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More Babylonian Lunar Theory in the Astronomical Papyrus P.Colker

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P.Colker, an unprovenanced papyrus paleographically dated to the first century AD in the private collection of Professor M. L. Colker (Charlottesville, Virginia, USA), was, when Neugebauer published a partial transcription and discussion of it in 1988, the first known example in Greek of a so-called System B lunar syzygy table, one of the most complex among the varieties of astronomical tables known from Late Babylonian cuneiform tablets from Babylon and Uruk.[1] Of the three columns of data in the fragment, Neugebauer identified col. iii as the component of System B known as Column G, which is an approximation of the length of the time interval from the preceding to the present conjunction or opposition of the Sun and Moon taking into account the periodic variation in the Moon's apparent speed.[2] In my subsequent edition of the papyrus, I conjectured that cols. i and ii together are Column J, which is a correction to be added to or subtracted from G to account for the periodic variation of the Sun's apparent speed.[3] Col. ii consists just of abbreviated words alternately signifying "additive" and "subtractive", and it was the presence of these indications, written at intervals of six lunar months (i.e. approximately half a year) that suggested that the numerals in col. i belonged to a sequence of J. I was unable to confirm this hypothesis, however, because the rules for computing J are rather complicated, while the numerals preserved in col. i – which are only the *ends* of the original numbers – were difficult to read from the black-and-white photograph on which my article, like Neugebauer's, was based.

The recent discovery of a papyrus in the Cairo Museum (P.Cair. Mus. S. R. 3059, part) containing a sequence of System B Column H, which provides the basis for computing Column J, suggested that it might be worthwhile to revisit the question of the identity of P.Colker cols. i and ii.[4] Through Prof. Colker's kindness I have been able to study the papyrus in person as well as to photograph it, resulting in the much improved transcription of the problem columns offered below.[5] (Col. iii, whose readings could be controlled by the known rules for computing Column G, did not require revision.) The new readings make it possible to confirm that cols. i and ii were J, as well as to reconstruct the values of this column and the lost Column H from which it was derived.

Transcription and translation of P.Colker cols. i and ii

In the translation, uncertain digits are represented by "x" and insecurely read digits are underlined.

]ζ̣ ̣		]$\underline{7}$ x	
]		]	
]		]	
] κ̣		] $\underline{20}$	
] ̣ ̣	πρ(οσθετικός)	]xx	additive
] ̣ ̣		]xx	
] ̣		]x	
] ̣		]x	
] ̣ ̣		]xx	
] ̣ κ ν⟦ ̣⟧		]x 20 50	

[1] Neugebauer 1988.

[2] For the arrangement and computational rules of the System B tables, see Neugebauer 1955, 1.69–85 and (on the specific columns with which the present article is concerned) Neugebauer 1975, 1.482–497.

[3] Jones 1997, 172.

[4] Aish–Jones 2016. See also *PSI* 15.1491, containing a terse description of a System B lunar table with columns recognizable as H, J, and G in that order.

[5] See Aish–Jones 2016, fig. 2. Additionally, a color photograph of P.Colker was published in Finkel–Seymour 2008, 193. I am grateful to John Steele and Irving Finkel for providing me with this photograph in higher resolution.

]ζ ν	ἀφ(αιρετικός)	]7 50 subtractive
] να μ		] 51 40
] μη ō		] 48 0
] ̣ς ν		] x6 50
] κ̣ λ		] $\underline{20}$ 30
]δ κ		]4 20
]θ κ	πρ(οσθετικός)	]9 20 additive
] ιη̣ ̣		] 1$\underline{8}$ x
] ̣ κθ λ		]x 29 30
] ̣		]x
]		]
]		]
] ̣	ἀφ(αιρετικός)	]x subtractive

5 ζ̣: faint | ̣: traces near baseline, apparently parts of an ascending and a descending stroke
8 κ̣: faint and indistinct
9 ̣ ̣: blurred traces
10 ̣ ̣: indistinct traces
11 ̣: blurred ink, no distinct strokes
12 ̣: faint vertical stroke
13 ̣ ̣: small ink traces near baseline
14 ̣: the right end of a horizontal stroke near top height, nearly touching the kappa, probably γ, ς, or ζ | ⟦ ̣⟧: indistinct blur of ink, probably not an intentional letter
18 ̣: vertical stroke, either ι or ν
19 κ̣: left half of letter stripped away; ε̣ or ς̣ also possible
22 η̣: left vertical and lower half of right vertical, the rest of the letter indistinct | ̣: blurred but possibly the upper part of a vertical
23 ̣: pairs of traces of ink at baseline and top letter height, suggesting four extremities of a squarish letter, remainder blurred
24 ̣: right end of slightly descending horizontal stroke near baseline
27 ̣: two specks of ink near baseline

Algorithms for Columns H and J

The following analytical sections of this article assume that the reader is familiar with the basic principles of Babylonian mathematical astronomy as described, for example, in the introduction of Neugebauer's *Astronomical Cuneiform Texts*.

As attested in numerous Babylonian System B lunar tablets, Column H is a linear zigzag function computed according to the following parameters and algorithm:[6]

Column H
Parameters:

minimum	$m_H = 0$
maximum	$M_H = 0;21$
step	$d_H = 0;6,47,30$
period	$P_H = 1008/163 = 6;11,2,34,36,\ldots$

[6] Neugebauer 1955, 1.78. To maintain consistency with the notations of the papyrus and P.Cair. Mus. S. R. 3059 (part), we will consider the units of Columns H and J to be "large hours", equivalent to 4 hours; in Babylonian texts the fundamental unit was the UŠ, equivalent to 4 minutes or 1/60 of a large hour.

Algorithm:
Let $H(n)$ designate the value of Column H in row n of a table. $H(n)$ is on either an increasing (↑) or a decreasing (↓) branch of the sequence.

If $H(n)$ ↑ and $H(n) + d \le M$, then $H(n+1) = H(n) + d$
else $H(n+1) = 2M - H(n) - d$
If $H(n)$ ↓ and $H(n) - d \ge m$, then $H(n+1) = H(n) - d$
else $H(n+1) = 2M - H(n) + d$

Column J is a non-linear zigzag function; instead of having a constant increment/decrement d, its values alternately increase and decrease between a defined minimum and maximum by a sequence of values of the linear zigzag function H. Two variants are known for J's numerical parameters:[7]

Column J
Parameters (abbreviated):
minimum $m_J = -0;32,28$
maximum $M_J = +0;32,28$
period[8] $P_J = 3896/315 = 12;22,5,42,51,\ldots$
Parameters (unabbreviated):
minimum $m_J = -0;32,28,6$
maximum $M_J = +0;32,28,6$
period $P_J = 2783/225 = 12;22,8$
Algorithm:
Let H(n) and $J(n)$ designate the values of Columns H and J in row n of a table. $J(n)$ is on either an increasing (↑) or a decreasing (↓) branch of the sequence.

If $J(n)$ ↑ and $J(n) + H(n+1) \le M_J$, then $J(n+1) = J(n) + H(n+1)$
else $J(n+1) = 2M_J - J(n) - H(n+1)$
If $J(n)$ ↓ and $J(n) - H(n+1) \ge m_J$, then $J(n+1) = J(n) - H(n+1)$
else $J(n+1) = 2\, m_J - J(n) + H(n+1)$

Synchronization
The purpose of defining Column J in this complicated way was to smoothe its periodic variation so that the rate of change would be slow around the mininum and maximum and fast around the mean (i.e. around zero).[9] Hence the maxima and minima of J must approximately coincide with the minima of Column H, whereas Column J's transitions from additive to subtractive and *vice versa* must approximately coincide with H's maxima.[10] To maintain this synchronization in the long term would require $P_J = 2P_H$. This relation does not hold exactly for either the unabbreviated or the abbreviated parameters of J, but the discrepancy and the resulting systematic phase drift are much smaller for the abbreviated parameters, so it is possible that the leaving off of the third sexagesimal place in M and m was motivated not only by arithmetical convenience but also by improved coherence. The unabbreviated parameters are attested only in System B tablets from Uruk,

[7] Neugebauer 1955, 1.78-79.

[8] P_J, calculated as $4(M_J - m_J)/(M_H - m_H)$, is an idealization since the dates of its maxima and minima are neither well defined nor equally spaced, as is the case with linear zigzag functions; see Neugebauer 1975, 1.492–496. For the unabbreviated parameters $P_J = 12;22,8$ is a well attested Babylonian parameter for the length of the year in mean lunar months, which shows that the inventors of the System B H–J scheme employed this definition of P_J.

[9] For further discussion of the exceptional computational basis of System B Column J see Aaboe 2002 and Britton 2003.

[10] For an illustration of the chaos that could result when an arithmetical error disturbed the synchronization, see Neugebauer 1955, 1.156 with Fig. 42.

whereas use of the abbreviated parameters appear in tablets from Babylon and occasionally from Uruk.[11]

In passing, an unobvious detail in the calculation of J as it is practiced in the cuneiform tablets deserves to be noted. As defined above, H is a zigzag function whose period is half a solar year and whose minimum is zero. One might be tempted to redefine it as a zigzag function with the solar year itself as period and with $m = -M = -0;21$, modifying the rules for J so that $H(n+1)$, which now can be either positive or negative, is always *added* to $J(n)$ with appropriate reflections at the maxima and minima. In the tablets, however, the situation sometimes arises that H has passed a zero minimum (or on the alternative definition, has changed sign) before J reaches one of the extrema, and in these cases it becomes clear that the criterion for whether to treat H as additive or subtractive was the current trend (increasing or decreasing) of J; H only switches from being treated as additive to subtractive or *vice versa* in the following row of the table.

First analysis of P.Colker cols. i and ii

Lines 15–20, which are comparatively well preserved in the papyrus, comprise a complete subtractive half-period of Column J. A subtractive half-period begins with a quarter-period of decelerating decrease followed by a quarter-period of accelerating increase, so the minimum must have been reached around lines 17–18. We can infer that Columns H and J were both decreasing in lines 15–17. Hence the last two places of $H(17)$ were 56,20, and we can use the securely read numerals together with the algorithm for H to extrapolate the last two places of Columns H and J back to line 14 by the following argument.

Either $H(16)$ is on the same decreasing branch as $H(17)$ or on the preceding increasing branch. In the former case, $H(16) = H(17) + 0;6,47,30$, so that the last places of $H(16)$ are 43,50 and the last places of $J(15)$ are 7,50, in agreement with the readings of the papyrus. In the latter case, $H(16)$ would end in 16,10, and the last places of $J(15)$ would be 35,30, which is obviously not the case. The same considerations allow us to deduce that $H(15)$ is on the increasing branch, that is, H passes its maximum between lines 15 and 16, so that $H(15)$ ends in 28,40 and $J(14)$ in 20,50, matching the papyrus's readings. $H(14)$ is also on the increasing branch, since a branch of H always comprises three or four successive values. Summarizing what we have so far:

14	0;xx,41,10 ↑	0;xx,20,50 additive ↓
15	0;xx,28,40 ↑	0;xx,7,50 subtractive ↓
16	0;xx,43,50 ↓	0;xx,51,40 subtractive ↓
17	0;xx,56,20 ↓	0;xx,48,0 subtractive ↓

Column J must have passed its minimum either between lines 17 and 18 or between 18 and 19; a minimum between 19 and 20 is excluded because it would result in a branch of J comprising just one value, which is impossible. We know that $H(18)$ is still on the decreasing branch, again because a branch of Column H always comprises three or four values; hence $H(18)$ ended in 8,50. If J passed its minimum between lines 17 and 18, the last places of $J(18)$ would have been either 59,10 (using the abbreviated parameters for J) or 59,22 (using the unabbreviated parameters). Instead the papyrus has x6,50, which is consistent with this being a normal decreasing step, so we restore:

18	0;xx,8,50 ↓	0;xx,56,50 subtractive ↓

The passage of J's minimum thus followed this line, and from the surviving final places of lines 18–20 we see that the abbreviated parameters were used, since otherwise $J(19)$ would have ended in 20,42.

19	0;xx,38,40 ↑	0;xx,20,30 subtractive ↑
20	0;xx,26,10 ↑	0;xx,54,20 subtractive ↑
21	0;xx,13,40 ↑	0;xx,19,20 additive ↑

At this stage, we have abundant confirmation that col. i is indeed J, with col. ii providing the indications of when it changes from additive to subtractive and *vice versa*.

[11] Neugebauer 1955, 1.169.

Reconstructing Columns H and J

To narrow down the possible reconstructions of col. i, an Excel spreadsheet was prepared to reproduce the calculated sequences of H and J assuming any hypothetical values of $H(14)$ and $J(14)$, which determine all values of H and J both before and after line 14. Paying attention only to the secure readings in cols. i and ii, lines 14 through 21, one finds as possible values for $H(14)$ and $J(14)$ the combinations marked ✓ in the Table 1.

	$J(14)$ = 0;12,20,50	$J(14)$ = 0;13,20,50	$J(14)$ = 0;14,20,50	$J(14)$ = 0;15,20,50	$J(14)$ = 0;16,20,50
$H(14)$ = 0;12,41,10			✓	✓ (1)	
$H(14)$ = 0;13,41,10	✓	✓ (2)	✓	✓ (3)	✓ (4)

Table 1. Candidates for the values of H and J in P.Colker line 14

The trace of a letter along the left edge of the papyrus on line 14, which would belong to the first place of $J(14)$ and hence to a letter whose numerical value is between 2 and 6, is compatible with γ, ϛ, or at a stretch ε. We may thus reduce the choices to those in the table for which the check mark is followed by a parenthesized number.

The sequences of J generated from these remaining four pairs of values have identical last places from row 12 onwards as far as the papyrus is preserved. In all the sequences, the change from additive to subtractive takes effect on row 28, whereas the papyrus indicates the change on line 27. This must reflect a mistake; either the scribe simply wrote the subtractive indication one line too high in col. ii, or an arithmetical error affected the last few rows of the papyrus. The only potential discriminant in lines 12–28 is the ambiguous traces of the second digit of the first place of J in line 23, which could be either ζ, in agreement with (1), or η, in agreement with (2), whereas they are unlikely to be ε or ϛ as required respectively for (4) and (3).

In rows 5 through 12, the last digits generated by (2), (3), and (4) are identical to each other, but differ from those of (2). Unfortunately the papyrus offers scarcely anything legible in these lines, but the uncertain zeta in line 5 would be consistent with $H(5)$ = 0;30,5<u>7</u>,40 as in (2), (3), and (4) but not with 0;30,42,40 as in (1). Combining this discriminant with that of line 23, we would be left with (2) as a unique solution for reconstructing the papyrus's sequence of J. Obviously this narrowing down of our original seven possibilities to one can at best be regarded as probable since it rests on just three insecurely read letters.

Table 2 gives a complete reconstruction of the H and J sequences that would have been in lines 1–28 according to our preferred choice of values. As Fig. 1 shows, the synchronization is excellent. (For clarity, the units shown in the graph are UŠ rather than large hours.)

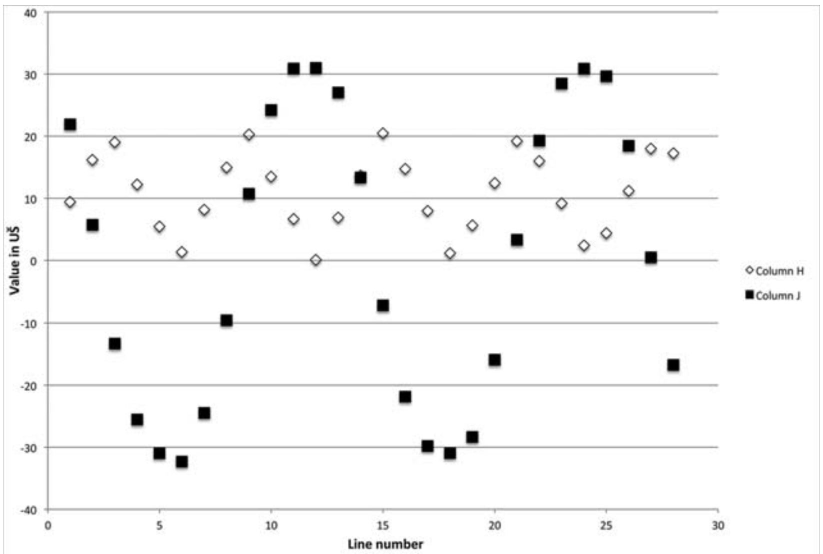

Fig. 1. Graph of reconstructed Columns H and J of P.Colker.

Line	H	J
1	0;9,23,40	0;21,55,0
2	0;16,11,10	0;5,43,50
3	0;19,1,20	0;13,17,30 subtractive
4	0;12,13,50	0;25,31,20
5	0;5,26,20	0;30,57,40
6	0;1,21,10	0;32,18,50
7	0;8,8,40	0;24,28,30
8	0;14,56,10	0;9,32,20
9	0;20,16,20	0;10,44,0 additive
10	0;13,28,50	0;24,12,50
11	0;6,41,20	0;30,54,10
12	0;0,6,10	0;31,0,20
13	0;6,53,40	0;27,2,0
14	0;13,41,10	0;13,20,50
15	0;20,28,40	0;7,7,50 subtractive
16	0;14,43,50	0;21,51,40
17	0;7,56,20	0;29,48,0
18	0;1,8,50	0;30,56,50
19	0;5,38,40	0;28,20,30
20	0;12,26,10	0;15,54,20
21	0;19,13,40	0;3,19,20 additive
22	0;15,58,50	0;19,18,10
23	0;9,11,20	0;28,29,30
24	0;2,23,50	0;30,53,20
25	0;4,23,40	0;29,39,0
26	0;11,11,10	0;18,27,50
27	0;17,58,40	0;0,29,10
28	0;17,13,50	0;16,44,40 subtractive

Table 1. Reconstructed Columns H and J of P.Colker. Single rulings indicate passage of a minimum; double rulings, passage of a maximum.

Neugebauer has provided a test for whether a sequence of H can be a continuation of another sequence, and a formula for determining the intervening number of rows when such continuity is possible.[12] Since a sequence of H repeats its values exactly after 1008 rows (which is equivalent to about 81 1/2 years), the formula's results are not unique, but only the smallest mathematically possible intervals will be plausible candidates for the actual span of time between the dates covered in a pair of tables that we hope to connect in this way. It is a matter of interest to see whether the sequence of H from which P.Colker's column J was computed could be a continuation of the sequence in the Cairo Museum papyrus or *vice versa*.

We have established above that P.Colker's sequence of H had, on line 14, either 0;12,41,10 or 0;13,41,10 on an ascending branch. The Cairo Museum papyrus, line 13, has 0;13,51,10 on an ascending branch. The criterion for whether the two sequences are joinable is that the difference between two values on the same

[12] Neugebauer 1955, 1.78.

kind of branch is an integer multiple of 0;0,2,30. Both candidate values for P.Colker satisfy this condition with respect to 0;13,51,10; in other words, a continuation of the Cairo Museum papyrus's column H will, if prolonged indefinitely, eventually reproduce both candidate sequences for P.Colker.

If the difference between two values of H on ascending branches is δ, a sequence starting with the smaller of the two values will yield the larger value after $338400\delta \pm 1008n$ rows, where n can be any integer. Using the candidate value 0;12,41,10, we have δ = 0;1,10,0, from which it results that a sequence starting with 0;12,41,10 will first yield 0;13,51,10 after 532 rows (about 43 years), and a sequence starting with 0;13,51,10 will first yield 0;12,41,10 after 476 rows (about 38 1/2 years). With the other candidate, 0;13,41,10, we have δ = 0;0,10,0, from which we find that a sequence starting with 0;13,41,10 will first yield 0;13,51,10 after 940 rows (about 76 years) while a sequence starting with 0;13,51,10 will yield 0;13,41,10 after just 68 rows (about 5 1/2 years). This last interval is so small that it is tempting to conjecture that the two papyri were part of a single series of computations of conjunctions or oppositions spanning several years. Since each papyrus covers about two years, they could easily have come from a single roll.

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