CALENDRICA I: NEW CALLIPPIC DATES

1. Introduction.

Callippic dates are familiar to students of Greek chronology, even though up to the present they have been known to occur only in a single source, Ptolemy's *Almagest* (c. A.D. 150).1 Ptolemy's Callippic dates appear in the context of discussions of astronomical observations ranging from the early third century B.C. to the third quarter of the second century B.C. In the present article I will present new attestations of Callippic dates which extend the period of the known use of this system by almost two centuries, into the middle of the first century A.D. I also take the opportunity to attempt a fresh examination of what we can deduce about the Callippic calendar and its history, a topic that has lately been the subject of quite divergent treatments.

The distinguishing mark of a Callippic date is the specification of the year by a numbered "period according to Callippus" and a year number within that period. Each Callippic period comprised 76 years, and year 1 of Callippic Period 1 began about midsummer of 330 B.C. It is an obvious, and very reasonable, supposition that this convention for counting years was instituted by Callippus, the fourthcentury astronomer whose revisions of Eudoxus' planetary theory are mentioned by Aristotle in *Metaphysics* Λ 1073b32–38, and who also is prominent among the authorities cited in astronomical weather calendars (parapegmata).2

The point of the cycles is that 76 years contain exactly four so-called Metonic cycles of 19 years. The Metonic cycle reflects the fact that 235 lunar months, reckoned (for example) from full moon to full moon or from new moon crescent to new moon crescent, are practically indistinguishable in length from 19 solar years, reckoned from solstice to the same kind of solstice, or equinox to the same kind of equinox. This relation provides a convenient basis for regulating a lunar calendar so that the beginning of the year will not drift outside a particular season. It is merely necessary to establish a repeating cycle of nineteen calendar years, twelve containing twelve lunar months, and seven containing thirteen months. A convention for naming the months is also needed, one method being to have twelve names in a fixed order, and to stipulate that in the thirteen-month years a particular month will occur twice in a row. The doubled month need not be the same in all thirteen-month years, but it should be the same for all years that have the same position in the nineteen-year cycles. By distributing the thirteen-month years evenly in the nineteen-year cycle one can keep the beginning of the lunar year as close as possible to a particular point in the solar year. Years can be named in any traditional way, for example according to magistrates or regnal years.

The beginnings of the lunar months can be determined by observation, for example by the sighting of the new moon crescent. If this is the practice, then the Metonic cycle only regulates the beginnings of years and the naming of months. Alternatively, one can set out a fixed pattern of thirty-day ("full") months and twenty-nine-day ("hollow") months covering the entire 235-month cycle. A Metonic cycle of this kind additionally regulates the beginnings of months and the naming of days. For it to work well,

 $¹$ The literature on the Callippic periods is extensive. Because for the sake of clarity I have eschewed a historiographical</sup> manner of presentation in this article, I should acknowledge that there is little in sections 1 and 2 that someone has not said before. Older discussions and reconstructions are reviewed in sufficient detail by Ginzel, *HMTC* v. 1, 409–419. Fotheringham (1924) is fundamental to more recent work; see also Samuel (1972) and van der Waerden (1960) and (1984). The interesting, complicated historical reconstruction of Goldstein & Bowen (1989) is based on a false premise, as I have shown in Jones (1997) 157–158 and 166 n. 25.

² See the parapegmata of pseudo-Geminus (Geminus ed. Manitius, 210–232) and Ptolemy (*Opera astronomica minora*, ed. Heiberg, 3–67) *passim*. Callippus is specifically credited with the principle of a 76-year calendrical cycle by Geminus (ed. Manitius, 120–122), who does not however mention the Callippic periods as such. Goldstein & Bowen (1989) elaborate a theory that the Callippic periods were instituted only late in the third century B.C.

one has to have the total number of days in the cycle correspond as nearly as possible to the number of days in an observed interval of 235 lunar months, or alternatively in an observed interval of nineteen solar years. Moreover the distribution of full and hollow months should be, broadly speaking, uniform through the cycle.

There is, however, no need to count the number of days in an actual Metonic cycle; it will suffice to have an estimate of the length of the average month or of the year. Taking the approximation

 $1 \text{ year} = 365 \frac{1}{4} \text{ days}$ (1) we obtain the relationship:

 235 months = 19 years = 6939 3/4 days (2)

To have a cycle comprising a whole number of days, we have to multiply these numbers by four:

940 months = $76 \text{ years} = 27759 \text{ days}$ (3)

This is the derivation of the Callippic periods. It stands to reason that a practice of naming years according to their position in 76-year periods ought to go along with a lunar calendar in which the sequence of months is regulated by a Metonic cycle repeated four times, and the distribution of full and hollow months is determined so as to satisfy relation (3). Without regulation of the lengths of the months and hence the total number of days in a cycle, the 76-year period would have no advantage over the 19-year period.

2. The oldest Callippic dates.

In the following we shall use the abbreviation CP1 for "first period according to Callippus," and so forth. The Callippic dates in Ptolemy's *Almagest* belong to CP1 (year 1 = 330/329 B.C.), CP2 (year 1 = 254/253 B.C.), and CP3 (year $1 = 178/177$ B.C.). All are dates of astronomical observations. The earliest four, a set of observations of the moon by a certain Timocharis,³ are as follows (*Alm.* VII 3, ed. Heiberg v. 2, 25–32):

[1] Again, Timocharis, who observed at Alexandria, says that in year 36 of the first period according to Callippus, on Poseideon 25, which is Phaophi 16, when the tenth hour was beginning, the moon was seen with great accuracy as having overtaken with its northern rim the northern one of the stars on the forehead of Scorpius. [According to Ptolemy, this was Era Nabonassar 454, Phaophi 16/17, i.e. 295 B.C. December 20/21; the date is confirmed by the astronomical content.]

[2] Again, Timocharis, who observed at Alexandria, records that in year 36 of the first period according to Callippus, on Elaphebolion 15, which is Tybi 5, as the third hour was beginning, the moon overtook Spica with the middle of the part of its rim that points towards the equinoctial rising, and Spica traversed it, cutting off exactly one third of its diameter on the north side. [According to Ptolemy, this was Era Nabonassar 454, Tybi 5/6, i.e. 294 B.C. March 9/10; the date is confirmed by the astronomical content.]

[3] Timocharis, who observed these things at Alexandria, records that in year 47 of the first 76 year period according to Callippus, on Anthesterion 8, which is Hathyr 29 according to the Egyptians, as the third hour was ending, the southern half part of the moon was seen to cover exactly either the trailing third or the trailing half of the Pleiades. [According to Ptolemy, this was Era Nabonassar 465 Hathyr 29/30, i.e. 283 B.C. January 29/30; the date is confirmed by the astronomical content.]

³ The *Almagest* also preserves a report of two observations of Venus in 272 B.C. by Timocharis (X 4, ed. Heiberg v. 2, 310–311), lacking any trace of Callippic dating, as well as undated measurements of stellar declinations (VII 3, ed. Heiberg v. 2, 19–23). A scholion to Aratus (*Scholia in Aratum Vetera* ed. Martin, 213) suggests that Timocharis wrote a description of the constellations or catalogue of stars. His name appears in broken context at the end of an arithmetical scheme for predicting Venus' motion in longitude, *P. Oxy.* LXI.4135 in Jones (1999).

[4] Likewise, in year 48 of the same [i.e. the first] period, he says that on Pyanepsion 6 waning $[\tau \hat{\eta} \zeta' \varphi \hat{\theta}$ (vov $\tau \circ \zeta$, i.e. the 25th], which is Thoth 7, when as much as half an hour of the tenth hour had passed and the moon was risen above the horizon, Spica was seen to touch the very northern part exactly. [According to Ptolemy, this was Era Nabonassar 466, Thoth 7/8, i.e. 283 B.C. November 8/9; the date is confirmed by the astronomical content.]

Ptolemy gives the date of each observation twice: once at the beginning of the report, and a second time in his ensuing analysis. The second version gives the date according to Ptolemy's own preferred convention, using Egyptian calendar months and days, with years counted serially from the first (Egyptian!) regnal year of the Babylonian king Nabonassar (reigned 747–734 B.C.).4 Since the observations are all nocturnal, Ptolemy avoids ambiguity by giving the day numbers of both the preceding and following days. Such dates are fully in our control, so that we know their exact equivalents in the Julian calendar of modern chronology. The date that appears as part of the observation report, on the other hand, has the year number in CP1, an Athenian month and day number, and also an Egyptian month and day number.5

Egyptian dates have no natural affinity with the Callippic periods. The Egyptian year, which is a constant 365 days, is neither lunar nor solar, and fails to coordinate with the Metonic cycles on which the Callippic reckoning is based. Athenian months, on the other hand, were at least notionally lunar, although there is extensive evidence that the actual civil calendar of Athens was often several days out of synchronization with the phases of the moon.6 Athenian months are in any case highly anomalous in an Alexandrian context; and our suspicions are further aroused by the circumstance that Timocharis' Athenian months are consistently in line with the moon's phases. There seems no alternative to supposing that the dates in the observation reports of Timocharis are not really according to the Athenian calendar that was in civil use in Athens contemporary with Timocharis' activity, but belong to a special lunar calendar employing the Athenian month names but regulated in such a way that it was possible for an astronomer in Alexandria to know precisely when each month began.7 This must be the calendar associated with the Callippic reckoning.

In the remainder of this article, I use the unqualified term "Athenian calendar" to refer to this regulated lunar calendar employing Athenian months names and associated with the counting of years in Callippic periods. "Athenian year," "Athenian month," and "Athenian day" will signify the chronological units of this artificial calendar.

The reports give evidence of when these Athenian days and months were supposed to begin. Since the Egyptian day number is always the same as the first of the two day numbers that Ptolemy later records, it is clear that here the night is treated as the continuation of the preceding Egyptian day. Each time, Ptolemy baldly equates the Egyptian date with the corresponding Athenian date, *before* specifying the time of the observation; the plain sense of the words is that the Athenian and Egyptian days were

⁴ For Ptolemy's dating convention and the regnal canon on which it is based, see Depuydt (1995).

⁵ The months of the Egyptian and Athenian calendars are listed here for the reader's convenience:

Egyptian: (I) Thoth, (II) Phaophi, (III) Hathyr, (IV) Choeac, (V) Tybi, (VI) Mecheir, (VII) Phamenoth, (VIII) Pharmuthi, (IX) Pachon, (X) Payni, (XI) Epeiph, (XII) Mesore, followed by Epagomenae 1–5.

Athenian: (I) Hekatombaion, (II) Metageitnion, (III) Boedromion, (IV) Pyanopsion [Pyanepsion], (V) Maimakterion, (VI) Poseideon, (VII) Gamelion, (VIII) Anthesterion, (IX) Elaphebolion, (X) Mounychion, (XI) Thargelion, (XII) Skirophorion.

 6 Pritchett & Neugebauer (1947) 14–23. Dunn (1998) is offered as a corrective to Pritchett's contention that the civil calendar was extensively subjected to capricious insertions of days on the part of the archons, but does not contradict the broad thesis that the civil calendar often lagged behind the moon's phases by a variable number of days.

 7 To my knowledge, only Neugebauer (1975) v. 2, 616–618 has maintained that the dates in these observation reports are in the civil calendar of Athens; see however the reservations in note 10a on p. 617. It is curious that in observation [4] the Athenian day number is expressed using the traditional backward count for the last decad of the month, whereas in the other three it is simply the ordinal number.

coextensive, both beginning at sunrise.8 Now if we extrapolate backward from the known equivalences of the Athenian dates with dates in the Julian calendar to find what Julian calendar date corresponded to the first day of each Athenian month, we find in all four cases that the conjunction of the sun and moon took place during that day, or late during the preceding night.

In the foregoing section of this paper we concluded that in the Callippic calendar the number of days in a month was determined by a rule, so that the distribution of 30-day and 29-day months was spread out evenly. The actual moments of conjunction and the dates of the moon's last and first visibility are less uniformly spaced. A schematic calendar cannot be constructed so that the first days of its months always coincide with, say, the new moon crescent; the best that can be achieved is to have coincidence more often than not. At the latitude of Athens the interval between the morning when the waning moon is last seen and the evening when the new moon appears is most often three and a half days, less often two and a half, occasionally four and a half, with the conjunction occurring towards the middle of the interval of invisibility.9 Fotheringham and van der Waerden conjectured that the Callippic months began with the day of mean conjunction, which differs from the true conjunction by at most about half a day.¹⁰ Whether it is plausible that a calendar in other respects modelled on a civil calendar would have tied its months to such an abstract, unobservable moment is perhaps open to doubt.¹¹ What is not in dispute is that months that tend to begin on the day of conjunction are unlikely to have been designed to coincide with the new moon crescent, though they might be meant to coincide with the first morning when the moon is *not* seen.

The years of the civil calendar of Athens began about midsummer, generally (though at least one exception is known) with the first month following the summer solstice.12 Now in 330 B.C., the year within which CP1 year 1 began, a conjunction took place on June 28 at 1:44 A.M. U.T. (about two hours later local time for the eastern Mediterranean), while the moment of solstice was close to midnight U.T., less than two hours earlier.13 We need not suppose that Callippus, or whoever instituted the Callippic reckoning, was able to determine the moment of solstice or conjunction with this precision. Nor do we have to assume that the beginning of the Callippic months was *defined* as the day of conjunction, merely inferring from the four examples in the *Almagest* that the two generally coincided. Then it seems obvious that the Callippic cycles were set up so that in year 1 the time interval between the beginning of the first month of the year and the preceding solstice was minimum, effectively zero.

⁸ It is obviously preferable to take the text on its own terms rather than to force a reading of it based on a prior assumption that we know when the "Greek day" began, especially given that reputable scholars have with equal conviction maintained both a sunset epoch and a sunrise epoch. See Pritchett (1982) 262–263 for discussion and references. Goldstein & Bowen (1989) 281–282 confuse the issue by analysing the dates assuming a sunrise epoch and then hypothesizing a sunset epoch.

⁹ A good elementary presentation of the astronomical criteria determining conjunctions and the visibility of the lunar crescent is Parker (1950) 1–8. For practical purposes dates of first or last visibility for a particular locality may be estimated by means of tables such as those in P. V. Neugebauer (1929) (reproduced and evaluated favourably against Babylonian astronomical records by Huber (1982) 24–26). Using these tables I obtained results similar to those of Dunn (1998) 217, who estimates that on average in ten lunations there will be three intervals of 2 1/2 days, six of 3 1/2 days, and one of 4 1/2 days from last visibility to first visibility. Conjunction can occur anywhere within about half a day of the midpoint of the interval of invisibility, as illustrated by Parker (1950) 14.

¹⁰ Van der Waerden (1960) 178.

 11 So Goldstein & Bowen (1989) 279.

 12 Pritchett (1970) 64–66. The only explicit ancient statement of the rule that the year begins with the first month after the summer solstice is Plato, *Laws* VI 767c, where Plato is describing the calendar of an ideal state, not Athens. The pattern of intercalations in the Athenian civil calendar from the late first century B.C. to the middle of the second century A.D. fits the solstice rule very well (see note 45 below).

¹³ Solstice dates can be obtained from Tuckerman (1962) and (1964). Dates and times of conjunctions are tabulated in Goldstine (1973). For a solstice and conjunction to occur as close together as they did in 330 B.C. is an exceedingly rare event, as remarked by Rawlins (1991) 49 n. 1; but I rather doubt whether Callippus could have known that.

From relation (2) or (3) we know that 1 solar year is approximately 12 7/19 months. Hence year 1 of a Callippic period has to have thirteen months if the next solstice is not to fall within the first month of year 2. Continuing in this way through subsequent years, we can establish which years of a Callippic period must have had thirteen months, assuming (a) that the solstice always falls in the last month of the year, and (b) that the solstice precedes the beginning of the year by a minimum in year 1. The pattern for the first 19 years of a Callippic period is of course repeated in the remaining three Metonic cycles that the period contains.14

Table 1. 13-month years in the Callippic cycle.

The four Callippic dates of Timocharis are consistent with this pattern. For example, observations [1] and [2] are respectively in months VI and IX of CP1 year 36. This ought itself to be a 13-month year, and is preceded by thirteen such years. If we count the lunar months between June 28, 330 B.C. and the conjunctions preceding the observation dates, we find that they are the expected numbers, presuming that thirteen intercalary months occurred in the intervening time. Hence the intercalary month of year 36 (and by implication 17, 55, and 74) would have to be a repetition of one of the last four calendar months. Similarly we find from observation [3] in CP1 year 47 month VIII and observation [4] in CP1 year 48 month IV that there must have been an intercalary month between them, and since year 47 should have had thirteen months, the intercalary month in years 9, 28, 47, and 68 would have to be one of the last five calendar months.

3. The new Callippic dates.

The foregoing seems to be the limit of what can be securely deduced about the Callippic calendar from the four Callippic dates of Timocharis' observations, taken together with what I consider to be a minimum of plausible supplementary assumptions. Ptolemy associates some seventeen later observation reports, all of which he derived directly or indirectly from various writings of Hipparchus, with specific years in CP1, CP2, and CP3, but none is given an Athenian month. Indeed it has been asserted that the Callippic calendar—as distinct from the mere counting of years by Callippic periods—was abandoned by Hipparchus if not by his predecessors.15 The fact that Ptolemy cites no Callippic dates of any kind after the last one reported for Hipparchus (CP3 year 50, 128 B.C.) also seems to suggest that reckoning by Callippic periods was obsolete by the end of the second century B.C.

The recently published *P. Oxy.* LXI.4137 shows how precarious such inferences can be.16 This manuscript, written in a first-century-A.D. hand, is a fragment of a lunar eclipse canon, that is, a list of future dates of certain full moons (called "lunar eclipse possibilities") when the moon is nearest to the plane of the sun's apparent orbit, so that an eclipse of the moon may occur.17 Lunar eclipse possibilities occur usually at intervals of six lunar months, sometimes of five, and about half of them will be the occasions of lunar eclipses visible in a particular locality. The papyrus preserves most of the detailed

¹⁴ Van der Waerden (1960) 176–177 deduces the intercalations in the Metonic and Callippic calendars (which he presumes to have been the same) from the attested dates, rather than by assuming *a priori* that the solstice was kept in the last month, as I have done following Fotheringham. He arrives at the same pattern, with possible alternatives at three stages in the cycle.

¹⁵ Toomer (1984) 13; Goldstein & Bowen (1989) 287.

¹⁶ Text and commentary in Jones (1999) v. 1, 87–94 and v. 2, 16–17.

¹⁷ On the importance of eclipse possibilities in ancient eclipse theory see Aaboe (1972) and Britton (1989).

forecast of a partial eclipse, followed by the date information of the subsequent eclipse possibility. I repeat here the lines (18–21) giving the date (*x* represents lost numerals):

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\delta[ι[ὰ δ]ὲ μηνῶν ζ, (ἔτει) γ΄ κ[ c. 5 letters κατὰ
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 δ έ Κάλλιπ(πον) της ζ πε[ριό(δου) (έτει) *x*, διχό-

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20 μηνος μηνὸς Θαργ[ηλιῶνος, μη-
   \vec{v} \vec{o} \vec{c} \vec{a} \vec{b} \vec{a} \vec{b} \vec{b} \vec{c} \vec{b} \vec{c} \vec{b} \vec{c} \vec{b} \vec{c} \vec{b} \vec{c} \vec{
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At an interval of 6 months, in the year 3? [according

to Callippus [the *x*th year] of period 6, 20 mid-month of the month Thargelion,

and in the Egyptian month Epeiph [(night of day) *x* leading into *x*

Thus we have a year specified in CP6, three periods later than the latest Callippic date in the *Almagest*, accompanied by an Athenian and Egyptian date just as in the reports of Timocharis' observations more than three centuries before. (I do not know what the damaged year number at the end of line 18 was.)

In spite of the loss of the year numbers in lines 18 and 19, we can determine the date of this eclipse possibility by combining the information in these lines with the eclipse forecast that precedes them. Extrapolating by 76-year intervals from the known beginnings of CP1–3, we find that CP6 year 1 began in A.D. 51, and CP6 year 76 ended in A.D. 127. If the eclipse possibility of lines 18–21 fell in Thargelion (month XI) of a Callippic year, the eclipse described in lines 1–20, which was six lunar months earlier, took place in the same Callippic year. We therefore have to look for a lunar eclipse within the interval A.D. 51–127 that fitted the forecast in the papyrus, within the range of accuracy that can be expected of an eclipse prediction in the early Roman period.18

According to the papyrus, during the eclipse in question the moon was going to be shadowed from its south side to an extent of less than two-thirds its diameter, and it would be situated in the constellation Gemini between two specific stars, one of which can be identified as Gem. Only one eclipse visible in Egypt during CP6 fits this description, namely the eclipse of magnitude 5.3 digits that took place on the night of December 10/11, A.D. 56, with mid-eclipse at about 1:40 A.M. local time. The conjunction six lunar months after this one, during the night of June 5/6, A.D. 57, was also, as it happens, the occasion of a total eclipse visible (except for its beginning) in Egypt. The equivalent date in the civil Egyptian calendar was Payni 11/12, and in the old (unintercalated) Egyptian calendar, Epeiph 1/2. Thus the Egyptian date in the papyrus is in the old calendar, retained as it often was in astronomical contexts because of its convenience for chronological calculations.

We can restore the date in line 19 as CP 6 year 6. This should have been a thirteen-month year according to the hypothetical reconstruction given above. Thargelion is the eleventh month of the Athenian year, but the full moon in question was the twelfth since the preceding summer solstice; so an intercalary month was evidently inserted duplicating one of the first ten months of the year. Combining this with what we know about the intercalations in years 9 and 17, we can infer that either the intercalated month was a repetition of the ninth or tenth month (Elaphebolion or Mounychion) or different months were intercalated in different years of the cycle. The Babylonian civil calendar, which was regulated by a Metonic cycle from the early fifth century on, had such a non-uniform pattern, with most intercalations repeating the twelfth month but one of them repeating the sixth.19 Later in this article I will show evidence that suggests that the intercalated month in year 6 (and 25, 44, and 63) of a Callippic

¹⁸ Reliable data on lunar eclipses in antiquity may be obtained from Liu & Fiala (1992). For full details on the dating summarized here, see my commentary in the edition of the papyrus.

¹⁹ Parker & Dubberstein (1956) 1–3 and 6. It is generally accepted now that the 19-year cycle was used to regulate the Babylonian calendar as far back as 498 B.C.; see Bowen & Goldstein (1988) 42 note 17.

period was the sixth month, Poseideon, which would leave it open for the intercalations in other years to have been of the twelfth month, Skirophorion.²⁰

"Mid-month" (δ ιχόμηνος) restored in lines 19–20 is probably intended as a day number in the Athenian month rather than merely as the tautologous observation that the eclipse happens at full moon. In some Greek calendars, conspicuously that of Rhodes, the fifteenth day of a month is named δ ιχομηνία, and I imagine that a variant of this nomenclature is in use here. On the supposition that the night belongs to the preceding day, as in the observation reports of Timocharis, Thargelion 1 would have corresponded to May 22, A.D. 57. The conjunction took place on the morning of that day.

The discovery of a Callippic date in the middle of the first century A.D. justifies us in looking for other Callippic dates that might have passed unnoticed. One document that may contain unrecognized Callippic dates is the planetary almanac *Tab. Amst.* inv. 1, published in 1977 by Neugebauer, Sijpesteijn, and Worp.²¹ This is a table of dates in the Egyptian calendar when each of the five planets Saturn, Jupiter, Mars, Venus, and Mercury were calculated as crossing from one zodiacal sign to a neighbouring sign. Two years of sign entries are recorded, the second of them being identified as year "2". In the original publication of the table, Neugebauer did not succeed in finding dates for which the information in the almanac fitted the planets' actual motions as calculated by modern theory. Subsequently I showed that the recorded positions for all the planets except Mercury were approximately correct for the years $26/25$ and $25/24$ B.C.²² I was then unable to explain why the year $25/24$ B.C., i.e. regnal year 6 of Augustus, should be numbered as year 2. But CP5 year 1 began in 26 B.C., so that it is very tempting to see the year numeration of the almanac as a Callippic numbering applied to Egyptian years. Whether the Egyptian years belong to the old or reformed Egyptian calendar cannot be determined, since the two calendars were either coincident or divergent by only one day in those years.

The recto of the demotic papyrus P. Berol. 13146+13147, published by Neugebauer, Parker, and Zauzich in 1981, is a lunar eclipse canon very similar in character to *P. Oxy.* LXI.4137, but covering dates between 85 and 74 B.C.²³ Neugebauer dated the eclipse possibilities securely using the astronomical information in the papyrus. In the papyrus they are dated by a year number and an Egyptian month and day (the Egyptian dates are correct). Neugebauer also noted two peculiarities about the year numbers. First, the numeration, such that year 18 corresponds to 85/84 B.C. and so on up to year 28 (75/74 B.C.), fits no king's regnal years nor any other epoch known to Neugebauer. Secondly, the year numbers fall out of synchronization with the Egyptian months and days. This phenomenon can be seen in the following list of date correspondences, in which the year numbers 24 through 26 seem to be written next to dates belonging to the preceding years:

Year	Egyptian Date	Julian Date
18	$[IV]$ 17	85 B.C. Dec 28
	X 14	84 Jun 23
[1]9	[IV] 6	84 Dec 17
	IX $[4?]$	83 May 14?
20	II 26	83 Nov 7
	[VIII 23?]	82 May 3?
$\lceil 21 \rceil$	$[II]$ 15	82 Oct 27
	VIII 12	81 Apr 21
22	II4[?]	81 Oct 15?

 20 Reduplications of five of the twelve months of the civil calendar of Athens are attested in inscriptions; see Pritchett (1970) 63. The most frequently intercalated month was Poseideon.

²¹ Neugebauer, Sijpesteijn, & Worp (1977).

²² Jones (1993).

²³ Neugebauer, Parker, & Zauzich (1981).

Table 2. Dates in P. Berol. 13146+13147.

Both anomalies disappear with the realization that these too are Callippic dates, counted from year 1 of CP4 (which began in 102 B.C.). Although no Athenian months are specified in the papyrus, the years are reckoned from the midsummer conjunction, and this explains why years 24–26 began before the beginning of the Egyptian calendar year. This document is very likely an abridgement of a canon, perhaps originally written in Greek, that give the dates in the double Athenian-Egyptian format.

4. Callippic dates in Hipparchus' work.

Now let us return to the remaining Callippic dates in the *Almagest*, all of which are connected in one way or another with Hipparchus (mid second century B.C.). As the demotic eclipse canon has shown, it is sometimes possible to find an indication of when a year was considered to have begun, even in the absence of the month and day. This is also true of several reports of observations of equinoxes and solstices in the *Almagest*:

[5] (*Alm.* III 1, ed. Heiberg v. 1, 206–207) Summer solstice observed by Aristarchus in CP1 year 50, "as the year was coming to an end" (τῷ ν' ἔτει λήγοντι). The year is equated by Ptolemy with the 44th year (inclusive) from the death of Alexander, i.e. 281/280 B.C. Since the beginning of the next Egyptian calendar year was November 1, 280 B.C., the Callippic year is evidently Athenian.

[6] (*Alm.* IV 11, ed. Heiberg v. 1, 344–346) Lunar eclipses observed at Alexandria in:

(a) CP2 year 54, Egyptian Mesore 16/17 in year 547 from Era Nabonassar = 201 B.C. September 22/23

(b) CP2 year 55, Egyptian Mecheir 9/10 in year 548 from Era Nabonassar = 200 B.C. March 19/20

(c) CP2 "same year" (as the foregoing), Egyptian Mesore 5/6 in year 548 from Era Nabonassar = 200 B.C. September 11/12

The Athenian year 54 of CP2 ended in July 200 B.C., while the Egyptian year ended on October 12. Ptolemy's text implies that a change of Callippic year fell between late September and late March, which does not fit the Athenian year but does fit the Egyptian year if we understand Callippic year *x* to mean the Egyptian year that begins *before* the beginning of the *x*th Athenian year counting from the beginning of the Callippic period. These eclipse observations were used by Hipparchus.

[7] (*Alm.* III 1, ed. Heiberg v. 1, 195–196) Autumnal and vernal equinoxes reported by Hipparchus:

(a) CP3 year 17, Egyptian Mesore 30 = 162 B.C. September 27

(b) CP3 year 20, Egyptian Epagomenae 1 = 159 B.C. September 27

(c) CP3 year 21, Egyptian Epagomenae 1 = 158 B.C. September 27

(d) CP3 year 32, Egyptian Epagomenae 3/4 = 147 B.C. September 26/27

(e) CP3 year 32, Egyptian Mecheir 27 = 146 B.C. March 24

(f) CP3 year 33, Egyptian Epagomenae 4 = 146 B.C. September 27

(g) CP3 year 36, Egyptian Epagomenae 4 = 143 B.C. September 26

(h) CP3 year 43, Egyptian Mecheir 29/30 = 135 B.C. March 23/24

(i) CP3 year 50, Egyptian Phamenoth $1 = 128$ B.C. March 23

The equivalences in the Julian calendar are established from Ptolemy's statement (ed. Heiberg v. 1, 204) that the year of observations (d) and (e) was the 178th year, inclusive, from the death of Alexander. Since CP3 year 1 began in 178 B.C., CP3 year 17 therefore began in 162 and ended in 161 B.C. The Egyptian calendar date given by Ptolemy for observation (a) is, however, very close to the end of an Egyptian year, which shows that the Callippic year was not the Egyptian year, but began earlier. The same conclusion follows from the other five observations of autumnal equinoxes.

[8] (*Alm.* VI 5 and 9, ed. Heiberg v. 1, 477–478 and 526) A lunar eclipse observed at Rhodes, probably by Hipparchus, in CP3 year 37, Egyptian Tybi 2/3 in year 607 from the Era Nabonassar = 141 B.C. January 27/28. The Callippic year could be either Athenian or Egyptian.

[9] (*Alm.* III 1, ed. Heiberg v. 1, 207) Summer solstice observed by Hipparchus in CP3 year 43 "as the year was coming to an end." The observation was 145 years after [5], which places it in 135 B.C. As in the case of [5] the Callippic year must be Athenian, since the Egyptian year did not end until September 25.

[10] (*Alm.* VII 2, ed. Heiberg v. 2, 15) Hipparchus' measurement of the elongation of Regulus from the summer solstitial point, made in CP3 "about" ($\kappa \alpha \tau \dot{\alpha}$) year 50, which according to Ptolemy was "about" $(\pi \circ \circ \circ)$ 265 years before the reign of Antoninus Pius (A.D. 137/138), i.e. 129/128 B.C. I assume that the qualification in the time interval reflects only the vagueness of the first date. This report yields no information about the beginning of the year.

[11] (*Alm.* V 3, ed. Heiberg v. 1, 363) Hipparchus' measurement of the longitudes of the sun and moon in CP3, year 50, Egyptian Epeiph 16 in year 620 from the Era Nabonassar = 128 B.C. August 5. The Athenian year 50 in CP3 ended in July, 128 B.C., but the end of the Egyptian year fell on September 23. Thus the date of Hipparchus' observation does not fall within the 50th Athenian year, but is within the Egyptian year that began *after* the beginning of the 50th Athenian year.

We may summarize the situation as follows. Hipparchus' investigation of solar phenomena using observations of solstices and equinoxes (i.e. items [5], [7], and [9]) was among his latest known works, completed after 128 B.C. In it he certainly employed Callippic years together with an Athenian Callippic calendar, although Ptolemy has suppressed the Athenian months. The same may be true of other Hipparchian observations (items [8] and [10]). The years of the eclipse observations [6] apparently begin in advance of the Athenian Callippic years, whereas the year of Hipparchus' lunar observation [11] apparently began later than the Athenian Callippic year. The year number for [11] could have been affected by a scribal error in the transmission of the *Almagest*, and was indeed emended to 51 by Ideler.24 No simple emendation can remove the discrepancy in [6].25 In both instances, however, there could be a mistake on Ptolemy's part, or even on Hipparchus'. I do not believe that either provides sufficient foundation for concluding that Hipparchus adopted a reckoning using Callippic periods with Egyptian calendar years, in the teeth of the contrary evidence from the solstice and equinox observations.

5. Athenian months without Callippic periods.

I turn now to a handful of citations of dates involving Athenian months concerning which there is good reason to suspect that the calendar in question is a schematic one rather than the civil calendar of Athens. Since Callippic periods do not appear in these passages, it is an open question whether the dates were determined by the Callippic calendar, extropolated if required to the time preceding CP1.

Dionysius of Halicarnassus (I 63) discusses the date of the fall of Troy: "Ilium was captured when the spring was already coming to an end, 17 days before the summer solstice, on the 8th waning ($\phi\gamma\delta\phi\eta\phi\theta\gamma$ fivov toc, i.e. the 23rd) of the month Thargelion, as the Athenians reckon time; and there remained 20 days to complete that year after the solstice." Elsewhere (I 74) Dionysius informs us that he follows Eratosthenes' chronology, according to which Troy fell 407 years before the first Olympiad, i.e. 1184 B.C. (*FGH* 241F1).

This curious exercise in ancient historical chronology combines three givens: (a) that Troy fell to the Greeks in 1184 B.C., (b) that the event occurred in the late Spring, and (c) that the moon rose at midnight on the night in question, i.e. that the moon was at its last quarter phase.26 The calendrical information given by Dionysius must be based on a computation of the date of the solstice in 1184 B.C. according to a schematic version of the Athenian calendar. The situation, with the summer solstice falling twenty days before the end of the year, resembles the end of year 1 of a Callippic period. As we saw above, year 1 should be a thirteen-month year, hence approximately 384 days long, and the solstice, having previously occurred on the last day of the preceding year, would fall on the 365th day, i.e. approximately twenty days before the first day of year 2. Extrapolating backwards from the end of CP1 year 1 (329 B.C.) by eleven 76-year periods plus one 19-year period, we arrive precisely at 1184 B.C., which was therefore year 39 in a Callippic period and a year closely resembling CP1 year 1 in the relationship between the solstice and the lunar months.27 An intercalary month has to be assumed before Thargelion; and the beginning of Thargelion must have been taken to coincide approximately with conjunction rather than the new moon crescent to get the moon's last quarter on the night of the 23rd.²⁸

A fragment of the prefatory text from a parapegma excavated at Miletus cites the "summer solstice that occurred under (the archonship of) Apseudes, Skirophorion 13, which was Phamenoth 21 according to the Egyptians."29 Diodorus (XII 36) reports the Athenian version of the date (archonship of Apseudes, Skirophorion 13) as the epoch of the nineteen-year cycle ($\dot{\epsilon}$ yve α καιδεκαετηρίδα) established by Meton son of Pausanias. Finally, Ptolemy (*Alm.* III 1, ed.

²⁴ Ideler (1806) 217–218; see also Toomer (1984) 224 note 13.

²⁵ Ideler (1806) 216–217 forced the text to agree with expectation by emending in two places; see also Toomer (1984) 214 note 72.

²⁶ The lunar phase comes from a line of the *Ilias parva*; see Bernabé (1996) 78–79.

²⁷ This explanation of the passage in Dionysius was discovered by Dodwell (1704) v. 1, 50 note (*non vidi*); see also Pritchett (1970) 41. In fact the summer solstice in 1184 B.C. was only about fifteen days before the next conjunction. The discrepancy, which is due to the small inaccuracy of the 19-year cycle as an equation of lunar months and solar (tropical) years, shows that Dionysius' information could *only* have been obtained by use of the 19-year (and 76-year) cycles.

 28 Geminus ch. 9 (Manitius p. 128) considers the 23rd to be the latest possible date for this phase in a lunar month that begins during the interval of the moon's invisibility.

²⁹ Diels & Rehm (1904).

Heiberg v. 1, 205–206) refers to the "summer solstice observed by those about $(\tau \hat{\omega} v \pi \epsilon \hat{\rho})$ Meton and Euctemon," dating it by the archonship of Apseudes and the *Egyptian* date, Phamenoth 21, adding that it fell in the morning of that day. Ptolemy reckons 152 years between this solstice and the solstice of Aristarchus ([5] above), which makes the year in question 432 B.C. This is in agreement with the date of the archonship of Apseudes according to the reconstructed Athenian archon list for this period.30

The usual understanding of these reports is that Meton and his associate Euctemon instituted a 19-year calendrical cycle in the summer of 432 B.C., and that the summer solstice of that year in some sense marked the beginning of the cycle. The Egyptian date associated with the solstice by Ptolemy and the Miletus parapegma was probably not due to Meton; at least, it would be surprising if anyone working in Athens in the fifth century B.C. would have had knowledge of the operations of the Egyptian calendar, whereas there would have been good motives for later astronomers to attempt to determine an Egyptian equivalent for Meton's Athenian date.

Now if the point of the solstice date was to help regulate a calendrical cycle, it would only make sense for the date to belong to that calendar, rather than to the rather irregular civil calendar.³¹ Could Meton's calendar have been essentially the same in structure as the Callippic calendar? If we extrapolate backwards from CP1 year 1, we find that 433/432 B.C. (the "archonship of Apseudes") was year 50 of a Callippic period. From our hypothesis that a solstice was supposed to fall on the day before the beginning of year 1 of a Callippic period, the solstice in year 50 ought to fall on the 12th or 13th of the final month, and that agrees with the Metonic date.

The Egyptian date Phamenoth 21 in 432 B.C. is equivalent to June 27. In relation to this date, we know that the actual solstice occurred on the morning of June 28, and the previous conjunction was in the late morning of June 16. We found for the Callippic dates of Timocharis that the first day of the Athenian month tended to coincide with the date of conjunction; if we make this assumption here, then June 27 would be Skirophorion 12. I do not know whether this discrepancy is serious enough to worry about; it could be eliminated along with the error in the solstice date if we imagined that whoever established the Egyptian equivalent of Meton's Athenian date made an error of one day.

In *Almagest* IV 11 (ed. Heiberg, v. 1 340–343) Ptolemy cites from a lost work of Hipparchus three lunar eclipse observations that were "among those that had been brought from Babylon, as having been observed there." Each is assigned to an Athenian archon-year and Athenian month, without day number, and to an Egyptian date in a year counted, according to Ptolemy's convention, from the Era Nabonassar. The equivalances are the following:

[a] Phanostratos archon, Poseideon = Nabonassar 366 Thoth 26/27 = 383 B.C. December 22/23.

[b] Phanostratos archon, Skirophorion = Nabonassar 366 Phamenoth 24/25 = 382 B.C. June 18/19.

[c] Euandros archon, First Poseideon = Nabonassar 367 Thoth 16/17 = 382 B.C. December 12/13.

In the original Babylonian documents from which these reports ultimately derive, the months and years must have been according to Babylonian reckoning. Somehow and at some stage in the transmission the Babylonian dates have been replaced by Athenian ones. Even if this was done in Athens, it is incon-

³⁰ Samuel (1972) 207.

³¹ Bowen and Goldstein (1988) 64–72 discuss Meton's solstice at length, concluding that Meton's Athenian date was in the civil calendar, and that the Egyptian equivalent is in error by several days. They further maintain that Meton used the 19 year cycle to correlate a solar parapegma-calendar with the Athenian civil calendar. Such an approach seems to me unworkable, given the irregularity of the intercalary months and the large deviations from the moon's phases that the civil Athenian calendar exhibited.

ceivable that there existed the chronological means (let alone the motive) for translating Babylonian dates of the early fourth century B.C. into true Athenian civil dates. Again the Athenian months must derive from a schematic calendar, probably either the one instituted by Meton or the Callippic calendar extrapolated backwards. Since the Babylonian calendar was also regulated by a 19-year cycle of intercalary months, it would have been easy to correlate the months with Athenian counterparts. Establishing the day numbers in the Athenian calendar would, on the other hand, have been difficult or impossible, and so it is probably significant that no Athenian day numbers appear for these observations.

Extrapolating backwards, the archon-years of Phanostratos and Euandros are years 24 and 25 respectively in a Callippic period. Year 25 is a thirteen-month year according to the hypothetical scheme, and the observation report states that the month is "First Poseideon," i.e. it was followed by an intercalary Poseideon. This year has the same position in its 19-year cycle as the Callippic year CP6 year 6 in *P. Oxy.* LXI.4137, for which we had found that an intercalary month must have been inserted before Thargelion. These results are consistent, so that it is possible that the calendar of the Babylonian eclipse reports is essentially the same as the Callippic calendar.32

The parapegma fragment from Miletus that gives the Athenian and Egyptian dates of Meton's solstice goes on to state that Meton's solstice was a certain (lost) interval of time before "the one that occurred under (the archonship of) <Pol>ykleitos, Skirophorion 14, which was Payni 11 according to the Egyptians."

"Polykleitos" is the only possible restoration of the broken name, unless (which seems improbable) the year is in this instance specified by someone other than the Athenian archon. The archon-year of Polykleitos is securely dated from two independent sources to 110/109 B.C.³³ In 109 B.C. the Egyptian date Payni 11 corresponds to June 26; and if this is equated with Skirophorion 14, then Skirophorion 1 would have fallen on June 13. Conjunction occurred on the morning of June 14, so the Athenian month is set to begin a day before the conjunction. This is the same situation as we found for the Athenian date of the Metonic solstice. The actual solstice took place in the late morning of June 25.

It is probably no accident that the interval between the Metonic solstice and the one of 109 B.C. is 323 years, i.e. seventeen 19-year cycles. What is more, 323 times a year of 365 1/4 days yields 117,975 3/4 days, which, disregarding the fraction, turns out to be the precise number of days between the two dates.34 It seems likely, therefore, that the second solstice was calculated, not observed. Its Egyptian date could have been found by extrapolating from the Egyptian date of the Metonic solstice. The Athenian equivalent of the resulting date would presumably be according to an artificial regulated calendar which it was possible for someone in Miletus to correlate with the Egyptian calendar. It could be the Callippic calendar, although one would then have expected the pertinent year number, CP 3 year 69, to be specified in the inscription.

6. Geminus and the counting of days in months.

Several modern accounts of the Callippic calendar take as their starting point chapter 8 of Geminus' *Isagoge*.35 Geminus' status as an informant about the Callippic system is, however, less straightforward than that of the attested Callippic dates, and I have preferred to bring in his testimony only after we have seen what can be done without it.

³² Fotheringham (1924) assumed that the calendar of the Babylonian reports, which he believed to be Meton's, intercalated *only* Poseideon, whereas the Callippic calendar reconstructed from the observations of Timocharis intercalated *only* Skirophorion, hence that the two were distinct.In fact it is not even established with certainty that the intercalary month between Timocharis' observations [3] and [4] was Skirophorion, though it was definitely not Poseideon.

³³ For details see Samuel (1972) 44–46.

³⁴ Toomer (1974) 338.

³⁵ See in particular Fotheringham (1924), Samuel (1972), and van der Waerden (1960) and (1984).

Geminus begins by defining the term "month" as the interval from conjunction to conjunction or from full moon to full moon, and states its value as 29 1/2 1/33 days. This is close to 29 1/2. From this approximation Geminus derives the length of the lunar year as 354 days, and the alternation of "full" (30-day) and "hollow" (29-day) months, which he tells us is used for "civil" (oι κατά πόλιν) months.

Geminus then sets out a problem that he alleges that the astronomers $(\dot{\alpha}\sigma\tau\rho o\lambda\dot{o}\gamma o\tau)$ set themselves, and its motivation. The Greeks, he tells us, wished to regulate years by the sun, and months by the moon. The former means that religious festivals (associated with particular days in particular months) should always fall at the same seasons from year to year; the latter, that the days in each month should be named according to the moon's phases. He gives examples of the latter: the first day, on which the moon appears as new, is called vouµnví α , the second $\delta \epsilon$ u $\tau \epsilon \rho \alpha$, the day midway through the month $\delta y_0 \mu \gamma \alpha$, and the thirtieth (and last) day $\tau \rho \alpha \kappa \dot{\alpha} \zeta^{36}$ In accordance with this, solar eclipses occur on the day called τ pt α $\kappa \dot{\alpha}$, and lunar eclipses on the night before the day called δ ₁ γ ⁰ α .

At this point Geminus digresses in order to contrast the Greek preference for regulating the calendar by sun and moon with the Egyptian calendar of fixed 30-day months and 365-day years. The Egyptians, he tells us, wanted to have their festivals migrate gradually from season to season.

Returning to his main argument, Geminus describes a sequence of successive schematizations that the Greeks supposedly adopted in order to regulate the calendar. First, they used constant 30-day months and intercalary (13-month) years. Then this was replaced by the *oktaeteris*, based on the equation of 8 solar years with 99 months (3 intercalary) and 2922 days. He says this was derived from the assumption of a solar year of 365 1/4 days and a lunar month of 29 1/2 days: eight non-intercalary years of alternating full and hollow months would fall short of eight times 365 1/4 days by exactly 90 days, or 3 full months. The intercalary months were allegedly "already" (ἤδη) placed at roughly equal spacing, in the third, fifth, and eighth years of the cycle.

Geminus then describes successive corrections to the *octaeteris*: first an addition of 3 days after two cycles in order to correct for the inaccuracy of the assumed length of the month; then a deduction of one intercalary month after ten of these 16-year double cycles in order to compensate for the distortion in the assumed length of the year resulting from the first correction. Then, in a rather unclear paragraph (part of which Geminus' editor Manitius treats as an interpolation), Geminus dismisses the octaeteris, corrected or not, as basically irreconcilable with the true phenomena of the sun and moon.

The unsatisfactory character of the *octaeteris* is now said to have motivated "the astronomers about Euctemon and Philip and Callippus" (ot περί Εὐκτήμονα καὶ Φίλιππον καὶ Κάλλιππον άστρολόγοι) to adopt the *enneakaidekaeteris* or 19-year period. This is based on the equation of 19 solar years, 235 months (7 intercalary), and 6940 days. Geminus shows how they supposedly derived from this equation the need to make 110 of the months hollow. These they distributed evenly through the 19 years by a peculiar rule: days are counted as if all months were full, but after every 63 days, the next day number in the count is omitted ($\dot{\epsilon} \xi \alpha \iota \rho \dot{\epsilon} \sigma \iota \mu \rho \varsigma$). Nothing is said about which years in the cycle were supposed to be intercalary.

Lastly, Geminus says that the "astronomers about Callippus" corrected the 19-year cycle to obtain agreement with the assumed 365-day year, by deducting one day from every four 19-year periods (making up a 76-year period comprising 940 months and 27759 days). "They make," he adds, "the same placement of intercalary months," presumably as was used in the 19-year cycle.

³⁶ The nomenclature is that of the calendar of Rhodes, where Geminus is presumed to have lived. In chapter 9, however, Geminus states that the new moon may actually appear on the voum α , the second day, or the third day of the month.

In reading this chapter, one has to keep in mind that Geminus is fond of presenting historical reconstructions to explain elements of the astronomy of his time.³⁷ The narrative of chapter 8 bears the hallmarks of a rationalization, leading us by neat logical stages from an original difficulty encountered by people in the vaguely remote past, through successively better solutions, to the situation in the author's own time. It is noteworthy that after telling us that the astronomers were motivated to devise calendrical cycles by the desire to control the dates of religious festivals, which amounts to saying that the cycles were intended for civil calendars, Geminus has no more to say about civil or cult calendars; nor in fact does he mention specifically any one of the many local calendars in the Greek world, or assert that any of the cycles that he describes was adopted anywhere for civic purposes.

We are entitled to question Geminus' *bona fides* as a historian. What is less clear is how far Geminus can be trusted concerning the structure of the calendrical cycles. We now know that astronomers were still using the Callippic dating conventions during the first century B.C., when Geminus wrote [Jones (1999a)]. But Geminus seems to know nothing about a specific calendar whose year reckoning begins with an established epoch; his cycles could begin and end anywhere.

Geminus ascribes the 19-year cycle to Euctemon, Philip, and Callippus. The omission of Meton's name is not too disturbing, since our ancient sources seem to have had considerable trouble differentiating between Meton and Euctemon, and cite one, the other, or both at will. "Philip" is presumably the Philip who is frequently cited in parapegmata, and whom Ptolemy alleges observed weather phenomena in the Peloponnese, Locris, and Phocis.38 More surprising is Callippus' appearance here, but it is probably accounted for by the fact that Geminus regards the 76-year cycle as a minor modification of the 19-year cycle.

In Geminus' story, the Greek 19-year cycle was from the outset an equation of fixed numbers of years, months, and days, not merely of years and months as in the Babylonian calendar. Specific years in the cycle were given intercalary months (Geminus forgets to tell us what the pattern was), and each of the 235 months in the cycle was fixed at either 29 or 30 days. Geminus does tell us the rule for how the lengths of the months were determined. One began by pretending that all months of the cycle were "full" (30-day) months. The resulting total number of days, 7050, is 110 days more than the 6940 days assumed for the cycle. Since 110 is just under 1/64 of 7050, we therefore delete every sixty-fourth day from the calendar to obtain a cycle of the desired length. One might expect that the days in the months that were thus made hollow would have been numbered from 1 to 29. But Geminus expressly states that this was not so. Rather, one would skip over the day numbered 4 in one month, and the day numbered 8 two months later, and so forth.

Geminus tells us that when four 19-year cycles minus one day were put together to make a 76-year cycle by "the astronomers about Callippus," the placement of the intercalary months was not changed. He does not, however, say what became of the rule for deducting days. There are several imaginable possibilities. (a) One could have treated each of the four 19-year cycles separately, that is, beginning to count off every 64th day starting from the first day of years 1, 20, 39, and 58; but then one would have to remove one further day, it is not clear where. (b) One could have counted off every 64th day continuously from the first day of year 1 right through to the end of year 76, again with one more day removed somewhere. (c) Or one could have altered the rule so that at twenty-four points in the cycle the 63rd day is deleted instead of the 64th, counting continuously from the beginning to the end of the 76 year cycle.

 37 Other instances of this habit of Geminus are found in chapters 17 (on the origin of the parapegmata) and 18 (on an arithmetical model for the moon's variable apparent speed). The latter is discussed by Neugebauer (1975) 585–587, and in painstaking detail by Bowen & Goldstein (1996).

³⁸ Neugebauer (1975) 739–740.

Fotheringham adopted the second hypothesis as a basis for analysing the day numbers of Timocharis' observations.39 Thus in effect he reconstructed a complete hypothetical 76-year Callippic period of 940 months, leaving out CP1 year 1 Boedromion 4, CP1 year 1 Maimakterion 8, and so forth through CP1 year 76 Skirophorion 20, as well as the final day, CP1 year 76 Skirophorion 30. Then for each of the Timocharidian observations he lined up the date attested in the *Almagest* with his reconstructed calendar, and worked backwards to find what date in the Julian calendar would correspond to CP1 year 1 Hekatombaion 1. In each case he found the same initial date, June 28, 330 B.C. Fotheringham considered this agreement to be satisfactory confirmation that Geminus' rule (as Fotheringham interpreted it) applied to the Callippic calendar. Following him, van der Waerden was so confident of this reconstruction that he was ready to redate the archon-year of Polykleitos in order to force the Athenian date in the Miletus parapegma into agreement with Fotheringham's Callippic calendar. 40

The case for Fotheringham's reconstruction of the calendar thus rests on two arguments. First, there is the description of Geminus, which we may judge on its own merits to be either well informed or not. Secondly, there is the test of whether Fotheringham's specific interpretation of Geminus leads to a consistent date for CP1 year 1 day 1. The test by itself is not even-handed. If it fails, it decisively refutes a candidate for the Callippic calendar. If it succeeds, it merely increases the probability that the candidate is the right one. Out of the field of all plausible reconstructions of the calendar, it eliminates some from consideration, but does not discriminate among those that remain.

Now by general consent the Callippic calendar was a schematic lunar calendar, and what that means is that the full and hollow months were spread out more or less evenly, in a ratio of about 22 hollow months to 25 full months. This means that if we take an interval comprising *n* months in two different schematic calendars, the number of days in the interval will come out the same in both calendars well above half the time. This probability is essentially the same whether *n* is large or small, unless the two calendars are founded on significantly values for the mean length of the lunar month. Because of this, it takes a rather large number of test dates to determine the detailed structure of a schematic lunar calendar, such as we suppose the Callippic calendar to have been, with a small probability of error. A sampling of four dates will probably rule out only about half the candidates for the Callippic calendar.⁴¹

Hence the question comes back to whether Geminus' account is inherently credible. Opinions of competent historians have differed sharply on this matter. Neugebauer, for example, castigates Fotheringham for "blindly accepting one of these oversimplified stories written ad usum Delphini," maintaining that a calendar omitting days haphazardly in the middle of months would be unworkable for the astronomical purposes for which it was ostensibly invented.42 Van der Waerden contradicts Neugebauer, accepting Geminus as a trustworthy informant, and arguing that the rule of deleting every 64th day would have made it a straightforward arithmetical problem to calculate the number of days between any two Callippic dates.43

³⁹ Fotheringham (1924).

⁴⁰ Van der Waerden (1960) 179–180. His argument is that the equation Skirophorion 14 = Payni 11 is impossible for 109 B.C., whether the Athenian month is regulated by observation or by Fotheringham's reconstructions of the calendars of Euctemon (i.e. of Meton) or Callippus. In this paper, and still in van der Waerden (1984) 124, he appears not to have been aware of the evidence that fixes the date of Polykleitos' archon-year.

 41 For example Fotheringham's test would have given the same positive result if he had ignored Geminus' instructions about deleting days in the middle of months, and counted the days in hollow months from 1 to 29. (On the other hand, the test fails if the counting of deleted days begins afresh at the start of each 19-year cycle.) Goldstein & Bowen (1989) 282–282 found that Timocharis' dates agreed perfectly with a reconstruction of the Macedonian calendar (but using Athenian month names!) based on a calendrical cycle of 25 Egyptian years. On this see Jones (1997) 157–160.

⁴² Neugebauer (1975) 617. Similarly Toomer (1984) 13 describes the passage in Geminus as a "fiction", and Bowen & Goldstein (1988) 43 call it "obviously a reconstruction and a poor one at that."

⁴³ Van der Waerden (1984) 124.

In my judgement Geminus' rule for the deletion of days should neither be dismissed out of hand nor embraced unreservedly. The rule is certainly peculiar, and has no obvious parallel in what we know about ancient calendars. Whether it is impractical is harder to say, since we do not have certain knowledge of the original purpose of the Callippic calendar, or for that matter, of what Timocharis was doing with his lunar observations. Elsewhere Geminus' information about contemporary astronomy is verifiable;44 and in the present particular the emphasis he places on the deletion of days in the middle rather than at the ends of months implies that he regards it himself as anomalous.

7. A suggested history of the Callippic calendar.

Three hypotheses can be put forward for the invention of the Callippic calendar, and its putative predecessor, the calendar of Meton and Euctemon. (a) They may have been intended as regulated civil calendars, or more specifically as a regulated Athenian civil calendar. (b) They may have been connected with a parapegma. (c) They may have been designed to provide a stable and controllable chronological framework for astronomical and historical research.

Hypothesis (a) was long upheld by writers on Greek chronology, but had to be abandoned as evidence accumulated of the irregular, not to say capricious, course of the Athenian calendar.⁴⁵ Hypothesis (c) fits the fact that all attested Callippic dates are dates of astronomical observations or predictions. The calendrical information relating to the fall of Troy reported by Dionysius of Halicarnassus shows how a schematic lunar calendar could be applied to purely chronographical, as opposed to astronomical, questions.

If, however, we look at the evidence for the activities of Meton, Euctemon, and Callippus, we find that all were conspicuously makers of parapegmata. Van der Waerden and Toomer have suggested that their calendars were meant to be used in conjunction with the parapegmata.46 I think that this is a very plausible hypothesis, and deserves a fuller working out than has been given.

So far as we can tell, the earliest Greek parapegmata were arranged according to a year beginning with the day of the summer solstice.⁴⁷ The year was divided into twelve parts, named by the twelve zodiacal constellations. Each of these "zodiacal months" contained a fixed number of days, generally in the neighbourhood of 30, adding up to a total of 365 days. Appearances and disappearances of stars and constellations, and changes in weather, were marked next to specific days. In their original form of public inscriptions, the parapegmata kept track of the current day by means of a peg that was moved daily along a sequence of holes corresponding to the days.

To start off the parapegma, one would need to know when the summer solstice takes place. Rather than depend on repeated observations, the most efficient way to do this would be to establish a pattern setting out the dates of successive solstices according to the civil calendar. This unfortunately also requires that the civil calendar have a high degree of predictability. In Babylon, where months began with the new moon crescent and intercalary months were strictly regulated by a nineteen-year cycle, it was possible to construct a simple arithmetical rule, the so-called "Uruk scheme," for assigning dates to the solstices and equinoxes in each year of the cycle.48 If Meton and Euctemon intended to do

⁴⁴ Neugebauer (1975) 581–587.

⁴⁵ Pritchett & Neugebauer (1947) 7–10; Pritchett (1970) 44–50. Müller(1991) and (1994) shows that the attested intercalations from 124/123 B.C. until the mid second century A.D. fit a regular 19-year cycle. The pattern differs from our hypothetical Callippic calendar (Table 1) only in that in place of the intercalations in Callippic years 11, 30, 49, and 68, civil intercalations were made in the year after. Müller's Athenian cycle is what one would obtain if one required that the summer solstice falls always in the last month of the year, but that the solstice is approximately two and a half days earlier than was apparently assumed in the Callippic cycle. Such a shift is roughly correct for the first century B.C.

⁴⁶ Van der Waerden (1960) 177; Toomer (1974) 338.

⁴⁷ For a brief description of the parapegmata and references see Neugebauer (1975) 587–589.

⁴⁸ Neugebauer (1975) 354–366.

something similar, they would have been compelled to devise a regulated version of the Athenian calendar in which to embed the calculated solstice dates. Ideally it would have been desirable to impose the 19-year cycle on the civil calendar if the parapegma was meant for public use, but this evidently did not happen. Nevertheless there is every reason to expect that Meton would have made his calendar operate as closely to the civil calendar as was compatible with the principles of the regulation, since any unnecessary innovations would have only made it harder to tell where one was in the schematic lunar calendar, and hence in the solar calendar of the parapegma. It is not clear that the Metonic calendar would have had to define the sequence of full and hollow months or the total number of days in the cycle. The first day of the Metonic lunar month appears to have been approximately the day of conjunction, although it does not necessarily follow that Meton adopted an artificial astronomical definition of when the month began. I suspect that if we knew more about the operation of the Athenian civil calendar and other Greek calendars, we would find a similar tendency of the months to be in advance of the new moon crescent.

The only attested date that can confidently be identified as Metonic is the summer solstice observation of 432 B.C., on Skirophorion 13. We have no evidence that Meton instituted a system of numbering years to be used with either the lunar or the solar calendar. Because this was not a chronographical system, there was no awkwardness in having two distinct beginnings of the year, namely the solstice for the parapegma and the first day of Hekatombaion for the lunar calendar.

If Meton's 19-year cycle began with the solstice of 432 B.C., as is generally admitted, the choice of year appears arbitrary. Callippus' cycle, on the other hand, began with an astronomically significant date, June 28, 330 B.C., when the summer solstice and the beginning of the lunar month nearly coincided.49 This circumstance strongly suggests that the Callippic calendar was designed using an arithmetical scheme analogous to the Uruk scheme so as to keep the summer solstice always within the last month of the lunar year. The numbering of years sequentially from the initial year is an interesting innovation, the more so because it implied from the start that the counting would end with year 76. We know from the solstices of Aristarchus and Hipparchus cited in the *Almagest* that, for the purposes of numbering, the year was considered to begin not at the solstice but with the first day of the following lunar month.

The use subsequent astronomers made of the Callippic periods and calendar is remarkably uniform over an interval of more than three centuries. Particularly noteworthy is the practice, already in evidence with Timocharis and still maintained in *P. Oxy.* LXI.4137, of dating an event in both the Callippic Athenian calendar and in the Egyptian calendar. Egyptian dates were useful for calculating the intervals in days between events, whereas the Athenian dates related the event to the moon, and facilitated the translation of dates into and out of other lunar calendars such as the Babylonian. What seems anomalous now is that Ptolemy did not perpetuate the Callippic reckoning, choosing instead to operate exclusively with the Egyptian calendar and the Era Nabonassar.

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⁴⁹ Goldstein & Bowen (1989) 279, disregarding the astronomical significance of the year, suggest that 330 B.C. was chosen because Alexander assumed the title of Great King in that year. It is not easy to see, however, why a Greek calendar would have employed this political era, which is so far as I know unattested accept to the extent that some documents from Babylon counted Alexander's regnal years from that date.

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