More Astronomical Tables from Tebtunis

During a recent reexamination of the box of papyri belonging to the Egypt Exploration Society in which the texts published as *P. Tebt. Tait* were found, Prof. W. J. Tait found several fragments of Greek astronomical tables, to which he kindly drew my attention. The papyri in this box were procured by Grenfell and Hunt under unknown circumstances, and are believed to be largely or entirely from Tebtunis, although this provenance cannot be proved for the individual fragments published below.[1] Two further fragments from the same box with black tabular ruling and apparently belonging to numerical tables are too abraded to be worth transcribing. I have numbered the present four tables consecutively with three that I recently published from another E.E.S. box of Tebtunis papyri.[2]

4. P. Fay. ined. s.n.

A small scrap, 3×3 cm, broken on all sides. The table is written in an upright hand along the fibres, with black ruling separating the columns. The back is blank.

Transcription.

i	ii	iii
]	ζ ι ζ	β̣ [. . α
].	ια ς	γ ιη̣ [β
].	θ ιζ ζ	δ ιβ̣ γ
].	ι κδ η	ε ς δ
].	ια ιη θ	λ ε
	]	ς̣ κδ̣ ς̣
		ζ] ιη [ζ

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Comment.

The table is a sign-entry almanac, listing computed dates when each of the five planets Saturn, Jupiter, Mars, Venus, and Mercury cross from one zodiacal sign to the next.[3] When a line within a column of the almanac consists of three numerals, the first indicates the month (starting from Thoth = 1), the second is the day number, and the third indicates the zodiacal sign being entered (counted from Virgo = 1). A line consisting of only two numerals, slightly indented from the left, gives the day and zodiacal sign for a second sign-entry occurring in the same month as the line above. Consecutive columns will usually contain sign-entries for consecutive years, although not all almanacs were arranged so that the data for a year always began at the top of a new column.

Our first problem is to identify the planet whose movements are described in each of the two legible columns. This may be done by examining the pattern of motion and the intervals of time between sign-entries. In col. ii the planet enters the seventh zodiacal sign (Pisces) on Phamenoth 10, and just one day later has crossed back into the preceding sign, Aquarius. Hence its first stationary point was very near

[1] W. J. Tait, *Papyri from Tebtunis in Egyptian and in Greek (P. Tebt. Tait)*, London, 1977, vii-viii.

[2] A. Jones, “Three astronomical tables from Tebtunis,” *Zeitschrift für Papyrologie und Epigraphik* 121 (1998) 211–218.

[3] A. Jones, “A Classification of Astronomical Tables on Papyrus,” *Ancient Astronomy and Celestial Divination*, ed. N. M. Swerdlow. Cambridge (U.S.A.), 2000, section V 2.

the beginning of Pisces, about Phamenoth 10. Then after an interval of retrograde motion it begins its direct motion, again reaching Pisces on Pachon 17, that is, about 67 days after the stationary point. This eliminates Mars, Jupiter, and Saturn from consideration, since these planets all take more than sixty days to make their retrogradations, and of course require more time to get back to their original positions; while Mercury always catches up with its first stationary points in less than sixty days. The interval in the papyrus does, however, fit a typical pattern of Venus' motion, since Venus always travels retrograde for about forty days. In col. iii the planet traverses five consecutive zodiacal signs (Libra through Aquarius) in uniform intervals of 24 days, which is again characteristic only of Venus' direct motion. Thus our fragment contains parts of Venus' sign-entries for, probably, two consecutive years.

Now we remark that the middle of the retrogradation in col. ii must have occurred about the beginning of Pharmuthi, and Venus must then have been at about Aquarius 23°. Since Venus travels retrograde when it passes between the earth and the sun, we can estimate that the sun too was at about Aquarius 23° on Pharmuthi 1. According to the Egyptian civil (or 'Alexandrian') calendar, which was kept in line with the solar year by regular intercalations, the sun is always close to Aquarius 23° already on Mecheir 12. This discrepancy of about fifty days is too large to attribute entirely to the inaccuracy of ancient astronomical theory. Instead, we have to conclude that this almanac, like a number of other astronomical tables of the Roman period, uses the old, unintercalated Egyptian calendar, and that the years that it covers are not very far from A.D. 175, when corresponding dates in the two calendars had drifted apart by fifty days.

Since Venus makes an almost exact repetition of its pattern of motion after eight solar years (i.e. eight unintercalated Egyptian years plus two days), if we find a year in which Venus' sign-entries corresponded closely to those in the papyrus, a dating eight years earlier or later will fit almost as well, or perhaps slightly better. For this reason it is impossible to determine the specific pair of years described in the almanac. I compared the papyrus with an almanac generated by computer from Ptolemy's *Handy Tables*, and found the best match for the years A.D. 136/137 and 137/138 (Table 1), with particularly close correspondences in the direct motion, where ancient planetary theories tended to be most accurate. We possess one roughly contemporary sign-entry almanac employing the old Egyptian calendar (*P. Oxy.* LXI.4186, for A.D. 142–146).[4] All later almanacs known at present use the civil calendar.

Papyrus	*Handy Tables*
col. i	A.D. 136/137
Phamenoth 10: Pisces	Mecheir 17: Pisces
Phamenoth 11: Aquarius (reentry)	Pharmuthi 21: Aquarius (reentry)
Pachon 17: Pisces	Pachon 7: Pisces
Payni 24: Aries	Payni 19: Aries
Epeiph 18: Taurus	Epeiph 19: Taurus
col. ii	A.D. 137/138
Hathyr 18: Libra	Hathyr 19: Libra
Choeac 12: Scorpio	Choeac 13: Scorpio
Tybi 6: Sagittarius	Tybi 7: Sagittarius
Tybi 30: Capricorn	Tybi 30: Capricorn
Macheir 24: Aquarius	Mecheir 24: Aquarius
Phamenoth 18: Pisces	Phamenoth 18: Pisces

Table 1. Comparison of sign-entries in papyrus with recomputation.

[4] Edition in A. Jones, *Astronomical Papyri From Oxyrhynchus*, Memoirs of the American Philosophical Society 233, Philadelphia, 1999.

5. P. Fay. ined. s.n.

8.5×24 cm (assembled from three pieces), broken on the left and right, but about 0.5 cm margin is preserved at the top and bottom. The table is written in an informal upright hand along the fibres, with horizontal black rulings separating sections of the columns, and no ruling separating the columns. Col. i is in a different hand from the others, with tighter spacing of lines. The back is blank.

Transcription. Since the spacing of the lines of writing in each column of the table is independent of the others, I have assigned the lines a continuous numeration as if we were dealing with successive columns of verbal text.

col. i (*m.* 1)

(2 lines lost)

α] ζ ε
.]. ς

α .]γ
β .]α̣ δ
γ .]η ε
. .]ς̣
ς .]. ζ
ζ .]α η
. .]ζ θ
ι . .]ϊ
ια .]ε ια

α .]α̣ γ
. .] . δ
. .]θ̇ γ
ε] .γ δ
ς .]β ε
ζ .]β ς
. . .]ζ̣
(approx. 20 lines lost)

col. ii (*m.* 2)

(2 lines lost)
ϛ . [η]

ϛ κζ ζ

α ιε ιβ
γ ια α
ϛ γ β
ζ ζ α
ϊ ε β
ιβ ιη γ

α α α
κβ β
β ιη γ
γ ιϛ δ
δ ζ ε
ε α ϛ
κϛ ζ̣
. . .
(approx. 25 lines lost)

col. iii

∟΄ ϊα

α α η
ια α θ

ζ ιγ η
ιβ ιδ θ̣

α κδ δ
γ ε ε
δ ιγ ϛ
ε κα ζ
ϛ α̣ η
η ια θ
θ κγ ϊ
ια ιδ ια
ιβ κη ιβ

α θ ιβ
β ι . [α
γ . [β
δ̣ . [γ
. . δ
ε κ̣ . [ε

ϛ] ιϛ ϛ̣
ζ] θ ζ̣
[η . η]
κ θ
θ̣ .]β̣ [ι
ι .] ια
ια̣ [.] ιβ̣
ι̣β̣ . α
κη̣ β̣

α α β
ϊ γ
δ̣ δ δ
κβ ε
ε ε ϛ
ϛ ζ ε
ιϛ ϛ
ζ ιγ ζ
λ η
η] ι̣[.] θ
θ γ ι
. δ ια
. β̣ ιβ̣
. . . .

col. iv

∟ $\overline{ιβ}$

β η [η
. [
[———]
α α [η
ζ̣ [. θ
ιβ [. . ι

α [α ιβ
β [. . α
δ̣ [. . β
ϛ [. . γ
ι . [. . δ

α ιη [γ
β ιγ [δ
γ [. . ε
(approx. 21 lines lost)

Comment.

This is another sign-entry almanac. In this instance each column of the almanac presents the sign-entries for each of the five planets in the standard order Jupiter, Saturn, Mars, Venus, Mercury, with a regnal year number at the head of the column, and horizontal rulings to mark the end of the section for each planet. The first line for a planet is sometimes merely an indication of which zodiacal sign the planet was in on the first day of the year. The date and calendar are easily and securely established from the preserved regnal years in cols. iii and iv and the recorded positions of the planets, which can be compared with computations made by modern astronomical theory or by Ptolemy's *Handy Tables*. The regnal years turn out to belong to Severus Alexander, and the calendar is the reformed Egyptian civil calendar. The specific years are A.D. 229/230 (col. i) through 232/233 (col. iv). This is a comparatively late date by the standards of the box of papyri in which the papyrus was found.[5] The change of hand between columns i and ii may indicate that the original set of tables extended only to year 9 of Severus Alexander, and that it was subsequently extended as positions on later dates came to be needed.

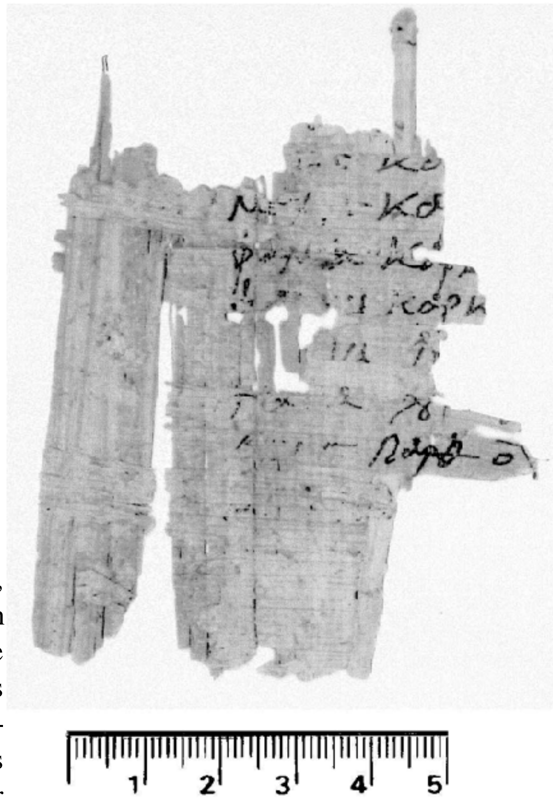
6. P. Fay. ined. Gc36

6.5×9 cm, broken on the top and right sides, but with 2 cm vacant on the left and 1.5 cm vacant on the bottom. The table is written in a slightly sloping informal hand along the fibres, without ruling. The back is blank.

Transcription.

— — — — — —

[φαω ̣]	κ̣[αρκ
[αθυ ̣	καρκ]
χοι]α̣ [̣]	κ̣[αρκ
τυ]β̣ ς	κα[ρκ
μεχ [̣]ε̣	κα[ρκ
φαμ α	καρκ [
φαρ̣ κβ	καρκ [
[παχ] κ̣α̣	̣ ̣[
παυ̣ α	λ̣ε[ον] ̣ [
επ̣ε̣ι̣ γ	παρθ ÷ [
μ̣[εc] ̣ ̣	̣[



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Comment.

This is part of a different format of planetary table, which I call a monthly almanac.[6] A planet's motion during the course of a year is tabulated line-by-line through the twelve months. The table may record dates of entry into zodiacal signs (as in a sign-entry almanac), critical points in a planet's motion such as its dates of first and last visibility and stationary points, or if nothing else occurred during a month, the planet's position at the beginning of the month.

In this fragment, the planet remains within Cancer for at least three months, and then traverses the whole of Leo in an interval of less than 71 days, reaching the beginning of Virgo on Epeiph 3. This is a plausible pattern for Mars, and for no other planet; moreover it can only be made to match an actual sequence of positions of Mars during a specific year if the calendar of the almanac was the Egyptian

[5] Tait viii-ix

[6] Jones, "Classification," section V 3.

civil (Alexandrian) calendar. For example, using Ptolemy's *Handy Tables*, I found the following dates and positions for the Egyptian year A.D. 194/195:

(Entry into Gemini, Epeiph 30 of preceding year)
Entry into Cancer, Thoth 17
First stationary point at Cancer 20°, Hathyr 23
Rising at sunset at Cancer 11°, Tybi 5
Second stationary point at Cancer 1°, Mecheir 15
Entry into Leo, Pachon 13
Entry into Virgo, Epeiph 4
Entry into Libra, Mesore 21

This would fit the papyrus rather well, with the recorded event on Tybi 6 being the sunset rising, the event on Mecheir 5 (or 15?) being the second station, and the events in Pachon and on Epeiph 3 being sign-entries. This is not the only possible dating on astronomical grounds, because Mars repeats its cycles of motion fairly closely after certain periodicities in years. A 79-year periodicity is particularly accurate, and so A.D. 115/116 is an equally plausible date for the almanac. *P. Tebt.* II 274 is a monthly almanac for all five planets covering years (A.D. 106–115) immediately preceding this earlier dating, but there is no resemblance in the hand or general appearance of the table.

7. P. Fay. ined. Gc36

4.5×16.5 cm, broken on the left, right, and bottom, but with 1.5 cm upper margin. The table is written in a neat upright hand across the fibres in a tabular framework ruled in black. On the other side is part of a mid second-century register (written along the fibres), which is mostly covered by what appears to be a repair patch bearing part of another contemporary register (written across the fibres).

Transcription.

i	ii	iii	iv	
]		π	λθ	[
]ε̣		ϙγ	λ	[
]ε		ρς	λη	[
].		ρκ	ε	[
]ε̣	ρκ	ρλγ	ν	[
]		ρμζ	νβ	[
]		ρξβ	ιγ [	
	]	ρος	να [	
	]	ρϙ̣α̣	μη̣ [	
	ρ]κε	cζ	β [	
	]	cκα	νθ [	
	]	cλς	λη [	
	]	cν	νη [	
	]	cξε	α [	
	ρ]λ	coη	μ̣ε̣ [	
	]	c̣ϙ̣β	ιβ [	
	]	τε	κ[α	
	]	τιη	ια̣ [	
	]	τλ	μ[δ	
	] ρλε	τμβ	νη̣ [	
	]	τνδ	ν[ε	

— — — — — — — —

Comment.

This piece is recognizable from its format as a ‘template’, a table presenting a pattern of daily progress of some heavenly body not for specific dates, but for the interval following all occurrences of some critical stage in the heavenly body’s cycles of motion.[7] The numerals in col. ii count the days since the epoch event (counting by fives, as in most known templates). Cols. ii and iii contain the degrees and minutes of progress to be added to the position at epoch, which the user would have to find in another kind of table.

The identity of the heavenly body, and the mathematical structure of the table, can be recognized by examining the line-to-line differences in the template:

day	*progress*	*difference*	*second difference*
116	80° 39'		
117	93° 30'	12° 51'	
118	106° 38'	13° 8'	17'
119	120° 5'	13° 27'	19'
120	133° 50'	13° 45'	18'
121	147° 52'	14° 2'	17'
122	162° 13'	14° 21'	19'
123	176° 51'	14° 38'	17'
124	191° 48'	14° 57'	19'
125	207° 2'	15° 14'	17'
126	221° 59'	14° 57'	–17'
127	236° 38'	14° 39'	–18'
128	250° 58'	14° 20'	–19'
129	265° 1'	14° 3'	–17'
130	278° 45'	13° 44'	–19'
131	292° 12'	13° 27'	–17'

Only the moon can advance by twelve to fifteen degrees per day through the zodiac. In reality the rate of change in the moon’s speed (revealed by the second differences) diminishes to zero as the speed reaches its maximum or minimum; but in our table the second differences remain effectively constant at 18' increase or decrease per day, the small fluctuations around this value evidently being due merely to the fact that the numbers are cut off at the first fractional place.

Such a sequence of numbers alternately increasing by constant differences between some minimum and maximum value is called a ‚linear zigzag function’. Zigzag functions were among the basic tools of Babylonian mathematical astronomy, and in a Greek text are usually indicative of Babylonian influence. As it turns out, the present template supplies a ‘missing link’ between a Babylonian method of computing daily positions of the moon attested in cuneiform tablets of the second and first centuries B.C. and a more refined Greek method attested in papyri of the first through the fourth centuries A.D.[8]

[7] Jones, “Classification,” section IV 2.

[8] A. Jones, “The Development and Transmission of 248-Day Schemes for Lunar Motion in Ancient Astronomy,” *Archive for History of Exact Sciences* 29 (1983) 1-36; A. Jones, “Studies in the Astronomy of the Roman Period. I. The Standard Lunar Scheme,” *Centaurus* 39 (1997) 1–36, with addendum, *Centaurus* 40 (1998) 41; Jones, *Astronomical Papyri From Oxyrhynchus* v. 1, 321–342.

daily motion	*day*	*total*	*daily motion*	*day*	*total*	*daily motion*	*day*	*total*	*daily motion*	*day*	*total*	*daily motion*	*day*	*total*
11° 06' 35"		11° 06' 35"	11° 14' 35"		19° 14' 55"	11° 22' 35"		27° 23' 15"	11° 12' 35"		24° 09' 00"	11° 20' 35"		32° 17' 20"
11° 24' 35"		22° 31' 10"	11° 32' 35"	30	30° 47' 30"	11° 40' 35"		39° 03' 50"	11° 30' 35"	85	35° 39' 35"	11° 38' 35"		43° 55' 55"
11° 42' 35"		34° 13' 45"	11° 50' 35"		42° 38' 05"	11° 58' 35"		51° 02' 25"	11° 48' 35"		47° 28' 10"	11° 56' 35"		55° 52' 30"
12° 00' 35"		46° 14' 20"	12° 08' 35"		54° 46' 40"	12° 16' 35"	60	63° 19' 00"	12° 06' 35"		59° 34' 45"	12° 14' 35"	115	68° 07' 05"
12° 18' 35"	5	58° 32' 55"	12° 26' 35"		67° 13' 15"	12° 34' 35"		75° 53' 35"	12° 24' 35"		71° 59' 20"	12° 32' 35"		80° 39' 40"
12° 36' 35"		71° 09' 30"	12° 44' 35"		79° 57' 50"	12° 52' 35"		88° 46' 10"	12° 42' 35"		84° 41' 55"	12° 50' 35"		93° 30' 15"
12° 54' 35"		84° 04' 05"	13° 02' 35"	35	93° 00' 25"	13° 10' 35"		101° 56' 45"	13° 00' 35"	90	97° 42' 30"	13° 08' 35"		106° 38' 50"
13° 12' 35"		97° 16' 40"	13° 20' 35"		106° 21' 00"	13° 28' 35"		115° 25' 20"	13° 18' 35"		111° 01' 05"	13° 26' 35"		120° 05' 25"
13° 30' 35"		110° 47' 15"	13° 38' 35"		119° 59' 35"	13° 46' 35"	65	129° 11' 55"	13° 36' 35"		124° 37' 40"	13° 44' 35"	120	133° 50' 00"
13° 48' 35"	10	124° 35' 50"	13° 56' 35"		133° 56' 10"	14° 04' 35"		143° 16' 30"	13° 54' 35"		138° 32' 15"	14° 02' 35"		147° 52' 35"
14° 06' 35"		138° 42' 25"	14° 14' 35"		148° 10' 45"	14° 22' 35"		157° 39' 05"	14° 12' 35"		152° 44' 50"	14° 20' 35"		162° 13' 10"
14° 24' 35"		153° 07' 00"	14° 32' 35"	40	162° 43' 20"	14° 40' 35"		172° 19' 40"	14° 30' 35"	95	167° 15' 25"	14° 38' 35"		176° 51' 45"
14° 42' 35"		167° 49' 35"	14° 50' 35"		177° 33' 55"	14° 58' 35"		187° 18' 15"	14° 48' 35"		182° 04' 00"	14° 56' 35"		191° 48' 20"
15° 00' 35"		182° 50' 10"	15° 08' 35"		192° 42' 30"	15° 12' 35"	70	202° 30' 50"	15° 06' 35"		197° 10' 35"	15° 14' 35"	125	207° 02' 55"
15° 10' 35"	15	198° 00' 45"	15° 02' 35"		207° 45' 05"	14° 54' 35"		217° 25' 25"	15° 04' 35"		212° 15' 10"	14° 56' 35"		221° 59' 30"
14° 52' 35"		212° 53' 20"	14° 44' 35"		222° 29' 40"	14° 36' 35"		232° 02' 00"	14° 46' 35"		227° 01' 45"	14° 38' 35"		236° 38' 05"
14° 34' 35"		227° 27' 55"	14° 26' 35"	45	236° 56' 15"	14° 18' 35"		246° 20' 35"	14° 28' 35"	100	241° 30' 20"	14° 20' 35"		250° 58' 40"
14° 16' 35"		241° 44' 30"	14° 08' 35"		251° 04' 50"	14° 00' 35"		260° 21' 10"	14° 10' 35"		255° 40' 55"	14° 02' 35"		265° 01' 15"
13° 58' 35"		255° 43' 05"	13° 50' 35"		264° 55' 25"	13° 42' 35"	75	274° 03' 45"	13° 52' 35"		269° 33' 30"	13° 44' 35"	130	278° 45' 50"
13° 40' 35"	20	269° 23' 40"	13° 32' 35"		278° 28' 00"	13° 24' 35"		287° 28' 20"	13° 34' 35"		283° 08' 05"	13° 26' 35"		292° 12' 25"
13° 22' 35"		282° 46' 15"	13° 14' 35"		291° 42' 35"	13° 06' 35"		300° 34' 55"	13° 16' 35"		296° 24' 40"	13° 08' 35"		305° 21' 00"
13° 04' 35"		295° 50' 50"	12° 56' 35"	50	304° 39' 10"	12° 48' 35"		313° 23' 30"	12° 58' 35"	105	309° 23' 15"	12° 50' 35"		318° 11' 35"
12° 46' 35"		308° 37' 25"	12° 38' 35"		317° 17' 45"	12° 30' 35"		325° 54' 05"	12° 40' 35"		322° 03' 50"	12° 32' 35"		330° 44' 10"
12° 28' 35"		321° 06' 00"	12° 20' 35"		329° 38' 20"	12° 12' 35"	80	338° 06' 40"	12° 22' 35"		334° 26' 25"	12° 14' 35"	135	342° 58' 45"
12° 10' 35"	25	333° 16' 35"	12° 02' 35"		341° 40' 55"	11° 54' 35"		350° 01' 15"	12° 04' 35"		346° 31' 00"	11° 56' 35"		354° 55' 20"
11° 52' 35"		345° 09' 10"	11° 44' 35"		353° 25' 30"	11° 36' 35"		1° 37' 50"	11° 46' 35"		358° 17' 35"	11° 38' 35"		6° 33' 55"
11° 34' 35"		356° 43' 45"	11° 26' 35"	55	4° 52' 05"	11° 18' 35"		12° 56' 25"	11° 28' 35"	110	9° 46' 10"	11° 20' 35"		17° 54' 30"
11° 16' 35"		8° 00' 20"	11° 08' 35"		16° 00' 40"				11° 10' 35"		20° 56' 45"			

daily motion	*day*	*total*	*daily motion*	*day*	*total*	*daily motion*	*day*	*total*	*daily motion*	*day*	*total*
11° 10' 35"		29° 05' 05"	11° 18' 35"		37° 13' 25"	11° 08' 35"		33° 59' 10"	11° 16' 35"		42° 07' 30"
11° 28' 35"	140	40° 33' 40"	11° 36' 35"		48° 50' 00"	11° 26' 35"	195	45° 25' 45"	11° 34' 35"		53° 42' 05"
11° 46' 35"		52° 20' 15"	11° 54' 35"		60° 44' 35"	11° 44' 35"		57° 10' 20"	11° 52' 35"		65° 34' 40"
12° 04' 35"		64° 24' 50"	12° 12' 35"	170	72° 57' 10"	12° 02' 35"		69° 12' 55"	12° 10' 35"	225	77° 45' 15"
12° 22' 35"		76° 47' 25"	12° 30' 35"		85° 27' 45"	12° 20' 35"		81° 33' 30"	12° 28' 35"		90° 13' 50"
12° 40' 35"		89° 28' 00"	12° 48' 35"		98° 16' 20"	12° 38' 35"		94° 12' 05"	12° 46' 35"		103° 00' 25"
12° 58' 35"	145	102° 26' 35"	13° 06' 35"		111° 22' 55"	12° 56' 35"	200	107° 08' 40"	13° 04' 35"		116° 05' 00"
13° 16' 35"		115° 43' 10"	13° 24' 35"		124° 47' 30"	13° 14' 35"		120° 23' 15"	13° 22' 35"		129° 27' 35"
13° 34' 35"		129° 17' 45"	13° 42' 35"	175	138° 30' 05"	13° 32' 35"		133° 55' 50"	13° 40' 35"	230	143° 08' 10"
13° 52' 35"		143° 10' 20"	14° 00' 35"		152° 30' 40"	13° 50' 35"		147° 46' 25"	13° 58' 35"		157° 06' 45"
14° 10' 35"		157° 20' 55"	14° 18' 35"		166° 49' 15"	14° 08' 35"		161° 55' 00"	14° 16' 35"		171° 23' 20"
14° 28' 35"	150	171° 49' 30"	14° 36' 35"		181° 25' 50"	14° 26' 35"	205	176° 21' 35"	14° 34' 35"		185° 57' 55"
14° 46' 35"		186° 36' 05"	14° 54' 35"		196° 20' 25"	14° 44' 35"		191° 06' 10"	14° 52' 35"		200° 50' 30"
15° 04' 35"		201° 40' 40"	15° 12' 35"	180	211° 33' 00"	15° 02' 35"		206° 08' 45"	15° 10' 35"	235	216° 01' 05"
15° 06' 35"		216° 47' 15"	14° 58' 35"		226° 31' 35"	15° 08' 35"		221° 17' 20"	15° 00' 35"		231° 01' 40"
14° 48' 35"		231° 35' 50"	14° 40' 35"		241° 12' 10"	14° 50' 35"		236° 07' 55"	14° 42' 35"		245° 44' 15"
14° 30' 35"	155	246° 06' 25"	14° 22' 35"		255° 34' 45"	14° 32' 35"	210	250° 40' 30"	14° 24' 35"		260° 08' 50"
14° 12' 35"		260° 19' 00"	14° 04' 35"		269° 39' 20"	14° 14' 35"		264° 55' 05"	14° 06' 35"		274° 15' 25"
13° 54' 35"		274° 13' 35"	13° 46' 35"	185	283° 25' 55"	13° 56' 35"		278° 51' 40"	13° 48' 35"	240	288° 04' 00"
13° 36' 35"		287° 50' 10"	13° 28' 35"		296° 54' 30"	13° 38' 35"		292° 30' 15"	13° 30' 35"		301° 34' 35"
13° 18' 35"		301° 08' 45"	13° 10' 35"		310° 05' 05"	13° 20' 35"		305° 50' 50"	13° 12' 35"		314° 47' 10"
13° 00' 35"	160	314° 09' 20"	12° 52' 35"		322° 57' 40"	13° 02' 35"	215	318° 53' 25"	12° 54' 35"		327° 41' 45"
12° 42' 35"		326° 51' 55"	12° 34' 35"		335° 32' 15"	12° 44' 35"		331° 38' 00"	12° 36' 35"		340° 18' 20"
12° 24' 35"		339° 16' 30"	12° 16' 35"	190	347° 48' 50"	12° 26' 35"		344° 04' 35"	12° 18' 35"	245	352° 36' 55"
12° 06' 35"		351° 23' 05"	11° 58' 35"		359° 47' 25"	12° 08' 35"		356° 13' 10"	12° 00' 35"		4° 37' 30"
11° 48' 35"		3° 11' 40"	11° 40' 35"		11° 28' 00"	11° 50' 35"		8° 03' 45"	11° 42' 35"		16° 20' 05"
11° 30' 35"	165	14° 42' 15"	11° 22' 35"		22° 50' 35"	11° 32' 35"	220	19° 36' 20"	11° 24' 35"		27° 44' 40"
11° 12' 35"		25° 54' 50"				11° 14' 35"		30° 50' 55"			

The Babylonian lunar scheme uses a zigzag function—called F* in modern discussions—for the moon's daily progress that has a maximum M = 15° 14' 35", a minimum m = 11° 6' 35", and a daily increment/decrement d = 18'.[9] With these defining parameters, the sequence exactly repeats after 248 days, having made nine cycles of increasing and decreasing speed around a mean value of 13° 10' 35". The sequence starts not exactly with the theoretical minimum value m but with a slightly higher value, 11° 7' 10", with the result that neither m nor M appears as an actual daily progress in the table, although the sequence 'reflects off' these limits periodically. The period and mean value F* are reasonably good approximations of the moon's period of anomaly and mean daily motion so long as one does not attempt to forecast the moon's motion for more than a few consecutive years without some sort of correction.

About the middle of the first century B.C., the Greek writer Geminus described the pattern of the moon's varying daily progress as following precisely this Babylonian zigzag function (*Isagoge* ch. 18).10 The only difference between Geminus' version and F* as it is found in the cuneiform texts is that Geminus starts precisely at the minimum value, m. This is the earliest known instance in a classical source of a modelling of the variation of the moon's motion, or indeed the motion of any heavenly body, in terms of a zigzag function. Geminus does not mention tables based on F*.

Our template proves to be an embodiment of Geminus' variant of F*, that is, a summation of the numbers generated by the sequence starting with m as the progress from day 0 to day 1. Table 2 gives a reconstruction of the template based upon this sequence. In each set of three columns, the middle column contains the day number, the leftmost column contains the corresponding value of the zigzag function representing the degrees of progress between the preceding day and this day, and the rightmost column is the running total, that is, the total progress since epoch. All legible numerals in the papyrus are in perfect agreement with the reconstruction of lines 116–136. I think it is probable that col. i of the papyrus, in which final epsilons can be read in three of the first five lines, gave the daily motion (which always ends in 35") as in the reconstruction. The template was undoubtedly accompanied by an epoch table supplying positions of the moon at 248-day intervals, representing dates when the moon's rate of progress was at a minimum. Positions on arbitrary dates would be found by adding together the position on the immediately preceding epoch date and, from the template, the accumulated progress corresponding to the days elapsed since the epoch.

Not later than the first century A.D., someone who was quite adept in handling zigzag functions and their mathematical properties (which are by no means trivial) took this epoch-and-template scheme and turned it into a much more refined set of lunar tables, which I refer to as the Standard Lunar Scheme. The elements that were carried over from the F* tables to the Standard Scheme included the representation of lunar daily motion as a zigzag function, the choice of its theoretical minimum as the initial value, and the spacing of epoch dates at (mostly) 248-day intervals. The defining parameters of the zigzag function, however, were all different from the Babylonian model, and evidently chosen in order to make the tables produce more accurate predictions over a much longer span of time. Unlike the F* tables, which with the present fragment make their first explicit appearance in a papyrus, the Standard Scheme as very well attested in papyri from the first through the fourth century A.D.

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[9] O. Neugebauer, *Astronomical Cuneiform Texts* 76-77 O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin, 1975, 480.

10 For Geminus' date see A. Jones, "Geminus and the *Isia*," *Harvard Studies in Classical Philology* 99 (in press).