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Working Paper #11-33

October 2011

**Multidimensional Heterogeneity and Platform Design**

Andre Veiga  
Toulouse School of Economics

E. Glen Weyl  
University of Chicago

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# MULTIDIMENSIONAL HETEROGENEITY AND PLATFORM DESIGN

André Veiga and E. Glen Weyl<sup>1</sup>

This version: September 30, 2011

ABSTRACT. One of the most salient issues faced by platforms like newspapers and credit card issuers is that users are heterogeneous in the value they bring to other users or to the platform. We develop a model with multi-dimensional heterogeneity where a monopoly platform chooses (price or non-price) *instruments*. Users play two roles: 1) they are users of the platform's services with heterogeneous preferences over instruments and platform *characteristics*; 2) they make heterogeneous contributions that endogenously determine these characteristics. The marginal (private or social) value of an instrument or characteristic includes the classical direct impact on profit and on (relevant) participants' utilities, but also includes a novel *sorting* effect of marginal users and consequent further impact on platform characteristics. The sorting effect is quantified by the *covariance, within the set of marginal users, between user preferences and user contributions towards characteristics*. The private optimum departs from efficiency by prescribing lower quantities and catering to the tastes of marginal (rather than average) users. Under reasonable conditions, optimal allocations may be implemented uniquely by allowing each instrument to be contingent on all characteristics. We discuss applications to newspapers, broadcast media, credit cards, and suggest simple extensions to the case of imperfect competition in insurance provision and college admissions.

**Keywords:** User Heterogeneity, Two-Sided Markets, Multi-Sided Platforms, Screening, Robust Implementation

**JEL Classification Codes:** D21, D42, D85, L12

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<sup>1</sup>Veiga: Toulouse School of Economics (TSE), Manufacture de Tabacs, 21 allées de Brienne, 31000 Toulouse, France; [andreveiga@gmail.com](mailto:andreveiga@gmail.com). Weyl: Department of Economics, University of Chicago, 1126 E. 59th Street, Chicago IL, 60637; [weyl@uchicago.edu](mailto:weyl@uchicago.edu). We thank the participants of Seminars at TSE, Universidad de Los Andes, Georgetown University, and University of Maryland, Jean Tirole, Bruno Jullien, Renato Gomes and especially Jacques Crémer for very helpful comments. We thank the NET Institute ([www.NETinst.org](http://www.NETinst.org)) for their generous financial support. All mistakes are our own.

## 1. INTRODUCTION

Radio stations famously introduced melodramatic “soap operas” in the 1920s to cater to the tastes of female listeners, since women had control over their families’ purse strings and were therefore particularly valuable to advertisers. Similarly, American Express’ premium travel services cater to the tastes of wealthy travelers, since these are the customers most valuable to the expensive stores that generate much of American Express’ revenues. As these examples illustrate, one of the most salient features of “platform” industries, in which a firm’s user base functions also as its selling point to other users, is the heterogeneity of the value generated by users. Yet the economic literature has struggled to analyze how platforms select and implement an optimal allocation when facing users heterogeneous in both *preferences* and *values*.

In this paper we present a general, yet simple, treatment of this multi-dimensional screening problem, based on empirically tractable moments of user heterogeneity, specifically the *covariance between marginal users’ preferences and their contributions to platform characteristics*. Our framework can be adapted to the analysis of several industries, illuminating common driving forces as well as crucial differences. It establishes a link between the literatures on multi-sided platforms and multi-dimensional screening, showing how issues in the two fields can be jointly understood, generalized and simplified.

We model a monopoly platform (say, a newspaper) that can directly choose the level of several *instruments*, possibly at a cost. For instance, these could be the prices charged to readers and advertisers, or a non-price instrument like the political slant of the newspaper. Users (readers and advertisers) may have heterogeneous preferences over these instruments, as some readers enjoy a conservative slant but others dislike it. Participating users make potentially heterogeneous contributions towards several platform *characteristics*, which are thereby endogenously determined. These may be, for instance, the number of readers (readers make homogeneous contributions of 1), or the total wealth of the readership, where heterogeneous contributions are more likely. These characteristics may be of some direct value to the platform, as is the case regarding the total number of paying readers. Additionally, users themselves may have heterogeneous preferences over platform characteristics, in which case these constitute consumption externalities. For instance, a change in the readership’s total wealth may be highly valued by an advertiser of luxury goods, but not as important to a seller of basic necessities. In sum, we allow for heterogeneity of users in their preferences for characteristics and instruments, as well as in their contributions towards determining the platform’s characteristics.

As in [Weyl \(2010\)](#) (henceforth, W10), users play two roles. On the one hand, they are consumers of the platform’s services, to which the standard theory of monopoly ([Cournot \(1838\)](#)) may be applied. On the other hand, users determine platform characteristics, making it useful to employ the analysis of a quality-choosing monopolist of [Spence \(1975\)](#) since, by determining its participants, the platform determines also the properties of the product it offers to participants. However, when users are heterogeneous in their contributions to characteristics, the platform cares not only about differences in preferences, but also about the relationship between preferences and contributions. For this reason, our analysis will also be in the spirit of [Rothschild & Stiglitz \(1976\)](#) (henceforth, RS76), in which platforms choose the levels of price and non-price instruments in order to attract the most valuable users.

In our model, the marginal value of an instrument or characteristic has three components. First, it impacts the revenues and costs of the platform. Second, it affects the utilities of participants, which is directly relevant for welfare maximization, but is also taken into account by a profit maximizer to the extent that it can capture users’ surplus. The key novel element is the third. Among users indifferent about participating (marginal users), a change in an instrument or characteristic will attract those who enjoy the change, but repel those who dislike it, thereby sorting participants. Depending on the contributions to characteristics of those attracted and those repelled, this sorting of participants will have a subsequent impact on characteristics. If those attracted make large positive contributions to some characteristic and those repelled make small or negative contributions, then the effect of sorting will be to increase the level of that characteristic. Formally, this will occur when there is positive covariance between marginal users’ preferences for the change, and users’ contributions to the characteristic of interest. We use a simplified environment to illustrate this mechanism in [Section 3](#).

Because of these feedback effects, the (private or social) marginal values of each of the relevant characteristics must be determined simultaneously. We show in [Section 4](#) that the general solution is characterized by a system of equations that can be described using three matrices. The first contains the marginal contributions to characteristics that result from changes in the instruments; the second captures the extent to which the feedback among characteristics causes these initial changes to be self-reinforcing or self-defeating; the final matrix consists of the value of the ultimate changes in the characteristics to users and the platform.

The private optimum distorts relative to the social optimum by having too little user participation, in the spirit of [Cournot \(1838\)](#), and by catering to the tastes of marginal, rather than average, users (as in [Spence \(1975\)](#)). In our simplest setting, the distortions described by our model are conceptually equivalent to those

obtained in W10, although different in magnitude as the [Spence \(1975\)](#) distortion is scaled by the sorting effect. When we allow utility to be imperfectly transferable, optimality conditions take into account the wedge that describes how efficiently the platform transforms user surplus into profit. In two simple extensions to the case of imperfect competition, the presence of asymmetric information reveals distortions and inefficiencies in the spirit of [Akerlof \(1970\)](#) and RS76.

Because users' decisions depend on their expectations about the decisions of other users, multiple equilibria may exist, as described by [Rohlf's \(1974\)](#). We consider the platform's implementation problem in [Section 5](#). In a multi-sided setting with quasi-linear utilities and homogeneous uni-dimensional values, W10 shows that prices contingent on the number of users on each side can ensure that participation on each side is unaffected by ("insulated" from) changes in every other side, thereby producing uniqueness. We offer a generalization where uniqueness is achieved by making each of the platform's instruments contingent on the levels of all characteristics. Intuitively, this is possible when the joint power of instruments to affect each of the characteristics is sufficiently large to counteract the self-reinforcing or self-defeating feedback among characteristics. When this is the case, any change in the expectation of users regarding the levels of the characteristics can be undone by the platform's appropriate use of its instruments, and therefore the platform can guarantee to users that its desired level of each characteristic will surely obtain. Formally, this requires appropriate bounds on the eigenvalues of the matrix of effects of instruments on characteristics, and the matrix of effects of characteristics on themselves.

In [Section 6](#), we illustrate the flexibility of our model by revisiting and generalizing a number of previous results. By modelling a newspaper faced with readers heterogeneous in wealth and in preference for political slant, we are able to interpret a robustness check of [Gentzkow & Shapiro \(2010\)](#) as meaning that the covariance between readers' preferences over slant and their wealth is close to zero. We model a broadcast media platform in the style of [Anderson & Coate \(2005\)](#), where users cannot be charged prices, but must instead be attracted by adjusting properties of the platform's media. Because utility is imperfectly transferable, the platform's profit maximizing conditions take into account the wedge between user surplus and the corresponding platform profit. We combine elements of [Bedre-Defolie & Calvano \(2010\)](#) and [Rochet & Stole \(2002\)](#) in modelling a credit card issuer's choice of a non-linear tariff to apply to card-carrying users, whose heterogeneous number of transactions depends endogenously on the tariff. The platform's per transaction fee caters to the tastes of marginal users but considers also the change in consumption by infra-marginals, thereby combining the logics of [Spence \(1975\)](#) and [Mussa & Rosen \(1978\)](#). Our fourth example is a simple extension to the case of imperfect

competition, in the spirit of RS76, where we quantify in an empirically tractable way that paper’s cream-skimming distortion and propensity for market failure. Our final example addresses the competitive matching of students to colleges in the spirit of [Gale & Shapley \(1962\)](#). We build upon the framework of [Azevedo \(2011\)](#) by allowing colleges’ preferences to consider properties of their entire student pool, rather than just individual students.

## 2. LITERATURE

Our paper draws from (and links) the literatures on multi-sided platforms and multi-dimensional screening. The majority of the theoretical and empirical literature on platforms assumes users are homogeneous in the value they generate ([Rysman \(2009\)](#) offers a thorough survey), although some recent papers have relaxed this assumption. On the empirical side, [Tucker \(2008\)](#), [Ryan & Tucker \(2010\)](#), [Cantillon & Yin \(2008\)](#) and [Lee \(2010\)](#) have found strong evidence for heterogeneity of externalities in video-conferencing technology, stock exchanges and video game markets, but have not considered optimal firm responses to this as we do. On the theoretical side, [Chandra & Collard-Wexler \(2009\)](#) and [Athey et al. \(2010\)](#) consider tightly parameterized models of platform competition in which one-dimensional heterogeneity plays a role, in contrast to our multi-dimensional approach. More closely related, [Hagiu \(2009\)](#), [Gomes \(2009\)](#) and [Jeon & Rochet \(2010\)](#), discuss the value of screening out low-value users, but consider explicit (“third-degree”) mechanisms, while we focus on implicit (“second-degree”) screening by means of product characteristics. [Gomes & Pavan \(2011\)](#) consider implicit screening in a multi-sided platform setting, but assume a single dimension of preference and value heterogeneity, which rules out the sorting effect we describe and makes their analysis closer to the classic theory of screening of [Mussa & Rosen \(1978\)](#).

By contrast, the literature on multi-dimensional screening ([Armstrong & Rochet \(1999\)](#), [Rochet & Chone \(1998\)](#), [Rochet & Stole \(2002\)](#), [Rochet & Stole \(2003\)](#)) has tended to allow for multi-dimensional heterogeneity of preferences and instruments. On the other hand, unlike our paper, it does not study environments with consumption externalities, nor does it emphasize comparative statics, and it often maintains strict assumptions about the parametrization of preferences. This literature also tends to be mathematically complex, in contrast to our reliance on simple primitives of user heterogeneity. However, our approach is more restrictive than this literature in that we only allow finite-dimensional instruments and focus on discrete user decisions. While these are important limitations, non-linear price schedules are typically well approximated by tariffs of only a few parts, which are possible in our setting (see [Wilson \(1993\)](#)).

Our paper can be seen in the tradition of [Rothschild & Stiglitz \(1976\)](#), since we allow firms to use both price and non-price instruments to screen users with heterogeneous (private or social) values. However, their analysis relies on uni-dimensional heterogeneity, with a single parameter determining both preferences and values, and relying on the classic [Spence \(1973\)](#)-[Mirrlees \(1971\)](#) single crossing condition to determine the relationship between the two. In our multi-dimensional environment, this condition is replaced by the covariance between users' preferences and their contributions to characteristics. Our flexible setup is able to quantify the effect of competition on the propensity for market shutdown and the size of the cream-skimming distortions present in that paper. Moreover, the sorting effect we describe requires multi-dimensional heterogeneity.

Three recent papers closely related to ours are [Einav et al. \(2010\)](#), [Einav et al. \(2011\)](#) and [Weyl & Tirole \(2011\)](#). In the first, users have multi-dimensional types but insurers choose only the prices charged for otherwise fixed contracts. Since welfare maximization depends on the marginal cost of coverage, while competitive prices are equated to average costs, inefficiencies arise from competition due to adverse or advantageous selection, and markets may be excessively or insufficiently covered, which generalizes the logic of [Akerlof \(1970\)](#). However, this paper does not allow insurers to determine their coverage and assumes the cost of coverage is fixed for each user.<sup>2</sup> [Einav et al. \(2011\)](#) model a firm screening clients with multi-dimensional heterogeneity, by use of multiple instruments, and derive profit maximizing conditions based on the elasticities of revenues and costs to changes in the instruments. Unlike our paper, they do not tie their solutions to the distribution of user heterogeneity, do not consider departures from efficiency by profit maximizers, nor do they consider the platform environment we focus on. [Weyl & Tirole \(2011\)](#) consider the use of market power to screen innovations, obtaining a characterization in terms of a covariance between innovation characteristics related to our formula, but do not consider the multi-sided feedback of characteristics that we describe. Their instruments, and thus their optimality conditions, are more specific than those we consider, so their paper can be thought of as providing a contract theoretical micro-foundation for a possible application of the sorting logic of our model.

### 3. A SIMPLE EXAMPLE

In this section, we present a simple illustration of the main ingredients of our approach. To that effect, we make several assumptions that are relaxed in [Section 4](#). We will consider the example of a cell phone service provider faced with users

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<sup>2</sup>[Einav & Finkelstein \(2011\)](#) offers an intuitive graphical analysis of the same phenomena.

heterogeneous in their propensity to make calls and in their preferences for the popularity of the network.

There is a continuum of potential phone users, with mass normalized to 1. Each user has a unique “type”  $\theta \in \mathbb{R}^{\mathcal{T}}$ , a  $\mathcal{T}$ -dimensional vector of user characteristics, which is each user’s private information. Types are distributed in the population according to a continuous and atomless probability density function  $f(\theta)$  (hereafter,  $\theta \sim f$ ), which is common knowledge.

A monopoly phone service provider chooses the level of its instrument, the uniform participation price  $P$ . Users decide whether or not to join the platform’s phone network based on  $P$  and on their expectation of a platform characteristic  $K$ . In this context,  $K$  can be the phone network’s overall popularity among users. Given  $(P, K)$ , if the user with type  $\theta$  (hereafter, “user  $\theta$ ”) participates, she obtains quasi-linear utility  $u \equiv v(K; \theta) - P$ . Outside options are normalized to zero, so the set of participating users is  $\Theta \equiv \{\theta : v(K; \theta) \geq P\}$ , assuming  $\Theta$  is measurable. The boundary of  $\Theta$  constitutes the set of marginal users, who are indifferent about participating, and it is denoted  $\partial\Theta \equiv \{\theta : v(K; \theta) = P\}$ .

The platform’s popularity ( $K$ ) is endogenously determined by the participants. User  $\theta$  makes a contribution  $k(\theta)$  to  $K$ . One can think of  $k(\theta)$  as user  $\theta$ ’s likeliness to make phone calls, or her individual popularity. The platform’s overall popularity is  $K \equiv \int_{\theta \in \Theta} k(\theta) f(\theta) d\theta$ .<sup>3</sup> Assuming  $k(\theta) : \mathbb{R}^{\mathcal{T}} \rightarrow \mathbb{R}$  is continuous and bounded insures the integral is well behaved.

The share of participants is  $N \equiv \int_{\theta \in \Theta} f(\theta) d\theta$ . It will be useful to define the density of marginal users as  $M \equiv \int_{\mathbf{t} \in \partial\Theta} f(\mathbf{t}) d\mathbf{t}$ , because it captures the sensitivity of the set of participants to changes in the  $P$  and  $K$ .<sup>4</sup> The platform’s profit is  $\Pi \equiv NP - C(N)$ , where  $dC/dN = C'$  is the marginal cost of an additional user. That is, the platform values popularity only to the extent that it attracts paying users. By quasi-linearity, social welfare is  $W \equiv \int_{\theta \in \Theta} v(K; \theta) f(\theta) d\theta - C(N)$ .

We assume that, if there are multiple equilibria, the platform can choose its most preferred allocation, and we focus on the necessary conditions under which a choice of  $P$  is welfare- or profit maximizing. We will also assume that a local change in  $P$  has a local impact on the corresponding level of  $K$ , thereby preventing discontinuous changes in the equilibrium. We will omit functional arguments without ambiguity (for instance,  $\int_{\theta \in \Theta} f(\theta) d\theta \equiv \int_{\Theta} f$ ).

<sup>3</sup>Since  $\Theta$  depends on  $K$ , then  $K$  is a fixed point of the function  $\int_{\Theta} kf$ . The implications of this are addressed in Section 5.

<sup>4</sup>One should think of  $M$  as (a component of) the derivative of the set of participants with respect to parameters, not a subset of participants per se. For instance,  $dN/dP = -M$ . Its analogue, in models of uni-dimensional heterogeneity, is the density ( $f$ ) evaluated at the (unique) marginal user. See [Flanders \(1973\)](#) for details.



First, we consider a platform's choice of a welfare maximizing price. Letting  $\lambda^W$  be the social value of an additional unit of popularity, the classical analysis of [Pigou \(1912\)](#) shows that the socially optimal price must be equated to the marginal cost of an additional user, net of the social value of the externality that user generates, or

$$P - C' = - \underbrace{\tilde{k}}_{\substack{\text{popularity of} \\ \text{typical marginal user}}} \cdot \underbrace{\lambda^W}_{\substack{\text{marginal social value} \\ \text{of popularity}}}$$

where  $\tilde{k} = E[k(\boldsymbol{\theta}) \mid \boldsymbol{\theta} \in \partial\Theta]$  is the popularity contributed by the typical marginal user.

In turn, the marginal social value of popularity is the externality this generates to all participants, in addition to the value of popularity in further attracting or repelling users, which then has a feedback effect on popularity itself. This effect will be positive when the attracted users are themselves popular and repelled users are unpopular. Formally, this means

$$\lambda^W = \underbrace{N\bar{v}_K}_{\substack{\text{externality to} \\ \text{all users}}} + \underbrace{\lambda^W M\sigma}_{\substack{\text{social feedback} \\ \text{value of popularity}}}$$

where  $\bar{v}_K = E[v_K(\boldsymbol{\theta}) \mid \boldsymbol{\theta} \in \Theta]$  is the marginal valuation for popularity of the average user, and  $\sigma = Cov(v_K, k \mid \boldsymbol{\theta} \in \partial\Theta)$  is the covariance, within the set of marginal users, between valuation of network popularity ( $v_K$ ) and individual contribution to popularity ( $k$ ). This term therefore captures the extent to which the popularity is self-reinforcing or self-defeating. Our first result follows immediately:

**Proposition 1.** *A welfare maximizing platform sets its price so that*

$$(1) \quad P - C' = - \underbrace{\tilde{k}}_{\substack{\text{popularity of} \\ \text{typical marginal user}}} \cdot \underbrace{\frac{1}{1 - M\sigma}}_{\substack{\text{sorting} \\ \text{multiplier}}} \cdot \underbrace{N\bar{v}_K}_{\substack{\text{externality to} \\ \text{all users}}}$$

*Proof.* We re-state the problem as  $\max_{P,K} W$ , subject to  $\int_{\Theta} kf = K$ . The Lagrangian is  $\mathcal{L} = W + \lambda^W (\int_{\Theta} kf - Q)$ . First Order Conditions (FOCs) are  $\frac{d\mathcal{L}}{dP} = -M(P - C') - \lambda^W M\tilde{q} = 0$  and  $\frac{d\mathcal{L}}{dK} = M\tilde{v}_K(P - C') + N\bar{v}_K + \lambda^W (M\tilde{v}_K\tilde{k} - 1) = 0$ . Eliminating  $\lambda^W$  and using  $\sigma = \tilde{v}_K\tilde{k} - \bar{v}_K\tilde{k}$  concludes.  $\square$

As expected, a welfare maximizing platform equates price to marginal cost, net of the social value of the externality generated by an additional (necessarily marginal) user. The value of this externality is intuitively determined by the contribution to popularity of the typical marginal user ( $\tilde{k}$ ), and by the extent to which overall users enjoy popularity ( $N\bar{v}_K$ ), the sign of  $\bar{v}_K$  determining if the externality is positive or negative.

The sorting multiplier  $1/(1-M\sigma)$  captures the sorting effect of an increase in popularity and it is proportional to  $\sigma$ , the covariance between appreciation for, and contribution to, popularity. Following an initial increase in popularity, among marginal users, those who enjoy popularity will be attracted into participating, those who dislike it being repelled. This way, increasing popularity will *sort* for marginal users who enjoy it. If these attracted users are themselves individually popular, and repelled users are unpopular, sorting will cause popularity to be self-reinforcing, so the social value of the popularity generated by an additional users will be scaled upwards, as the sorting multiplier will be greater than 1. Conversely, if attracted users make low contributions, while repelled users make large positive contributions, the sorting effect would cause an initial increase in popularity to be self-defeating, so the social value of additional popularity is scaled down by  $1/(1-M\sigma) < 1$ . The effect of sorting will therefore intuitively depend on the relationship between appreciation and generation of  $K$  among marginal users and, formally, on the covariance between these quantities within the set of marginal users. Intuitively, the effect will also be larger when the density of marginal users ( $M$ ) is larger.

For concreteness, if users are either “hot shots” (popular users who enjoy a popular network) or “loners” (unpopular users with little use for a popular network), then  $\sigma > 0$  and  $1/(1-M\sigma) > 1$ . An initial increase in popularity attracts “hot shots” and repels “loners” causing popularity to be self-reinforcing. If instead users are either “telemarketers” (who enjoy a popular network, but disturb other users), and “customer service reps” (whom everyone wants to call, but who dislike being bothered), then  $\sigma < 0$  and  $1/(1-M\sigma) < 1$ . In this case, an initial increase in popularity would attract “telemarketers” but repel the “customer service reps,” so an initial increase in  $K$  is partly undone by the sorting effect.

The sorting multiplier

$$\frac{1}{1-M\sigma} = 1 + \underbrace{M\sigma}_{\text{first round of sorting}} + \underbrace{M^2\sigma^2}_{\text{second round of sorting}} + \dots$$

can be understood as an the infinite series of effects, the first term corresponding to the weight of the initial change in  $K$ , and each successive term corresponding to the weight of the changes in  $K$  produced by the successive rounds of sorting.<sup>5</sup> When  $\sigma > 0$ , an initial increase in  $K$  will generate, by sorting, some additional popularity proportionally to  $M\sigma$ , which in turn will produce some additional popularity, this time proportionally to  $M^2\sigma^2$ . When  $\sigma < 0$ , the initial increase in  $K$  is partly offset by a decrease in  $K$  in the first round of sorting, but that in turn will produce a small increase in  $K$ , captured by  $M\sigma^2 > 0$ .

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<sup>5</sup> $|M\sigma| < 1$  will be true for any stable equilibrium.

Let us now address the price chosen by a profit maximizer. Let  $\lambda^\Pi$  be the private value of an additional unit of popularity to this platform. Classical theory prescribes that the profit maximizing price must be such that marginal revenue is equated to marginal cost, net of the value to the platform of the externality generated by a marginal user. If  $P - N/M = P - \mu$  is marginal revenue, then

$$\underbrace{P - \mu}_{\text{marginal revenue}} - C' = - \underbrace{\tilde{k}}_{\text{popularity of typical marginal user}} \cdot \underbrace{\lambda^\Pi}_{\text{marginal private value of popularity}}$$

Moreover, the private value of an additional unit of popularity ( $\lambda^\Pi$ ) must equal the value of the externality this generates to users, to the extent that this can be captured by the platform, in addition to the continuation value of  $K$  in generating additional popularity, as above. By using uniform prices, the platform is able to extract, from all participants ( $N$ ), the increase in surplus experienced by marginal users, so the former is equal to  $N\tilde{v}_K$ . The latter is again determined by the sorting effect, which now has a value to the platform of  $\lambda^\Pi$  per unit:

$$\lambda^\Pi = \underbrace{N\tilde{v}_K}_{\text{externality captured by platform}} + \underbrace{\lambda^\Pi M\sigma}_{\text{private feedback value of popularity}}$$

The second result follows:

**Proposition 2.** *A profit maximizing platform sets its price so that*

$$(2) \quad P - \underbrace{\mu}_{\text{Cournot distortion}} - C' = -\tilde{k} \frac{1}{1 - M\sigma} N \cdot \underbrace{\tilde{v}_K}_{\text{Spence distortion}}$$

*Proof.* We set up the problem as a constrained maximization (as above), with Lagrangian  $\mathcal{L} = \Pi + \lambda^\Pi (\int_{\Theta} qf - Q)$ . The FOCs are  $\frac{d\mathcal{L}}{dP} = -M(P - C') + N - \lambda M\tilde{q} = 0$  and  $\frac{d\mathcal{L}}{dK} = M\tilde{v}_K(P - C') + \lambda^\Pi (M\tilde{v}_K\tilde{k} - 1) = 0$ . Eliminating  $\lambda^\Pi$  using the definition of  $\sigma$  and  $\mu = N/M$  concludes.  $\square$

A profit maximizing monopoly equates marginal revenue to marginal cost net of externalities, the latter being determined by the preferences of marginal users. The term  $-\mu = N \cdot dP/dN$  illustrates the classical distortion of quantities downward from the social optimum by a profit maximizer described by [Cournot \(1838\)](#).

Moreover, the welfare maximizing platform of [Equation 1](#) considers the externalities to all participants ( $N\bar{u}_K$ ), but a profit maximizer considers only the preferences of marginal users ( $N\tilde{u}_K$ ), because their surplus can be entirely extracted by the adjustment of  $P$ . The catering to the tastes of marginal users, rather than average users, by a profit maximizer is as described by [Spence \(1975\)](#).

Unlike the Cournot distortion, the direction of the Spence distortion is not fixed. If marginal users enjoy popularity more than the average users ( $\widetilde{u}_K > \overline{u}_K$ ), a profit maximizer will tend to be inefficiently popular, and vice-versa. Moreover, the Spence distortion is scaled by the sorting effect. If marginal users enjoy popularity less than average users ( $\widetilde{u}_K < \overline{u}_K$ ), the platform will tend to have an inefficiently low level of popularity. However, if  $\sigma < 0$ , when the platform attempts to set this low level of popularity, it attracts marginal users who generate a great deal of popularity, thereby increasing overall popularity and mitigating the Spence distortion. This is formally illustrated by the fact that  $\sigma < 0$  implies  $1/(1-M\sigma) < 1$ . Notice however, that it is certainly possible that the Spence distortion is exacerbated.

In this simple example, all distortions are conceptually akin to those described by W10, and indeed our results reduce to those of that paper when users are homogenous ( $\sigma = 0$ ). Additional effects will arise in the more complex environments of Sections 4 and 6.

#### 4. THE GENERAL CASE

The model presented in Section 3 contains a number of important limitations:

- There is a unique instrument available to the platform.
- A unique platform characteristic enters users' utilities.
- Users respond homogeneously to changes in the instrument.
- Individual user contributions to characteristics are fixed.
- Utility is perfectly transferable between users and the platform.
- The platform's preferences does not depend on any characteristic that users care about.

In this section, we present a model that preserves the logic of Section 3 but relaxes these assumptions. The usefulness of this generality is illustrated in the applications of Section 6. However, we maintain two crucial assumptions. First, user decisions are discrete. This can be relaxed in simple cases, as illustrated in Sub-Section 6.3, but not generally.<sup>6</sup> Second, platform characteristics are linear aggregations of individual contributions, although individual contributions may respond to changes in the instruments or characteristics. Although our specification covers a large number of scenarios, we assume it is with some loss of generality.

**4.1. Setup.** There is a continuum of users, of mass normalized to 1. Each user has a type  $\theta \in \mathbb{R}^T$  that is her private information, and types are distributed in the population of users according to a continuous and atomless probability density function  $f(\theta)$ , which is common knowledge.<sup>7</sup>

<sup>6</sup>See Fabinger & Weyl (2012) on this issue.

<sup>7</sup>These assumptions are primarily technical, but also economically plausible in that they suggest that no two users are exactly alike.

A monopoly platform chooses  $\boldsymbol{\rho} = (\rho^1, \dots, \rho^l, \dots) \in \mathbb{R}^{\mathcal{P}}$ , a vector of  $\mathcal{P}$  instruments with components indexed by  $l \in \{1, 2, \dots, \mathcal{P}\}$ . Users decide whether to join the platform based on  $\boldsymbol{\rho}$  and on their expectation of  $\mathbf{K}$ , a vector of  $\mathcal{K}$  platform characteristics determined endogenously by the participants. Given  $(\boldsymbol{\rho}, \mathbf{K})$ , if users  $\boldsymbol{\theta}$  joins the platform, she obtains utility  $u(\boldsymbol{\rho}, \mathbf{K}; \boldsymbol{\theta})$ , assumed continuously differentiable in  $(\boldsymbol{\rho}, \mathbf{K})$ .

Non-participants obtain zero utility. Then  $\Theta \equiv \{\boldsymbol{\theta} : u(\boldsymbol{\rho}, \mathbf{K}; \boldsymbol{\theta}) \geq 0\}$  is the set of participants, assumed measurable. The set of marginal users is  $\partial\Theta \equiv \{\boldsymbol{\theta} : u(\boldsymbol{\rho}, \mathbf{K}; \boldsymbol{\theta}) = 0\}$ . The share participating users is  $N \equiv \int_{\Theta} f$  and the density of the margin is  $M \equiv \int_{\partial\Theta} f$ . We also define  $\mu = N/M$ .

Let  $\mathbf{K} = (K^1, \dots, K^i, \dots) \in \mathbb{R}^{\mathcal{K}}$  be the vector of  $\mathcal{K}$  characteristics, with components indexed by  $i, j \in \{1, 2, \dots, \mathcal{K}\}$ . Given  $(\boldsymbol{\rho}, \mathbf{K})$ , if user  $\boldsymbol{\theta}$  participates, she makes an individual contribution  $k^i(\boldsymbol{\rho}, \mathbf{K}, \boldsymbol{\theta})$  to characteristic  $K^i$ . We define  $K^i$  as  $K^i \equiv \int_{\Theta} k^i(\boldsymbol{\rho}, \mathbf{K}, \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}$ . Assuming  $k^i : \mathbb{R}^{\mathcal{P}+\mathcal{K}+\mathcal{T}} \rightarrow \mathbb{R}$  is continuous in all arguments, bounded, and continuously differentiable with respect to  $(\boldsymbol{\rho}, \mathbf{K})$  assures the convergence and differentiability of  $K^i$ .

We assume that, in equilibrium, user expectations of  $\mathbf{K}$  are correct, so  $\mathbf{K}$  is a fixed point of the function  $\kappa(\boldsymbol{\rho}, \mathbf{K}) : \mathbb{R}^{\mathcal{P}+\mathcal{K}} \rightarrow \mathbb{R}^{\mathcal{K}}$ , the  $\mathcal{K}$  components of which are the integrals defined above for each  $K^i$ . The fixed-point nature of  $\mathbf{K}$  illustrates the potential multiplicity of equilibria common in environments with consumption externalities. In this section, we assume that the platform can choose its preferred equilibrium among all feasible equilibria, and discuss the conditions necessary for uniqueness in Section 5. We also assume the instruments  $(\boldsymbol{\rho})$  do not permit individual contributions  $k^i$  to be contractible by the platform.<sup>8</sup>

The vector  $\mathbf{K}$  includes all characteristics that enter users' preferences. For a characteristic (say,  $L$ ) that enters only the platform's preferences, then  $L(\boldsymbol{\rho}, \mathbf{K})$  so the platform's profit can always be expressed as  $\Pi(\boldsymbol{\rho}, \mathbf{K})$ . Social welfare is  $W = \int_{\Theta} u f + \Pi$ .

We focus on the necessary FOCs for social and private optimality, and assume that the platform can select its preferred equilibrium in the case of multiplicity. Section 5 addresses the conditions necessary for unique implementation, under which the necessary conditions are sufficient. We continue to assume that local changes in  $\boldsymbol{\rho}$  imply local changes in the corresponding equilibrium levels of  $\mathbf{K}$ .

We also use the following notation:

<sup>8</sup>Contributions  $k^i$  are usually unobservable, much like preferences are. Alternatively, the platform might make observations of  $k^i$  with idiosyncratic noise, or there might be resales of the platform's product among users, making discrimination ineffective.

- $\Sigma_{\mathbf{u}_\rho, \mathbf{k}}$  is the matrix of covariances between valuations for instruments and contributions to characteristics. It has dimension  $\mathcal{P} \times \mathcal{K}$  and generic element  $\sigma_{u_{\rho^l}, k^i} = \text{Cov}(u_{\rho^l}, k^i \mid \partial\Theta)$ .
- $\Sigma_{\mathbf{u}_K, \mathbf{k}}$  is the matrix of covariances between valuations for characteristics and contributions to characteristics. It has dimension  $\mathcal{K}^2$  and the element in row  $i$  and column  $j$  is  $\sigma_{u_{K^i}, k^j} = \text{Cov}(u_{K^i}, k^j \mid \partial\Theta)$ .
- $\overline{\mathbf{k}_\rho}$  is the matrix of effects of instruments on individual contributions. It has dimension  $\mathcal{P} \times \mathcal{K}$  and generic element  $\overline{k_{\rho^l}^i} = E \left[ dk^i/d\rho^l \mid \boldsymbol{\theta} \in \Theta \right]$ .
- $\overline{\mathbf{k}_K}$  is the matrix of effects of characteristics on individual contributions. It has dimension  $\mathcal{K}^2$  and the element in row  $i$  and column  $j$  is  $\overline{k_{K^j}^i} = E \left[ dk^i/dK^j \mid \boldsymbol{\theta} \in \Theta \right]$ .
- $\overline{\mathbf{u}_\rho}$  is the diagonal matrix of average valuations for instruments. It has dimension  $\mathcal{P}^2$  and generic element  $\overline{u_{\rho^l}}$ .
- $\overline{\mathbf{u}_K}$  is the diagonal matrix of average valuations for characteristics. It has dimension  $\mathcal{K}^2$  and generic element  $\overline{u_{K^i}}$ .
- $\widetilde{\mathbf{u}_\rho}$  is the diagonal matrix of valuations for the instruments by the typical marginal user. It has dimension  $\mathcal{P}^2$  and generic element  $\widetilde{u_{\rho^l}}$ .
- $\widetilde{\mathbf{u}_K}$  is the diagonal matrix of valuations for the characteristics by the typical marginal user. It has dimension  $\mathcal{K}^2$  and generic element  $\widetilde{u_{K^i}}$ .
- $\widetilde{\mathbf{k}}$  is the  $\mathcal{K}$ -dimensional *row* vector of contributions to the characteristics. It has generic element  $\widetilde{k^i}$ .
- $\mathbf{1}_A$  is the column vector of dimension  $A \in \mathbb{N}$ , where every entry is 1.
- $\Pi_K$  is the  $\mathcal{K}$ -dimensional column vector of marginal impacts of each of the characteristics on platform profit. Its generic element is  $\Pi_{K^i} = d\Pi/dK^i$ .
- $\Pi_\rho$  is the  $\mathcal{P}$ -dimensional column vector of marginal impacts of each of the instruments on platform profit. Its generic element is  $\Pi_{\rho^l} = d\Pi/d\rho^l$ .

**4.2. Welfare Maximization.** We consider a platform choosing  $(\boldsymbol{\rho}, \mathbf{K})$  to maximize  $W$ , subject to the  $\mathcal{K}$  constraints  $\int_{\Theta} k^i f = K^i$ , for  $i \in \{1, \dots, \mathcal{K}\}$ . Let  $\lambda^{W^i}$  be the Lagrange multiplier on  $K^i$ , and  $\boldsymbol{\lambda}^W$  the column vector containing these multipliers. The Lagrangian is  $\mathcal{L}^W = W + \sum_{i=1}^{\mathcal{K}} \left( \int_{\Theta} k^i f - K^i \right) \lambda^{W^i}$ .

The FOC for  $\rho^l$  is  $d\mathcal{L}^W/d\rho^l = N\overline{u_{\rho^l}} + M\widetilde{u_{\rho^l}}u + \Pi_{\rho^l} + \sum_{i=1}^{\mathcal{K}} \left( M\widetilde{u_{\rho^l}}\widetilde{k^i} + N\overline{k_{\rho^l}^i} \right) \lambda^i = 0$ .<sup>9</sup> Since  $\int_{\partial\Theta} u_{\rho^l} u f = 0$  and  $\widetilde{u_{\rho^l}}\widetilde{k^i} = \sigma_{u_{\rho^l}, k^i} + \widetilde{u_{\rho^l}}\widetilde{k^i}$ , we obtain

$$\frac{1}{\widetilde{u_{\rho^l}}} \left( N\overline{u_{\rho^l}} + \Pi_{\rho^l} \right) + \frac{1}{\widetilde{u_{\rho^l}}} \sum_{i=1}^{\mathcal{K}} \left( M\sigma_{u_{\rho^l}, k^i} + N\overline{k_{\rho^l}^i} \right) \lambda^{W^i} = -M \sum_{i=1}^{\mathcal{K}} \widetilde{k^i} \lambda^{W^i}$$

<sup>9</sup>Notice that, in taking each of these derivatives, only  $\mathbf{K}$  (characteristics entering user preferences) are held fixed, the effect of  $\rho^l$  on characteristics entering only the platform's preferences subsumed in  $\Pi_{\rho^l}$ .

The covariance between marginal users' preferences and contributions ( $\sigma_{u_{\rho^i}, k^i}$ ) now plays a role because users have heterogeneous preferences over instruments, not just characteristics. These  $\mathcal{P}$  FOCs can then be expressed as the matrix equation

$$\widetilde{\mathbf{u}}_{\rho}^{-1} (N\overline{\mathbf{u}}_{\rho}\mathbf{1}_{\mathcal{P}} + \mathbf{\Pi}_{\rho}) + \widetilde{\mathbf{u}}_{\rho}^{-1} (M\mathbf{\Sigma}_{u_{\rho}, k} + N\overline{\mathbf{k}}_{\rho}) \boldsymbol{\lambda}^W = -M\mathbf{1}_{\mathcal{P}}\widetilde{\mathbf{k}}\boldsymbol{\lambda}^W$$

Similarly, the FOC with respect to  $K^i$  is  $\frac{d\mathcal{L}^W}{dK^i} = N\overline{u}_{K^i} + M\widetilde{u}_{K^i} + \mathbf{\Pi}_{K^i} + \sum_{j=1}^{\mathcal{E}} \lambda^j \left( M\widetilde{u}_{K^i} k^j + N\overline{k}_{K^i}^j - 1 \right) = 0$ , and these  $\mathcal{K}$  equations can be expressed as

$$\widetilde{\mathbf{u}}_{\mathbf{K}}^{-1} (N\overline{\mathbf{u}}_{\mathbf{K}}\mathbf{1}_{\mathcal{K}} + \mathbf{\Pi}_{\mathbf{K}}) + \widetilde{\mathbf{u}}_{\mathbf{K}}^{-1} (M\mathbf{\Sigma}_{u_{\mathbf{K}}, k} + N\overline{\mathbf{k}}_{\mathbf{K}} - \mathbf{I}_{\mathcal{K}}) \boldsymbol{\lambda}^W = -M\mathbf{1}_{\mathcal{K}}\widetilde{\mathbf{k}}\boldsymbol{\lambda}^W$$

where  $\mathbf{I}_{\mathcal{K}}$  is the  $\mathcal{K}^2$  identity matrix.

We obtain a system with  $\mathcal{P} + \mathcal{K}$  equations, from which we must eliminate  $\mathcal{K}$  unknowns (the elements of  $\boldsymbol{\lambda}^W$ ) and then solve for  $\mathcal{P}$  unknowns (the elements of  $\boldsymbol{\rho}$ ). This is always possible, but is especially straightforward when  $\mathcal{P} = \mathcal{K}$ . The vectors on the right hand side (RHS) of both equations have the scalar  $M\widetilde{\mathbf{k}}\boldsymbol{\lambda}^W$  for every entry and, when  $\mathcal{P} = \mathcal{K}$ , the two vectors have the same dimension and the two left hand sides (LHSs) can be identified to solve for  $\boldsymbol{\lambda}^W$ . We focus on this case for clarity. When  $\mathcal{P} > \mathcal{K}$ , we can simply consider  $\mathcal{P} - \mathcal{K}$  times an irrelevant characteristic (denoted  $K^0$ ), which enters no preferences and to which all agents make zero contributions, as we illustrate in Section 6. Using the result for  $\boldsymbol{\lambda}^W$  on the first matrix equation and using the definition of the ‘‘excess covariance’’ matrix  $\mathbf{\Sigma}_{u_{\mathbf{K}} - u_{\rho}, k} = \mathbf{\Sigma}_{u_{\mathbf{K}}, k} - \widetilde{\mathbf{u}}_{\mathbf{K}}\widetilde{\mathbf{u}}_{\rho}^{-1}\mathbf{\Sigma}_{u_{\rho}, k}$  and the ‘‘intensive effect’’ matrix  $\overline{\mathbf{k}}_{\mathbf{K} - \rho} = \overline{\mathbf{k}}_{\mathbf{K}} - \widetilde{\mathbf{u}}_{\mathbf{K}}\widetilde{\mathbf{u}}_{\rho}^{-1}\overline{\mathbf{k}}_{\rho}$ , we obtain the first general result:

**Proposition 3.** *The welfare maximizing choice of  $\boldsymbol{\rho}$  satisfies*

$$(3) \quad \underbrace{\frac{1}{M}\widetilde{\mathbf{u}}_{\rho}^{-1} (N\overline{\mathbf{u}}_{\rho}\mathbf{1}_{\mathcal{P}} + \mathbf{\Pi}_{\rho})}_{\substack{\text{marginal net} \\ \text{social benefit}}} = - \underbrace{\Delta}_{\substack{\text{marginal} \\ \text{contribution}}} \underbrace{\Gamma^{-1}}_{\substack{\text{feedback} \\ \text{multiplier}}} \underbrace{\mathbf{E}^W}_{\substack{\text{social value} \\ \text{of externalities}}}$$

where

$$\begin{aligned} \Delta &= \underbrace{\mathbf{1}_{\mathcal{P}}\widetilde{\mathbf{k}} + \widetilde{\mathbf{u}}_{\rho}^{-1}\mathbf{\Sigma}_{u_{\rho}, k}}_{\substack{\text{extensive} \\ \text{marginal contribution}}} + \underbrace{\mu\widetilde{\mathbf{u}}_{\rho}^{-1}\overline{\mathbf{k}}_{\rho}}_{\substack{\text{intensive} \\ \text{marginal contribution}}} \\ \Gamma &= \mathbf{I}_{\mathcal{K}} - \underbrace{M\mathbf{\Sigma}_{u_{\mathbf{K}} - u_{\rho}, k}}_{\substack{\text{extensive feedback} \\ \text{(sorting)}}} - \underbrace{N\overline{\mathbf{k}}_{\mathbf{K} - \rho}}_{\substack{\text{intensive feedback}}} \end{aligned}$$

$$E^W = \underbrace{\Pi_{\mathbf{K}} - \widetilde{\mathbf{u}}_{\mathbf{K}} \widetilde{\mathbf{u}}_{\rho}^{-1} \Pi_{\rho}}_{\text{externality to platform}} + N \underbrace{\left( \overline{\mathbf{u}}_{\mathbf{K}} \mathbf{1}_{\mathbf{K}} - \widetilde{\mathbf{u}}_{\mathbf{K}} \widetilde{\mathbf{u}}_{\rho}^{-1} \overline{\mathbf{u}}_{\rho} \mathbf{1}_{\rho} \right)}_{\text{externality to users}}$$

Although complex, the structure of this formula is that of standard Pigouvian pricing. Let us consider first the LHS. In Section 3, welfare maximization equated the direct net social benefit of including an additional user ( $P - C'$ ) to the negative of the externality generated by that user. Similarly here, the  $l$ -th entry of the column vector on the LHS is the direct social benefit of a marginal increase in instrument  $\rho^l$  that attracts an additional user:  $(N\overline{u_{\rho^l}} + \Pi_{\rho^l}) / M\widetilde{u_{\rho^l}}$ . The denominator is clearly the impact on users and the platform of a change in  $\rho^l$ , whereas the numerator captures the change in  $\rho^l$  necessary to attract an additional user. Presenting the solution in terms of an additional marginal user (rather than, for instance, in terms of a unit increase in  $\rho^l$ ) allows for clear comparisons with Section 3 and with most of the economic literature. For instance, when the instrument is uniform price  $P$  and utilities are quasilinear, we recover the result of Section 3:  $\frac{-N + N + (-M)(P - C')}{-M} = P - C'$ .

The RHS of the equation captures 1) the marginal contributions to all characteristics of the typical user attracted by a change in  $\rho$ , 2) the feedback effect among characteristics and 3) the social value of the ultimate changes in characteristics for both users and the platform. The matrix  $\Delta$  captures the first effect. The  $l$ -th entry of the column vector  $\Delta \mathbf{1}_{\rho}$  captures the contributions to all characteristics by the average user attracted by an increase in  $\rho^l$ :  $\sum_{i=1}^{\mathcal{K}} \left\{ \widetilde{k}^i + \sigma_{u_{\rho^l}, k^i} + N\overline{k_{\rho^l}^i} \right\}$ . The average attracted user contributes  $\widetilde{k}^i$  to characteristics  $K^i$  but, since users are heterogeneously attracted by  $\rho^l$ , this effect must be correct by  $\sigma_{u_{\rho^l}, k^i}$ . One must then also consider that a change in  $\rho^l$  may affect the individual contributions of all ( $N$ ) users to  $K^i$ , hence the term  $N\overline{k_{\rho^l}^i}$ . When users respond homogeneously to price and individual contributions are fixed, then  $\sigma_{u_{\rho^l}, k^i} = \overline{k_P} = 0$  and we obtain the familiar result  $\widetilde{k}$ .

The second term ( $\Gamma^{-1}$ ) captures the feedback effects among characteristics, although the matrix  $\Gamma$  is easier to analyze in this context. The  $j$ -th entry of the vector  $\Gamma \mathbf{1}_{\mathbf{K}}$  is an (inverse) measure of the feedback effect of some characteristic  $K^j$  on all other characteristics  $K^i$ , including the effect of  $K^j$  on itself:  $1 - M \sum_{i=1}^{\mathcal{K}} \left\{ \sigma_{u_{K^j}, k^i} - \widetilde{u_{K^j}} / \widetilde{u_{\rho^j}} \cdot \sigma_{u_{\rho^j}, k^i} \right\} - N \sum_{i=1}^{\mathcal{K}} \left\{ \overline{dk^i / dK^j} - \widetilde{u_{K^j}} / \widetilde{u_{\rho^j}} \cdot \overline{dk^i / d\rho^j} \right\}$ . The first summation captures the sorting (or “extensive”) effect of  $K^j$ , stemming from users being attracted into or out of participation along the margin, and is therefore proportional to  $M$ . Every characteristic  $k^i$  increases proportionally to  $\sigma_{u_{K^j}, k^i}$ . However, as discussed above, the FOC is expressed in terms of the impact of an additional marginal user, so the platform’s envelope conditions for its instrument  $\rho^l$



prescribe that it adjusts to maintain the number of participants fixed. This adjustment is intuitively proportional to  $\widetilde{u_{K^j}}/\widetilde{u_{\rho^j}}$  and, because users react heterogeneously to instruments, it has an additional impact on  $k^i$  of  $\sigma_{u_{\rho^j}, k^i}$ . The “excess covariance” matrix  $\Sigma_{\mathbf{u}_K - \mathbf{u}_\rho, \mathbf{k}}$  therefore captures the extent to which the preferences of the average user attracted by  $\mathbf{K}$  covaries with her contributions towards  $\mathbf{K}$ . The “intensive” effect of  $K^j$  on the characteristics consists of its impact on the individual contributions ( $\overline{dk^i/dK^j}$ ) of all users ( $N$ ). Again, instruments adjust in the manner described above which results in the additional term  $\widetilde{u_{K^j}}/\widetilde{u_{\rho^j}} \cdot \overline{dk^i/d\rho^j}$ . When users respond homogeneously to an instrument and contributions are fixed, as in Section 3, then  $\sigma_{u_{\rho^j}, k^i} = \overline{k^i_{\rho^j}} = \overline{k^i_{K^j}} = 0$  and we obtain  $1 - M\sigma_{u_K, k}$ . The infinite series  $\mathbf{\Gamma}^{-1}$  converges when the matrix  $\mathbf{\Gamma}$  is positive definite. Intuitively, this means that there is no characteristic that, by means of the feedback between all characteristics, is self-reinforcing to the point of being “explosive.” This will be true at any stable optimum, otherwise an arbitrarily small initial change in  $\mathbf{K}$  would generate an unbounded response.

The column vector  $\mathbf{E}^W$  captures the marginal value of the ultimate change in  $\mathbf{K}$  to both users and the platform. This depends on their valuations ( $N\overline{\mathbf{u}_K} + \mathbf{\Pi}_K$ ) but also on how  $\rho$  changes to fix the number of participants ( $-\widetilde{\mathbf{u}_\rho}^{-1}\widetilde{\mathbf{u}_K}(N\overline{\mathbf{u}_\rho} + \mathbf{\Pi}_\rho)$ ). In Section 3, this becomes  $N\overline{u_K} + M\widetilde{u_K}(P - C') + \frac{-\widetilde{u_K}(-N + N - M(P - C'))}{-1} = N\overline{u_K}$ .

**4.3. Profit Maximization.** Consider now a platform’s choice of  $(\rho, \mathbf{K})$  to maximize  $\Pi$ , subject to the  $\mathcal{K}$  constraints  $\int_{\Theta} k^i f = K^i$ , for  $i \in \{1, \dots, \mathcal{K}\}$ . Let  $\lambda^{\Pi i}$  be the Lagrange multiplier associated with  $K^i$  and  $\boldsymbol{\lambda}^{\Pi}$  be the  $\mathcal{K}$ -dimensional column vector of these multipliers. The Lagrangian is  $\mathcal{L}^{\Pi} = \Pi + \sum_{i=1}^{\mathcal{K}} (\int_{\Theta} k^i f - K^i) \lambda^{\Pi i}$ .

Proceeding as above, one can express the  $\mathcal{P}$  FOCs with respect to  $\rho$  and the  $\mathcal{K}$  FOCs with respect to  $\mathbf{K}$ , respectively, as

$$\widetilde{\mathbf{u}_\rho}^{-1} \mathbf{\Pi}_\rho + \widetilde{\mathbf{u}_\rho}^{-1} (M \Sigma_{\mathbf{u}_\rho, \mathbf{k}} + N \overline{\mathbf{k}_\rho}) \boldsymbol{\lambda}^{\Pi} = -M \mathbf{1}_{\mathcal{P}} \widetilde{\mathbf{k}} \boldsymbol{\lambda}^{\Pi}$$

$$\widetilde{\mathbf{u}_K}^{-1} \mathbf{\Pi}_K + \widetilde{\mathbf{u}_K}^{-1} (M \Sigma_{\mathbf{u}_K, \mathbf{k}} + N \overline{\mathbf{k}_K} - \mathbf{I}_{\mathcal{K}}) \boldsymbol{\lambda}^{\Pi} = -M \mathbf{1}_{\mathcal{K}} \widetilde{\mathbf{k}} \boldsymbol{\lambda}^{\Pi}$$

Assuming  $\mathcal{P} = \mathcal{K}$ , we can eliminate  $\boldsymbol{\lambda}^{\Pi}$  as above and obtain the second general result:

**Proposition 4.** *The profit maximizing choice of  $\rho$  satisfies*

$$(4) \quad \frac{1}{M} \underbrace{(\widetilde{\mathbf{u}_\rho}^{-1} \mathbf{\Pi}_\rho)}_{\substack{\text{marginal net} \\ \text{private benefit}}} = - \underbrace{\Delta}_{\substack{\text{marginal} \\ \text{contribution}}} \underbrace{\mathbf{\Gamma}^{-1}}_{\substack{\text{feedback} \\ \text{multiplier}}} \underbrace{\mathbf{E}^{\Pi}}_{\substack{\text{private value} \\ \text{of externalities}}}$$

where  $\Delta, \mathbf{\Gamma}$  are defined as above and

$$\mathbf{E}^\Pi = \mathbf{\Pi}_K - \widetilde{\mathbf{u}}_K (\widetilde{\mathbf{u}}_\rho)^{-1} \mathbf{\Pi}_\rho$$

Here, the  $l$ -th entry of the vector on the LHS is the private direct net gains to the platform from increasing  $\rho^l$  enough to attract an additional user. In the environment of Section 3 this becomes  $\frac{-(N-M(P-C'))}{M} = P - \mu - C'$ . Regarding the RHS, the marginal contribution to each of the characteristics by the typical marginal user attracted by a change in  $\rho$  ( $\mathbf{\Delta}$ ) and the feedback multiplier matrix ( $\mathbf{\Gamma}^{-1}$ ) are unchanged between Equations 3 and 4. However, a profit maximizer considers the private value of a change in  $\mathbf{K}$  ( $\mathbf{E}^\Pi$ ), which ignores the effects on the utility of users. In the simpler environment, this term becomes  $M\widetilde{u}_K(P - C') - \frac{\widetilde{u}_K(N-M(P-C'))}{-1} = N\widetilde{u}_K$ .

**4.4. A Multi-Sided Platform.** The results above can describe a typical multi-sided platform faced with users with heterogeneous values. We do this by partitioning the set of possible users (with types  $\theta \sim f$  and  $\theta \in \mathbb{R}^T$ ) into  $\mathcal{S}$  sub-sets, called “sides,” and indexed by  $s \in \{1, 2, \dots, \mathcal{S}\}$ . We normalize the mass of each side to 1. We let  $\theta = (\theta^1, \dots, \theta^s, \dots, \theta^{\mathcal{S}})$ , such that, for users on side  $s$ , only the component  $\theta^s \in \mathbb{R}^{\mathcal{T}^s}$  is relevant (and  $\mathcal{T} = \sum_{s=1}^{\mathcal{S}} \mathcal{T}^s$ ). Then  $\theta^s \sim f^s$  where  $f^s$  is the PDF of  $\theta$  conditional on side  $s$ .

Users on side  $s$  are charged a uniform participation price  $P^s$  by the platform, and participating users obtain utility  $u^s = v^s(\mathbf{K}; \theta^s) - P^s$ . The vector  $\mathbf{K} = (K^1, \dots, K^{\mathcal{S}})$  consists of  $\mathcal{S}$  endogenously determined platform characteristics. The set of participants is  $\Theta = \cup_{s=1}^{\mathcal{S}} \Theta^s$ , where  $\Theta^s = \{\theta^s : v^s(\mathbf{K}; \theta^s) \geq P^s\}$  and  $\partial\Theta^s = \{\theta^s : v^s(\mathbf{K}; \theta^s) = P^s\}$ . User  $\theta^s$  makes a contribution  $k^s(\theta^s)$  to  $K^s$  and we define  $K^s = \int_{\Theta} k^s f = \int_{\Theta^s} k^s f^s$ . That is, only users on side  $s$  respond to changes in  $P^s$  and make non-zero contributions to  $K^s$ . We define  $N^s \equiv \int_{\Theta^s} f^s$ ,  $M^s \equiv \int_{\partial\Theta^s} f^s$  and  $V^s \equiv \int_{\partial\Theta^s} v^s f^s$ . Here, for instance,  $N\overline{u}_K^i = \sum_{s=1}^{\mathcal{S}} N^s \overline{u}_K^i$ .

Let  $\mathbf{M}$  be the  $\mathcal{S}^2$  diagonal matrix with generic element  $M^s$ , and let  $\mathbf{N}, \mathbf{P}, \mathbf{C}', \boldsymbol{\mu}, \mathbf{V}$  be the  $\mathcal{S}$ -dimensional column vectors with generic elements  $N^s, P^s, dC/dN^s, N^s/M^s$ , and  $V^s$  respectively. The (non-diagonal) matrix  $\overline{\mathbf{u}}_K$  has, on its  $i$ -th row and  $j$ -th column, the element  $\overline{u}_{K^j}^i$ , and  $\widetilde{\mathbf{u}}_K$  is defined similarly. The platform’s profit is  $\Pi = \mathbf{N}^T \mathbf{P} - C(\mathbf{N})$  and social welfare is  $W = \mathbf{V}^T \mathbf{1}_{\mathcal{S}} - C(\mathbf{N})$ .

To apply Equations 3 and 4, notice first that  $\Sigma_{\mathbf{u}_\rho, \mathbf{k}} = \overline{\mathbf{k}}_K = \overline{\mathbf{k}}_\rho = \mathbf{0}$ , so  $\mathbf{\Gamma} = \mathbf{I}_K - \mathbf{M}\Sigma_{\mathbf{u}_K, \mathbf{k}}$ . Moreover,  $\mathbf{\Pi}_\rho = \mathbf{N} - \mathbf{M}(\mathbf{P} - \mathbf{C}')$  and  $\mathbf{\Pi}_K = \mathbf{M}\widetilde{\mathbf{u}}_K(\mathbf{P} - \mathbf{C}')$ . Then, the welfare and profit maximizing conditions are, respectively,

$$\mathbf{P} - \mathbf{C}' = -\mathbf{1}_{\mathcal{S}} \widetilde{\mathbf{k}} \mathbf{\Gamma}^{-1} \overline{\mathbf{u}}_K \mathbf{N}$$

$$P \underbrace{-\mu}_{\text{Cournot distortion}} - C' = -\mathbf{1}_S \tilde{\mathbf{k}} \Gamma^{-1} \underbrace{\tilde{\mathbf{u}}_K}_{\text{Spence distortion}} N$$

These are easily recognizable extensions of Section 3, including the [Cournot \(1838\)](#) and [Spence \(1975\)](#) distortions. Notice that, in W10,  $k^i = 1$  and  $K^i = N^i$ , so  $\Sigma_{\mathbf{u}_K, \mathbf{k}} = \mathbf{0}$  and  $\Gamma = \mathbf{I}_K$ .

## 5. IMPLEMENTATION

The presence of consumption externalities tends to produce multiple equilibria, since the decision of each user depends on her expectations about the decisions of other users, as pointed out by [Rohlfis \(1974\)](#) and emphasized by [Katz & Shapiro \(1985\)](#). W10 addresses this issue in a quasi-linear multi-sided setting, where the only relevant characteristic is the number of users. Uniqueness can then be achieved by allowing each price to be contingent on the number of users on every side, which can be thought of as a reduced-form way of modeling dynamic pricing behavior, as discussed in [White & Weyl \(2011\)](#) and [Cabral \(2011\)](#). For instance, a credit card issuer may charge low fees to merchants until it secures a critical mass of users, raising fees afterwards. Contingent pricing can ensure uniqueness by making the number of users on each side invariant to (“insulated” from) changes in the number of users on other sides. This also addresses the “chicken-and-egg” problem of [Caillaud & Jullien \(2003\)](#), where a platform needs a critical mass of users to turn a profit, but users do not join unless this critical mass is already present. The “insulating tariff” of W10 prescribes that, for any profile of users expectations about participation on each side,  $P^I$  will adjust so that the realized profile of participation is always the one chosen by the platform. Since users are correct about participation in equilibrium, the only equilibrium is the platform’s desired allocation. Formally, for two sides  $I$  and  $J$ , the contingent price function  $P^I(\cdot)$  is chosen such that  $dN^I/dN^J = \partial N^I/\partial N^J + dN^I/dP^I \cdot dP^I/dN^J = 0$ , and  $dN^I/dN^I = 1$ .

The technique is not immediately applicable here for three reasons. First, users in our model respond heterogeneously to instruments, and a given subset of users may respond to several instruments. We also allow users to make heterogeneous contributions and to contribute to several characteristics. In particular, W10 assumes that contributions are always positive and fixed, and that a user’s response to the instrument (price) is negative and fixed, which implies that an increase in price always strictly reduces the number of users on a given side. The third difference, related to these two but deserving emphasis, is that in W10, for each characteristic (number of participants on a given side), there was a single instrument (price on that side) affecting its level, and vice-versa.

Nonetheless, the notion of insulation is still intuitive in our environment. A newspaper may increase prices to advertisers after it secures a wealthy readership, and a search engine may offer higher quality searches to users if it starts accepting advertisers which are particularly distracting.

Formally, insulation in our setting requires choosing the derivatives of each instrument with respect to all characteristics (the matrix  $d\rho/d\mathbf{K}^T$ ), such that  $d\mathbf{K}/d\mathbf{K}^T = \mathbf{I}_{\mathcal{K}}$ . This implies

$$(5) \quad \frac{d\rho}{d\mathbf{K}^T} = \frac{d\mathbf{K}}{d\rho}^{-1} \left( \mathbf{I}_{\mathcal{K}} - \frac{\partial \mathbf{K}}{\partial \mathbf{K}^T} \right) = \left( M\widetilde{\mathbf{u}}_{\rho}\mathbf{k} + N\overline{\mathbf{k}}_{\rho} \right)^{-1} \left( \mathbf{I}_{\mathcal{K}} - M\widetilde{\mathbf{u}}_{\mathbf{K}}\mathbf{k} - N\overline{\mathbf{k}}_{\mathbf{K}} \right)$$

where  $\widetilde{\mathbf{u}}_{\rho}\mathbf{k}$  has dimension  $\mathcal{P} \times \mathcal{K}$  and generic entry  $\widetilde{u}_{\rho^i k^i}$ , and similarly for  $\widetilde{\mathbf{u}}_{\mathbf{K}}\mathbf{k}$ . Notice that all instruments  $\rho$  insulate jointly, rather than each instrument being responsible for a particular characteristic, as in W10. Intuitively, this formula can be understood as meaning that instruments must react to changes in user expectations ( $d\rho/d\mathbf{K}^T$ ) such that the effect of user expectations on characteristics ( $\mathbf{I}_{\mathcal{K}} - \partial\mathbf{K}/\partial\mathbf{K}^T$ ) is “undone” by the effect of instruments on characteristics  $(d\mathbf{K}/d\rho)^{-1}$ .

Immediately, the invertibility of the  $\mathcal{K} \times \mathcal{P}$  matrix  $d\mathbf{K}/d\rho$  requires  $\mathcal{P} = \mathcal{K}$ . That is, the number of instruments must equal the number of characteristics relevant to users.<sup>10</sup> It is easy to see that insulation is impossible when  $\mathcal{P} < \mathcal{K}$ . For instance, consider characteristics  $K^1$  and  $K^2$  but only instrument  $\rho^1$ . Then, insulation ( $dK^1/dK^1 = 1, dK^2/dK^1 = 0$ ) requires  $d\rho^1/dK^1 = (1 - M\widetilde{u}_{K^1 k^1})/M\widetilde{u}_{\rho^1 k^1} = -M\widetilde{u}_{K^1 k^2}/M\widetilde{u}_{\rho^1 k^2}$ . When  $\mathcal{P} > \mathcal{K}$ , the platform may be able to ignore some of its instruments and still implement its desired  $\mathbf{K}^*$  and, in fact, there might be multiple ways to achieve insulation. For instance, consider a market with sides  $i \in \{A, B\}$ , where users on side  $i$  have types  $\theta^i$  and care about the number of users on the opposite side,  $j \neq i$ . The platform charges a participation fee  $P^i$  to each side  $i$  but can also set a standard of quality  $\rho^i$  that all users have positive valuations for:  $u^i = v^i(\rho^i, N^j; \theta^i) - P^i$ , and  $v_{\rho^i}^i > 0$ . Then, to insulate  $N^i$ , the platform can choose an unconditional level of  $\rho^i$  and a contingent schedule  $P^i(N^j)$ , or it can set an unconditional level of  $P^i$  and use a conditional schedule of  $\rho^i(N^j)$  to insulate. It is possible, of course, that when  $\mathcal{P} > \mathcal{K}$ , using a particular set of  $\mathcal{K}$  instruments permits insulation, whereas a different set of  $\mathcal{K}$  instruments does not.

A number of additional conditions are required for  $d\rho/d\mathbf{K}^T$  to be always well defined, although their precise formulation is something we can only conjecture about for the time being. First,  $d\mathbf{K}/d\rho$  must be invertible for every value of  $\mathbf{K}$ , which means that its eigenvalues can never be zero. Since  $d\mathbf{K}/d\rho$  captures the effect of

<sup>10</sup>In W10,  $\mathcal{P} = \mathcal{K}$  is always satisfied and the insulating tariff becomes  $\frac{d\mathbf{P}}{d\mathbf{N}} = \widetilde{\mathbf{u}}_{\mathbf{N}} - \frac{1}{M}\mathbf{I}_{\mathcal{D}}$ , which exists and is unique for each  $N$ .

instruments on characteristics, and  $\partial \mathbf{K} / \partial \mathbf{K}^T$  captures the effect of user expectations on the realizations of the characteristics, we should intuitively require that instruments have sufficient power to undo any effect that a change in expectations might have on the characteristics. That is, we require that  $d\mathbf{K}/d\rho$  is large, in some sense, relative to  $\partial \mathbf{K} / \partial \mathbf{K}^T$ . We conjecture that the necessary condition involves bounding the eigenvalues of  $d\mathbf{K}/d\rho$  away from zero, as well as assuring that those of  $\partial \mathbf{K} / \partial \mathbf{K}^T$  are not too large, relative to those of  $d\mathbf{K}/d\rho$ .

## 6. APPLICATIONS

In this section, we use our framework to re-visit a number previous models and consider them in the light of richly heterogeneous users. In doing so, we clarify and illustrate the applicability of the results derived in Section 4. We assume all quantities are well behaved and defined as above unless stated otherwise.

**6.1. Newspaper Slant.** [Gentzkow & Shapiro \(2010\)](#) test whether US newspapers' political slants are profit maximizing. In their main model, the authors assume all readers have the same value to the newspaper and, in a robustness check, they do not reject this hypothesis. We offer an interpretation of this robustness check using the logic of our model. This application extends our analysis to a concrete multi-sided setting, illustrates what is lost when types are uni-dimensional and shows how the model is tractable when the number of instruments and characteristics differ ( $\mathcal{P} \neq \mathcal{K}$ ).

A monopoly newspaper determines its amount of (say, right-wing) slant  $s$ , its price to readers ( $P^R$ ) and to advertisers ( $P^A$ ). There is a mass 1 of potential readers, with types  $\theta^R \sim f^R$ . Participating readers obtain utility  $u^R = v^R(s; \theta^R) - P^R$ , since they have heterogeneous preferences over slant. Readers are also heterogeneous regarding their wealth, with reader  $\theta^R$  having wealth  $w(\theta^R)$ . If participating readers are the set  $\Theta^R$ , the total wealth of the readership is  $W = \int_{\Theta^R} w f^R$ . There is also a mass 1 of potential advertisers, with uni-dimensional types  $\theta^A \sim f^A$ . Participating advertisers have heterogeneous preferences over wealth and obtain utility  $u^A = \theta^A W - P^A$ . For instance, wealthy readers may be particularly important to sellers of luxury goods, but less important to sellers of basic necessities. Non-participants obtain zero utility. The uni-dimensionality of advertiser types implies all marginal advertisers are equal, so the corresponding covariance term vanishes and there is no sorting on the advertiser side.<sup>11</sup> It also implies  $\widetilde{u_W^A} = P^A/W$ . Letting  $i \in \{A, R\}$ , we define  $N^i = \int_{\Theta^i} f^i$ ,  $M^i = \int_{\partial \Theta^i} f^i$  and  $\mu^i = N^i/M^i$ . The newspaper's profit is  $\Pi = \sum_i N^i P^i - C(N^R, N^A, s)$ , where  $N^A(P^A, W)$ ,  $N^R(P^R, s)$  and the cost

<sup>11</sup>This is the case, for instance, in [Gomes & Pavan \(2011\)](#).

function  $C(\cdot)$  may include, for instance, the disutility of slant to the newspaper's owner.

We focus on profit maximization. To derive the solution directly from Equation 4 we need  $\mathcal{P} = \mathcal{K}$ , so we twice consider an irrelevant characteristic, denoted  $K^0$ , that enters no preferences and to which all contributions are zero. Then, the newspaper's instruments are  $\boldsymbol{\rho} = (P^A, P^R, s)$ , and  $\mathbf{K} = (W, K^0, K^0)$  are the endogenously determined characteristics. Since contributions are fixed,  $\overline{\mathbf{k}_\rho} = \overline{\mathbf{k}_K} = \mathbf{0}$ . Readers (the only agents making heterogeneous contributions) do not care about any characteristics, so  $\boldsymbol{\Sigma}_{\mathbf{u}_K, \mathbf{k}} = \mathbf{0}$ . For the same reason  $\widetilde{\mathbf{u}_K} \boldsymbol{\Sigma}_{\mathbf{u}_\rho, \mathbf{k}} = \mathbf{0}$ , so  $\boldsymbol{\Gamma} = \mathbf{I}$ . Finally,  $d\Pi/dW - \widetilde{u}_W^A / \widetilde{u}_{P^A}^A \cdot d\Pi/dP^A = P^A N^A / W$ . Equation 4 becomes

$$\begin{bmatrix} P^A - C_{N^A} - \mu^A \\ P^R - C_{N^R} - \mu^R \\ P^R - C_{N^R} - \frac{C_s}{M^R \widetilde{u}_s^R} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & 0 \\ \widetilde{w} & 0 & 0 \\ \widetilde{w} + \frac{\sigma_{u_s, w}^R}{\widetilde{u}_s} & 0 & 0 \end{bmatrix} \mathbf{I} \begin{bmatrix} \frac{P^A}{W} N^A \\ 0 \\ 0 \end{bmatrix}.$$

Advertisers produce no externalities, so  $P^A$  equates marginal revenue to marginal cost:  $P^A - \mu^A = C_{N^A}$ . When setting  $P^R$ , the newspaper considers the externality of marginal readers to advertisers, to the extent that this can be captured by the newspaper, thereby distorting in the spirit of Spence (1975):  $P^R - \mu^R - C_{N^R} = -\widetilde{w} N^A P^A / W$ . The optimal amount of slant satisfies

$$\underbrace{C_s}_{\text{disutility of slant}} = \underbrace{N^R \widetilde{u}_s^R}_{\text{Spence distortion to readers}} + \underbrace{M^R \sigma_{u_s, w}^R}_{\text{sorting for wealth by slant}} \underbrace{\frac{P^A}{W} N^A}_{\text{Spence distortion to advertisers}}$$

The disutility to slant is equated to the benefit it brings by attracting paying readers, to the extent that their surplus can be captured ( $N^R \widetilde{u}_s^R$ ). Moreover, slant sorts marginal readers and thereby affects wealth proportionally to  $M^R \sigma_{u_s, w}^R$ . This is internalized by the platform proportionally to the surplus of marginal advertisers, illustrating the Spence distortion on the advertiser side.

Although one might conjecture that more conservative readers tend to be wealthier and therefore of greater value to advertisers ( $\sigma_{u_s, w}^R > 0$ ), the robustness check of Gentzkow & Shapiro (2010) shows that one cannot reject that  $\sigma_{u_s, w}^R = 0$  in their data. This suggests that the assumption of homogenous readers is a good approximation for the newspaper industry.

**6.2. Broadcast Media.** Firms are often restricted in the prices they can charge to certain users, instead competing for those users along other dimensions. For instance, in Anderson & Coate (2005), a TV network chooses an amount of advertising but users cannot be charged for viewing while, in White (2008), a search engine

charges advertisers, but users respond only to the quality of the search results.<sup>12</sup> We model a two-sided platform restricted to a non-price instrument on one side, which illustrates the importance of allowing for such instruments and clarifies the role of the “excess covariance” term introduced in Section 4. This application also introduces the “conversion factor” or “wedge” between user surplus and platform profits, which becomes crucial when utility is imperfectly transferable. To fix ideas, we will use the example of melodramatic radio soap operas mentioned in Section 1.

There is a mass 1 of advertisers and of listeners. A radio station charges price  $P^A$  to advertisers but cannot charge listeners. Instead, it attracts listeners by adjusting the level of melodrama ( $m$ ) in its programming. Listeners have types  $\theta^L \sim f^L$  while advertisers have uni-dimensional types  $\theta^A \sim f^A$ . Listener  $\theta^L$  controls family disposable income in the amount of  $w(\theta^L)$ . Advertiser  $\theta^A$  generates distraction  $d(\theta^A)$  towards users. For  $i \in \{A, L\}$ , we define  $\Theta^i$ ,  $N^i$ ,  $M^i$  and  $\mu^i$  as above. Then the total wealth of listeners is  $W = \int_{\Theta^L} w f^L$ , and the overall distraction of listeners is  $D = \int_{\Theta^A} d f^A$ . Advertiser utility is  $u^A = \theta^A W - P^A$ , while participating listeners obtain  $u^L(m, D; \theta^L)$ . The radio station’s profit is  $\Pi = N^A P^A - C(N^A, N^L, m)$ , where  $N^A(W, P^A)$ ,  $N^L(D, m)$  and  $C(\cdot)$  is the cost function.

We focus on profit maximization. The platform’s instruments are  $\rho = (P^A, m)$ , while characteristics  $\mathbf{K} = (W, D)$  are determined endogenously. Equation 4 becomes

$$\begin{bmatrix} P^A - \mu^A - C_{N^A} \\ -\frac{C_m}{M^L u_m^L} - C_{N^L} \end{bmatrix} = - \begin{bmatrix} 0 & \tilde{d} \\ \tilde{w} + \frac{\sigma_{u_m, w}^L}{u_m^L} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ M^L \left( \sigma_{u_D, w}^L - \frac{\sigma_{u_m, w}^L \tilde{u}_D^L}{u_m^L} \right) & 1 \end{bmatrix} \begin{bmatrix} \frac{N^A P^A}{W} \\ \frac{\tilde{u}_D^L C_m}{u_m^L} \end{bmatrix}$$

The optimal level of  $P^A$  satisfies

$$\frac{P^A - \mu^A - C_{N^A}}{-\tilde{d}} = \underbrace{\frac{N^L \tilde{u}_D^L}{N^L u_m^L}}_{\text{externality to listeners}} \cdot \underbrace{\frac{C_m}{N^L u_m^L}}_{\text{conversion into platform profits}} + \underbrace{M^L \left( \sigma_{u_D, w}^L - \sigma_{u_m, w}^L \frac{\tilde{u}_D^L}{u_m^L} \right)}_{\text{sorting of listeners for wealth}} \cdot \underbrace{\frac{P^A N^A}{W}}_{\text{externality to advertisers}}$$

Marginal revenue ( $P^A - \mu^A$ ) is equated to marginal cost ( $C_{N^A}$ ) net of the amount of externality generated by an additional advertiser ( $\tilde{d}$ ) multiplied by the value of that externality to the platform, the RHS. Increasing  $D$  repels listeners proportionally to  $\tilde{u}_D^L$ . Because the FOC is expressed in terms of an additional marginal user, the platform adjusts melodrama ( $m$ ) to maintain the number of listeners fixed. The necessary adjustment in  $m$  is inversely proportional to the responsiveness of listeners ( $u_m^L$ ), but implies incurring the marginal cost  $C_m = dC/dm$ . Here, as in [White \(2008\)](#), a platform adjusts its instrument on the user side to compensate for

<sup>12</sup>See also [White & Jain \(2010\)](#).

the presence of less valuable advertisers. The conversion factor  $C_m/N^L \widetilde{u}_m^L$  captures the wedge between the surplus of listeners and the platform's profit. When prices are used, this conversion factor is equal to 1, as is the case on the advertiser side.

The excess covariance  $\sigma_{u_D,w}^L - \sigma_{u_m,w}^L \cdot \widetilde{u}_D^L/\widetilde{u}_m^L$  captures the effect of distraction  $D$  on wealth  $W$ , considering that the platform adjusts  $m$  to hold fixed the number of listeners. The adjustment is proportional to  $\widetilde{u}_D^L/\widetilde{u}_m^L$  for the reason mentioned above. The overall impact on  $W$  is therefore captured by the extent to which the users repelled by distraction are wealthier than the users attracted by the compensating change in melodrama. This term arises only in environments where instruments affect users heterogeneously, otherwise the second covariance vanishes.

The optimal level of melodrama satisfies

$$\underbrace{0}_{\text{price}} - \underbrace{\frac{C_m}{M^L \widetilde{u}_m^L}}_{\text{pseudo market power}} - C_{NL} = - \left( \widetilde{w} + \underbrace{\sigma_{u_m,w}^L}_{\text{housewives' spending power}} \right) \underbrace{\frac{P^A N^A}{W}}_{\text{externality to advertisers}}$$

The expression is analogous to the typical equating of marginal revenue to marginal cost net of externalities. Clearly, the marginal cost of an additional listener is  $C_{NL}$ . Typically, marginal revenue is price added to market power ( $N/M \widetilde{u}_P^L = -N/M$ ), which captures the cost of changing the instrument by the amount required to attract an additional user. Here, there are no direct revenues from changing  $m$ , by construction. However, increasing  $m$  to attract an additional listener implies incurring the cost  $C_m$ . As in the case of prices, the necessary change is inversely proportional to  $M^L \widetilde{u}_m^L$  for the reasons mentioned above.

Since users respond to changes in melodrama heterogeneously, the impact on wealth of a change in melodrama that attracts an additional listener is the wealth of the typical marginal listener, corrected by the covariance  $\sigma_{u_m,w}^L$ . One might interpret this term as the spending power of housewives, to the extent that they are the ones with greater preference for melodrama. We conjecture that this term is positive, which would explain radio station's use of melodrama to attract female listeners, since they would be the most valuable listeners to advertisers. The final term, capturing the externality to advertisers, illustrates the familiar Spence distortion.

When price enters the preferences of both users and the firm, these can be easily compared. Determining the social optimum when utility is not transferable requires additional assumptions, which are beyond the scope of this paper.

**6.3. Credit Cards With Non-Linear Pricing.** The large body of literature on the credit card industry has tended to assume that all users have the same value to merchants, and that this value is fixed for each user. For instance, [Bedre-Defolie &](#)



Calvano (2010) find that, when homogenous users are charged a two-part tariff, the card issuer acts as a welfare maximizer towards users. We combine their framework with elements Rochet & Stole (2002), and find that this result depends crucially on the homogeneity assumption. This application illustrates the importance of allowing user contributions to vary which, in this simple setup, offers a way to model user decisions which are not discrete.

There is a mass 1 of potential consumers with types  $\theta^C \sim f^C$ , and of retailers with uni-dimensional types  $\theta^R \sim f^R$ . A credit card issuer charges consumers a per-transaction fee  $p$  and a participation price  $P^C$  (that is, a two-part tariff).<sup>13</sup> It also charges retailers a participation price  $P^R$ . User  $\theta^C$  has a demand for transactions  $q(p; \theta^C)$ , naturally depending on the fee  $p$ .<sup>14</sup> If all retailers participate ( $N^R = 1$ ),  $\theta^C$  obtains a surplus of  $S(p; \theta^C) = \int_p^\infty q(\hat{p}; \theta^C) d\hat{p} - pq(p; \theta^C)$ . When a fraction  $N^R$  of retailers participate, she obtains  $u^C = N^R S(p; \theta^C) - P^C$ . Total demand for transactions is endogenously determined and defined as  $Q = \int_{\Theta^C} q(p; \theta^C) f^C$ . Retailers value the total demand for transactions, and obtain utility  $u^R = \theta^R Q - P^R$ . Letting  $i \in \{C, R\}$ , we define  $\Theta^i$ ,  $N^i$  and  $M^i$  in the usual way. A total of  $QN^R$  transactions occur, at a unit cost of  $c$ . Profit is  $\Pi = \sum_i N^i P^i + (p - c) Q N^R$ , where  $N^C(P^C, N^R, p)$ . By the envelope theorem,  $du^C/dp = -N^R q$ . Moreover,  $\widetilde{u}_Q^R = P^R/Q$  and  $\widetilde{u}_{N^R}^C = P^C/N^R$ .

Consider first welfare maximization. The platform's instruments are  $\rho = (P^R, P^C, p)$ , and  $\mathbf{K} = (N^R, Q, K^0)$ , where  $K^0$  is an irrelevant characteristics as above. Here  $\overline{\mathbf{k}}_{\mathbf{K}} = \mathbf{0}$ . Equation 3 becomes

$$\begin{bmatrix} P^R \\ P^C \\ P^C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \tilde{q} & 0 \\ 0 & \tilde{q} + \frac{V(\tilde{q})}{v_p^C} + \frac{\mu^C}{-N^R \tilde{q} \overline{q}_p} & 0 \end{bmatrix} \begin{bmatrix} (p - c)Q + N^C \overline{S} \\ (p - c)N^R + N^R \overline{v}_Q^R \\ 0 \end{bmatrix}$$

The welfare maximizing choices of  $\rho$  satisfy

$$P^C = 0$$

$$P^R = \underbrace{cQ}_{\text{social cost}} - \underbrace{N^C (\overline{S} + p\overline{q})}_{\text{externality to all consumers}}$$

<sup>13</sup>Extending this model to more complex tariffs would offer no additional insight, and few-part tariffs are often a good approximation (see Wilson (1993)).

<sup>14</sup>In a more detailed model, consumer  $\theta^C$  decides the number of transactions ( $t$ ) and obtains surplus  $q^{-1}(t; \theta^B) > 0$  from her  $t$ -th transaction, for  $q^{-1}$  decreasing in  $t$ . Utility is  $u^C = \max_t N^R \left( \int_0^t q^{-1}(\hat{t}; \theta^C) d\hat{t} - pt \right) - P^C$ , consumer  $\theta^C$  will perform transactions while  $p \leq q^{-1}$ , and  $q(p; \theta^B)$  is the solution to her maximization problem.

$$p = c - \underbrace{\overline{\theta^R}}_{\substack{\text{externality to} \\ \text{average retailer}}}$$

Equating  $P^C$  to zero maximizes welfare because there is no cost or externality of an additional user (for  $Q$  held fixed). The efficient level of  $P^R$  is equal to the marginal social cost of an additional retailer ( $cQ$ ) net of the externality to all users ( $N^C(\bar{S} + p\bar{q})$ ). Finally, the transaction fee is set such that  $pN^R = cN^R - N^R\overline{\theta^R}$ , where  $(p - cN^R)$  is the profit from an increase in  $Q$  and  $N^R\overline{\theta^R}$  is the externality generated towards all retailers.

Consider now a profit maximizer. Equation 4 becomes

$$\begin{bmatrix} P^R - \mu^R \\ P^C - \mu^C \\ P^C - \frac{Q}{\tilde{q}M^C} \end{bmatrix} = - \begin{bmatrix} 1 & 0 & 0 \\ 0 & \tilde{q} & 0 \\ 0 & \tilde{q} + \frac{V(\tilde{q})}{\tilde{q}} + \frac{\mu^C}{-N^R\tilde{q}} \frac{d\tilde{q}}{dp} & 0 \end{bmatrix} \begin{bmatrix} (p-c)Q + N^C\tilde{S} \\ (p-c)N^R + N^R\tilde{v}_Q^R \\ 0 \end{bmatrix}$$

The optimal level of  $P^R$  and  $P^C$  are

$$\begin{aligned} P^R - \mu^R &= \underbrace{(c-p)Q}_{\substack{\text{net private} \\ \text{transactional cost}}} - \underbrace{\frac{N^C P^C}{N^R}}_{\substack{\text{externality to} \\ \text{consumers}}} \\ \frac{P^C - \mu^C}{\tilde{q}} &= \underbrace{(c-p)N^R}_{\substack{\text{net private} \\ \text{transactional cost}}} - \underbrace{\frac{N^R P^R}{Q}}_{\substack{\text{externality to} \\ \text{retailers}}} \end{aligned}$$

The marginal revenue from an additional retailer ( $P^R - \mu^R$ ) is equated to the net marginal transaction costs incurred by the platform, net of externalities to all users ( $N^C$ ), as experienced by marginal users ( $S(p; \theta^C) = P^C/N^R$ ), as in [Spence \(1975\)](#). The reasoning is similar for the profit maximizing  $P^C$  except all costs, revenues and externalities are proportional to the transactions performed by the typical marginal user ( $\tilde{q}$ ).

The optimal level of the transaction fee  $p$  satisfies

$$p = c - \underbrace{\frac{P^R}{Q}}_{\substack{\text{Spence distortion} \\ \text{for retailers}}} + \frac{\overbrace{1 - \frac{\tilde{q}}{q}}^{\substack{\text{Spence distortion} \\ \text{for consumers}}}}{\underbrace{\frac{N^R}{p} \epsilon_X \frac{Var(\tilde{q})}{\tilde{q}}}_{\substack{\text{extensive effect} \\ \text{(sorting)}}} + \underbrace{\frac{1}{p} \bar{\epsilon}_I}_{\substack{\text{intensive effect} \\ \text{(Mussa-Rosen-Wilson)}}}}$$

where  $\bar{\epsilon}_I = -\frac{d\tilde{q}}{dp} \cdot p/\tilde{q}$  and  $\epsilon_X = \tilde{q}M^C p/Q$ .

The marginal cost of a transaction is  $c$  and  $\widetilde{\theta}^R = P^R/Q$  is the externality experienced by the typical marginal retailer from an additional transaction. The final term refers to the revenue obtained from transaction fees, and reflects the heterogeneity of user demands for transactions and in the effect of  $p$  on user demands. Concerning the numerator, the platform increase  $p$  for larger values of the number of transaction of infra-marginal users ( $\bar{q}$ ), because these users are “captive.” However,  $p$  decreases with  $\bar{q}$  because higher fees will cause marginal users to leave the platform. The platform therefore caters to the tastes of marginal users in the spirit of [Spence \(1973\)](#), although considering both marginal users (positively) and infra-marginals (negatively).

These effects are weighted then by the impact of  $p$  on marginal users leaving the platform, and on infra-marginal users reducing their consumption, as captured by the numerator. The “exiting” elasticity of  $Q$  is  $\epsilon_X = \bar{q}^{M^C} p/Q$ , where  $\bar{q}^{M^C}$  is the change in  $Q$  caused by departing marginal users. However, a unit increase in  $p$  does not affect all marginal users equally, since they execute different numbers of transactions. The exiting effect is therefore proportional to  $\sigma_{u,p,q}^C = -N^R \text{Var}(\bar{q})$ , by the envelope theorem. It is in this sense that our model resembles [Rochet & Stole \(2002\)](#). When marginal users have disparate levels of consumption ( $\text{Var}(\bar{q})$  large), increasing  $p$  will repel most strongly those marginal users who most contribute to  $Q$ , so the profit maximizing level of  $p$  is inversely proportional to this variance. The second mechanism reflects the fact that infra-marginal users will decrease their number of transactions flowing an increase in  $p$ . The term  $\bar{\epsilon}_I = -\overline{dq/dp} \cdot p/\bar{q}$  is the average “intensive” elasticity of transactions, capturing the extent to which the average user changes her consumption with response to a change in  $p$ . The optimal level of the transaction fee is intuitively inversely proportional to this quantity. This consideration of the effects of per unit prices on the consumption of infra-marginal users is typical of non-linear pricing, in the tradition of [Mussa & Rosen \(1978\)](#) and [Wilson \(1993\)](#).

Notice that, in [Bedre-Defolie & Calvano \(2010\)](#), user contributions are homogeneous, so  $\bar{q} = \bar{q}$  and the last term vanishes. In that case, the optimal level of  $p$  is not distorted based on the user’s preferences, so the platform acts like a welfare maximizer towards users, as that paper concludes.

**6.4. Insurance Cream-Skimming.** The remain two applications depart from our general framework in that they discuss a simple duopoly. We show that introducing competition is straightforward in the case where characteristics do not enter user preferences, by revisiting the classical model of insurance competition of RS76.<sup>15</sup> We allow for flexible user heterogeneity and, in particular, allow the individual cost

<sup>15</sup>When this is not the case, a number of complications arise, as discussed in [White & Weyl \(2011\)](#).

of coverage to change, which provides a simple way to model moral hazard. We identify the cream-skimming distortion of RS76 (absent from models of platform competition like [White & Weyl \(2011\)](#)), quantify the propensity for market failure, and identify a pricing distortion in the spirit of [Akerlof \(1970\)](#).

Two insurers, indexed by  $i, j \in \{A, B\}$ , each offer a policy defined by a price-coverage pair  $(P^i, \rho^i)$ . A mass 1 of potential users have types  $\theta \sim f$ . The parameter  $\theta$  will typically include the user's relative preference over insurers, her risk aversion, propensity for moral hazard and coverage cost. If user  $\theta$  joins insurer  $i$ , she obtains  $u = v(\rho^i; \theta) - P^i = v^i - P^i$ . Outside options are zero, so the participants of insurer  $i$  are  $\Theta^i = \{\theta : v^i - P^i \geq \text{Min}\{v^j - P^j, 0\}\}$ . User  $\theta$ 's expected cost of provision by an insurer with coverage  $\rho$  is  $c(\rho, \theta)$ , that is, cost depends on the level of coverage but not on the insurer. The effect of  $\rho$  on  $c(\cdot, \theta)$  can be understood as the cost or providing additional coverage, as well as the cost of the associated increase in user moral hazard. We will focus on the second interpretation for concreteness. Insurer  $i$ 's total cost is  $C^i = \int_{\Theta^i} c(\rho^i, \theta) f$ , and its profit is  $\Pi^i = N^i P^i - C^i$ .

Because of competition, we must differentiate between the ‘‘expansion margin’’ of  $i$  (users indifferent between joining  $i$  and remaining uninsured) and the ‘‘switching margin’’ of  $i$  (users indifferent between  $i$  and  $j$ ). The former is  $\partial\Theta^{X^i} = \{\theta : v^i = P^i, v^j < P^j\}$ , while the latter is  $\partial\Theta^S = \{\theta : v^i - P^i = v^j - P^j > 0\}$ . We define  $N^i$ ,  $M^{X^i}$  and  $M^S$  in the usual way. We focus on the symmetrical equilibrium and optimum, where both insurers offer the same price-coverage pair (heterogeneous preferences for an insurer still typically imply  $\Theta^i \neq \Theta^j$ ).

Consider a welfare maximizing insurer choosing  $\rho^i = (P^i, \rho^i)$  to maximize  $W = \sum_i \{\int_{\Theta^i} v(\rho^i; \theta) f - C^i\}$ . By symmetry, the effect of users switching along  $\partial\Theta^S$  are welfare-neutral. Since  $\rho^i$  affects only the participants of  $i$ , welfare maximization considers only its participants ( $\Theta^i$ ) and the expansion margin  $\partial\Theta^{X^i}$ . Equation 3 becomes

$$\begin{bmatrix} -N^i \\ N^i \overline{v_{\rho^i}^i} \end{bmatrix} + \begin{bmatrix} N^i - M^{X^i} P^i + M^{X^i} \widetilde{c}^{X^i} \\ M^{X^i} \widetilde{v_{\rho^i}^i}^{X^i} P^i + M^{X^i} \widetilde{v_{\rho^i}^i}^{X^i} c - N^i \overline{c_{\rho^i}^i} \end{bmatrix} = \mathbf{0}$$

where  $\widetilde{c}^{X^i} = E[c | \partial\Theta^{X^i}]$  and similarly for  $\widetilde{v_{\rho^i}^i}^{X^i}$ . The welfare maximizing  $\rho^i$  satisfies

$$P^i = \underbrace{\widetilde{c}^{X^i}}_{\text{social marginal cost}}$$

$$\underbrace{N^i \overline{v_{\rho^i}^i}}_{\text{social marginal benefit}} - \underbrace{N^i \overline{c_{\rho^i}^i}}_{\text{moral hazard}} = \underbrace{M^{X^i} \sigma_{u_{\rho^i}, c^i}^{X^i}}_{\text{efficient cream-skimming}}$$

where  $\sigma_{u_{\rho^i}, c^i}^{X^i} = \text{Cov}(u_{\rho^i}, c^i \mid \partial\Theta^{X^i})$ . Price is equated to the marginal cost of the typical user admitted from the expansion margin, for the reasons discussed above. The welfare maximizing coverage considers the benefit of additional coverage to all users ( $N^i \overline{v_{\rho^i}^i}$ ) net of the additional cost from exacerbated moral hazard ( $N^i \overline{c_{\rho^i}^i}$ ). Additionally, the optimal level of coverage takes into consideration the extent to which greater coverage sorts for high costs participants along the expansion margin ( $M^{X^i} \sigma_{u_{\rho^i}, c^i}^{X^i}$ ). Selection is adverse if  $\sigma_{u_{\rho^i}, c^i}^{X^i} > 0$  and advantageous otherwise. That is, if the marginal users most attracted by coverage are particularly unhealthy, this increase in costs must be taken into account when setting  $\rho^i$ . Alternatively, if  $\partial\Theta^{X^i}$  contains health-conscious users who enjoy coverage but are quite healthy, selection will be advantageous. Notice that even a welfare maximizer engages in some amount of “cream skimming,” along the expansion margin.

Conversely, a profit maximizing oligopolist considers its entire margin  $\partial\Theta^i = \partial\Theta^S \cup \partial\Theta^{X^i}$ . Then, Equation 4 is

$$\left[ \begin{array}{c} N^i - M^{X^i} (P^i - \widetilde{c}^{X^i}) - M^S (P^i - \widetilde{c}^S) \\ M^{X^i} \left( P^i \widetilde{v_{\rho^i}^{X^i}} - \widetilde{c v_{\rho^i}^{X^i}} \right) + M^S \left( P^i \widetilde{v_{\rho^i}^S} - \widetilde{c v_{\rho^i}^S} \right) - N^i \overline{c_{\rho^i}^i} \end{array} \right] = \mathbf{0}$$

The levels of  $\rho^i$  that maximize profit satisfy

$$\begin{array}{c} P^i - \underbrace{\mu^{S+X^i}}_{\text{Cournot distortion}} = M^{X^i} \widetilde{c}^{X^i} + \underbrace{M^S \widetilde{c}^S}_{\text{Akerloff distortion}} \\ \\ N^i \underbrace{\widetilde{v_{\rho^i}^{S+X^i}}}_{\text{Spence distortion}} - \underbrace{N^i \overline{c_{\rho^i}^i}}_{\text{moral hazard}} = \underbrace{M^{X^i} \sigma_{u_{\rho^i}, c^i}^{X^i}}_{\text{efficient cream-skimming}} + \underbrace{M^S \sigma_{u_{\rho^i}, c^i}^S}_{\text{Rothschild-Stiglitz distortion}} \end{array}$$

where  $\sigma_{u_{\rho^i}, c^i}^S = \text{Cov}(v_{\rho^i}^i, c^i \mid \partial\Theta^{S+X^i})$ . A profit maximizer’s price takes into account its market power on both margins ( $\mu^{S+X^i} = N^i / (M^S + M^{X^i})$ ). Such an insurer also departs from efficiency by considering the socially neutral costs along the switching margin ( $\widetilde{c}^S$ ). Since users infra-marginal to the market as a whole are likely to be more costly than marginal users (when selection is adverse), considering the price incurred from the former leads to higher prices and underprovision, as described in [Akerlof \(1970\)](#) and [Einav et al. \(2010\)](#). A profit maximizer’s optimal coverage considers the benefits as measured by its marginal users ( $\widetilde{v_{\rho^i}^{S+X^i}}$ ), as in [Spence \(1975\)](#). It also considers  $\rho^i$ ’s sorting for high-cost users along both the market expansion margin ( $\sigma_{u_{\rho^i}, c^i}^{X^i}$ ), and the switching margin ( $\sigma_{u_{\rho^i}, c^i}^S$ ). As in RS76, a competitive firm does not take into account the extent to which it “cream-skimms” from its competitors and the impact this has on their profits.

The results highlight the impact of competition on the various distortions. In the competitive limit all users are on some switching margin so  $M^S \rightarrow \infty$ . Switching users are marginal to the market as a whole, so they tend to be more similar to average users than to marginal users. Therefore, competition mitigates the Spence distortion but exacerbates the Akerloff distortion. Moreover,  $\mu^{S+X^i} \rightarrow 0$  with competition, so the Cournot distortion is reduced competition. Finally, when  $M^S = \infty$ , the profit maximizing condition for  $\rho^i$  has no solution and the market collapses, as in RS76.

**6.5. College Admissions.** The matching of students to colleges has been the topic of a large literature, from Gale & Shapley (1962) and Roth & Sotomayor (1989) to Azevedo & Leshno (2011). We depart from our main model by introducing competition and also by assuming user types are not private information. This will imply that a change in a college's instruments will attract students proportionally to their value to the college, not proportionally to student preferences. We build upon the framework of Azevedo (2011) by using a competitive matching model where, in equilibrium, platforms accept all users with values to the platform above a chosen threshold. We extend this model by allowing users to have multi-dimensional types, and allowing colleges to have preferences over the composition of their class, rather than over individual students.

There is a mass 1 of potential students with types  $\theta \sim f$ , which are observable by the colleges (through their admissions processes). Student  $\theta$  has humanities talent  $h(\theta)$  and technical talent  $t(\theta)$ . There are two colleges, indexed by  $i, j \in \{A, B\}$ . If  $\Theta^i$  is the set of students joining college  $i$ , its overall humanities and technical talents are  $H^i = \int_{\Theta^i} hf$  and  $T^i = \int_{\Theta^i} tf$ , respectively. College  $i$ 's payoff is its reputation,  $R^i(H^i, T^i)$ . We assume  $R_{H^i}^i > 0$  and  $R_{T^i}^i > 0$ , and also assume  $R^i(x, y) = R^j(y, x)$ , that is, colleges are symmetrically differentiated in their specialization. Let  $r^i(\theta) = R_{H^i}^i h(\theta) + R_{T^i}^i t(\theta)$  be the marginal contribution of student  $\theta$  to the reputation of college  $i$ . In the spirit of Azevedo (2011), each college  $i$  decides its admissions threshold,  $a^i$ , such that it accepts only those students for whom  $r^i(\theta) \geq a^i$ .<sup>16</sup> College  $i$  can also choose its humanities emphasis  $e^i$ , that is, the share of its total budget (normalized to 1) that it allocates to humanities programs, with  $1 - e^i$  being allocated to technical programs. Therefore, college  $i$ 's instruments are  $\rho^i = (a^i, e^i)$ .<sup>17</sup> Student  $\theta$  joining college  $i$  obtains utility  $u(e^i; \theta) = u^i > 0$ , and outside options are zero. That is, students may have a preference for a given college, they may have heterogeneous preferences over the emphasis of the college

<sup>16</sup>We assume that each college  $i$  is truthful, in the sense that it would not reject a student with high  $r$  but accept one with lower  $r$ . This could trigger a rejection chain that could ultimately benefit college  $i$ , but we abstract from this possibility as does most of the literature.

<sup>17</sup>We assume tuition payments are of second order importance to students and colleges.

they attend ( $e^i$ ) and they always prefer to attend some college rather than none. However, students do not care about the characteristics of the student body of the college they attend ( $H^i$  and  $T^i$ ). Student  $\theta$  may be rejected by both colleges ( $r^i < a^i, r^j < a^j$ ), be accepted by just one college, or be accepted by both and attend whichever college she prefers. Therefore, the participants of college  $i$  is the set  $\Theta^i = \{\theta : r^i \geq a^i, r^j < a^j\} \cup \{\theta : r^i \geq a^i, r^j > a^j, u^i > u^j\}$ .<sup>18</sup>

It is useful to partition the margin of  $i$  into 3 subsets. The “expansion” margin,  $\partial\Theta^{Xi} = \{\theta : r^i = a^i, r^j < a^j\}$ , is composed of students who would stop participating altogether if  $a^i$  is raised. Then,  $\partial\Theta^{Ri} = \{\theta : r^i = a^i, r^j > a^j, u^i > u^j\}$  is the “rejected” margin, composed of students who would switch to  $j$  if  $a^i$  is raised. Neither of these groups is affected by changes in the emphasis  $e^i$ . The “voluntary” margin,  $\partial\Theta^V = \{\theta : r^i > a^i, r^j > a^j, u^i = u^j\}$  is common to both colleges and composed of those students accepted by both colleges and indifferent between them, who would respond to a change in  $e^i$ . We define  $N^i, M^{Xi}, M^{Ri}, M^V$  in the usual way. College  $i$ 's payoff is  $\Pi^i = R^i - cN^i$ , where accommodating each student costs  $c$ . This allows colleges to have preferences about the overall composition of their student bodies.

We focus on the symmetrically differentiated equilibrium where  $l^i = 1 - l^j$  and  $a^i = a^j$ . We also assume that students are symmetrically distributed in the joint permutation of colleges and contributions.<sup>19</sup> This implies  $M^{Xi} = M^{Xj}$ ,  $M^{Ri} = M^{Rj}$ ,  $H^i = T^j$ ,  $H^j = T^i$ ,  $R_H^i = R_T^j$  and  $R_T^i = R_H^j$ . This setup being significantly different from our main model, we will not use Equations 3 and 4.

Consider college  $i$  choosing  $\rho^i$  to maximize  $W = \sum_i \{\int_{\Theta^i} uf + R^i - cN^i\}$ . The choice of emphasis  $e^i$  affects all of its participants ( $\Theta^i$ ) and may cause students along  $\partial\Theta^V$  to switch colleges. By symmetry,  $u^i = u^j$  within  $\partial\Theta^V$ , and  $\tilde{h}^V = \tilde{t}^V$ . The welfare maximizing level of  $e^i$  satisfies

$$\underbrace{M^V (R_H^i - R_T^i) (\sigma_{u_e, h}^V - \sigma_{u_e, t}^V)}_{\text{socially efficient college diversification}} = - \underbrace{N^i \overline{u_e^i}}_{\text{benefit to all students in } i}$$

The humanities talent attracted to  $i$  by a shift in the college's emphasis ( $e^i$ ), has an impact on  $i$ 's reputation proportional to  $R_H^i \sigma_{u_e, h}^V$  and, similarly, the technical talent attracted has a value of  $R_T^i \sigma_{u_e, t}^V$ . By the assumption of symmetry, students switching colleges along  $\partial\Theta^V$  will have the same effects on the reputations of both colleges. That is, if  $i$  chooses  $e^i$  to sort for technically gifted students valuable to

<sup>18</sup>Notice that sets defined by two equalities, such as  $\{r^i = a^i, r^j = a^j\}$ , have dimension  $\mathcal{T} - 2$  and therefore zero measure even in the context of first derivatives.

<sup>19</sup>For every student  $\theta$  with contributions  $h(\theta), t(\theta)$  and preferences  $u(l^i, \theta), u(l^j, \theta)$ , there is a student  $\theta'$  with symmetrical contributions  $h(\theta') = t(\theta)$  and  $t(\theta') = h(\theta)$ , and symmetrical preferences  $u(l^i, \theta) = u(l^j, \theta')$  and  $u(l^j, \theta) = u(l^i, \theta')$ , and  $f(\theta) = f(\theta')$ .

$i$ , then students with large humanities talent sorted into  $j$  will be equally valuable to  $j$ . Of course, a change in  $e^i$  also affects all students of  $i$  ( $N^i \bar{u}_e^i$ ).

A small change in the admissions criteria  $a^i$  affects the sets  $\partial\Theta^{X^i}$  and  $\partial\Theta^{R^i}$ , where  $\pi^i = a^i$ . The efficient level of  $a^i$  satisfies

$$\underbrace{M^{X^i} \tilde{u}^{X^i}}_{\text{inclusion benefits to students}} + \underbrace{M^{X^i} (\tilde{r}^{X^i} - c)}_{\text{net inclusion benefits to } i} + \underbrace{M^{R^i} (\tilde{u}^{R^i} - \tilde{u}^{j^{R^i}})}_{\text{allocation benefits to students}} = \underbrace{M^{R^i} (R_{H^i}^i - R_{T^i}^i) (\tilde{h}^{R^i} - \tilde{t}^{R^i})}_{\text{net allocation costs to colleges}}$$

Since marginal users do not have zero utility, relaxing the admissions strictness of  $i$  allows students along  $\partial\Theta^{X^i}$  to attend some college, adding  $M^{X^i} \tilde{u}^{X^i}$  to social welfare. Moreover, college  $i$  obtains the reputational benefit from these additional students ( $\tilde{r}^{X^i}$ ), despite having to pay the cost of  $c$  per student. Then, students along  $\partial\Theta^{R^i}$  are now able to attend their preferred college, producing a social gain of  $\tilde{u}^{R^i} - \tilde{u}^{j^{R^i}} > 0$ . However, when  $i$  poaches students from  $j$ , these are infra-marginal to  $j$  but marginal (and therefore of lower value) to  $i$ . This implies a cost to the colleges of allowing students to be allocated according to their own preferences, rather than the colleges': the RHS equals  $M^{R^i} (\tilde{r}^{j^{R^i}} - \tilde{a}^{R^i}) < 0$ .

Consider now college  $i$  choosing  $\rho^i$  to maximize  $\Pi^i = R^i - cN^i$ . Its choice of emphasis  $e^i$  satisfies

$$\underbrace{R_{H^i}^i \sigma_{u_e, h}^V + R_{T^i}^i \sigma_{u_e, t}^V}_{\text{privately efficient college diversification}} = \underbrace{\tilde{u}_e^V}_{\text{Spence distortion}} \underbrace{(c - \tilde{r}^{i^V})}_{\text{net private cost of diversification}}$$

A reputation maximizing college does not take into account that it's own diversification has a symmetrical impact on the reputation of its competitors and therefore is under-diversified. The Spence distortion can be seen in the fact that such a college does not take into account the surplus of infra-marginal students, instead considering the preferences of students on the voluntary margin.

The reputation maximizing level of  $a^i$  satisfies

$$\underbrace{\tilde{r}^{X^i+R^i}}_{\text{private benefit of inclusion}} = c$$

A reputation maximizer considers neither the welfare of non-participating students, nor of students not attending their preferred college. It also does not take into account the effect that poaching (infra-marginal) students from its competitor decreases its payoff. Finally, a payoff maximizer considers private costs, which include those of students on the rejected margin.



## 7. CONCLUSION

We have set up a simple and flexible framework to analyse private and social optimality conditions in platform industries, while allowing users to be heterogeneous in their preferences and in their value to other users and to the platform. We also allow platforms to have flexible payoff functions and to use price or non-price instruments to screen users. Finally, we discuss conditions under which the platform can implement its desired allocation uniquely and illustrate the applicability of the model through a series of short applications.

Our optimality conditions depend crucially on the sorting effect of changes in instruments or characteristics among marginal users. This effect is quantified by the covariance, within that set, between user preferences and contributions to the platform's endogenously determined characteristics. We thus provide a characterization that is general, simple, and empirically tractable.

Promising directions for future work include the full generalization of the approach to the case of competition. While we show this is straightforward to do when platform characteristics do not enter user preferences, relaxing that assumption would require addressing the coordination problems between users, as discussed by [White & Weyl \(2011\)](#) in a more restricted environment.

Allowing users to make non-discrete decisions is also an exciting path for future work. While our framework can accommodate simple reaction rules as those illustrated in Sub-Section 6.3, generalizing this aspect of the model would require allowing for a broader class of envelope conditions on the part of users.

The empirical calibration of the model is also an important direction in which to proceed. Our hope is that our reliance on simple moments of the distribution of user heterogeneity will facilitate future empirical research on industries from insurance to broadcast media.

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