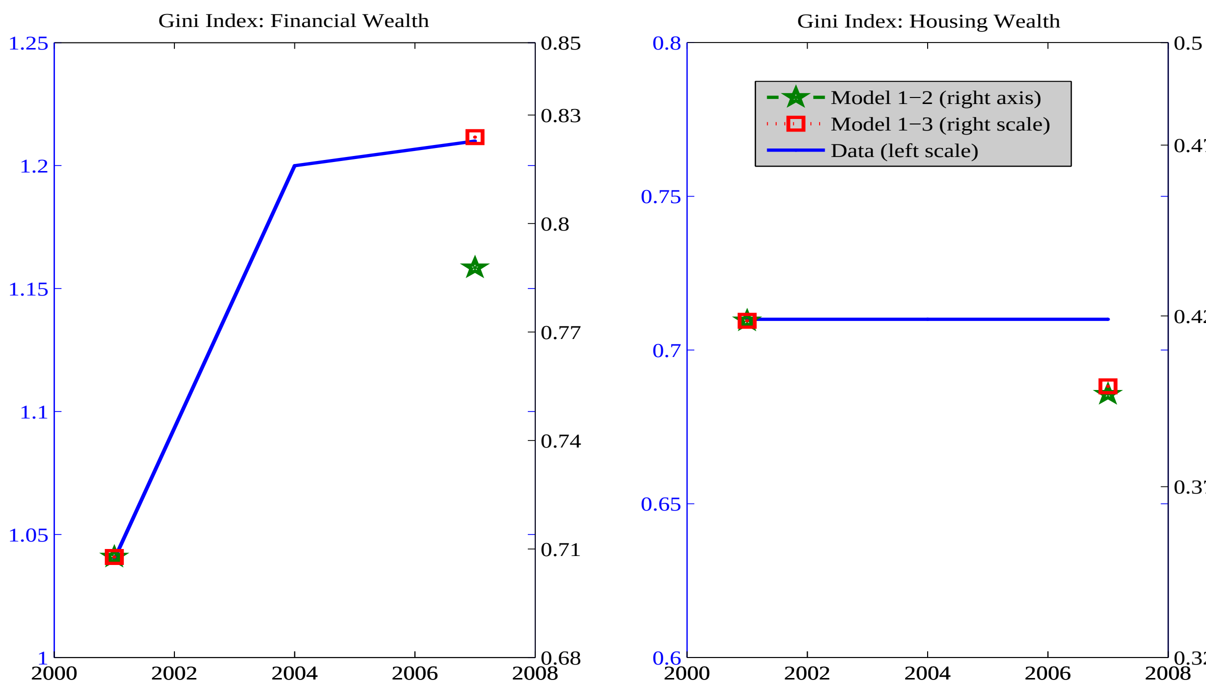


Table 1: Calibration

This table reports the parameter values of our model. The baseline "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP. Our benchmark model is the model without land in the production function for housing ($\phi = 1$), but we also consider a model with a land share of 10% ($\phi = 0.9$).

	Parameter	Description	Baseline, Model 1	Model 2	Model 3
			Production		
1	$\{\phi_C(\cdot), \phi_H(\cdot)\}$	adj. cost	$\left\{\varphi\left(\frac{I}{K}-\delta\right)^2, \varphi\left(\frac{I}{K}-\delta\right)^2\right\}$		
2	δ	deprec., K_C, K_H	12% p.a.		
3	δ_H	depreciation, H	2.5% p.a.		
4	α	capital share, Y_C	0.36		
5	ν	capital share, Y_H	0.30		
6	ϕ	non-land share, Y_H	{1,0.9}		
			Preferences		
7	σ	risk aversion	8		
8	β	time disc factor	0.923		
9	ε	elast of sub, C, H	1		
10	χ	weight on C	0.70		
			Demographics and Income		
11	G_a	age earnings profile	SCF		
12	$\pi_{a+1\|a}$	survival prob	mortality tables		
13	σ_E	st. dev ind earnings, E	0.0768		
14	σ_R	st. dev ind earnings, R	0.1298		
			Transactions Costs		
15	$\overline{F}$	participation cost, K	$\approx 1\% \ \overline{C}^i$		
16	ψ_0	fixed trans cost, H	$\approx 3.2\% \ \overline{C}^i$	$\approx 2.2\% \ \overline{C}^i$	$\approx 2.2\% \ \overline{C}^i$
17	ψ_1	variable trans cost, H	$\approx 5.5\% \ p_t^H H^i$	$\approx 3.5\% \ p_t^H H^i$	$\approx 3.5\% \ p_t^H H^i$
18	ϖ	collateral constr	25%	1%	1%
			Foreign Supply		
19	B^F	foreign capital	0	0	18% $\overline{Y}$

Table 2: Real Business Cycle Moments

Panel A denotes business cycle statistics in annual post-war U.S. data (1953-2008). The data combine information from NIPA Tables 1.1.5, 3.9.5, and 2.3.5. Output ($Y = Y_C + p^H Y_H + C_H$) is gross domestic product minus net exports minus government expenditures. Total consumption (C_T) is total private sector consumption (housing and non-housing). Housing consumption ($C_H = \mathcal{R} * H$) is consumption of housing services. Non-housing consumption (C) is total private sector consumption minus housing services. Housing investment ($p^H Y_H$) is residential investment. Non-housing investment (I) is the sum of private sector non-residential structures, equipment and software, and changes in inventory. Total investment is denoted I_T (residential and non-housing). For each series in the data, we first deflate by the disposable personal income deflator, We then construct the trend with a Hodrick-Prescott (1980) filter with parameter $\lambda = 100$. Finally, we construct detrended data as the log difference between the raw data and the HP trend, multiplied by 100. The standard deviation (first column), correlation with GDP (second column), and the first-order autocorrelation are all based on these detrended series. The autocorrelation AC is a one-year correlation in data and model. The share of GDP (fourth column) is based on the raw data. Panel B denotes the same statistics for the Model 1 with normal transaction costs and collateral constraints. Panel C reports on Model 2 with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. The model is the benchmark model without land ($\phi = 1$).

	Panel A: Data (1953-2008)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.78	1.00	0.46	1.00
C_T	1.78	0.91	0.62	0.80
C	1.89	0.91	0.60	0.68
C_H	1.64	0.62	0.74	0.12
I_T	8.01	0.93	0.36	0.20
I	8.66	0.80	0.37	0.14
$p^H Y_H$	12.77	0.71	0.49	0.06

	Panel B: Model 1			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.77	1.00	0.14	1.00
C_T	2.14	0.97	0.17	0.72
C	1.88	0.95	0.11	0.45
C_H	2.95	0.87	0.31	0.27
I_T	4.73	0.96	0.12	0.28
I	4.37	0.89	0.09	0.23
$p^H Y_H$	14.87	0.51	0.13	0.05

	Panel C: Model 2			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.71	1.00	0.12	1.00
C_T	1.85	0.99	0.14	0.73
C	1.79	0.94	0.12	0.49
C_H	2.30	0.92	0.12	0.25
I_T	5.21	0.99	0.09	0.27
I	5.19	0.81	0.08	0.21
$p^H Y_H$	13.83	0.61	0.15	0.06

Table 3: Correlations House Prices and Real Activity

The table reports the correlations between house prices p^H and house price-rent ratios $p^H/\mathcal{R}$ with GDP and the correlation of house prices with residential investment $p^H Y_H$. Panel A is for the data. The house price and price-rent ratio are measured three different ways. In the first row (Data 1), the housing price is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds). The price-rent ratio divides this housing wealth by the consumption of housing services summed over the four quarters of the year (NIPA). In Data 2, the housing price is the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac). The price-rent ratio divided this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. In Data 3, the housing price is the repeat-sale Case-Shiller National House Price index. The price-rent ratio divided this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1987, equal to the one in Data 1. The price and price-rent ratio values in a given year are the fourth quarter values. The annual price index, GDP, and residential investment are first deflated by the disposable personal income price deflator and then expressed as log deviations from their Hodrick-Prescott trend. Panels B and C are for the models without ($\phi = 1$) and with land/permits ($\phi = 0.9$). The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP.

Correlations	(Y, p^H)	$(p^H Y_H, p^H)$	$(Y, p^H/\mathcal{R})$
		Panel A: Data	
Data 1 (1953-2008)	0.23	0.43	0.23
Data 1 (1973-2008)	0.33	0.50	0.27
Data 2 (1973-2008)	0.33	0.52	0.29
Data 3 (1987-2008)	0.36	0.75	0.10
		Panel B: $\phi = 1$	
Model 1	0.95	0.28	0.17
Model 2	0.91	0.28	0.62
Model 3	0.87	0.39	0.60
		Panel C: $\phi = 0.9$	
Model 1	0.93	0.25	0.21
Model 2	0.92	0.17	0.49
Model 3	0.89	0.41	0.55

Table 4: Housing Wealth Relative to Total Wealth

The first column reports average housing wealth of the young (head of household is aged 35 or less) divided by average total wealth (i.e., net worth) of the young. The second column reports average housing wealth of the old divided by average net worth of the old. The third column reports average housing wealth of the young plus average housing wealth of the old divided by average net worth of the young plus average net worth of the old. The fourth (fifth) [sixth]column reports average housing wealth of the low (medium) [high]earners divided by average net worth of the low (medium) [high] earners. Low (medium) [high] earners are those in the bottom 25% (middle 50%) [top 25%] of the income distribution, relative to the cross-sectional income distribution at each age. The data in Panel A are from the Survey of Consumer Finance for 1998-2007. The last two rows report the model. In the model, housing wealth is $P_H * H$ and total wealth is $W + P_H * H$. Panels B and C are for the models without ($\phi = 1$) and with land/permits ($\phi = 0.9$). The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 15% of GDP.

	young	old	all	low earn	medium earn	high earn
			Panel A: Data			
1998	0.67	0.44	0.46	0.43	0.63	0.40
2001	0.67	0.43	0.44	0.44	0.58	0.40
2004	1.14	0.53	0.55	0.49	0.70	0.51
2007	0.92	0.52	0.54	0.51	0.71	0.50
			Panel B: $\phi = 1$			
Model 1	1.50	0.48	0.52	0.44	0.49	0.56
Model 2	1.83	0.52	0.56	0.49	0.54	0.60
Model 3	1.78	0.54	0.59	0.50	0.56	0.64
			Panel C: $\phi = .9$			
Model 1	1.50	0.47	0.50	0.42	0.48	0.55
Model 2	1.97	0.51	0.56	0.48	0.54	0.60
Model 3	1.85	0.52	0.57	0.48	0.54	0.62

Table 5: Return Moments

The table reports the mean and standard deviation of the return on physical capital, on a levered claim to physical capital, and on housing, as well as their Sharpe ratios. The Sharpe ratios are defined as the average excess return, i.e., in excess of the riskfree rate, divided by the standard deviation of the excess return. It also reports the mean and standard deviation of the riskfree rate. The last column is the change in the price-rent ratio, measured as the percentage change between 2000 and 2006 in the data and the percentage change relative to Model 1 in the model. Panel A reports the data. The housing return and price-rent ratio are measured three different ways. In the first row (Data 1), the housing return is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds) plus the consumption of housing services summed over the four quarters of the year (NIPA) divided by the value of residential real estate in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities due to population growth. In Data 2, the housing return uses the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac) and the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. We subtract realized CPI inflation from realized housing returns to form monthly real housing returns. We construct annual real housing returns by compounding monthly real housing returns over the year. The levered physical capital return in the data is measured as the CRSP value-weighted stock return. We subtract realized annual CPI inflation from realized annual stock returns between 1953 and 2008 to form real annual stock returns. The risk-free rate is measured as the yield on a one-year government bond at the start of the year minus the realized inflation rate over the course of the year. The data are from the Fama-Bliss data set and available from 1953 until 2008. Panels B and C are for the models without ($\phi = 1$) and with land/permits ($\phi = 0.9$). The leverage ratio (debt divided by equity) we use in the model is 2/3: $R_E = R_f + (1 + B/E)(R_K - R_f)$. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP.

	$E[R_K]$	$Std[R_K]$	$E[R_E]$	$Std[R_E]$	$E[R_H]$	$Std[R_H]$	$E[R_f]$	$Std[R_f]$	$SR[R_E]$	$SR[R_H]$	$\Delta p^H/\mathcal{R}$
Panel A: Data											
Data 1 (53-08)			7.86	19.11	9.89	4.91	1.62	2.49	0.34	1.49	31.2%
Data 1 (72-08)			6.60	19.43	9.78	5.87	1.66	3.01	0.27	1.22	31.2%
Data 2 (72-08)			6.60	19.43	9.11	4.32	1.66	3.01	0.27	1.36	30.7%
Panel B: $\phi = 1$											
Model 1	4.02	6.49	5.62	11.40	13.02	6.20	1.63	3.50	0.31	1.52	−−
Model 2	5.71	7.88	7.15	13.86	10.42	6.71	3.56	4.31	0.23	0.80	23.4%
Model 3	4.66	8.72	7.82	15.41	9.90	7.84	0.00	4.92	0.44	1.01	31.0%
Panel C: $\phi = .9$											
Model 1	3.39	5.73	4.91	10.17	14.17	6.17	1.10	3.06	0.33	1.78	−−
Model 2	5.36	7.14	6.63	12.58	11.04	6.12	3.45	3.70	0.22	0.98	27.5%
Model 3	4.19	8.57	6.73	15.07	10.60	7.42	0.39	4.65	0.37	1.09	33.7%

Table 6: Predictability

Panel A reports the the coefficients, t-stats, and R^2 of real return and real dividend growth predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r^i_{t+j} = \alpha + \kappa^r pd^i_t + \varepsilon_{t+k}$, where k is the horizon in years, r^i is the log housing return (left panel) or log stock return (right panel), and pd^i_t is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). The dividend growth predictability specification is similar: $\frac{1}{k}\sum_{j=1}^{k} \Delta d^i_{t+j} = \alpha + \kappa^d pd^i_t + \varepsilon_{t+k}$, where Δd^i is the log rental growth rate (left panel) or log dividend growth rate on equity (right panel). Panel B reports the the coefficients, t-stats, and R^2 of *excess* return predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r^{i,e}_{t+j} = \alpha + \kappa^{r,e} pd^i_t + \varepsilon_{t+k}$, where k is the horizon in years, $r^{i,e}$ is the log real housing return in excess of a real short-term bond yield (left panel) or the log real stock return in excess of a real short-term bond yield (right panel), and pd^i_t is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). In the model, we use the return on physical capital for the real return on equity and the return on the one-year bond as the real bond yield. The model objects are obtained from a 1150-year simulation, where the first 150 periods are discarded as burn-in. The model is the benchmark Model 1 without land/permits ($\phi = 1$). In the data, we use the CRSP value-weighted stock return, annual data for 1953-2008. The housing return in the data is based on the annual Flow of Funds data for 1953-2008. We subtract CPI inflation to obtain the real returns and real dividend or rental growth rates. The real bond yield is the 1-year Fama-Bliss yield in excess of CPI inflation.

Panel A: Raw Returns and Dividends/Rents

	Housing - Model							Equity - Model					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	−0.26	−16.72	13.46	−0.06	−5.75	1.83	1	−0.14	−18.18	25.46	0.48	19.45	29.22
2	−0.20	−20.60	24.75	−0.04	−4.44	1.67	2	−0.09	−20.03	33.96	0.30	20.84	37.36
3	−0.17	−24.50	35.86	−0.02	−3.30	1.21	3	−0.06	−20.75	36.37	0.22	22.71	41.84
5	−0.13	−30.55	54.36	−0.01	−1.70	0.48	5	−0.04	−23.59	38.48	0.13	24.32	44.13
10	−0.09	−34.72	71.45	−0.00	−0.31	0.02	10	−0.02	−24.67	47.25	0.07	27.03	50.76
20	−0.05	−29.51	75.83	0.00	0.79	0.19	20	−0.01	−27.51	53.91	0.04	34.14	58.06
30	−0.03	−29.52	75.43	0.00	1.18	0.40	30	−0.01	−24.52	57.90	0.02	34.67	65.26
	Housing - Data							Equity - Data					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.12	-2.2	5.3	0.00	-0.1	0.0	1	-0.14	-2.4	9.3	-0.07	-2.9	4.6
2	-0.12	-3.0	8.1	0.00	0.1	0.0	2	-0.12	-2.4	13.3	-0.03	-1.9	3.5
3	-0.11	-4.3	9.4	0.01	1.0	0.4	3	-0.09	-3.1	14.4	-0.01	-0.6	0.4
5	-0.09	-5.4	11.7	0.03	2.4	4.0	5	-0.07	-4.2	16.0	0.01	0.7	0.7

Panel B: Excess Returns

	Housing - Model				Equity - Model		
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	−0.16	−5.76	2.63	1	−0.09	−7.90	6.03
2	−0.12	−5.61	4.27	2	−0.06	−6.75	6.25
3	−0.10	−5.72	5.98	3	−0.04	−5.64	5.02
5	−0.08	−5.97	9.16	5	−0.02	−3.74	2.38
10	−0.06	−6.33	14.94	10	−0.01	−1.96	1.00
20	−0.04	−6.77	20.52	20	−0.00	−0.67	0.15
30	−0.02	−6.90	21.99	30	−0.00	−0.86	0.29
	Housing - Data				Equity - Data		
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	-0.15	-1.8	7.8	1	-0.16	-2.4	11.7
2	-0.15	-2.0	11.4	2	-0.11	-2.4	12.9
3	-0.15	-2.7	14.0	3	-0.08	-3.3	13.1
5	-0.16	-4.6	20.8	5	-0.06	-3.4	14.6

Table 7: Risk Sharing

This table reports various measures of cross-sectional risk sharing: the cross-sectional standard deviation of the consumption share $C^i_{T,a,t}/C_{T,t}$, the cross-sectional standard deviation of the intertemporal marginal rate of substitution, the Gini coefficient of consumption, and the variance of log consumption. The first two measures are reported for all ages, as well as for various age groups. All numbers are multiplied by 100. Panel A is for the model without land/permits ($\phi = 1$), while Panel B is for the model with land/permits ($\phi = 0.9$). We simulate the model for $N = 2400$ households and for $T = 1150$ periods (the first 150 years are burn-in and discarded). We calculate cross-sectional means and standard deviations of individual consumption share or consumption growth within each age group for each period, and then average over periods. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the model with foreign holdings of bonds to the extent of 19% of GDP.

	Panel A: $\phi = 1$					
	Cross-sectional St. Dev. Consumption Share					Gini cons.
	all	≤ 35	36-50	51-65	>65	
Model 1	79.63	49.44	55.74	70.72	81.56	37.63
Model 2	77.30	47.86	54.08	68.38	76.82	36.42
Model 3	78.33	49.01	55.33	69.69	79.66	36.73
	Cross-sectional St. Dev. IMRS					Var of log cons.
	all	≤ 35	36-50	51-65	>65	
Model 1	60.35	64.88	57.90	66.43	33.08	45.05
Model 2	55.14	62.96	54.89	55.80	28.35	42.37
Model 3	62.50	68.75	60.63	65.42	35.71	42.81
	Panel B: $\phi = 0.9$					
	Cross-sectional St. Dev. Consumption Share					Gini cons.
	all	≤ 35	36-50	51-65	>65	
Model 1	78.68	49.75	55.51	70.43	80.14	37.44
Model 2	75.72	48.66	54.51	68.26	75.50	36.34
Model 3	79.50	50.05	56.44	72.11	82.28	37.31
	Cross-sectional St. Dev. IMRS					Var of log cons.
	all	≤ 35	36-50	51-65	>65	
Model 1	59.46	62.23	57.48	67.65	31.84	44.65
Model 2	54.65	60.83	55.37	56.50	28.22	42.30
Model 3	62.64	68.86	62.55	66.86	34.81	44.26