

Studies in the Astronomy of the Roman Period

III. Planetary Epoch Tables

by

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To the memory of B. L. van der Waerden (1903–1996)

Until very recently, almost all the known examples of planetary tables recovered by archeology from Roman Egypt have been varieties of almanac, which give dates and longitudes either at regular intervals or on the days when a planet crosses the boundary between two zodiacal signs. The earliest of these to come to light, and still the most extensively preserved examples, are the demotic Egyptian papyrus *P. dem. Berlin* 8279, covering –16 to 12, and the *Stobart Tablets*, a set of wooden boards also inscribed in demotic, covering parts of the interval 63 to 140.¹ Both are a format of table that I call ‘sign entry almanacs’ because they list computed dates when each of the five planets makes its entries, both direct and retrograde, into the zodiacal signs. Nineteen examples of sign entry almanacs on papyrus are at present known, and three on wooden boards, making this the most common type of table; we also have almanacs that give planetary positions at intervals of one month, five days, or one day.² In contrast to the impression given by the earliest discovered specimens, the corpus of astronomical papyri as now known is predominantly written in Greek, and

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it is usually supposed that the Egyptian tables are an offshoot of a Greek tradition rather than the other way around.³

None of these tables provided in themselves the means of computing the planetary longitudes that they contain, and it is a difficult problem to identify the methods by which they were generated unless we actually happen to possess the primary tables. Nevertheless van der Waerden sought in a series of papers to establish that some of the planetary data in the *Stobart Tablets* and the Berlin papyrus were computed according to arithmetical methods, and specifically according to the Babylonian so-called 'ACT' planetary schemes.⁴ This is not the place for a detailed review of van der Waerden's arguments or the criticisms levelled against them by Neugebauer and Parker.⁵ Suffice it to say that, in my judgement, van der Waerden successfully demonstrated the following theses: (a) that the positions of Venus in the *Stobart Tablets* were computed according to an arithmetical scheme – not known to be Babylonian – in which Venus' synodic cycle is supposed to be composed of intervals traversed at constant velocities; (b) that the positions of Mars in the *Stobart Tablets* were computed according to the Babylonian System A, or a scheme closely allied to it; and (c) that the positions of Jupiter were computed according to a scheme resembling the Babylonian System A' in structure.⁶

Another indirect argument for Greek knowledge of the Babylonian planetary schemes can be made from the presence of similar schemes in early Sanskrit astronomical works, especially the *Pañcasiddhāntikā* of Varāhamihira (early sixth century).⁷ These Indian schemes are of the System A variety, that is, they assume a uniform distribution of occurrences of planetary phenomena within defined zones of the ecliptic, and their Babylonian ancestry is particularly obvious for Mars and Mercury. The only plausible channel for this transmission is by way of Greek astronomy, and indeed certain elements are demonstrably of Greek origin.⁸

It was already apparent, then, that the basic principles of the Babylonian System A planetary schemes were known in Greek (or Greco-Egyptian) astronomy, and that some of the methods used to calculate planetary almanacs in the Roman period were fairly close to specific Babylonian schemes.⁹ But because of the oblique character of the evidence, it has not been possible to estimate how

much of the Babylonian theory passed over intact into Greek, nor the ways in which the schemes were modified, for example to take account of the difference in calendar. To answer these questions, one needs to have the primary tables by means of which the planetary positions in the almanacs were obtained; that is, tables corresponding in function to the 'ACT' planetary tables among the cuneiform material rather than to the Babylonian 'Almanacs'.

Not the least important among the contributions that the new astronomical papyri from Oxyrhynchus make to our understanding of Greek astronomy is that they provide us for the first time with significant numbers of such primary tables.¹⁰ The planetary tables fall into two main classes: kinematic tables from Ptolemy's *Handy Tables* or variants of that tradition, and tables that express a planet's motion in terms of the principal stages of its synodic cycle. The tables of the second class are of two varieties: 'epoch tables' that list the dates and positions of the synodic phenomena of a planet, and 'templates' that describe the pattern of motion of a planet between two phenomena.¹¹

The present article is concerned solely with the planetary epoch tables. Its main objects are to determine the structure and contents of the known tables, and to find out the methods by which they were computed. The text edition necessarily presents these results as *faits accomplis*. The fuller treatment in this paper is justified, in the first instance, by the fact that the development of Greek planetary theory before Ptolemy and its debt to Babylonian sources is a controversial topic, which makes it essential to show just how secure are those conclusions that have bearing on the question. Secondly, more fragments of epoch tables are certain to turn up sooner or later among the great uninventoried collections of papyri. I hope that the examples of analyses discussed below will be both an encouragement and a help to other explorers in this field.

Since all the texts discussed below are from the Oxyrhynchus corpus, we will for brevity's sake refer to them by their publication number alone, in bold face (e.g. **4159**). In the full form of citation, the number should be prefixed by '*P. Oxy.* LXI'.

1. Planetary phenomena and the ACT schemes

The format of a planetary epoch table on papyrus was very much like that of a typical Babylonian planetary table: each line of the table represents a successive synodic period of the planet in question, while the columns contain the elements of the date and longitude of the tabulated phenomenon. In the surviving fragments one tends to get vertical strips, that is, a sequence of dates or a sequence of longitudes or, sometimes, both. Moreover, the information may be incompletely preserved, for example one may have the months and days but not the years, or one may have longitudes curtailed to the whole degrees or even to just the zodiacal signs. Our first problem is usually to identify the planet concerned. This is easily done by recognizing the characteristic synodic period of the planet, its synodic arc (i.e. the progress in longitude between successive phenomena of the same kind), or the number of synodic periods required for the longitude and date of the phenomenon to return very roughly to the initial value (Table 1). None of these parameters is constant, but the ranges of values are small in comparison with the differences between one planet and another.

Because of the need for frequent reference to the Babylonian planetary theories, it will be helpful to have here a brief summary of the concepts and parameters. For further details the reader should consult the introduction to the second volume of Neugebauer's edition of the cuneiform texts.¹² This remains the best introduction to the elements of Babylonian mathematical astronomy as well as to methods of analysis and dating of the documents.

Planet	Mean synodic period	Synodic arc	Rough recurrence
Mercury	116d	95°–145°	3 syn. periods
Venus	584d=1y+219d	205°–225°	5 syn. periods
Mars	780d=2y+50d	30°– 90°	7 syn. periods
Jupiter	398d=1y+33d	28°– 38°	11 syn. periods
Saturn	378d=1y+13d	11°– 14°	29 syn. periods

Table 1. Characteristic parameters of planetary phenomena.

For the three planets Mars, Jupiter, and Saturn, the phenomena that are predicted in Babylonian tables, and the ‘Greek letter’ notations we will use for them, are:

- Γ First visibility (morning)
- Φ First station (morning)
- Θ Acronychal (sunset) rising
- Ψ Second station (evening)
- Ω Last visibility (evening)

The patterns of distribution of these phenomena are sufficiently alike for each planet so that we normally cannot tell which kind of phenomenon is being tabulated unless we can relate the planet’s position to the approximate position of the sun.

For the remaining planets Venus and Mercury the phenomena are:

- Γ First morning visibility
- Φ Morning station (predicted for Venus only)
- Σ Last morning visibility
- Ξ First evening visibility
- Ψ Evening station (predicted for Venus only)
- Ω Last evening visibility

There is a more pronounced variation between the distributions of the different phenomena for these planets, so that for Mercury in particular it is often possible to identify which of the four possible phenomena is predicted in a table by correlating the synodic arcs or times with the longitude at the beginning of each synodic period.

In the variety of Babylonian predictive scheme referred to as System A, the synodic arc from one occurrence of a phenomenon to the next is functionally dependent only on the planet’s longitude at the first occurrence. The ecliptic is notionally divided into two or more fixed zones of length a_i . Within any zone i the synodic arc is always a prescribed value w_i . But whenever the addition of w_i to an initial longitude would result in progress past the boundary with the next zone $i+1$, the part of the synodic arc beyond the boundary is scaled by the factor w_{i+1}/w_i . If after this correction

the longitude is past the boundary between zones $i+1$ and $i+2$, the part past this boundary is again scaled by the factor w_{i+2}/w_{i+1} . The effect of these rules is that for each System A scheme the graph of synodic arc as a function of initial longitude is composed of several linear stretches, some of which are ‘plateaux’, i.e. intervals within which the synodic arc does not vary. For any value of synodic

Planet	phenomena	i	a_i	w_i	boundaries	w_{i+1}/w_i
Mercury	Γ (System A ₁)	1	165°	106°	121°–286°	1;20
		2	134°	141;20°	286°– 60°	0;40
		3	61°	94;13,20°	60°–121°	1;7,30
	Ξ (System A ₁)	1	110°	160°	96°–206°	0;40
		2	134°	106;40°	206°–340°	0;54
		3	116°	96°	340°– 96°	1;40
	Σ (System A ₂)	1	90°	107;46,40°	90°–180°	1;12
		2	96°	129;20°	180°–276°	0;45
		3	89°	97°	276°– 5°	1;20
		4	85°	129;20°	5°– 90°	0;50
	Ω (System A ₂)	1	180°	108;30°	90°–270°	1;6,40
		2	60°	120;33,20°	270°–330°	0;54
3		60°	108;30°	330°– 30°	1;15	
4		60°	135;37,30°	30°– 90°	0;48	
Mars	Γ, Φ, Ω	1	60°	45°	30°– 90°	0;40
		2	60°	30°	90°–150°	1;20
		3	60°	40°	150°–210°	1;30
		4	60°	60°	210°–270°	1;30
		5	60°	90°	270°–330°	0;45
		6	60°	67;30°	330°– 30°	0;40
Jupiter	all (System A)	1	205°	36°	240°– 85°	0;50
		2	155°	30°	85°–240°	1;12
	all (System A')	1	120°	30°	99°–219°	1;7,30
		2	53°	33;45°	219°–272°	1;4
Saturn	all	1	200°	11;43,7,30°	130°–330°	1;12
		2	160°	14;3,45°	330°–130°	0;50

Table 2. Parameters of the principal System A planetary schemes.

arc that is not a plateau value, there will be only a small number of initial longitudes (usually two, at most four) that generate that value. The defining parameters for the principal System A schemes for Mercury, Mars, Jupiter, and Saturn are given in Table 2, and the resulting patterns of synodic arcs are shown in Figs. 1–7.¹³

Because the Babylonian calendar was lunar, the inventors of the planetary schemes found it convenient to measure the synodic times between phenomena in units of thirtieths of a lunar month ('*tithis*'), which could with negligible error be regarded as a whole number of lunar months plus a remainder treated as actual calendar days. The theoretical assumption that appears to underly the calculation of dates of phenomena is that the synodic arc in degrees is equal to the synodic time measured in units of 1/360 of a sidereal year, which amounts to assuming that the elongation of the planet from the (mean) sun is constant for any particular phen-

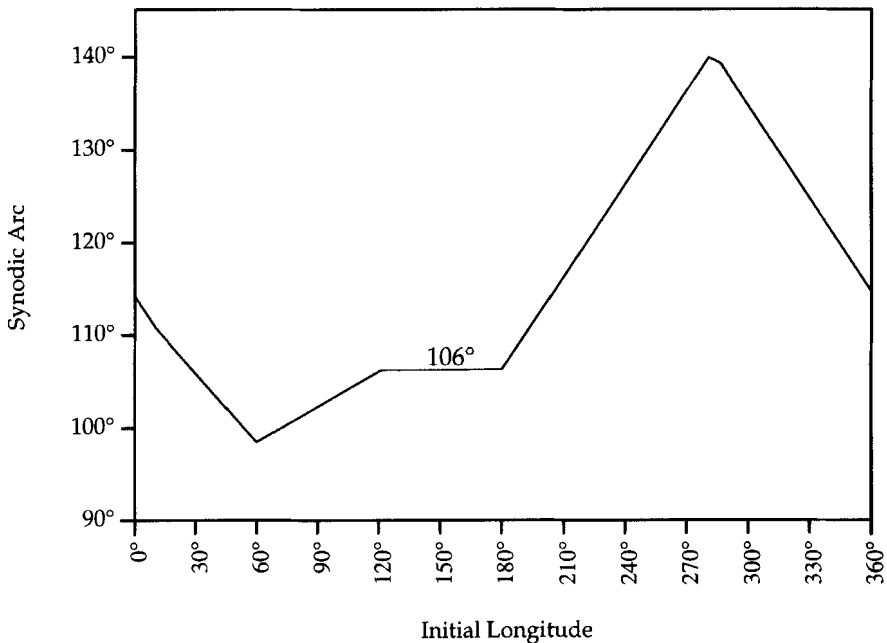


Fig. 1. Mercury, System A₁ for Γ .

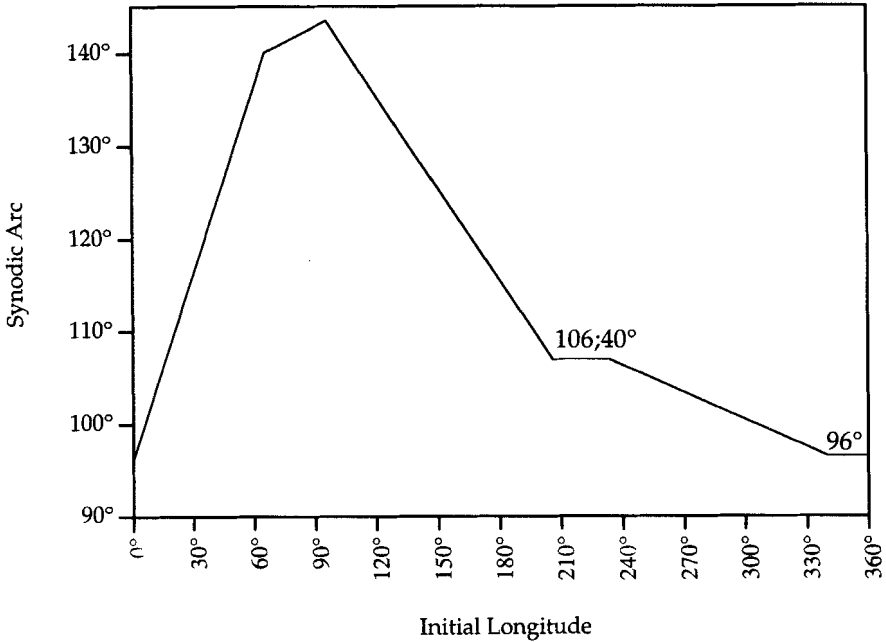


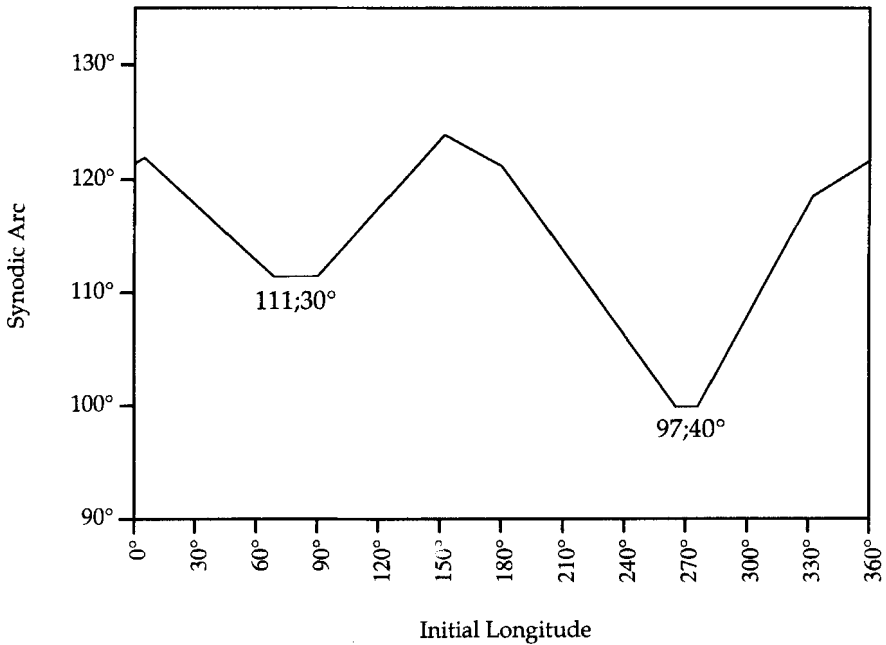
Fig. 2. Mercury, System A₁ for \mathcal{E} .

omenon. To facilitate the calculation of synodic times in *tithis*, however, one makes the simplifying approximation that the difference between the synodic time in these *tithis* and the synodic arc in degrees is a constant c obtained as the difference between the *mean* synodic time in *tithis* and the *mean* synodic arc. Thus if the scheme in question has a periodicity of Π synodic cycles and Z revolutions around the ecliptic in Y years (where 1 year is assumed to be 12;22,8 lunar months):

$$c = 30 \times 12;22,8^Y / \Pi - 360 \times Z / \Pi . \quad (1)$$

Adding this constant to the synodic arcs yields the corresponding synodic times, and the positions and dates of the consecutive phenomena are simply the running totals of these quantities.

This method of calculating the dates is well suited to a lunar calendar. One can imagine two plausible ways that one might

Fig. 3. Mercury, System A₂ for Σ .

adapt the scheme to work with a civil calendar, such as the Egyptian, operating with real days in months and years of fixed duration. The first, which is the way hypothesized by van der Waerden for the Egyptian almanacs, is that one determines independently the dates of new moons in the civil calendar, and then translates the calculated lunar dates into civil dates. But this is a needlessly roundabout procedure. The approximate relation (1) is acceptable because the *tithi* is only a little less than $1/360$ of a sidereal year. The true day, however, is even closer to that unit; and hence there is no obstacle to replacing (1) with

$$c = a \times Y / \Pi - 360 \times Z / \Pi \quad (2)$$

where a is the length of the sidereal year in days. This constant can be added to the synodic arcs to obtain directly the synodic times in days.

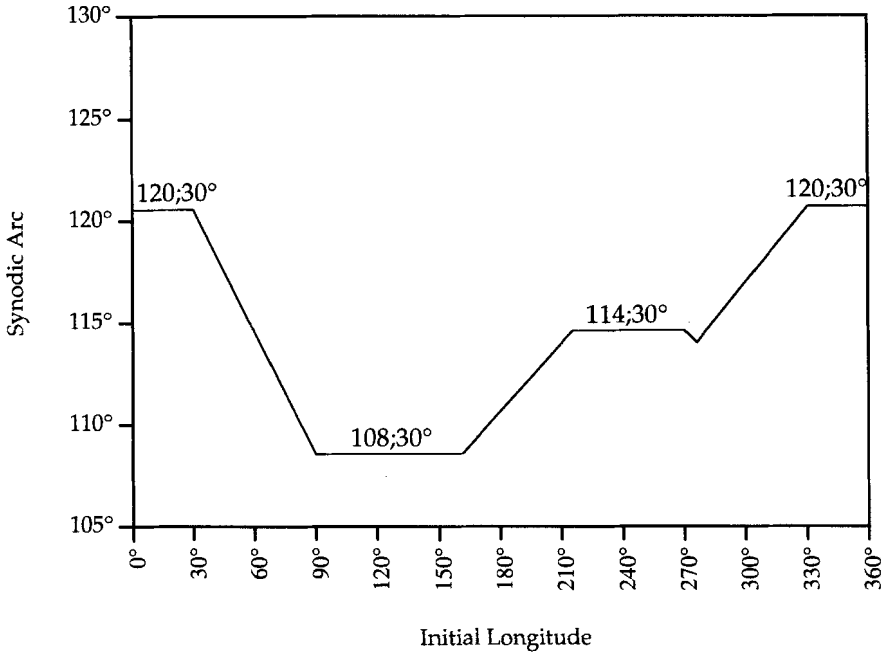


Fig. 4. Mercury, System A_2 for Ω .

Presented with a fragment of an epoch table, we can check whether it was computed according to one of the known System A schemes if we have any of the following data. (a) From a single longitude we can extrapolate a series of longitudes forward or backward according to the scheme. If we have a single date as well (which need not be from the same line as the selected longitude) we can also extrapolate a series of dates. These series can be compared with the readings in the fragmentary table. (b) From a single synodic arc we can obtain the corresponding longitude according to the scheme, unless the arc is one of the plateau values. This reduces the problem to the previous case. On the other hand the occurrence of plateau values is in itself a probable symptom of the System A scheme. (c) From a single synodic time we can subtract the constant c to obtain the synodic arc, reducing the problem to the preceding case. (d) From two consecutive dates we can obtain

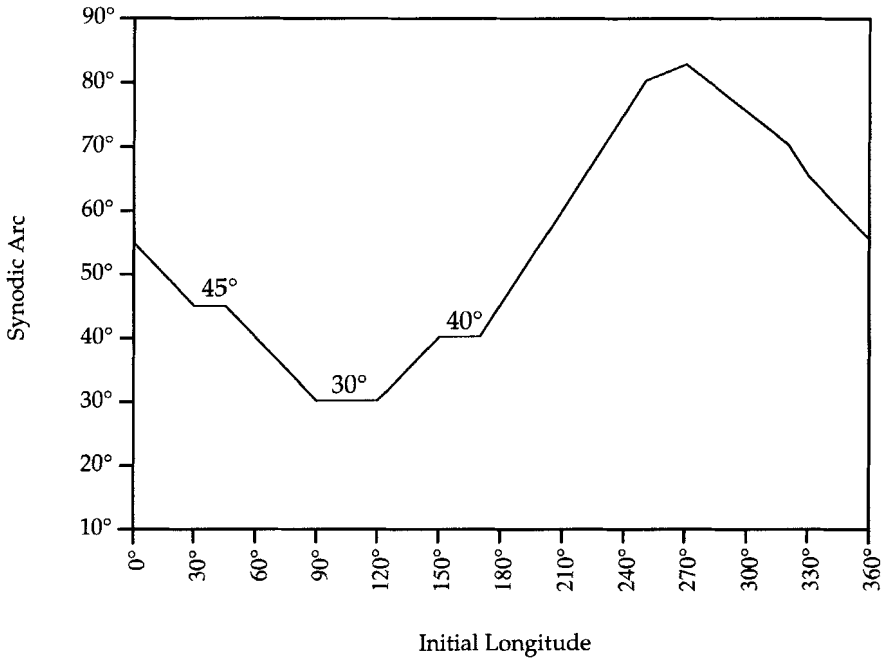


Fig. 5. Mars, System A for Γ , Φ , Ω .

the synodic time, reducing the problem to the preceding case. If we are working from truncated data, the initial longitude will be determined only within a range, but we should expect to find at least one value in that range that gives rise to the preserved contents of the table.

Besides the System A schemes, Babylonian astronomy embraced schemes for Mars, Jupiter, and Saturn based on a different hypothesis ('System B'). We can defer the discussion of these schemes until section 6 below.

2. Mercury

Fragments of six epoch tables for Mercury have turned up among the new Oxyrhynchus papyri, making this the best represented

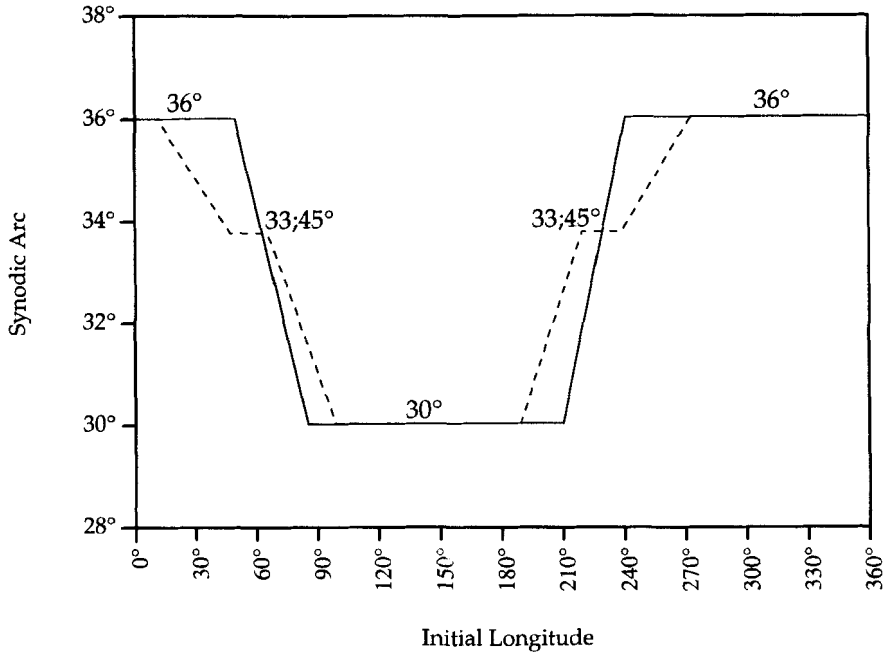


Fig. 6. Jupiter, Systems A (solid) and A' (broken).

planet. This may be a mere accident of preservation, although it is worth recalling that Mercury's brief synodic period means that its epochs over a given span of years will occupy much more writing surface than any other planet.

a. The first table that we will discuss, **4152**, contains on its front part of the columns giving the longitude of a phenomenon. To the translation in Table 3 I have added a column for the line-to-line differences, i.e. the synodic arcs, which are not present in the preserved fragment. The symbol 'x' stands for a digit that is lost or illegible. The synodic arcs in lines 1 and 3 are plateau values in System A₁ for Ξ (first evening visibility), and broadly speaking the large synodic arcs in Gemini and small ones in Aquarius and Pisces are symptomatic of this phenomenon (cf. Fig. 2). Recomputation according to System A₁ from an initial longitude of χ 12;36° turns out to reproduce precisely the readings of the papyrus,

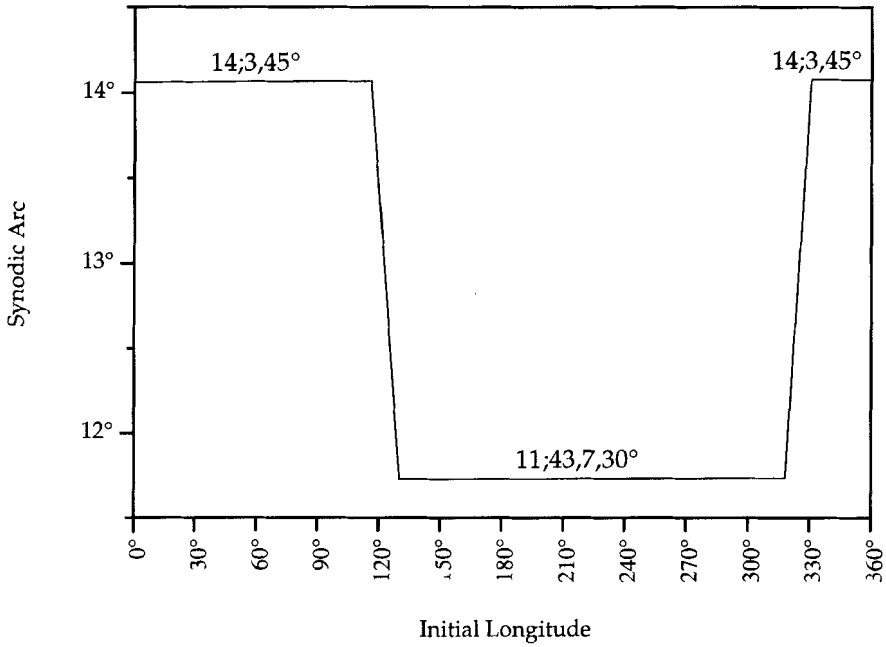


Fig. 7. Saturn, System A.

line	longitude	synodic arc
	♃ 12;36°	96;0°
	♄ 18;36°	141;24°
	♅ 10;0°	106;40°
	♆ 26;40°	97;20°
5	♁ 4;0°	138;40°
	♂ 22;40°	107;46,xx°
	♆ 10;26,xx°	98;57,xx°
	♃ 19;24°	128;56°
	♄ 28;20°	
10	♃ 24;xx,xx°	
	♃ 4;4x	

Table 3. Translation of 4152, front.

line	longitude	approx, syn, arc	System A ₂
	𐎠 2x°		𐎠 23;20°
	𐎡 1x°		𐎡 14;40°
	𐎢 12°	111°	𐎢 12°
	𐎠 3°	112°	𐎠 3;40°
5	𐎡 25 ⁷⁰	122°	𐎡 25;50°
	𐎢 27 ⁷⁰	107°	𐎢 27;15°
	𐎠 14°	115°	𐎠 14°
	𐎡 9°		𐎡 9;26,40°
	𐎢		𐎢 12;30°
10	𐎡		𐎡 24;20°
	𐎢		𐎢 23;3,20°

Table 4. Translation of 4152, back, and recomputation by System A₂ for Mercury.

and allows us to restore $\approx 10;26,40^\circ$ in line 7, $\text{𐎢 } 24;13,20^\circ$ in line 10, and $\text{𐎠 } 4;48^\circ$ in line 11. A clearer demonstration of the use of a Babylonian planetary scheme could hardly be desired.

The back of the same fragment was not originally used, but it bears the mirror-reversed traces of an ink offset from another part of the same set of tables. Again only the columns for longitude are preserved, and unfortunately all fractions of degrees are lost (Table 4, first three columns). Comparison with Figs. 1–4 shows that Σ (last morning visibility) is the only phenomenon for which the synodic arcs approximately fit the pattern, i.e. with values around 110° and decreasing in Taurus and Gemini, around 110° and increasing in Capricorn and Aquarius, and about 122° in Virgo. Applying the System A₂ scheme for Σ to an initial longitude of $\approx 12^\circ$ for line 3, we obtain the sequence in the last column of Table 4. This reconstruction matches all the securely legible traces, and is at least compatible with the uncertain ones. This makes it appear very probable that System A₂ was used to compute the positions in the papyrus table. Two objections may be raised, firstly that System A₂ is rarely found among the Babylonian cuneiform texts and only at an exceptionally early date (early third century B.C.), secondly that the known cuneiform tables that employ Mercury's System A₁ for the planet's first visibilities compute the last visibilities not by System A₂, but rather by a scheme of so-called

'pushes' that correlate the longitudinal progress during the period of visibility to the longitude at first visibility. We shall see presently that System A_2 was certainly known to the people who calculated the papyrus epoch tables, which dispenses with the former objection. To address the latter, we can use the known Babylonian scheme of pushes from Γ to Σ to conjecturally reconstruct a sequence of approximate longitudes of Γ from the attested longitudes of Σ , and check whether such a sequence could have been produced using System A_1 for Γ .¹⁴ Thus from lines 4–6 one obtains the sequence $\nearrow 29^\circ$, $\curvearrowright 19^\circ$, $\Omega 27^\circ$, $\nearrow 13^\circ$, all subject to possible errors of about 2° . The synodic arcs follow a pattern appropriate for Γ (cf. Fig. 1), but cannot be reproduced by System A_1 within the required tolerances. This accords with Neugebauer's observation that in general the pushes of System A_1 only roughly reproduce the behaviour of System A_2 and *vice versa*.¹⁵

b. Our second Mercury table, **4154**, preserves parts of the columns for the dates of the phenomena. In the translation in Table 5 I replace the Egyptian month names by Roman numerals, e.g. I=Thoth, and again I add a column for the line-to-line differences (synodic times), assuming that all calendar years are of 365 days (Table 5). The Babylonian schemes for Mercury all use period relations approximating the relation:

$$145 \text{ synodic periods} = 46 \text{ sidereal years}$$

so that, assuming a sidereal year between 365;15 and 365;16 days, we obtain from (2):

$$c = 1;40$$

to the nearest minute. Subtracting c from the synodic times of the papyrus, we find that the synodic arcs must have cycled through the three values $108;30^\circ$, $114;30^\circ$, $120;30^\circ$. These are precisely the plateaux of System A_2 for Ω .

Because all the reconstructible synodic arcs are plateau values, we cannot obtain an exact value for any longitude, but the requirement that all longitudes from line 3 to line 13 fall within the plateaux limits us to a narrow range. It is not difficult to show that if a longitude between $\mathbb{M} 6;30^\circ$ and $11;30^\circ$ is assigned to the epoch in line 3, all dates in the papyrus (except of course the scribal

	line	date	synodic time
		VI xx	
		X xx	
		I 28;42	110;10
		V 18;52	116;10
	5	IX 15;2	122;10
		I 12;12	110;10
		V 2;22	116;10
		VIII 28;32	122;10
		XII 30;42	110;10
	10	IV 15;52	116;10
		VIII 12;2	122;10
		XII 14;12	110;10
		III 29;22	116;10
		VII 25;32	122;10

Notes: Line 9, date originally written as Mesore 0;42 but apparently corrected. Line 11, the minutes of the date are written with an unrecognizable symbol; from the numerical pattern 2 is clearly meant. Line 12, date written as 11;12, which is certainly a scribal error for 14;12 (alpha for delta). In several lines the minutes are followed by traces of writing, which does not appear to be numerals.

Table 5. Translation of **4154**.

errors) will be reproduced. Since these are last evening visibilities, the sun's longitude must be roughly 15° less than Mercury's; for example in line 3 the sun must be within a few degrees of $\Omega 24^\circ$ on I 28. Now in the civil 'Alexandrian' calendar of the Roman period, which added an intercalary day after every four years, the sun's sidereal longitude on I 28 was always close to $\simeq 6^\circ$. In the unintercalated Egyptian calendar, however, the solar longitude on a given day of the year regresses, and a longitude $\Omega 24^\circ$ would correspond to a date about A.D. 170. One should allow several decades' tolerance to allow for our uncertainty about both the longitudes of Mercury and the corresponding elongation from the sun.

We may conclude (1) that this is a table for last evening visibilities (Ω); (2) that the longitudes were computed according to the Babylonian System A_2 ; (3) that the dates are in the Egyptian calen-

dar, and belong to the second or early third century; (4) that the synodic times *in days* were obtained from the synodic arcs by the addition of a constant $c=1;40$. This last is a point of divergence from what we know of Babylonian practice, not only in the obvious respect that the rule in the papyrus is adapted to work with real days in the civil calendar rather than *tithis*, but also because the only surviving Babylonian tables based on System A_2 do not use the usual principle of a constant difference between synodic time and synodic arc.¹⁶

c. In **4153** it is again the columns for the dates that are extant. The four columns, separated by ruled lines, contain (i) the accumulated number of days' divergence between equivalent Alexandrian and Egyptian calendar dates; (ii) the regnal years of an emperor, whose name is lost; (iii) the Egyptian calendar month; and (iv) the

line	divergence	year	month	day syn.	syn. arc time	
			VI			
			X			
		18	II			
10			V			
			IX			
		19	I			
			V			
			IX	6; 3x	127;7±0;5	125;27°±0;5°
15	59	20	I	8; 42	103;48±0;30	102;8°±0;30°
			IV	22; xx	113;0±1;0	111;20°±1;0°
			VIII	15; xx	134;0±1;0	132;20°±1;0°
			XII	29; xx	102;5±0;35	100;25°±0;35°
		21	IV	6; 3x	107;55±0;35	106;15°±0;35°
20			VII	24; xx		
			XII			
		22	III			
			VII			
			XI			
25		23	III			
			VI			

Table 6. Translation of **4153**, lines 7 to end.

day and sexagesimal fraction of a day within the month. Only from line 7 on is anything other than column i preserved. I have added to the translation in Table 6 columns for the approximate synodic times and for the corresponding synodic arcs, assuming $c=1;40$. The survival of parts of columns i and ii makes it possible to establish the range of years covered by the fragment. The divergence between the Alexandrian and Egyptian calendars reached 59 days in A.D. 211/212, which was the 20th regnal year of Septimius Severus. Moreover, the inclusion of column i shows that the calendar of the table is the Egyptian, since if it was Alexandrian the user of the table would have had no need for calendrical conversions. A comparison with modern theory (e.g. Tuckerman's tables) for any epoch date in the papyrus, say (line 14) Severus 19 IX 6 Egyptian = A.D. 211 March 4, shows that Mercury was always near its morning station, so that the phase in question must be Γ .

The synodic arc known within the narrowest limits is that between the epochs of lines 7–8, $125;27^\circ \pm 0;5^\circ$. Mercury was in Aquarius on the date of line 7's epoch, so we wish to determine what initial longitude in Aquarius yields this synodic arc according to the System A_1 scheme for Γ . The possible range turns out to be between $\approx 26;4^\circ$ and $26;34^\circ$. If we select, say, $\approx 26;30^\circ$ for the longitude of line 7's epoch and the attested date Severus 20 I 8;42 for the date of line 8's epoch, we obtain with System A_1 the sequence shown in Table 7. All legible digits in the fragment are reproduced, confirming the hypothesis that the longitudes of Γ in this table were computed according to System A_1 , and the dates were obtained by the same rule as in **4154**.

line	longitude	syn. arc	syn. time	date
7	326;30°	125;23,20°	127;3,20	19 IX 6;38,40
8	91;53,20°	102;21,40°	104;1,40	20 I 8;42
9	194;15°	110;45°	112;25	20 IV 22;43,40
10	305°	132;33,20°	134;13,20	20 VIII 15;8,40
11	87;33,20°	100;34,10°	102;14,10	20 XII 29;22
12	178;7,30°	106°	107;40	21 IV 6;36,10
13	284;7,30°			21 VII 24;16,10

Table 7. Recomputation of **4153** by System A_1 for Mercury.

line	col. ii	longitude	synodic arc	System A ₁
		♃		♃ 0;30°
		♄		♄ 4;33,20°
	x2	♅		♅ 17;15°
	3x	♆ 9°	136°±50	♆ 9°
5	25	♇ 2x°	96°±5°	♇ 20;13,20°
	x9	♈ 1°		♈ 1;7,30°
	xx	♉		♉ 17;30°

Notes: Line 4, col. ii, the second digit could be either 7 (zeta) or 2 (beta). Line 7, the second digit seems to resemble 8 (eta), but this is very uncertain.

Table 8. Translation of **4155**, fragment 2, and recomputation by System A₁, for Mercury.

d. **4155** is in some respects the most interesting of the epoch tables for Mercury. There are four fragments, which we may provisionally refer to as 1a, 1b, 1c, and 2. In each fragment there is part of a column for zodiacal signs, and fragments 1c and 2 have the left edge of the next column to the right, containing the longitude of the epoch in degrees within the zodiacal sign. To the left of the zodiacal signs is a column between double ruled lines, containing whole numbers; the preserved values range between 12 and 35. Still further to the left, fragments 1b and 2 have almost illegible traces of another column, apparently containing numbers with sexagesimal fractions. So little remains of this column that we will ignore it in the following translations and discussion.

We may begin by considering the longitudes in fragment 2 and their line-to-line differences (Table 8, first four columns). Comparison with Figs. 1–4 shows that a synodic arc greater than 130° starting in Aquarius is only plausible if the phenomenon in question is *I*. Specifically, assuming an initial longitude of $\approx 9^\circ$ for line 4, System A₁ yields the sequence in the last column of Table 8. We can confidently regard the phenomenon as identified; computation by System A₁ is also clearly compatible with the traces, but not proved at this stage of our argument.

Fragment 1c has just parts of four lines (Table 9c). In the (remaining two fragments there are no preserved degrees at all (Table 9a–b). Surprisingly, it is not difficult to show that Fragment 1a

line	col. ii	longitude
		II
	17	⊂
	13	⊂
	35	⊂
5	x8	⊂
	14	⊂
	34	⊂
	18	⊂
	15	⊂
10	30	⊂
		⊂
		⊂

Table 9a. Translation of **4155**, fragment 1a.

	22	⊂
	17	⊂
	15	⊂
	26	II
5	17	⊂
	12	⊂
	32	II
	18	⊂

Table 9b. Translation of **4155**, fragment 1b.

	⊂
	⊂ 1x°
	⊂ 1x°
	⊂ 1x°

Table 9c. Translation of **4155**, fragment 1c.

cannot have been computed using System A_1 for either Γ or Ξ , nor by System A_2 for Σ . For if one uses System A_1 for Γ and assumes the lowest possible longitude in line 2 ($\subseteq 0^\circ$), one finds already in line 4 that the longitude is $\text{II } 10;13,20^\circ$, in conflict with the table. Similarly using System A_1 for Ξ and starting with $\text{II } 0^\circ$ in line 1, one gets an irresolvable conflict in line 3, and using System A_2 for Σ and starting with $\subseteq 0^\circ$ in line 2 one gets a conflict in line 3. The same kind of argument allows us to eliminate System

line	longitude	syn. arc	syn. time	date
1	♃ 12°	112;6°	113;46	0
2	♄ 4;6°	111;0;40°	112;40,40	113;46
3	♅ 25;6,40°	116;15;50°	117;55,50	226;25,40
4	♆ 21;22,30°	116;13,30°	117;53,30	344;22,30
5	♇ 17;36°	109;10,40°	110;50,40	462;16
6	♈ 6;46,40°	113;58,20°	115;38,20	573;6,40
7	♉ 0;45°	120;21°	122;1	688;45
8	♊ 1;6°	108;30°	110;10	810;46
9	♋ 19;36°	114;30°	116;10	920;56
10	♌ 14;6°	120;30°	122;10	1037;6
11	♍ 14;36°	108;30°	110;10	1159;16
12	♎ 3;6°	114;30°	116;10	1269;26
13	♏ 27;36°	120;30°	122;10	1385;36
14	♐ 28;6°	108;30°	110;10	1507;46
15	♑ 16;36°	114;30°	116;10	1617;56
16	♒ 11;6°	120;30°	122;10	1734;6
17	♓ 11;36°	108;30°	110;10	1856;16
18	♈ 0;6°	113;54°	115;34	1966;26
19	♉ 24°	119;52,30°	121;32,30	2082;0
20	♊ 23;52,30°	109;43,30°	111;23,30	2203;32,30
21	♋ 13;36°	112;4°	113;44	2314;56
22	♌ 5;40°	117;35°	119;15	2428;40
23	♍ 3;15°	113;51°	115;31	2547;55
24	♎ 27;6°			2663;26

Table 10. 4155, recomputation of Ω by System A_2 for Mercury, incorporating fragments 1a–c.

A_1 (both Γ and Ξ) from consideration for fragment 1b. Fragment 1c is too small for independent analysis.

We are still left with several possibilities: the tables might not have been computed using the known System A schemes; fragment 1a might be Ω and 1b Σ ; or both might be Ω . This last is the most economical as well as the most testable hypothesis. It turns out not to be difficult to construct a sequence of longitudes computed according to System A_2 for Ω that reproduces the contents of fragment 1a and then, after six intervening lines, the contents of fragment 1b (i.e. lines 1–8 of 1b become 17–24 of the whole series). To this reconstruction (Table 10) I add columns giving the synodic

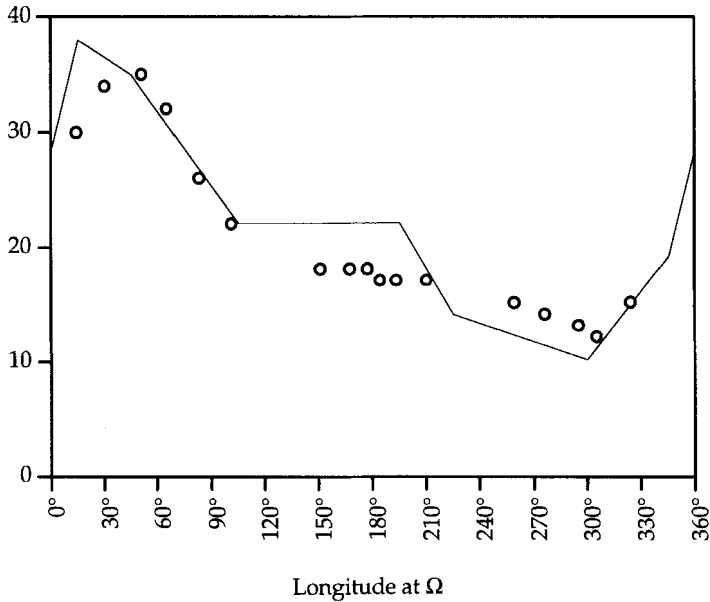


Fig. 8. Column ii of 4155 compared with pushes of System A_2 for Mercury.

arcs, the corresponding synodic times (assuming $c=1;40$), and the running total of the synodic times. Fragment 1c can probably be fitted into this sequence in lines 14–17, so that it directly joins 1b. Also, since the motion of Mercury between Ω and Γ is a small direct or retrograde arc, we can conjecture that line 1 of fragment 2 lined up with line 16 of the now united ‘fragment 1’.

And now we turn to the numbers in the column preceding the longitudes. If we graph these numbers against the reconstructed longitudes of the epochs in the same lines, we find (Fig. 8, small circles) a characteristic pattern shared by both fragments 1a and 1b – strong confirmation that they are pieces of the same sequence of epochs – whereas the quantity in the 30s corresponding to a longitude in Aquarius in fragment 2 shows that the pattern there is different.

We have already noted that in the Babylonian tables for Mercury one computed independently either just the first appearances (System A_1) or just the last appearances (System A_2). The intervals

in longitude and time from each computed phenomenon to the subsequent phenomenon, i.e. from computed first visibility to last visibility or from computed last visibility to the next reappearance, were found using schemes of so-called 'pushes'. The pushes defined the numbers of degrees and *tithis* between the pairs of phenomena as a function of the longitude at the former of each pair. The complete patterns of pushes are known exactly for the phase transitions Γ - Σ and Ξ - Ω , and at least approximately for Σ - Ξ and Ω - Γ .¹⁷ Each has an outline quite distinct from the others. Now if we compare the pushes in time (*tithis*) for the transition Ω - Γ (Fig. 8, continuous line) with the numbers in column ii of fragments 1a and 1b of the papyrus, we find that the pattern, although not identical, is strikingly similar. We can conclude from this (i) that this column represents the time intervals (in days, presumably) from a tabulated Ω epoch to the following Γ , and consequently (ii) that our tentative identification of the epochs in these fragments with Ω was correct. We might also conjecture by analogy that the numbers in column ii of fragment 2 are the time intervals from the epochs Γ tabulated in this fragment to the subsequent Σ .

In the recomputation in Table 10 of fragment 1 according to the rules of System A₂, I include columns leading to the calculation of the dates of the epochs relative to the date of the epoch of line 1 (day 0). We can do the same (Table 11) for the recomputation of fragment 2, extrapolated backward to give the whole sequence that we believe was lined up with fragment 1 (we will accordingly use the same continuous numbering of lines henceforth for the extrapolated fragment 2 as for fragment 1). Since we want to compare the calculated dates for the two sequences, we need to assign to the epoch of line 1's Γ a date about a month later than the Ω of the same line. The initial value used here, 30;13,30, was chosen by trial and error, but the precise value is not important. The last two columns are, respectively, the difference between the calculated dates of Γ and Ω , for those lines of the table where a number is at least partially legible in fragment 1, column ii, and the contents of that column.

The agreement between the attested numbers and the recomputed time intervals (truncated to whole numbers) is exact in all but five instances, where a difference of a fraction of a day would

line	longitude	syn. arc	syn. time	date	diff.	text
1	♈ 12°	99;52,30°	101;32,30	30;13,30		
2	♉ 21;52,30°	106°	107;40	131;46	18;0	17
3	♊ 7;52,30°	138;37,30°	140;17,30	239;26	12;59,20	13
4	♋ 26;30°	99;15°	100;55	379;43,30	35;21	35
5	♌ 5;45°	106°	107;40	480;38,30	18;22,30	x8
6	♍ 21;45°	133;15°	134;55	588;18,30	15;11,50	15
7	♎ 5°	104;37,30°	106;17,30	723;13,30	34;28,30	34
8	♏ 19;37,30°	106°	107;40	829;31	18;45	18
9	♐ 5;37,30°	127;52,30°	129;32,30	937;11	16;15	15
10	♑ 13;30°	110°	111;40	1066;43,30	29;37,30	30
11	♒ 3;30°	106°	107;40	1178;23,30		
12	♓ 19;30°	122;30°	124;10	1286;3,30		
13	♈ 22°	116;53,20°	118;33,20	1410;13,30		
14	♉ 18;53,20°	104;29,10°	106;9,10	1528;46,50		
15	♊ 3;22,30°	117;7,30°	118;47,30	1634;56		
16	♋ 0;30°	124;3,20°	125;43,20	1753;43,30		
17	♌ 4;33,20°	102;41,40°	104;21,40	1879;26,50	22;10,50	22
18	♍ 17;15°	111;45°	113;25	1983;48,30	17;22,30	17
19	♎ 9°	131;13,20°	132;53,20	2097;13,30	15;13,30	15
20	♏ 20;13,20°	100;54,10°	102;34,10	2240;6,50	26;34,20	26
21	♐ 1;7,30°	106;22,30°	108;2,30	2332;41	17;45	17
22	♑ 17;30°	138;23,20°	140;3,20	2440;43,30	12;3,30	12
23	♒ 5;53,20°	99;6,40°	100;46,40	2580;46,50	32;51,50	32
24	♓ 15°			2681;33,30	18;7,30	18

Table 11. 4155, recomputation of Γ by System A₁ for Mercury, incorporating fragment 2.

account for the discrepancy. These deviations can be ascribed to our uncertainty about the precise initial values for the longitudes and dates in both sequences of epochs, and about the method of determining the time difference (e.g. by subtraction of day numbers including fractions, or of merely the integer days). This result is decisive, because if the contents of the papyrus fragments had had different meanings from the ones we are assuming, or were computed by a different set of procedures, the deviations would certainly have been large and irregular. Hence we have a secure example of an epoch table for Mercury containing longitudes, and presumably also dates, of both Ω computed by System A₂ and Γ computed by System A₁. The recorded time intervals are not true

line	longitude	diff.	text
18	♃, 29;40°	42;25°	x2 (42?)
19	♃ 15;45°	36;45°	x2 or x7 (37?)
20	♃ 15;33,20°	25;20°	25
21	♃, 10°	38;52,30°	x9 (39?)
22	♃ 1°	43;30°	x8?

Table 12. 4155, recomputation of Σ by System A₂ for Mercury.

pushes in the Babylonian sense, since they are derived from the independently computed dates rather than from the longitude of Ω alone.

It may further be suspected that the original table had columns for all four of the phenomena of Mercury. Fragment 2 has part of a column that, by analogy with fragment 1, should contain the numbers of days from Γ to Σ . With enough of this column, we could reconstruct within a margin of one day the dates of Σ , and see whether they are compatible with computation according to the System A₂ scheme. Unfortunately we have only one complete and secure reading in line 5 (i.e. line 20 of the extrapolated series), 25 days corresponding to a longitude that we know must have been approximately $\text{II } 20^\circ$. Still it is possible to proceed on the following lines. If all dates of phenomena were computed using the hypothesis of a constant difference between synodic arc and synodic time, then the time intervals between consecutive Γ and Σ must also differ by some constant from the differences in longitude. This constant must in fact be almost zero because Mercury's overall motion during its intervals of visibility parallels the sun's motion, i.e. roughly 1° per day. So we can estimate that the longitude of Σ on this line of the table was near 15° . On this basis we use System A₂ to reconstruct the sequence of longitudes of Σ for lines 18–22 of the table, and find the differences in degrees between these longitudes and the reconstructed longitudes of Γ (Table 12). The last column gives the numbers in col. ii of fragment 2.

The result of the comparison is, needless to say, unsatisfactory. The readings of the last digits in lines 18 and 21, of which I am fairly confident, are in close enough agreement with the expected

values. On the other hand, the small trace in line 22 does not look like part of a 3 or 4, and definitely resembles the right side of an 8. But the evidence is so poor that I hesitate to draw any conclusions from them. [see postscript]

e. There remain two small bits of epoch tables for Mercury to be considered. **4156a** has parts of three lines from a table, as follows:

0	19	☾
24	29	☿
24	39	♁

It is probable that the numbers in the first two columns represent the day and sexagesimal fraction of a day within the calendar month corresponding to each epoch. If so, then the synodic times must have been 114;10 (or 119;10) days and 120;10 (or 125;10) days, where the larger values in parentheses would apply if the interval contained the end of an Egyptian year. The corresponding synodic arcs are therefore $112;30^\circ$ (or $117;30^\circ$) and $118;30^\circ$ (or $123;30^\circ$). None of the System A schemes can generate these precise synodic arcs from an initial longitude in Libra, but using System A₂ for Ω we can obtain the sequence ☾ $17;30^\circ$, ☿ 10° , ♁ $8;7,30^\circ$, hence synodic arcs $112;30^\circ$ and $118;8,30^\circ$. The table was therefore probably for Ω but not computed by the familiar System A₂ scheme. [see postscript]

4156b, also from the bottom of a table, has parts of five columns, separated by double ruled lines. Columns iv and v contain the zodiacal sign and degrees of the epoch longitude. The preceding columns may contain the date, in the form month (i), day (ii), and sexagesimal fraction of a day (iii), but this is far from certain. Only slight traces of columns i and v survive.

<i>ii</i>	<i>iii</i>	<i>iv-v</i>
12	9?	♁
29	36	♁
x	50	♁
26	47	♁ 9°

Progress from Capricorn to Gemini implies a synodic arc of at least 120° , and from Figs. 1–4 it is clear that this can only happen if the phenomenon in question is Γ . The computation does not seem to have been made following the System A_1 rules, however. System A_1 yields a constant synodic arc of 106° (and hence a constant synodic time of 107;40 days) for all initial longitudes in Virgo, whereas the synodic time between the first two preserved dates – if that is indeed what they are – cannot be more than 107;36 days.

3. Venus

Our only fragment of an epoch table for Venus so far, **4157**, preserves parts of just two lines from the bottom of the table. Columns iii and iv are the only ones with legible contents:

<i>iii</i>	<i>iv</i>
5xx	
610;7	month X

The two columns presumably represent, respectively, the synodic time leading up to the date of epoch, and the date itself. 610 days would be close to the upper limit for the synodic time between two consecutive phenomena of the same kind for Venus. In the known Babylonian schemes for Venus – which are themselves only poorly understood – the longest synodic time is 614 *tithis*, i.e. less than 604 days.¹⁸

4. Mars

Our first epoch table for Mars, **4158**, is broken into several disconnected fragments. We begin with two pieces that we may provisionally refer to as fragments 1a (from the bottom of the table) and 1b (Table 13). We can explain part of the structure of the table at once. Columns ii–iii contain the dates of a phenomenon, which is recognizable from its periodicity as pertaining to Mars. Column i gives the line-to-line differences, i.e. the synodic times leading up

line	i	ii	iii
<i>fragment 1a</i>			
	xx;27,xx		
	xx;34,4		
	xx;xx,4		
	47;14,[4]	V	15;3[4],[48]
5	[36];54,4	VI	22;2[8],52
	[31];[1]4,4	VII	23;42,56
	[37];27,24	IX	1;10,20
<i>fragment 1b</i>			
			xx;55,20
	[xx,24,4]		15;19,24
	[x1,14,4]		x6;33,28
	[x9,7,24]		25;40,52
5	[x7,0,44]		x2;41,36

Table 13. Translation of two fragments from 4158.

to each phenomenon. Using this knowledge we can restore many of the digits (enclosed in brackets in the translation above).

The recurring patterns in the last digits of the synodic times are obviously symptomatic of an arithmetical function. Moreover we note that the same synodic time, 31;14,4 days, seems to occur in both line 6 of fragment 1a and line 3 of 1b. Now in the Babylonian System A scheme for Mars the minimum synodic arc, 30° , must occur once in every zodiacal revolution of the phenomenon (i.e. every seven or eight lines of the table). We can hypothesize that 31;14,4 days is the synodic time corresponding to a synodic arc of 30° , so that $c=1;14,4$. And this is clearly correct, because the period relation underlying the Babylonian theory of Mars equates 284 sidereal years with 133 synodic periods and 18 zodiacal revolutions of any phenomenon, so that by (2) above:

$$c \approx 365;15,30 \times \frac{284}{133} - 360 \times \frac{18}{133} \approx 731;14 = 2 \text{ Egyptian years} + 1;14$$

The value 1;14,4 would correspond to a sidereal year of approximately 365;15,40 days.

We can therefore obtain an exact synodic arc from any known synodic time, say 36;54,4 days in line 5 of 1a, and deduce from the System A rules what the initial longitude must have been (in this instance \odot 18;40°). This leads us to the complete reconstruction of a sequence of 26 computed longitudes and dates of a phenomenon of Mars, into which we can fit the majority of the fragments with exact correspondence in every digit (Table 14). This proves, firstly, that the table was produced using the familiar System A scheme for longitudes; secondly, that the synodic times were ob-

line	longitude	syn. arc	syn. time	date
1	Π 20;30°			year 0 V 10;55,20
2	\odot 23;40°	33;10°	34;24,4	2 VI 15;19,24
3	Ω 23;40°	30°	31;14,4	4 VII 16;33,28
4	\sphericalangle 1;33,20°	37;53,20°	39;7,24	6 VIII 25;40,52
5	\mathcal{M} 17;20°	45;46,40°	47;0,44	8 X 12;41,36
6	ζ 26°	68;40°	69;54,4	10 XII 22;35,40
7	Υ 12°	76°	77;14,4	13 III 4;49,44
8	Π 3°	51°	52;14,4	15 IV 27;3,48
9	\odot 12°	39°	40;14,4	17 VI 7;17,52
10	Ω 12°	30°	31;14,4	19 VII 8;31,56
11	\mathcal{M} 16°	34°	35;14,4	21 VIII 13;46,0
12	\sphericalangle 26°	40°	41;14,4	23 IX 25;0,4
13	ζ 24°	58°	59;14,4	25 XI 24;14,8
14	χ 15;45°	81;45°	82;59,4	28 II 12;13,12
15	Υ 15;30°	59;45°	60;59,4	30 IV 13;12,16
16	\odot 0;20°	44;50°	46;4,4	32 V 29;16,20
17	Ω 0;20°	30°	31;14,4	34 VI 30;30,24
18	\mathcal{M} 0;26,40°	30;6,40°	31;20,44	36 VIII 1;51,8
19	\sphericalangle 10;26,40°	40°	41;14,4	38 IX 13;5,12
20	ζ 0;40°	50;13,20°	51;27,24	40 XI 4;32,36
21	\approx 16°	75;20°	76;34,4	43 I 16;6,40
22	Υ 27°	71°	72;14,4	45 III 28;20,44
23	Π 13°	46°	47;14,4	47 V 15;34,48
24	\odot 18;40°	35;40°	36;54,4	49 VI 22;28,52
25	Ω 18;40°	30°	31;14,4	51 VII 23;42,56
26	\mathcal{M} 24;53,20°	36;13,20°	37;27,24	53 IX 1;10,20

Table 14. **4158.** recomputation of phenomena by System A for Mars, incorporating fragments 1a and 1b. Synodic arcs and times are those preceding the epochs on the same line.

tained by adding exactly 1;14,4 to the synodic arcs; and thirdly that the dates are in the Egyptian calendar (since no intercalations are involved).

All the fragments come from the part of the table containing the synodic times and the dates within the year. A column must also have existed giving the years themselves, probably as regnal years, but this has not survived. The problem of establishing the dates covered by the table is related to that of identifying the relevant phenomenon. In the Babylonian System A tables, only the three synodic phenomena Γ , Φ , Ω are computed using the System A zone scheme, while the remaining two, Θ and Ψ , are obtained from Φ by one of several known 'retrogradation schemes' that work in a way analogous to the 'pushes' for Mercury. The retrogradation schemes are necessary to make the retrogradations vary through the ecliptic in the right way, and we have no reason to suppose that they were not still used in the Roman period. We also know that the phenomenon of our papyrus occurred, to select one instance (line 1), about the calendar day V 11 at about Υ 20°. During an acceptable range of dates for the papyrus (e.g. the first through fourth centuries) neither Γ nor Ω occurred in month V anywhere near this position. On the other hand there are a half dozen years in the third and fourth centuries in which Φ fell within a few days of the desired day and longitude.

To evaluate these prospective datings, I used Ptolemy's *Handy Tables* to compute the dates and sidereal longitudes of all the occurrences of Φ in the third and fourth centuries.¹⁹ Exact agreement between Babylonian methods and Ptolemy's theory is of course not to be expected, but one ought to find discrepancies that are as small as possible and that are roughly the same for both date and longitude, since otherwise there would have had to be large errors in Mars' longitudes during the planet's direct motion. The only dating that satisfied this criterion at all well equated line 1 with A.D. 271; with this alignment, the dates of Φ computed by Ptolemy's tables were between 7° behind and 1° ahead of those in the papyrus table, and the longitudes were between 12° behind and 2° ahead. I conclude that the papyrus table covered the years 271–325.

The table, when complete, contained columns for some or all of

line	longitude	syn. arc	syn. time	date
1	≈ 6°			year 0 month x 16;14,12
2	Υ 19;30°	73;30°	74;44,4	2 $x+2$ 30;58,16
3	Π 8°	48;30°	49;44,4	4 $x+4$ 20;42,20
4	Ϟ 15;20°	37;20°	38;34,4	6 $x+6$ 29;16,24
5	Ω 15;20°	30°	31;14,4	8 $x+8$ 30;30,28 or 9 $x-4$ 25;30,28
6	Ⓜ 0;26,40°	35;6,40°	36;20,44	10 $x+10$ 6;51,32 or 11 $x-2$ 1;51,32

Table 15. Translation of **4158**, fragment 3, with recomputation according to System A for Mars. Synodic arcs and times are those preceding the epochs on the same line.

Mars' other phenomena. We know this because two fragments cannot be fitted into the restored sequence. One of these, which I call fragment 3, comes from a column of dates computed according to the System A rules in the same way as the dates of Φ already discussed (Table 15). Using a method of testing 'connectability' devised by Neugebauer, it is easy to show that the sequence of longitudes in Table 14, extrapolated either forward or backward, will eventually take on the values in Table 15, but the corresponding dates will not match those in the fragment.²⁰ These were presumably the dates of a parallel column for either Γ or Ω . The other remaining piece, fragment 2, is from the bottom of the table, that is either to the right or left of fragment 1a. The writing is much abraded, but one can read parts of dates that seem to be consistently about 35 days later than the dates of the corresponding lines

line	date
	[5] VIII 2[5]
	[7] XI[9]
	[9] XII 28
	[12] II 1;1,1x
5	Maximinus
	[1] III 2;15

Table 16. Translation of **4159**.

line	date	synodic time	synodic arc	longitude
	4 [IV xx]			
	5 [V] 14;24	30;44	30°	150°
	6 VI 15;8	30;47±0;5	30;3°±0;5°	180°
	7 VII 15;5x	33;40±0;10	32;56°±0;10°	210;3°
5	8 VIII 19;3x	34;30±0;10	33;46°±0;10°	242;59°
	9 IX 24;x	36;30±0;10	35;46°±0;10°	276;45°
	10 XI 0;3x	36;50±0;10	36;6°±0;10°	312;31°
	11 XII 7;2x	36;42±0;5	35;58°±0;5°	348;37°
	13 I 9;7	34;38±0;5	33;54°±0;5°	24;35°
10	Galba [1] II 13;4x	34;30±0;10	33;46°±0;10°	58;29°
	Vespasian III 18;1x	32;40±0;10	31;56°±0;10°	92;15°
	2 IV 20;5 ^o x	30;35±0;35	29;51°±0;35°	124;11°
	3 V 21;xx;xx	} (96;0)	(93;48°)	154;2°
	[4] VI xx;xx			
15	[5] VII xx;xx			
	[6] VIII 27;xx	34;25±0;35	33;41°±0;35°	247;50°
	[7] X 1;5x	36;50±0;10	36;6°±0;10°	281;31°
	[8] XI 8;4x	36;40±0;10	35;56°±0;10°	317;37°
	[9] XII 15;2x	37;5±0;35	36;21°±0;35°	353;33°
20	[Titus I] I 17 ^o ;xx			29;54°
	[2] II 2x;xx			

Table 17. Translation of **4160** with reconstructed synodic times, synodic arcs, and longitudes.

in the Φ sequence. This would fit Θ well. Unfortunately too little of the day numbers can be read to allow us to reconstruct the method of computation.

The second known fragment of an epoch table for Mars, **4159**, preserves parts of five consecutive dates of a phenomenon (Table 16). Again the characteristic synodic time of 31;14 days appears between the fourth and fifth epochs, and it is clear that the method of computation was essentially the same as for **4158**. This time, however, there is no difficulty with the date or the identity of the phenomenon. The name of Maximinus written above the fifth epoch indicates that it occurred in that emperor's first or second regnal year (A.D. 234/235 or 235/236), and Mars' I indeed oc-

curred close to Maximinus 1, Hathyr 2 and about Ω 16° as required.²¹

5. *Jupiter*

The single known table for Jupiter, **4160** (Table 17), presents no difficulties of dating or identifying the tabulated phenomenon. From the preserved regnal years and dates it is obviously a table for Jupiter's Φ covering the years Nero 4 through Titus 2, i.e. A.D. 57–80. Only the columns for the dates are preserved, and most of these are missing part or all of the fractions of days. We can recover most of the synodic times within a tolerance of a fraction of a degree, and convert these to approximate synodic arcs using $c=0;44$ (obtained from the period relation 391 synodic cycles and 36 zodiacal revolutions of a phenomenon in 427 sidereal years). The reconstructed longitudes in Table 17 are based on a hypothetical initial longitude of Virgo 0° for line 2. I have omitted the tolerances for this column because they will not significantly affect the following discussion; but it should be noted that all longitudes could be greater or less than the ones assumed in the papyrus by some small constant.

Many of the synodic arcs hover around the three plateau values of the Babylonian System A', which are 30° , $33;45^\circ$, and 36° . However, if we graph the synodic arcs against initial longitudes and superimpose the pattern of System A', (Fig. 9), we find that the data from the papyrus cannot be reconciled with the Babylonian scheme. The intermediate plateaux at $33;45^\circ$ seem to be broader, and the extreme plateaux seem to be narrower, which would imply that the papyrus table was computed from a variant of System A' (which I call A*) that used the same division of the ecliptic into four zones, and the same w value for each zone, but lengthened the two intermediate zones at the expense of the fast and slow zones. It is in fact possible to redistribute the zones arbitrarily without affecting the periodicity of the scheme so long as one deducts 1° from the slow zone and 2° from the fast zone for every 3° that one adds to the total of the intermediate zones. By trial and error I found good agreement assuming consecutive zones of

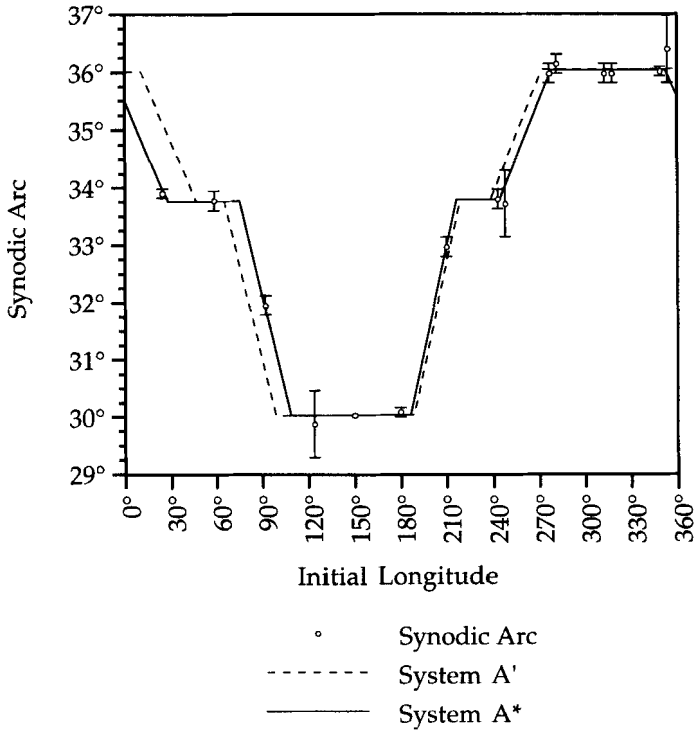


Fig. 9. Synodic arcs in **4160** compared with System A' for Jupiter.

length $107;30^\circ$ ($w=30^\circ$), $61;30^\circ$ ($w=33;45^\circ$), 110° ($w=36^\circ$), and 81° ($w=33;45^\circ$), such that the beginning of the slow zone is 41° before the longitude of the epoch in line 2 (whatever that was). The recomputed dates (Table 17) differ in only three cases from the numbers in the papyrus, and lowering these values by a mere $0;1$ day would produce complete agreement. These discrepancies may be due to our ignorance of the precise value of c used in the computations and the exact alignment of the longitudes.

6. Saturn

4161 has a different appearance from the other epoch tables discussed above. Except for the names of emperors, all the infor-

mation in the table is numerical. Each epoch is expressed by five whole numbers in parallel columns: the regnal year, the number of the month in the Egyptian calendar, the day of the month (no fractions), the number of the zodiacal sign counted from Virgo=1, and the whole number of degrees within the sign.²² In the translation in Table 18 these notations are replaced by the standard symbols.

Because we know the dates (ranging through the years Tiberius 9 – Gaius 3 and in the next column Domitian 2–12, i.e. A.D. 23–92), we can identify the phenomenon as Saturn's Γ . Moreover, from Saturn's period relation of 256 synodic cycles and 9 zodiacal revolutions in 265 sidereal years, we have approximately $c=0;26$. Thus the synodic arcs obtained as line-to-line differences of the epoch longitudes, which have a tolerance of $\pm 1^\circ$ around a whole number, can be supplemented by synodic arcs deduced from the synodic times, which have a tolerance of $\pm 1^\circ$ around a value $0;34^\circ$ greater than a whole number. Thus the pattern of synodic arcs plotted against initial longitude (Fig. 10) is sensitive to variations of about half a degree.

The synodic arcs were obviously not calculated by the Babylon-

line	date	longitude
		[Υ] 21°
5	[13] X 8	[Υ] 5°
	[14] X 23	Υ 19°
	[15] XI 7	Υ 3°
	[16] XI 21	Υ 17°
	[17] XII 5	Υ 1°
10	[18] XII 19	Υ 14°
	[19] XIII 2	Υ 27°
	[21] I 11 [?]	Υ xx°
	[22] I 2x	
	Gaius	
15	1 II 4	
	2 II 20 [?]	Υ xx°
	3 III 2	

Table 18. Translation of 4161, lines 4–17.

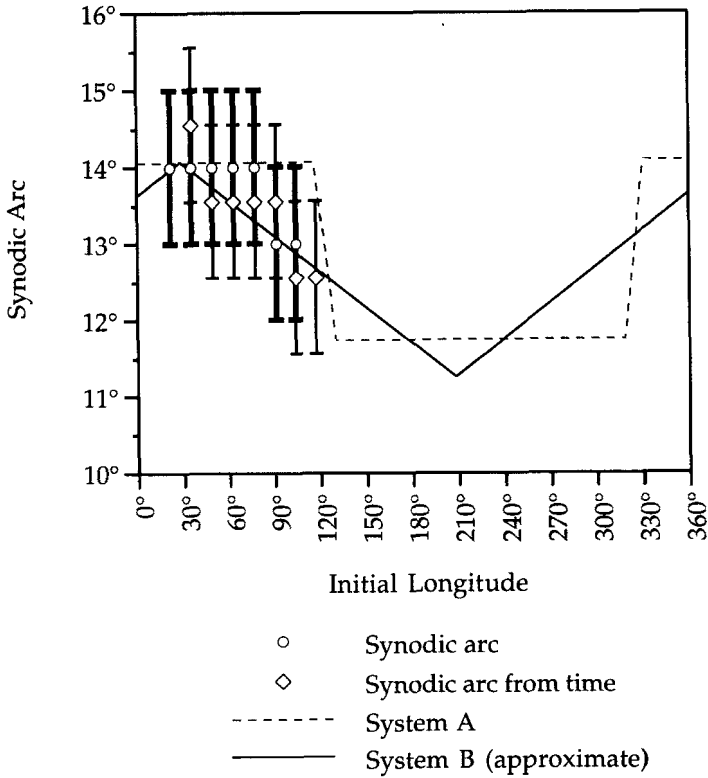


Fig. 10. Synodic arcs in **4161** compared with System A and System B for Saturn.

ian two-zone System A scheme for Saturn. In fact the pattern exhibits a sharp peak around the middle of System A's fast plateau, followed by what seems to be a steady decline heading for a minimum somewhere in the slow plateau. This suggests that the synodic arcs are varying continuously and more or less linearly between the maximum and minimum values.

Now Saturn's System A is not common among the Babylonian texts; it is known only from procedure texts and from so-called template tables that list a full period of computed longitudes of phenomena without dates. The dozen known tablets containing dated phenomena of Saturn use a System B scheme, according to

which the synodic arcs and times are consecutive values of linear zigzag functions, alternately increasing and decreasing by constant differences between a prescribed maximum and minimum value. System B schemes for Mars and Jupiter are also known in the cuneiform texts. The defining parameters for Saturn's zigzag function for synodic arc are a minimum $m=11;14,2,30^\circ$, a maximum $M=14;4,42,30^\circ$, and a constant increment/decrement $d=0;12^\circ$. The function for synodic time has the same d and amplitude, with the maximum and minimum increased by c . I had no difficulty in finding by trial and error a sequence of longitudes and dates computed according to System B (Table 19) that reproduced all data in the papyrus except for the date in line 15, which is probably a scribal error (delta for zeta).

7. General remarks

In analysing eleven planetary epoch tables written over a span of more than two hundred years (after A.D. 80 to after A.D. 325), we have been able to demonstrate the probable or certain use of the following Babylonian predictive schemes:

line	synodic time	date	synodic arc	longitude
4		12 IX 24;28		Υ 21;58°
5	14;16	13 X 8;44	13;50°	Υ 5;48°
6	14;28	14 X 23;12	14;2°	Υ 19;50°
7	14;21,25	15 XI 7;33,25	13;55,25°	Υ 3;45,25°
8	14;9,25	16 XI 21;42,50	13;43,25°	Υ 17;28,50°
9	13;57,25	17 XII 5;40,15	13;31,25°	Υ 1;0,15°
10	13;45,25	18 XII 19;25,40	13;19,25°	Υ 14;19,40°
11	13;33,25	19 XIII 2;59,5	13;7,25°	Υ 27;25,5°
12	13;21,25	21 I 11;20,30		
13	13;9,25	22 I 24;29,55		
14	12;57,25	23 II 7;27,20		
15	12;45,25	24 II 20;12,45		
16	12;33,25	25 III 2;46,10		

Table 19. **4161**, recomputation of dates and longitudes of Υ by System B for Saturn.

Mercury, System A₁ for Γ (**4153, 4155**)
 Mercury, System A₁ for Ξ (**4152**)
 Mercury, System A₂ for Σ (probable: **4152**)
 Mercury, System A₂ for Ω (**4154, 4155**)
 Mars, System A (**4158, 4159**)
 Jupiter, variant of System A' (**4160**)
 Saturn, System B (probable: **4161**)

This list accounts for most of the known Babylonian System A planetary schemes (not counting variants, only Jupiter's System A and Saturn's System A are not represented) as well as one of the three known System B schemes. The lack of duplication suggests that if we had more epoch tables, more of the schemes would turn up. But even without recourse to extrapolation we can see that most of the Babylonian mathematical planetary theory was accessible to, and used by, Greek-speaking astrologers and astronomers in the Roman Empire. What is more, the procedures are scarcely altered from the form in which they appear in cuneiform texts, the principal change being in the adaptation of the computation of dates to the non-lunar Egyptian calendar.

This fact has important consequences for the development of both astronomy and astrology, which we can only hint at here. Students of the history of astrology have tended to emphasize the profound conceptual differences between Mesopotamian and Greek astrology, and the much greater technical detail of Greek horoscopy as compared with its Babylonian counterpart, although it is also unquestionably true that certain essential principles in Greek astrology, including the idea and basic contents of a horoscopic document itself, are fundamentally Babylonian. Now we find that, notwithstanding the metamorphoses undergone by the interpretative apparatus of astrology during the Hellenistic period, the computational apparatus shows a remarkable continuity. Our hypotheses about the way that Babylonian astral science passed to the classical world need to be able to account for the transmission of a much larger baggage of mathematical astronomy than we formerly thought.

From the point of view of Greek astronomy it would be easy to dismiss the appearance of the Babylonian schemes in the papyri of the Roman period as a phenomenon of little significance except as

an instance of the survival of crude and outmoded methods alongside the superior kinematic Greek astronomy of the Hipparcho-Ptolemaic tradition. I think such a stance would be mistaken. In the first place although few would deny that there existed a predictive kinematic planetary astronomy before Ptolemy (i.e. tables relying on trigonometrical functions to represent models of uniform circular motion), we know practically nothing about it; and unless we embrace the naive notion that a kinematic model is inherently superior to an arithmetical one, how can we be sure that the users of the papyrus tables did not make the best choices available to them?

Secondly, the Babylonian planetary schemes would have been a valuable resource in the very establishment of kinematic models. They expressed in the most direct way the patterns of incidence of the conspicuous planetary phenomena in the form of mathematical generalizations. As such, they could serve either as guides to the selection of observations, or even as a substitute for observations. As an instance of how this could have worked may be cited G. Grasshoff's recent argument that Ptolemy based his theory of planetary visibility in the *Almagest* on Babylonian theoretical parameters rather than direct observations.²³ It goes without saying that Grasshoff's hypothesis becomes much more persuasive now that we know that these theoretical parameters were part of a living tradition in Ptolemy's milieu.

Postscript

Further investigation of the unpublished Oxyrhynchus papyri in 1996 brought to light several new fragments relevant to the foregoing articles. Full texts and commentary will appear in *Astronomical Papyri from Oxyrhynchus*.

Article I: (i) *P. Oxy.* 4164a gives us for the first time the end of the Standard Scheme template. The daily increments for days 249–303 turn out to have been the *final* 55 values of the 3031 days of the zigzag function, not the 55 values following day 248. Consequently the total progress in longitude on day 303 is exactly the 32;33,44,51° assumed in the epoch tables. (ii) Dr R. A. Coles and Dr J. R. Rea inform me that the handwriting of the procedure text

P. Oxy. 4136 is definitely of the first century, i.e. much older than the horoscopes on the other side. This obviously pushes the date of invention of the Standard Scheme at least this far back.

Article III: (i) Both the Mercury tables *P. Oxy.* 4155 and 4156a have been augmented by new fragments. In 4155 fr. 2 we now have five consecutive longitudes complete to two fractional places, which as expected were computed by System A_1 for Γ . More time intervals in the preceding column can now be read, and it is clear that they were derived from a lost sequence of Σ computed by System A_2 . 4156a now has both dates and longitudes of Ω , computed by System A_2 . (ii) New planetary epoch tables include 4157a, a table of Venus' inferior conjunctions (!); 4159a, a table of Mars' Ψ ; and 4160a, a table of Jupiter's Γ . The last of these adds System B for Jupiter to the repertoire of Babylonian schemes attested in the papyri.

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NOTES

1. Most recent editions: Neugebauer & Parker, *EAT* v. 3, 225–240 and plates 66–78.
2. For a detailed description of the formats and lists of the texts, see Jones [Forthcoming].
3. Neugebauer [1975] 565; Clagett, *AES* v. 2, 129. I have suggested a possibly greater role for the Egyptian language: Jones [1994] 44–48.
4. Van der Waerden [1947], [1960], [1972].
5. Neugebauer & Parker, *EAT* v. 3, 236–240.
6. Neugebauer was in the end satisfied by the demonstration of thesis (b); see Neugebauer [1975] 456.
7. Neugebauer & Pingree [1971] v. 2, 109–128.
8. Neugebauer & Pingree [1971] v. 1, 9–14.
9. A Greek papyrus table, *P. Mich.* III.151 + *P. Heid. inv.* 4144, has been interpreted by Neugebauer and myself as part of a scheme for Mars modified from the System A scheme: Jones [1990].
10. Full texts, translations, and textual commentary will appear in Jones, *APO*.
11. Two planetary templates were already known: *P.S.I.* XV.1492 (Saturn) and the demotic *P. Carlsberg* 32 (Mercury).
12. Neugebauer, *ACT* 279–315.
13. There exist variant schemes for Jupiter and Mercury (Neugebauer [1975] 445 and 468–471), as well as an as yet unpublished System A scheme for Venus. None of these is relevant to the texts discussed in this paper.
14. For details of the scheme, see Neugebauer, *ACT* 293–294.
15. Neugebauer [1954] 77–78 and fig. 7.
16. Neugebauer, *ACT* 298.
17. Neugebauer, *ACT* 293–295 with Figs. 56–57; 297–299 with Figs. 57a–b.
18. Neugebauer, *ACT* 301–302.
19. The sidereal longitudes were obtained by Theon's formula, i.e. adding 8° and subtracting 1° for every 80 years after 158 B.C.
20. Neugebauer, *ACT* 304–305.
21. The *Handy Tables* yield an epoch about 13 days and 8° earlier.
22. The convention of representing months and zodiacal signs by numerals is common in Greco-Egyptian almanacs: Neugebauer [1975] 788–789.
23. Grasshoff [1993].