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Another Greek Papyrus Concerning Babylonian Lunar Theory

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P.Cair. Mus. H.13.7 × W.2.9 Oxyrhynchus
S. R. 3059 (part) I AD?

Description

The papyrus (Fig. 1), whose provenance is Oxyrhynchus according to the Cairo Museum's archive, is light brown, broken on all sides, with four partial vertical breaks. At the top are a dark spot and some small holes. Some fibers are stripped off, especially on the right side. The papyrus is written across the fibers in black ink, continuing on the layer of horizontal fibers exposed by the stripping on the right. The letter strokes vary considerably in thickness, likely responding to the uneven surface. The papyrus contains remains of three lines of text at the top left (the third is extremely faint):[1]

]ρ[
]προφ̣α ̣[
] ̣ο̣υ̣// ̣[

Below this is an astronomical table consisting of 24 partially preserved rows separated by horizontal rulings in black ink; the ruling above the first row is approximately 1.5 cm below the top of the papyrus. There are also remains of a single vertical ruling in black running along the left edge of the papyrus, immediately to the left of the numerals. (The rulings are omitted in our transcription.) So far as we can tell, the writing at the top had no connection with the astronomical table; these remains are not counted in the line numbering of our transcription. The hand and layout are very similar – if not identical – to P.Colker (Fig. 2), an unprovenanced astronomical table of related contents paleographically dated to the first century AD; compare, e.g., the forms of alpha, zeta, eta, and kappa.[2] A new analysis of the contents of P.Colker renders it probable that the two fragments originally came from a single roll.[3] Unfortunately the recto of the Cairo fragment is mostly concealed by some sort of modern backing, which was perhaps applied to protect holes in the papyrus from further damage, though there are some very slight traces of writing visible.

Transcription and translation

ō	ις	κα	ι	0	16	21	10
ō	ιη	να	κ	0	18	51	20
ō	ιβ	γ⟦ ̣⟧	ν	0	12	3	50
ō	ε	ις	κ	0	5	16	20
ō	α	λα	ι	0	1	31	10
ō	η	ιη	μ	0	8	18	40
ō	ιε	ς̣	ι	0	15	6	10
ō	κ:	ς	κ	0	20	6	20
ō	ιγ	ιη	ν	0	13	18	50
ō	ς	λα	κ	0	6	31	20
ō	ō	ις	ι	0	0	16	10
ō	ζ	γ	μ	0	7	3	40
ō	ιγ	να	ι	0	13	51	10
ō	κ	λη	μ	0	20	38	40

[1] We are grateful to Prof. Cornelia Römer for transcribing these traces, as well as for confirming that the hand of the astronomical table is probably the same as that of P.Colker as discussed below.

[2] Neugebauer 1988; Jones 1997. Both papyri have the table on the back side, with similar ruling, and the line spacing in both (which is somewhat irregular) averages approximately 5.0 mm. An uncommon notation that they share is discussed in the note to line 8 of the present papyrus.

[3] See the accompanying article on P.Colker, Jones 2016.

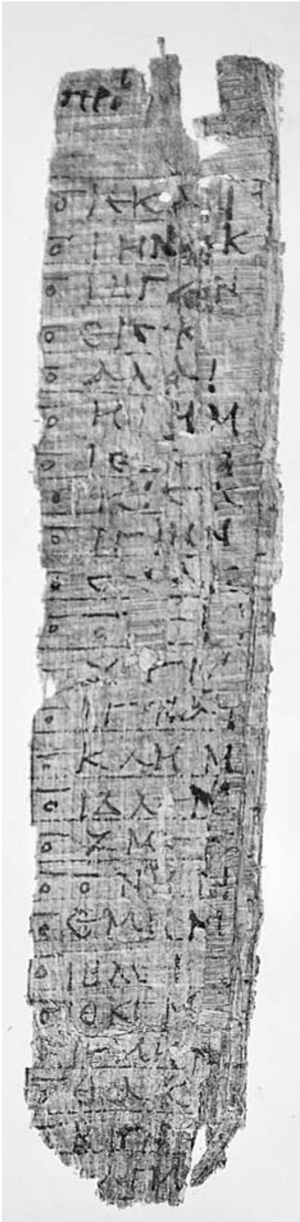

Fig. 1. P.Cair. Mus. S. R. 3059 (part)

Fig. 2. P.Colker

ō	ιδ	λγ	ν	0	14	33	50
ō	ζ	μς	κ	0	7	46	20
ō	ō	νη	ν	0	0	58	50
ō	ε	μη	μ	0	5	48	40
ō	ιβ	λς	ι	0	12	36	10
ō	ιθ	κγ	μ	0	19	23	40
ō	ιε̣	μη	ν	0	15	48	50
ō	θ̣	α	κ	0	9	1	20
ō	β	ιγ	ν	0	2	13	50
[ō]	δ̣	λγ	μ	[0]	4	33	40

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Notes

3. Between the gamma and nu there are strokes of ink that do not resemble any letter: an oblique stroke starting in the upper right and descending leftwards to the baseline, where it loops back to the right, running into a large ink blob out of which another small loop emerges on the right.

8. The colon-like pair of dots following kappa are a notation whose purpose is to indicate that the preceding letter, having a value in tens, is to be read as a separate numeral from the following letter which has a value in units, thus in this instance 20,6 rather than 26. The same notation occurs in P.Colker iii 12 and 29. Symbols with the same function, but different appearance, are attested in other astronomical papyri, for example a small raised circle in *P.Oxy. astr.* 4228 iii 3. It is tempting to trace the notation to an analogous practice found in Babylonian astronomical texts from Uruk, where the symbol employed was the same sign (GAM) that otherwise stood for zero as a sexagesimal place.[4]

Analysis and identification of the table

Any table consisting of rows of numerals never exceeding sixty and including the zero symbol (a small circle with a horizontal stroke above it)[5] is immediately identifiable as astronomical in purpose, with each sequence of four numerals to be interpreted as a whole number – here always zero – followed by a sexagesimal (base 60) fraction. This Greek sexagesimal notation was an adaptation of the sexagesimal notation of Babylonian mathematics and astronomy, though the Babylonians did not employ a conceptual or notational distinction between units and fractions.[6] In modern commentaries it is customary to separate the whole number from the fractional places by a semicolon, and the fractional places from each other by commas. For example we write the quantity in the first row as 0;16,21,10, and interpret this as $0 + 16/60 + 21/60^2 + 10/60^3$, which is approximately 0.272546….[7]

In the absence of any column heading or other explicit indication of what the numbers mean, we have to look for clues in the pattern they form. The values are alternately increasing and decreasing in stretches of six to seven rows, so they stand for some quantity that varies periodically in some sense. This may mean a periodic variation over time, since in many ancient astronomical tables successive rows represent steps forward in time, which may be at constant intervals (e.g. days) or less regular ones (e.g. dates of a planet's first morning visibility); in other tables, however, the vertical dimension represents a more abstract quantity, such as an angle in a theoretical planetary model.

Often the first thing to do when confronted with a column of sexagesimal quantities of unknown meaning in an ancient astronomical table is to calculate the line-to-line differences. (Incidentally it is best to do this in base 60 rather than converting everything to decimal, among other things because conversion results in many nonterminating decimals.) We do this for the values in the papyrus in Table 1.

Row	*Quantity*	*Difference*
1	0;16,21,10	
2	0;18,51,20	+0; 2,30,10
3	0;12, 3,50	–0; 6,47,30

[4] Neugebauer 1941; Ossendrijver 2012, 18–19.

[5] For variations of the symbol see Jones 1999, 1.62 fig. 16.

[6] Ossendrijver 2012, 17–19. The essential difference is that in a Greek astronomical text the first numeral is practically always the units part of the number, which may be anything from zero up to something in the hundreds or thousands written in standard Ionian non-place-value decimal notation, whereas in a Babylonian astronomical text the entire number is represented in base 60, and the first written numeral is usually the highest order non-zero place, whatever that may be. In other words, Babylonian sexagesimals are "floating point", and Greek sexagesimals are "fixed point". It is also worth noting that the Greek use of the zero symbol in astronomical texts is both more consistent than the Babylonian (since in Babylonian texts an empty sexagesimal place may simply be omitted or represented by a small vacant space) and more generalized (since in a Greek text a quantity of zero value can be represented by the zero symbol, whereas in Babylonian texts the symbol is employed only for empty sexagesimal places).

[7] For a general introduction to sexagesimal notation and arithmetic and the mathematical methodology of Babylonian astronomy, see Ossendrijver 2012, 17–54.

4	0; 5,16,20	–0; 6,47,30
5	0; 1,31,10	–0; 3,45,10
6	0; 8,18,40	+0; 6,47,30
7	0;15, 6,10	+0; 6,47,30
8	0;20, 6,20	+0; 5, 0,10
9	0;13,18,50	–0; 6,47,30
10	0; 6,31,20	–0; 6,47,30
11	0; 0,16,10	–0; 6,15,10
12	0; 7, 3,40	+0; 6,47,30
13	0;13,51,10	+0; 6,47,30
14	0;20,38,40	+0; 6,47,30
15	0;14,33,50	–0; 6, 4,50
16	0; 7,46,20	–0; 6,47,30
17	0; 0,58,50	–0; 6,47,30
18	0; 5,48,40	+0; 4,49,50
19	0;12,36,10	+0; 6,47,30
20	0;19,23,40	+0; 6,47,30
21	0;15,48,50	–0; 3,34,50
22	0; 9, 1,20	–0; 6,47,30
23	0; 2,13,50	–0; 6,47,30
24	0; 4,33,40	+0; 2,19,50

Table 1. Line-to-line differences of the quantities in the papyrus.

An arithmetical pattern begins to emerge: the majority of the tabulated values are either exactly 0;6,47,30 greater than the values in the immediately preceding rows, or exactly 0;6,47,30 less. The exceptions occur when the trend changes from increase to decrease or *vice versa*. This is a behavior characteristic of a so-called linear zigzag function, a type of numerical sequence especially characteristic of Babylonian astronomy.[8] In a linear zigzag function, a quantity is made to alternately increase and decrease by constant steps d between a minimum m and a maximum M. If the quantity is currently increasing and the next step would take it beyond M, the next value is exactly as much less than M as it would have been greater than M according to the normal increment. Conversely, if in a decreasing stretch the next value would be below m, the next value is as much greater than m as it would have been less than m. Graphing the resulting sequence results in a series of equally spaced points along a zigzag line bouncing back and forth between m and M, hence the name.

One of these bounces happens between rows 1 and 2. Before row 1, the quantity was increasing, so the expected value in row 2 would have been:

0;16,21,10 + 0;6,47,30 = 0;23,8,40

Instead we have 0;18,51,20, so M must be exactly halfway between 0;18,51,20 and 0;23,8,40, namely 0;21. We get the same value for M from looking at the pairs of rows 7–8, 14–15, and 20–21, which is as it should be if this is a linear zigzag function.

For row 5, we would expect 0;5,16,20 – 0;6,47,30, which would be a negative quantity, –0;1,31,10. Except for the minus sign, this is precisely what the table gives. The same thing occurs between lines 10–11, 17–18, and 23–24. We can think of this in two ways. We can say that m is exactly zero, so that the values make a cycle of alternate increase and decrease between 0 and 0;21 every six or seven rows. Alternatively, we can think of the transition from row 4 to row 5 as a change from functionally positive to functionally

[8] Neugebauer 1955, 1.30–32; Ossendrijver 2012, 42–47.

negative (or perhaps from negative to positive).[9] According to this interpretation, the bounce at the minimum m does not occur at rows 4–5 (or 10–11 etc.) but rather at rows 7–8, where m would be –0;21, equal to M but with opposite sign. This gives a linear zigzag function with a longer cycle of twelve to thirteen rows.

We can calculate the exact period or "wavelength" P of the zigzag function from the consideration that in this period the tabulated value has to increase from m to M and decrease again to m by steps of d, so that (adopting the interpretation of the sequence as alternately positive and negative):

$$P = 2(M - m)/d = 2 \times (0;42 / 0;6,47,30) = 2016/163 \approx 12;22,5,\ldots \approx 12.3680\ldots$$

If we interpret the sequence as having $m = 0$, on the other hand, the period is exactly half this long, i.e. 6.1840.... (This interpretation actually turns out to be the more legitimate one, for technical reasons that it will not be necessary to go into in the present article.)

At this point we have reached the limit of what a purely mathematical analysis can tell us about the numbers in the papyrus. However, if one is familiar with basic astronomical relations widely known in antiquity, one might recognize that 12.3680... is very near to the average number of lunar months in a solar year. (For example, according to the Metonic cycle which equates 19 solar years with 235 lunar months, the ratio is approximately 12.3684....) So perhaps the table in the papyrus represents some quantity that varies depending on the stage of the solar year, or equivalently, the Sun's position in the zodiac, tabulated at intervals of one lunar month. This suggests that one should look for something similar in the part of the Babylonian mathematical astronomy of the last several centuries BC that was devoted to calculating the circumstances of consecutive conjunctions or oppositions of the Sun and Moon.

The fundamental edition and study of the Babylonian tablets of mathematical astronomy is Neugebauer's *Astronomical Cuneiform Texts*, the first volume of which contains a survey of the methods of calculating lunisolar conjunctions and oppositions.[10] These existed in two distinct classes, which Neugebauer designated Systems A and B, each of which calculated multi-column spreadsheet-like tables in which each row represented one of a series of consecutive conjunctions or oppositions. Browsing through the part of Neugebauer's work describing System B, one comes across a column that Neugebauer named H, which is a linear zigzag function with precisely the parameters d, m, and M of our papyrus, except that the units in the Babylonian texts are one-sixtieth of the units in the papyrus, i.e. the numbers as written in the cuneiform tablets do not have the initial zero.[11]

Column H is a component involved in the calculation of the dates and times of the conjunctions or oppositions.[12] These are given in Column L, in units of time-degrees (designated by the Sumerogram UŠ in Babylonian texts, χρόνοι in Greek) such that 360 time-degrees equal a complete day and night.[13] Column L is calculated as the running totals of Column K, which represents the time elapsed from the preceding conjunction or opposition to the present one. K in turn is the sum of two components, Columns J and G. G is a linear zigzag function that varies (as we would say from the perspective of modern astronomy) according to the Moon's position on its elliptical orbit relative to the apogee of the orbit. J is a function that varies according to the Sun's position in the zodiac (or, from a modern perspective, according to the position of the Earth on its elliptical orbit around the Sun). Column J too is a kind of zigzag function, with values alternately increasing and decreasing between a minimum and a maximum value, but it is not linear;

[9] For the role of additive/subtractive quantities in Babylonian astronomy see Ossendrijver 2012, 29–31. Employing the terms "positive" and "negative" with respect to such quantities, though sometimes convenient, is anachronistic, and one must be careful not to associate with them the much richer and more general conceptualization of signed quantities in modern mathematics.

[10] Neugebauer 1955.

[11] Neugebauer 1955, 1.78.

[12] Neugebauer 1955, 1.78–80.

[13] Ossendrijver 2012, 32–33. In older discussions of Babylonian astronomy, e.g. Neugebauer 1955, the temporal columns that we now understand as being expressed in time-degrees were commonly interpreted as being in a hypothetical "large-hour" unit equivalent to 60 time-degrees (or 6 of our hours). Interestingly, our present papyrus and P.Colker effectively operate with large-hours.

instead of having a constant increase and decrease *d*, its line-to-line differences are themselves a linear zigzag function, namely Column H. The two columns are synchronized in such a way that H is close to zero when J is near its extreme values and *vice versa*, with the result that a graph of J follows a sinusoidal curve instead of a zigzag with sharp inflections. It should be stressed that H has no purpose other than in the calculation of J. Thus we can conclude that our papyrus is a fragment from a System B table that would also certainly have contained the other columns J, G, K, and L, that is, the complete calculation of the dates of the conjunctions or oppositions.

Although the zigzag function for H in the papyrus has identical numerical parameters to the standard function in the cuneiform tablets, there is a difference of practice apparent in the lowest order digits of the tabulated values. In the Babylonian tablets, the initial value for any sequence of Column H was always chosen so that the last fractional place of every value of H is always either 0 or 30. In the papyrus, the last place is alternately 20 or 50 on one side of the zero-crossing, and alternately 10 or 40 on the other side. Since the temporal value of the lowest order digits is on the order of a few seconds, the difference in the adopted pattern would have had no astronomical significance whatsoever, but it shows that the sequence in the papyrus did not arise as an unbroken continuation of any of the extant Babylonian tablets.

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