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Ptolemy's First Commentator

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CONTENTS

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BIBLIOGRAPHICAL ABBREVIATIONS

Aaboe [1955]	A. Aaboe, "On the Babylonian Origin of Some Hipparchian Parameters," *Centaurus* 4 (1955): 122–25.
Almagest [Heiberg]	*Claudii Ptolemaei Opera quae exstant omnia.* Vol. I, *Syntaxis Mathematica*, ed. J. L. Heiberg (2 vols., Leipzig: 1898–1903).
Almagest [Toomer]	G. J. Toomer, trans., *Ptolemy's Almagest* (London, etc.: 1984).
CCAG	*Catalogus Codicum Astrologorum Graecorum* (12 vols., Brussels: 1898-1953).
Cumont [1910]	F. Cumont, "Comment les Grecs connurent les tables lunaires des Chaldéens," *Florilegium . . . dédiés à M. le marquis Melchior de Vogüé. . . .* (Paris: 1910), 159–65.
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Paul [Boer]	*Pauli Alexandrini Elementa Apotelesmatica*, ed. Ae. Boer (Leipzig: 1958).

Pingree, *Hephaestio*	D. Pingree, ed., *Hephaestionis Thebani Apotelesmaticorum libri tres*, 2 vols. (Leipzig: 1973–74).
Ptolemy, *Almagest*	See *Almagest*.
Ptolemy, *OAO*	*Claudii Ptolemaei Opera quae exstant omnia.* Vol. II, *Opera Astronomica Minora*, ed. J. L. Heiberg (Leipzig: 1907).
Rome [1927]	A. Rome, "L'Astrolabe et le Météoroscope d'après le commentaire the Pappus sur le 5e livre de l'Almageste," *Annales de la Société Scientifique de Bruxelles* A 47 (1927) 1ère partie: 77–102.
Rome [1931,1]	A. Rome, "Sur une loi empirique des éclipses de lune," *Annales de la Société Scientifique de Bruxelles* A 51 (1931) 1ère partie: 94–103.
Rome [1931,2]	A. Rome, "Sur la date d'Artémidore," *Annales de la Société Scientifique de Bruxelles* A 51 (1931) 1ère partie: 104–12.
Rome, *CA*	A. Rome, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almagest* vol. 2 (Vatican: 1936, Studi e Testi 72).
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Tihon [1987]	A. Tihon, "Le livre V retrouvé du *Commentaire à l'Almageste* de Théon d'Alexandrie," *L' Antiquité Classique* 56 (1987): 201–18.
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Toomer [1985]	G. J. Toomer, "Galen on the Astronomers and Astrologers," *Archive for History of Exact Sciences* 32 (1985): 193–206.
Toomer	See also *Almagest* [Toomer].
Van der Waerden [1958]	B.L. van der Waerden, "The Astronomical Papyrus Ryland 27," *Centaurus* 5 (1958): 177–89.
Van der Waerden [1972]	B. L. van der Waerden, "Die 'Ägypter' und die 'Chaldäer'," *Sitz. Heidelberger Akad.* (Math.-nat.) 1972, Abh. 5.
Vettius Valens [Kroll]	*Vettii Valentis Anthologiarum Libri*, ed. W. Kroll (Berlin: 1908).
Vettius Valens [Pingree]	*Vettii Valentis Antiocheni Anthologiarum Libri Novem*, ed. D. Pingree (Leipzig: 1986).

I. INTRODUCTION

The early recognition of Ptolemy's major astronomical writings, the *Almagest* (or *Syntaxis*, finished between A.D. 147 and 161)[1] and the later *Handy Tables*, has become a commonplace in histories of ancient astronomy. As Neugebauer writes, "The eminence of these works, in particular the *Almagest*, had been evident already to Ptolemy's contemporaries. This caused an almost total obliteration of the prehistory of the Ptolemaic astronomy."[2] Certainly by the fourth century, when Pappus and Theon of Alexandria wrote huge didactic commentaries on Ptolemy's works, the writings of even his greatest predecessor, Hipparchus (fl. ca. 150-130 B.C.), were relegated to merely antiquarian status, and already some of these seem to have become scarce even in Alexandria. As for those who followed Hipparchus, and whom Ptolemy dismisses with contemptuous allusions or still more disdainful silence, it is only through a handful of contemporary *testimonia* that some meager knowledge of their works—indeed of their very names—is preserved. But the crucial century and a half between Ptolemy and Pappus, during which Ptolemy's works were first circulated and gained preeminence, is for mathematical astronomy as nearly barren of documents as the three centuries between Hipparchus and Ptolemy.

The fragmentary text with which this monograph is concerned casts some light on both these murky periods. Apparently written in the early third century, it shows how Ptolemy's works had already begun to be expounded, criticized, and even revised within half a century of their publication. Moreover it preserves quotations from Artemidorus, a still earlier critic of Ptolemy's innovations, and Apollinarius, a prominent astronomer from the time before Ptolemy.

The fragment was discovered by the editors of the *Catalogus Codicum Astrologorum Graecorum* in the thirteenth-century manuscript *Par. gr.* 2841 and its sixteenth-century copy *Par. gr.* 2415; an edition by F. Cumont was included in their eighth volume in 1911.[3] Although Cumont made some necessary emendations to the very corrupt text, and incorporated several more that J. L. Heiberg communicated to him, his edition still goes only part of the way toward restoring the fragment, and hardly at all toward explicating it. Moreover, it was prepared and printed carelessly.[4] Historians have thus had to struggle with a text that for the most part makes no astronomical, or even

[1] See Toomer in *Almagest* [Toomer], 1.
[2] Neugebauer, *HAMA*, 5.
[3] *CCAG* vol. 8 part 2, 125–34. Cumont is credited with the edition only in the corrigenda (p. 178); as a result, it has sometimes been mistakenly ascribed to the volume's editor, C. E. Ruelle.
[4] The textual apparatus is entirely untrustworthy. On p. 133, for example, ten lines of apparatus contain four mislineations, three wrong readings of the manuscript, two redundant notes referring to the same word, and an irrelevant scrap of elementary paleographical notes.

grammatical, sense. Not surprisingly, therefore, little has been written about the fragment since 1911.[5]

This monograph provides the first translation and complete annotation of the fragment, in order to make its contents more accessible and intelligible, and to correlate them to what we know from other ancient sources. I include also a new edition of the Greek text, based on a fresh transcription of *Par. gr.* 2841 and incorporating more than fifty new editorial corrections.

1. Contents and Genre of the Fragment

As it is preserved in *Par. gr.* 2841 the fragment begins abruptly in mid-sentence, and while it ends with the end of a sentence, the contents indicate that more must once have existed. The sequence of topics is as follows:

1	§§1-7	On the periods of lunar mean motions used by Kedenas, Hipparchus, and Ptolemy.
2	§§8-14	On the relative lengths of the synodic, draconitic, anomalistic, and sidereal months.
3	§§15-18	On the moon's latitude.
4	§§19-26	On the moon's anomaly. Return to topic 2.
5	§§27-33	Quotation from Artemidorus on Ptolemy's mean motions.
6	§§34-46	On a discrepancy between the *Almagest* and *Handy Tables*, with a computed example for A.D. 213.
7	§§47-53	On the mean motion tables in the *Handy Tables*.
8	§§54-55	On the table of lunar anomaly in the *Handy Tables*.
9	§§56-63	On the names of the mean motions.
10	§§64-88	Quotation from Apollinarius on the lunar periods.
11	§§89-93	Return to topic 9. Return to topic 8?

We know from references in the text that when complete it also discussed the tables for solar longitudes in the *Handy Tables*—but apparently not the solar and lunar tables in the *Almagest*—(cf. §§47-53), the theory of the sun's motion (cf. §15), probably also Ptolemy's lunar model (cf. §20), and eclipses (cf. §59). In sum, we seem to have an excerpt from a commentary on the *Handy Tables*, explaining, though not very well, both their use and their theoretical basis in the *Almagest* and earlier astronomy. Although there were several manuals in antiquity describing how to use the *Handy Tables* (starting with Ptolemy's own), the only other attempt that we know of to set out the theoretical derivation of the tables is Theon's *Greater Commentary on the Handy Tables*.[6]

Whereas Theon's commentary is immediately recognizable as a highly organized literary treatise, the genre of the work from which our fragment comes

[5] Most notable are Rome [1931,1] and [1931,2]. See also Neugebauer, *HAMA*, 948–49, and Jones [1983], 30–33.

[6] Theon, *GC* (only Book I of three and a fraction extant books has been edited so far). Theon writes in his preface (p. 94) that his undertaking was unprecedented.

is less obvious. The order of its exposition is digressive to the point of confusion, and the assumed level of technical competence is far from consistent. To take one conspicuous example, a lunar longitude is computed in §§34-46 although the procedure for carrying this out cannot yet have been explained. The author also shows a too great readiness to plunder other writers when he wishes to explain some important point, as when he passes on the burden of describing Ptolemy's innovations in lunar theory to the muddled and perhaps hostile Artemidorus, or when, to clarify the nomenclature of Ptolemy's lunar mean motions, he extracts from the works of Apollinarius page after page of contorted disquisition on the lunar periods, replete with expressions and concepts foreign to Ptolemy. It may be, then, that we are reading a draft of an unfinished work, or the class notes of a student (whose teacher may be alluded to in §40 and §44).

2. Date

The example in §§36-45 of a computation for 24 April, A.D. 213 probably indicates when the fragment was written, since ancient and medieval astronomers almost always, and quite naturally, picked examples with dates near the time of writing to illustrate their computations.[7] Rome was uncertain whether this computation was not part of the passage quoted from an otherwise unknown writer named Artemidorus, which begins at §28[8]; it would then supply Artemidorus' approximate date, and thus only a *terminus post quem* for the date of the fragment. There is, however, no topical connection, or only the most tenuous, between Artemidorus' criticism of Ptolemy's allegedly inconsistent handling of the lunar mean motions in the *Almagest* (§§28-32) and the ensuing demonstration and justification of a different discrepancy between lunar longitudes computed according to the tables in the *Almagest* and the *Handy Tables*.[9] Suppose that these two topics had been discussed consecutively by Artemidorus: then the author of our fragment, having occasion to quote Artemidorus on the first topic, would hardly, one would think, go on copying after the subject had changed.[10] On the other hand, such abrupt transitions of subject occur elsewhere in our fragment, and seem to be a habit of its author or a consequence of the way in which the commentary was compiled. For this reason, I believe that our fragment itself was written about 213, and that Artemidorus therefore wrote shortly before this date.

[7] A rare exception is found in Theon's commentary on the *Almagest*, Book 3 (Rome, *CA* vol. 2, 907) and the recently discovered Book 5 (Tihon [1987]). Theon demonstrates how to calculate the sun's and moon's longitude for a date in A.D. 323, long before Theon's career (ca. 360–380). These two passages seem to be excerpted from a freestanding computation of the positions of sun, moon, and planets for that date according to the *Almagest* and *Handy Tables*, composed perhaps by Pappus (fl. ca. 320) – or maybe it is Theon's own horoscope?

[8] Rome [1931,2], 111–12.

[9] The inferential partical that connects these passages (in §34) in the *CCAG* text is a mistaken insertion by Heiberg.

[10] But see the quotation from Apollinarius, §§64–88, which unquestionably goes on longer than our author needs.

But even if this computation was taken from the work of Artemidorus or some other earlier follower of Ptolemy, the fragment could not have been composed at a much later time. A dating to about 213, within half a century of Ptolemy's own career, is consistent with the author's familiarity with pre-Ptolemaic writings and terminology: the commentators of the fourth and later centuries, to judge by the several extant examples, would not readily have turned to the obsolete work of Apollinarius for definitions of basic terms, and they pointedly eschewed such non-Ptolemaic expressions as "depth" (*bathos*) for a planet's motion in anomaly. Greco-Egyptian papyri from the third century similarly testify that at this time pre-Ptolemaic and Ptolemaic methods and data were competing, sometimes being found together in one document.[11] From the fourth century on, at least in such culturally central places as Egypt and Constantinople, Ptolemy's tables and variations on them seem to have become the exclusive tools of all who would calculate the motions of the heavenly bodies, from the teacher of philosophy to the professional astrologer.[12]

3. Models, Periods, and Tables

Our fragment touches on many aspects of the pre-Ptolemaic and Ptolemaic theories and tables for solar and lunar motion, but in a haphazard and often allusive way. Brief explanations of some of these elements are given in this section.

a) Notation. Except for degrees, minutes, and seconds of arc, sexagesimal fractions are expressed here in the standard modern notation, in which a semicolon follows the integer part, and commas separate the fractional digits. Thus

$$13;10,35 = 13\,\tfrac{10}{60} + \tfrac{35}{3600}$$

b) Ptolemy's solar model.[13] Ptolemy's theory of the sun's motion, which was substantially the same as Hipparchus', is discussed only incidentally in the extant fragment. It is sufficient to observe that, for Ptolemy, the sun moves with uniform speed along a circle that is slightly eccentric with respect to the earth and inclined at an angle of 23° 51′ with respect to the plane of the equator (Fig. 1).

The ecliptic, that is the projection of the sun's eccentric circle on the celestial sphere, is a great circle intersecting the celestial equator in the two equinoctial points (Aries 0° and Libra 0°). The apsidal line through the earth and the center of the sun's eccenter is fixed with respect to the equinoctial points. The sun's longitude (λ), like all celestial longitudes, is reckoned from the vernal equinoctial point, Aries 0°. The "mean sun" ($\bar{\lambda}$), which is the longitude that the sun would appear to occupy if observed from the center of the sun's path, will be significant for Ptolemy's lunar model.

[11] See *Almagest* [Toomer], 2 note 2, and Neugebauer, *HAMA*, 808–13 and 944–48.
[12] See Neugebauer, *HAMA*, 973–75 and 1055–58.
[13] Neugebauer, *HAMA*, 53–61.

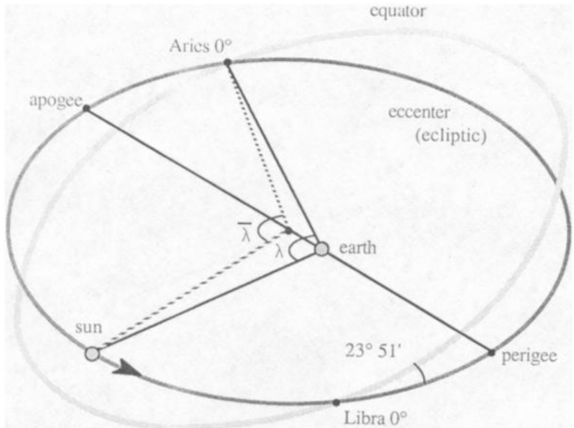

Figure 1. Eccentric model for solar motion (oblique view from north, not to scale)

c) The Hipparchian lunar model.[14] Hipparchus' theory of the moon assumed a simple epicyclic model for its motion, although it is possible that Hipparchus himself came to be aware that such a model could not yield consistently accurate predictions of lunar positions.[15] The moon (Fig. 2) is assumed to travel uniformly along a circle, the "epicycle," whose center travels uniformly in the opposite direction along a "deferent" circle concentric with the earth. The plane that contains these circles is inclined at a fixed angle of 5° with respect to the plane of the ecliptic; their intersections, the lunar nodes, make a slow uniform retrograde motion about the ecliptic. The node through which the moon passes as it moves northward across the ecliptic is called the "ascending" node, the other the "descending," and the point of the deferent halfway between the ascending and descending nodes is called the "northern limit." The moon's mean ($\bar{\lambda}$) and true (λ) longitudes are reckoned from Aries 0°, ignoring the inclination of the moon's plane with respect to the ecliptic.

d) The period relations.[16] The three uniform motions in this epicyclic lunar model are the motion of the epicycle along the deferent, that of the moon along the epicycle, and that of the nodes along the ecliptic. These uniform (or "mean") motions account for three conspicuous periodic phenomena in the moon's apparent motion: its longitudinal revolutions through the zodiacal signs, the fluctuations of its apparent speed, and its latitudinal devia-

[14] Neugebauer, *HAMA*, 68-69 and 315.
[15] An eccentric model (like the sun's, but with the center of the eccenter revolving about the earth) can be made to produce geometrically identical results to the epicyclic model. Hipparchus seems to have wavered between the two models. For simplicity's sake only the epicyclic model will be considered here. On Hipparchus' possible doubts, see Toomer's note, *Almagest* [Toomer], 217 note 2.
[16] Neugebauer, *HAMA*, 308–12 and 523, and Toomer [1980].

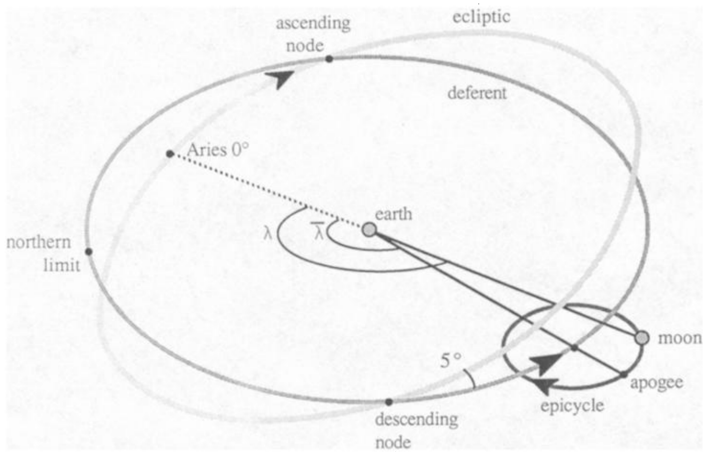

Figure 2. Hipparchus' epicyclic model for lunar motion (oblique view from north, not to scale)

tions from the ecliptic. The periods of these phenomena are respectively the sidereal month (called "longitudinal revolution" in our text), the anomalistic month (or "restitution in anomaly"), and the draconitic month (or "restitution in latitude"); to which may be added the synodic month (called simply "month" in our text), which is the interval between the moon's successive conjunctions or oppositions with the sun. Except for the anomalistic month, which (so far as ancient astronomy knew) is constant, these periods vary slightly in length because of the moon's changing speed.

The mean relative lengths of these various months can be approximated by establishing some interval of time in which very nearly an integer number of each kind of month is completed. One such period relation,

$$(1)\quad \text{223 syn. m.} = \text{239 anom. m.}$$
$$= \text{242 drac. m.}$$
$$= \text{241 long. rev.} + 10\tfrac{2}{3}^{\circ},$$

is called the "periodic" (*periodikos*) by Ptolemy (*Almagest* IV 2), who ascribes it to the "even more ancient" astronomers. In fact relation (1) is a component of Babylonian lunar theory, and it was known to Greek astronomers perhaps as early as Aristarchus, that is in the early third century B.C.[17] Because it will bring the moon from a situation of lunar eclipse (opposition with the sun while near a node) back to the same situation, relation (1) is an eclipse period: the pattern of occurrences of lunar eclipses will nearly repeat after 223 synodic months.

[17] Neugebauer, *HAMA*, 603–604, summarizing work of P. Tannery [1888]. Period (1) is often referred to as the "Saros" in modern discussions, although the application of this name is historically inaccurate. In early Greek astronomy the quantities in this relation were tripled to make a period called the *exeligmos* ("turn of the wheel"). The longitudinal component in relation (1) is not attested in Babylonian texts, and so may be a Greek innovation.

A more accurate relation relating only the synodic and anomalistic month,

(2) 251 syn. m. = 269 anom. m.,

was known to Hipparchus; it too was originally Babylonian. In order to verify the accuracy of relation (2), Hipparchus took the smallest multiple (by 17) of these intervals that would bring the moon roughly from a node to a node, so that lunar eclipses could be repeated at this longer interval. Incorporating other Babylonian values for the length of the year in synodic months and the length of the synodic month in days, he finally obtained the relation:

(3) 126007 days + 1 hour = 4267 syn. m.
= 4573 anom. m.
= 4612 long. rev. - $7\frac{1}{2}^{\circ}$,

which he was able to show to be accurate by comparing pairs of observed lunar eclipses at this interval. By a similar method, Hipparchus also confirmed to his satisfaction another Babylonian period relation,

(4) 5458 syn. m. = 5923 drac. m.,

so that he was able to determine accurate periods for all three of the uniform motions in his lunar model.

That this is what Hipparchus actually did, has been deduced only during this century from the newly accessible Babylonian astronomical texts and careful scrutiny of the information that Ptolemy gives in the *Almagest*.[18] Ptolemy's account in *Almagest* IV, 2, which the opening of our fragment paraphrases, gives only a compressed and distorted version that implies that Hipparchus first determined relation (3) from observations, and that his values for the lengths of the synodic and anomalistic months, as well as relation (2), followed from it. Ptolemy himself adopted the mean motions derivable from (2) and (3) with slight corrections to which our fragment alludes (see also section 4 below).

e) Ptolemy's lunar model.[19] Ptolemy's lunar theory incorporates a major modification of the epicyclic model, in order to correct the inaccurate lunar positions that it predicted. The discrepancy, as Ptolemy shows, could be explained as an increase in the anomalistic variation of the moon's apparent motion when the moon is not in conjunction or opposition with the sun. In the Ptolemaic epicycle-and-eccenter model (Fig. 3), the moon's deferent circle no longer has the earth for center; instead its center revolves uniformly around a new circle centered on the earth, in the opposite direction to the revolution of the moon's epicycle along the deferent. The angle between the epicycle's center and the deferent's center as seen from the earth is always to be twice the elongation $\bar{\eta}$ of the mean sun from the epicycle's center (the inclination of the moon's plane with respect to the ecliptic is ignored as negligible). This

[18] See Toomer [1980], developing discoveries by Kugler [1900] and Aaboe [1955].
[19] Neugebauer, *HAMA*, 84–93.

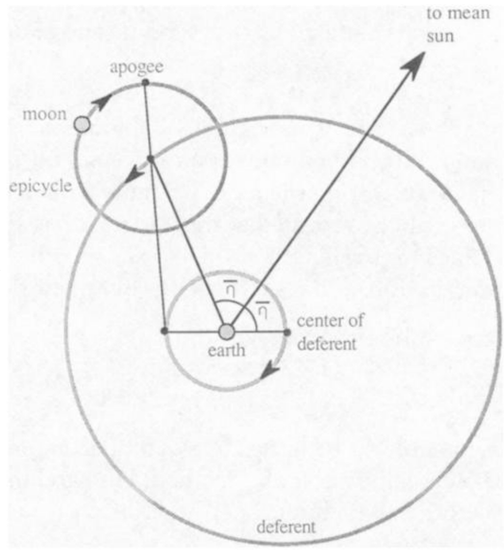

Figure 3. Ptolemy's model for lunar motion (view from north, not to scale)

change in the model results in the epicycle's being at its greatest distance from the earth at both syzygies ($\bar{\eta}$ = 0° or 180°), while it comes closest to the earth near the quadratures with the sun ($\bar{\eta}$ = 90° or 270°), so that the apparent anomaly caused by the moon's revolving about the epicycle is greater near quadrature than near syzygy. The moon's motion along the epicycle is uniform, not (as in the Hipparchian model) with respect to the line joining the earth to the epicycle's center, but instead with respect to a line joining the epicycle's center to a moving point, on the concentric that bears the deferent's center, that is always diametrically opposite to the deferent's center.

f) Ptolemy's solar and lunar tables.[20] In both the *Almagest* and the *Handy Tables* Ptolemy sets out a pair of tables for computing the position of each of the sun, moon, and five planets. In the first table the uniform (or "mean") motions of the various components of the model are tabulated for the intervals of time out of which a given date is composed. These mean motions determine the instantaneous geometrical disposition of the model. The second or "anomaly" table has columns of functions which are used to derive from the mean motions the planet's apparent position as seen from the earth.

The mean motion tables share the same arrangement for each planet, although this arrangement is different in each of the two treatises. In the *Almagest* Ptolemy tabulates the increments in mean motion corresponding to a hierarchy of intervals: groups of eighteen Egyptian 365-day years, single years

[20] Neugebauer, *HAMA*, 55, 58–61, 93–98, and 983–89. A scientifically usable edition of the *Handy Tables* has yet to appear (N. Halma's rare edition, Paris: 1822–25, is hopelessly unreliable). I have used photographs of two ninth-century manuscripts: *Vat. gr.* 1291 (for which see Neugebauer, *HAMA*, 970–73 and 977–78) and *Leid.* B.P.G. 78.

(up to eighteen), Egyptian 30-day months, days (up to thirty), and equinoctial hours (up to twenty-four). The interval between a given date and the tables' epoch date (1 Nabonassar, month Thoth 1 = 26 February, 747 B.C.) is decomposed into these intervals, and the sum of the increments in mean motion corresponding to each is added to the value for the epoch date in order to obtain the value for the given date. The *Handy Tables* use a different era, the era Philip (1 Philip, Thoth 1 = 12 November, 324 B.C.), and take as arguments the actual components of the date expressed in the Egyptian calendar, instead of the elapsed intervals. For example, to compute the mean motions using the *Almagest* tables for the date 960 Nabonassar, month Payni 28 at midnight, one adds to the epoch value the tabular entries for 810 (= 45·18) years, 144 years, 5 years, 270 (= 9·30) days, 27 days, and 12 hours. Using the *Handy Tables* for the equivalent date, 536 Philip, Payni 28, one adds the entries for 526 Philip (a base year, tabulated at 25-year intervals), 10 years, Payni, the 28th day, and 12 hours; the epoch value is already incorporated in the figure for the base year.

The sun's eccentric model calls for a single mean motion and a single column in the anomaly table. In the *Almagest* the mean motion tabulated is the longitude of the mean sun ($\bar{\lambda}$). From this one subtracts the longitude of the eccenter's apogee, Gemini 5° 30′, and enters the remainder, i.e., the mean elongation of the sun from the apogee, in the anomaly table to obtain the "equation." The equation is then added to (or subtracted from, depending on whether the mean elongation is more or less than 180°) the mean sun to yield the sun's true longitude (λ). In the *Handy Tables* the mean elongation from the apogee is itself tabulated in the mean motion table; this can therefore be entered in the anomaly table directly to obtain the equation. The equation is added to, or subtracted from, the mean elongation, and the result is added to the longitude of the apogee to obtain the sun's true longitude.

The moon's mean motion table in the *Almagest* tabulates four mean motions (Fig. 4): the mean motion in longitude ($\bar{\lambda}$), in anomaly ($\bar{\alpha}$, measuring the moon's motion on its epicycle), in latitude ($\bar{\omega}$, the elongation of the mean moon from its northern limit, or "argument of latitude"), and the elongation of the mean moon from the mean sun ($\bar{\eta}$).

The anomaly table in the Almagest has four columns for computing the longitudinal equation, and one for the latitude. Let c_1, c_2, c_3, c_4, and c_5 represent the functions tabulated in these columns. We first find the "true anomaly" α, reckoned from the line through the earth and the epicycle's center:

$$\alpha = \bar{\alpha} \pm c_1(2\bar{\eta}),$$

subtracting if the double elongation, $2\bar{\eta}$, is less than 180°, but otherwise adding. The equation c is then approximated by an interpolation between the extreme values $c_2(\alpha)$ and $c_2(\alpha) + c_3(\alpha)$, which are respectively valid at the greatest and least distances of the epicycle from the earth:

$$c = c_2(\alpha) + c_3(\alpha) \cdot c_4(2\bar{\eta}),$$

The equation c is added to or subtracted from the mean longitude and mean

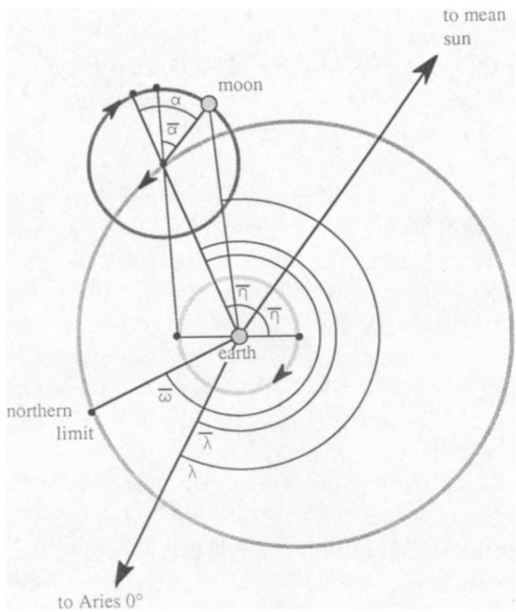

Figure 4. Mean motions in Ptolemy's lunar model

motion in latitude, depending on whether the true anomaly is greater or less than 180°, to get the moon's true longitude (λ) and true argument of latitude (ω). The column for latitude in the anomaly table, c_5, gives the latitude as a function of ω.

The *Handy Tables* differ from the *Almagest* in tabulating as the lunar mean motions the longitude of the apogee of the moon's eccenter (i.e., $2\bar{\eta}$-$\bar{\lambda}$), the double elongation ($2\bar{\eta}$), the mean motion in anomaly ($\bar{\alpha}$), and the longitude of the northern limit. The order of the columns in the anomaly table is changed to c_1, c_4, c_2, c_3, c_5; but their use is not changed, except that the true longitude is obtained by adding the equation c to the double elongation $2\bar{\eta}$, and subtracting from their sum the longitude of the eccenter's apogee ($2\bar{\eta}$-$\bar{\lambda}$); while the true longitude added to the longitude of the northern limit gives the argument of latitude.

For an illustration of how solar and lunar longitudes are computed by both sets of tables, see the notes to §36 below.

4. Artemidorus

Our fragment quotes two astronomical writers besides Ptolemy. §27 introduces a passage from a work by a certain Artemidorus. No other ancient reference to this man is known.[21] His date is fixed between Ptolemy's publication of the *Almagest*, which was later than A.D. 147, and A.D. 213.

[21] There seems little point in identifying him with the Artemidorus mentioned by the astrological "Anonymous of A.D. 379" as an authority on so-called "Chaldean" theories of fixed-star phases (*CCAG* vol. 5 part 1, 204).

Artemidorus reveals himself in §§28-32 as a not very intelligent critic of Ptolemy's methodology in establishing his lunar theory in the *Almagest*. Ptolemy began by assuming the same period relations (3) and (4) that Hipparchus had tested by comparing observations of lunar eclipses. On the basis of the mean motions derived from (3) and (4) and further eclipse observations, Ptolemy determined the other necessary elements of a simple epicyclic lunar model, namely the relative dimensions of the deferent and epicycle, and the epochs of the mean motions. With these data, however, Ptolemy was able to show (*Almagest* IV, 7 and 9) that Hipparchus' mean motions in anomaly and latitude required very slight corrections. Ptolemy was aware that it would not be strictly legitimate to assume his corrected mean motions at the start of a line of mathematical reasoning that led to these very corrections, and so in his working out of the dimensions of the lunar model from eclipse observations in IV, 6, he uses the Hipparchian mean motions, while taking care to point out that the discrepancies between these mean motions and his final approximations are insignificant over the time intervals that he uses in this chapter.[22]

Artemidorus' version of these circumstances seems somewhat confused. He begins by setting out some of the elements of Ptolemy's lunar model, specifically the Hipparchian period relation (4) initially assumed by Ptolemy, a dubious parameter for longitudinal motion, the maximum lunar equations at the epicycle's least and greatest distance from the earth, and Ptolemy's definition of the epicycle's apogee. Artemidorus then asserts (§30) that Ptolemy established the period of restitution (*apokatastasis*) of anomaly through these assumptions, and that, although he made a correction to Hipparchus' mean motions, he did not use the corrected values in his analyses of eclipse observations.[23] So, he concludes, the theory in the *Almagest* will not predict syzygies in agreement with fact. Artemidorus thus not only ignores Ptolemy's valid statement that it makes no difference which set of mean motions one assumes for the computations of *Almagest* IV, 6, but also misrepresents the reason behind Ptolemy's corrected values. As we have seen, Ptolemy corrected the mean motions in anomaly and latitude on the basis of his preliminary, simple epicyclic model. After he has derived the parameters of his epicycle-and-eccenter model in *Almagest* V, 3-5, he shows at length in V, 10 that the difference between this model and the simple epicyclic model is negligible for the analyses of eclipse observations in *Almagest* IV.

There remain three curious points to be noted in the quotation from Artemidorus. First, he ascribes to Ptolemy a motion in longitude associated with period relation (4); this is discussed below in the note to §28. Second, he states that the armillary sphere that Ptolemy describes in *Almagest* V, 1 for observing

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[22] *Almagest* [Heiberg], 304–305, [Toomer], 192 with Toomer's note 34.

[23] I am driven to believe, in spite of Artemidorus' vague expression, that in §31 the "operation of the syzygies" refers to the analyses of lunar eclipses in *Almagest* IV, 6–9, which is the only section of Ptolemy's lunar theory where the question of different values for the mean motions arises. It is probably just a coincidence that Artemidorus' words resemble the title of *Almagest* VI, 2, where Ptolemy gives instructions for computing dates of syzygies; this chapter makes no use of the preliminary values for the mean motions.

lunar positions had a diameter less than one foot, a datum not given by Ptolemy in the *Almagest*. Did Artemidorus have more knowledge of Ptolemy's equipment than we? Considering his early date (before A.D. 213), it is possible that he had actually seen Ptolemy's instruments, but it is more likely that he consulted a lost work of Ptolemy's that gave specific dimensions of a similar observational instrument.[24] Third, he refers only to the new definition of the lunar epicycle's apogee as Ptolemy's innovation, in a way that seems to imply that the epicycle-and-eccenter model itself was not original; but since Artemidorus' accusations in the quotation as a whole are inaccurate, to draw historical conclusions from his silence at this point would be hazardous.[25]

5. Apollinarius

Except in citing observations, Ptolemy names no astronomers later than Hipparchus in the *Almagest*. The necessary inference is not that theoretical astronomy stood still during the three centuries after Hipparchus, but that Ptolemy considered his own theoretical work to owe nothing to the astronomers of this period. Concerning the theory of the five planets, for example, Ptolemy tells us that Hipparchus went no farther than to compile a list of observations and to refute by their means the models of planetary motion that prevailed in his time.[26] Yet Ptolemy goes on in this passage to mention certain planetary tables devised by other astronomers based on eccentric, epicyclic, or epicyclic-eccentric hypotheses, attempts that he dismisses as entirely wrong-headed.[27] Even Hipparchus' work on lunar theory probably did not reach the point where he could publish tables for predicting lunar motion, since he was unable to deduce a consistent value for the magnitude of the moon's maximum equation of anomaly according to a simple epicyclic or eccentric model.[28] The lunar tables that seem to have been in almost universal use in Ptolemy's own time can be reconstructed from three Greco-Egyptian papyri discovered in this century, with some help from the second-century astrologer Vettius Valens; they turn out to represent a compromise between a simple Babylonian procedure for predicting lunar longitudes on successive days, and theoretical elements that must date from after Hipparchus.

The Babylonian procedure assumes that the moon's longitudinal advance on successive days can be represented by a linear "zigzag" function, that is an alternation of equal time-intervals in which the function linearly increases and decreases between a maximum and minimum value.[29] The particular zigzag function used for the lunar daily motion has a period of 248/9 days, so that the period relation

(5) 9 anom. m. = 248 days

[24] See the note to §29.
[25] See however §67 and note.
[26] *Almagest* IX, 2 [Heiberg] vol. 2, 210–11, [Toomer], 421–22.
[27] For these "Aeon-tables," see Toomer's note, *Almagest* [Toomer], 422 note 12.
[28] *Almagest* IV, 11 [Heiberg], vol. 1, 338–39, [Toomer], 211. See Neugebauer, *HAMA*, 317–19.
[29] Jones [1983], 2–11.

is implicit in the scheme. The upper and lower limits of the lunar daily motion are 15° 14′ 35″ and 11° 6′ 35,″ resulting in a mean motion of 13° 10′ 35″ per day. These parameters, which are sufficient to define the zigzag function for daily motion, were transmitted into Greek astronomy as discoveries of the "Chaldeans," and period relation (5), at least, was used by Hipparchus to supply an index of the moon's anomalistic motion over short intervals.[30]

At some uncertain date after Hipparchus, period relation (5) and the concept of a zigzag function for lunar daily motion fitted to it were made into components of the more elaborate scheme for predicting lunar longitudes and latitudes that seems to have been popular in Ptolemy's time.[31] The equipment for this scheme consisted of two tables. The first gave the moon's longitude and argument of latitude (λ and ω) at a succession of epoch dates. The interval between epoch dates was usually 248 days, but eleven such intervals were followed by an interval of 303 days, making a larger period of 3031 days. Because both 248 days and 3031 days are approximate anomalistic periods (3031 days $\approx$ 110 anom. m.), the consecutive epoch values of λ and ω increase by constant differences; the epochs are in fact dates of the moon's least daily motion at apogee. The second table listed increments in λ and ω over 248 consecutive days starting with least motion, using two zigzag functions for the daily motion in longitude and latitude. The current longitude and argument of latitude for any date could thus be obtained by adding two pairs of numbers: the values at the preceding epoch date taken from the first table, and the subsequent increments from the second.

In an often discussed passage of his astrological treatise, Vettius Valens tells us that he did not have the time to compile lunar and solar tables for himself, and so (as the only manuscript of this section reads):

> I decided to use Hipparchus for the sun, and Sudines and Kidynas and Apollonius for the moon—and moreover Apollonius for both kinds,[32] so long as one uses the addition of 8°,[33] as I choose to do. But though he worked out the tables according to the theories of the [astronomical] phenomena, he admits, as being human, to being off by one or two degrees; for only with the gods is anything constant and unambiguous.[34]

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[30] Jones [1983], 23–27. It is remarkable that the evidence for Hipparchus' use of a crude 248-day anomalistic cycle comes from an observation (quoted in *Almagest* V, 3 [Heiberg], 363; [Toomer], 224) dating from 128 B.C., one of the last known observations by Hipparchus, and years later than the observations by which he confirmed the accurate period relation (3). This is clearly no *péché de jeunesse!*

[31] Jones [1983], 14–30.

[32] "Both kinds" certainly means "both solar and lunar tables." The notion that Vettius Valens is alluding to tables for both solar and lunar eclipses (e.g., Neugebauer, *HAMA*, 263) comes from a misinterpretation of Valens' claim a few lines earlier that he tried to compose a table for the sun and moon "πρὸς τὰς ἐκλείψεις" ("to fit the [observed] eclipses," not "for [predicting] eclipses").

[33] This refers to the Babylonian norm placing the spring equinoctial point at Aries 8° instead of Aries 0° (the norm of Hipparchus and Ptolemy). Tables based on the Babylonian norm ought (all other things being equal) to give longitudes eight degrees greater than tables based on the 0° norm.

[34] Vettius Valens IX, 11 [Pingree], 339 (= [Kroll] 353). Pingree's text (a considerable improvement on Kroll's) requires further emendation in this passage: l. 24, do not add ὡς; same line, read ὁμολογεῖ; l. 25, read διαφέρειν.

Sudines and Kidynas are referred to elsewhere in Greek sources as authorities on Babylonian astronomy,[35] but who is this Apollonius? He has variously been identified as the geometer of Perge (known to have studied the geometrical properties of epicyclic models) or more plausibly as a certain Apollonius of Myndos who seems to have written about Babylonian astronomy.[36] But elsewhere Vettius Valens writes:

> For even Apollinarius, who worked out [tables] in accordance with the phenomena using the ancient observations and demonstrations of complicated periodic restitutions [?] and spheres, and who brought censure upon many, admits to erring by one degree or even two.[37]

Vettius Valens is certainly quoting the same person in both passages; hence I have proposed that at least the second "Apollonius" in the first passage is a scribal error for "Apollinarius."[38] And since Valens seems always to have used lunar tables of the type described above, it seems highly probable that one redaction of these tables was made by Apollinarius.

There are in fact several ancient allusions to Apollinarius, testifying to his importance as an astronomer.[39] The references by Vettius Valens (writing ca. A.D. 160) and Galen (late second century) give an upper bound to his date. If ours is the Apollinarius who wrote an astrological work cited by Porphyry and Paul of Alexandria, his career probably falls in the first or early second century of our era.[40] Both Paul and Porphyry write that Apollinarius, like Ptolemy, used geometrically derived values for the ascensional arcs of the equator that rise simultaneously with the zodiacal signs. Porphyry groups Apollinarius and Ptolemy together as "moderns" (*neoteroi*) in this respect, as opposed to the "ancients" such as Thrasyllus (died A.D. 36?) and the apocryphal Egyptian Petosiris who used Babylonian-style arithmetical methods to obtain the ascensional arcs; and indeed the theorems in spherical geometry

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[35] On Kidynas (or Kedenas), see §24 and note. References by Greek and Latin authors to Kidynas, Sudines, and other Babylonian astronomers are discussed by Neugebauer, *HAMA*, 610.

[36] Identification as Apollonius of Myndos by Cumont [1910]. Of three ancient references to this man, that of the astrological "Anonymous of A.D. 379" is the most relevant here: "The Babylonians and Chaldeans were pretty well the first to discover the knowledge of the [astronomical] phenomena, as we learn from our predecessors; for Apollonius of Myndos and Artemidorus report thus." (*CCAG* vol. 5 part 1, 204.)

[37] Vettius Valens VI, 3, [Pingree], 239 (= [Kroll], 250). Pingree's emendation of ἀνακαθάρσεων ("eclipse emersions," not meaningful in this context) to ἀποκαταστάσεων ("periodic restitutions") is attractive.

[38] Jones [1983], 31. Pingree adopts this conjecture, for the second occurrence of "Apollonius" only. The garbled list of names of "writers of tables" discovered by E. Maass in *Vat. gr.* 381 (Maass, *Aratea*, 140, reprinted in Vettius Valens [Pingree], 455) is certainly extracted from a lost manuscript of Valens IX, 12, and confirms that Apollinarius' name appeared there, whereas no Apollonius is cited.

[39] These are collected by Neugebauer, *HAMA*, 601 note 2. To his list may be added Vettius Valens IX, 11 as emended, and Galen's commentary on Hippocrates' *Airs, Waters, Places* (for which see Toomer [1985], 199 and 203–204).

[40] Porphyry, Introduction to Ptolemy's *Tetrabiblos* (*CCAG* vol. 5 part 4, 212); Paul [Boer], 1–2.

necessary to compute the ascensional arcs correctly first appear in the *Spherics* of Menelaus (ca. A.D. 100).[41]

Apollinarius' specific contributions to solar and lunar theory remain unclear. In the second passage quoted above, Vettius Valens says that Apollinarius made a highly critical revision of his predecessors' lunar theories and tables, based on reports of earlier observations and computations involving periods (?) and cinematic models. This account is perfectly consonant with the passage that our fragment quotes from an unnamed writing by Apollinarius (§§65-86). This appears to be an excerpt (possibly abridged by our fragment's author) from a work that concerned the mean motions of the moon. In it Apollinarius first defines the four fundamental periods of the moon's motion (the longitudinal revolution, and the anomalistic, draconitic, and synodic months) and describes the way in which the moon's anomaly introduces variations in the length of the synodic and draconitic months. He then examines in some detail how the anomaly interferes with an attempt such as Hipparchus' to establish a period of lunar latitude (containing whole numbers of draconitic and synodic months) from pairs of eclipse observations. The quotation ends with Apollinarius' declaration that the ideal conditions for establishing such a period cannot occur within a reasonable range of years of observation. Unfortunately we are given no hint of what compromises with this unattainable ideal Apollinarius considered acceptable. Nevertheless it is interesting to compare Apollinarius' doubts about Hipparchus' confirmation of his latitudinal period with Ptolemy's subsequent approach to the same problem. We know that Ptolemy found fault with Hipparchus' procedure both on the same grounds as Apollinarius (because Hipparchus wrongly treated the effect of lunar anomaly as negligible) and for a further reason that Apollinarius seems not to have considered, that Hipparchus had ignored the varying distance of the moon from the earth.[42] Ptolemy first attempted to get around these difficulties by modifying Hipparchus' procedure with corrections for the moon's anomaly and distance.[43] He subsequently discovered that his assumptions concerning the moon's distance and the apparent sizes of the moon's disc and the earth's shadow, taken from Hipparchus, were incorrect; and so in the *Almagest* (IV, 9) he used a pair of eclipses at which the moon's distance was predicted by theory to be equal, and made a correction only for anomaly.

Apollinarius presumably revised the numerical parameters of his solar and lunar tables on the basis of his investigation of the periods of mean motion. We do not know, however, whether the version of the "248-day" lunar scheme represented in the three papyri (our only source of exact parameters used in

[41] In IX, 11 ([Pingree], 339) Vettius Valens apparently refers to geometrical derivations of the rising arcs: "For let us pass over speaking of how great a discrepancy, both geometrical and arithmetical, the compilers of schemes of ascensions have wrought . . ." (my rendering of a textually dubious passage – Pingree punctuates differently, but it is hard to see what his text would mean). In practice Valens always uses quasi-Babylonian arithmetical schemes for the ascensions.

[42] Almagest VI, 9. See the note to §77.

[43] Ptolemy "published" the mean latitudinal motion resulting from this procedure in his *Canobic Inscription* (A.D. 146/7). For detailed discussion, see Hamilton et al. [1987].

the scheme) are the "Apollinarian" recension.[44] With our present knowledge we can only list the innovations in the lunar scheme of the papyri (compared to the simple Babylonian scheme), which can be attributed to a Hellenistic astronomer (or astronomers) who *may* have been Apollinarius. These innovations include a fundamental mean lunar motion in longitude of 13;10,34,52° per day, and in latitude of 0;52,55,2,45 "steps" (13;13,45,41,15°) per day; an anomalistic month of 27;33,16,21 days; an intermediate period relation equating 3031 days and 110 anomalistic months; and zigzag functions for lunar motion in both longitude and latitude that produce a maximum lunar equation of approximately 5° 4′ 30″.[45] A maximum lunar equation of about 5° looks like a parameter derived from eclipse observations, since this is about the maximum equation at conjunctions and oppositions (compare Ptolemy's value at syzygy, 5° 1′), but not at other times; the zigzag function of the Babylonian scheme has a much greater amplitude, giving a maximum equation of about 7° 7′. Hipparchus invented the method of extracting the size of the lunar epicycle (or equivalently the eccentricity in a simple eccentric model) from observations of lunar eclipses, but because of errors he arrived at inconsistent and inaccurate results.[46] It appears therefore that someone after Hipparchus made a new calculation of the dimensions in the lunar model, and that this supplied the new amplitude of the zigzag functions in the lunar tables. The fact that the argument of latitude is made to vary anomalistically in the tables is itself significant; this effect was surely deduced theoretically, from consideration of the epicyclic or eccentric lunar models, and has no precedent in Babylonian lunar theory.[47] Is the stress that Apollinarius puts on the anomalistic component of latitudinal motion in §§79-86 a hint that he introduced this

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[44] Vettius Valens never cites data from his tables to more than one sexagesimal place.

[45] All the numbers tabulated in the table of epochs represent *true* positions, based on the mean daily motions given in the text above but incorporating a small correction for the inaccuracy of the 3031-day and 248-day anomalistic periods (Van der Waerden [1958], 182); this explains the apparently different "mean" motions derivable from the different periods in the epoch table. Neugebauer, *HAMA*, 809, has not realized this fact, nor is my former account (Jones [1983], 16) satisfactory. For the reconstructed zigzag functions (Jones [1983], 18–19), I would now conjecture the following parameters:

daily longitudinal motion:
maximum = 14;38,59,18,40°,
minimum = 11;42,10,25,20°,
mean = 13;10,34,52°;

daily latitudinal motion:
maximum = 0;58,48,40,31,40 steps,
minimum = 0;47,1,24,58,20 steps,
mean = 0;52,55,2,45 steps.

These changes do not affect the truncated values in Table 4 (pp. 20-21).

For completeness it should be remarked that the Sanskrit *Pañcasiddhāntikā*, based ultimately on Greek sources through several streams of transmission, reports substantially the same lunar scheme as described above, but with variants in the parameters (Jones [1983], 11–14, 23, 33–34). The extent to which these variations are due to roundings and other accidental causes is not clear.

[46] Hipparchus' two results, quoted by Ptolemy (*Almagest* IV, 11), would produce maximum equations of about 4° 33′ and 5° 58′.

[47] See §§80 and note.

feature in the lunar tables? The mean motions in longitude, latitude, and anomaly underlying the scheme of the papyri cannot be derived from the Babylonian period relations (1), (2), or (4),[48] and so probably point to a post-Hipparchian attempt to establish more accurate mean motions, such as Apollinarius appears to have made.

Thus while it is possible that Apollinarius only tinkered in small ways with the structure and parameters of a scheme of lunar tables that was already established in his time, what evidence we have is also consistent with the hypothesis that Apollinarius was responsible for some or all of the major changes by which the Babylonian scheme was transformed into the scheme of the papyri. At least, whoever made these changes seems to have been doing the same kinds of things that we believe Apollinarius did. Either way, Apollinarius' lunar tables would have been characterized by a paradoxical combination of scrupulously precise numerical parameters and a Babylonian methodology of arithmetic functions imperfectly representing the behavior of geometrical models.

As a postscript to this discussion of Apollinarius' contribution to lunar theory, it may be mentioned that Achilles, a third-century commentator on Aratus, lists Apollinarius along with Hipparchus, Ptolemy, and one Orion as having studied solar eclipses "in the seven climata," that is, the intervals at which solar eclipses can be seen, and at what terrestrial latitudes.[49] But of his work on this complicated problem we know no more.

6. Manuscripts

Two manuscripts preserve the fragment, but one is merely a copy of the other and thus of no value as a witness to the text. The only substantive copy is in the parchment codex *Par. gr.* 2841, which has been described by Omont in his inventory of the *Parisini graeci* and by Ruelle in the *CCAG*.[50] It is a palimpsest, of which the original writing (parts of the *Septuagint*) has been dated by C. Ruelle and A. Olivieri to the eleventh century, and less definitely by A. Jacob to the tenth or eleventh[51]; we are concerned only with the later, thirteenth-century hand's work.[52] The first part of the manuscript, ff. 1-25^{v}, contains Aratus' *Phaenomena*, while the remainder is devoted to an incom-

[48] The mean daily motion in longitude *could* be a rounding of the value derivable from (1).

[49] Maass, *Aratea*, 143 note 52, and *CAR*, 13–14.

[50] Omont, *Inventaire* vol. 3, 48. *CCAG* vol. 8 part 2, 25–26.

[51] The rewritten leaves run from f. 17 (not 16 *pace* Ruelle) to the end of the manuscript (f. 66). The original writing was minuscule, in two narrow (about 5 cm.) columns of about 26 lines, and is mostly illegible. A. Jacob, who first drew attention to the palimpsest (Jacob [1895] 769–70) reported that ff. 18 and 25^{v} had been treated with a reagent. I think rather that someone has run over these pages with a blue pencil to bring out the impressions of the washed-off writing. Jacob also identified the text on f. 25^{v} as the last verses of *Job*. A. Olivieri ([1898], 8) merely alludes to writing in an eleventh-century hand, while Ruelle [1909] also identifies the passage from *Job*, in ignorance of Jacob's article. The text legible on f. 18 is *Zephaniah* 3:14–15.

[52] Dated by Omont, *Inventaire* vol. 3, 48.

plete and disordered copy of the astrological treatise of Hephaestio of Thebes.[53] Hephaestio's Book 3 begins on f. 26 and ends in mid-sentence in the middle of line 11 of f. 32. The writing continues without interruption on this line, but with our astronomical fragment; the break is indicated only by sense. The fragment stops on f. 34^v, line 23, with the remainder of the page (i.e. about space for eight lines) left blank. On f. 35 the text of Hephaestio begins again, now from the beginning of Book 1, and it continues to the bottom of f. 66^v, where Book 2 is interrupted, again in mid-sentence. Although the present binding makes examination of the manuscript's physical composition difficult, breaks between quires can be discerned before ff. 26 and 35; hence it is probable that the misordering of Hephaestio's books and the loss of the end of Book 2 occurred originally through damage to this manuscript. On the other hand, the exemplar from which *Par. gr.* 2841 was copied must itself have been defective, since the specious continuity of the astronomical fragment with the mutilated Book 3 of Hephaestio points to the loss of some quires not noticed by the scribe of *Par. gr.* 2841. The fragment itself may have been preserved on a few stray leaves bound in place of the lost end of the exemplar, since it seems too brief to have taken up a whole quire.

Par. gr. 2415, a sixteenth-century manuscript in a hand identified by Omont as that of Nicolas Sophianos, is a copy of the text of Hephaestio in *Par. gr.* 2841 (with our fragment following the incomplete Book 3 on ff. 50^v-55), as was determined by A. Engelbrecht.[54] Cumont nevertheless cited its readings in his apparatus, and often preferred them to those of *Par. gr.* 2841, not always with reason.

7. Editorial Conventions

The present edition of the fragment was prepared from photographs of *Par. gr.* 2841 (A), and subsequently corrected by direct examination of the manuscript. *Par. gr.* 2415 (B) was also collated at that time, but its variant readings are cited in the apparatus only when they have been adopted in this edition or Cumont's as emendations. I report all emendations adopted by Cumont, but not all errors in his text.

In the translation I have attempted to follow the conventions for rendering technical terminology of Toomer's translation of the Almagest[55]; in particular, my explanatory glosses are indicated as such by being enclosed in brackets. For ease of reference the fragment has been divided into numbered sentences, indicated in the margin of text and translation. The beginning of a new page in *Par. gr.* 2841 is indicated in the text by a vertical bar and in the margin by the folio number preceded by the letter A. Pages of the *CCAG* text are similarily indicated by the page number preceded by the letter C in the margin.

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[53] See Pingree, *Hephaestio* vol. 1, xi–xii.

[54] Engelbrecht [1887], 9; cf. Pingree, *Hephaestio* vol. 1, xii. Descriptions of *Par. gr.* 2415 in Omont, *Inventaire* vol. 2, 256, and *CCAG*, vol. 8 part 2, 9.

[55] *Almagest* [Toomer], 17–24.

II. TEXT AND TRANSLATION

... προκειμένων χρόνων, ἀποδείκνυται ὑπὸ τοῦ Ἱππάρχου ἀεὶ ἀπὸ ἐκλείψεως ἐπὶ ἑτέραν ὁμοίαν ἔκλειψιν ἀποκατάστασις τοὺς ἴσους μῆνας περιέχουσα καὶ ταῖς περιδρομαῖς ͵δχια ἴσας ἐπιλαμβάνουσα μοίρας τνβ L′, ἀκολούθως ταῖς πρὸς τὸν ἥλιον συζυγίαις. C 126 §1

§2 ἡ μέντοι ἀνταπόδοσις τῶν ἐκλείψεων πρὸς τὰς διαστάσεις μόνον τοῦ τε χρόνου καὶ τῶν κατὰ μῆκος περιόδων φαίνεται σῴζουσα τὰς ἰσότητας, οὐκέτι δὲ πρὸς τὰ μεγέθη καὶ τὰς ὁμοιότητας τῶν ἐπισκοτήσεων.

§3 ὅλως δέ, εἰ μή τις πολυπραγμονοίη τὸν ἀπὸ ἐκλείψεως ἐπὶ ἔκλειψιν ἀριθμόν, ἀλλὰ τὸν ἀπὸ ἁπλῶς συζυγίας ἐπὶ τὴν ὁμοίαν, εὕροι ἂν τὸν ἀποκαταστατικὸν χρόνον τῶν τε μηνῶν καὶ τῆς ἀνωμαλίας, τὸ μὲν κοινὸν μέτρον λαβὼν αὐτῶν, ιζ, ὃ συνάγει μῆνας μὲν σνα, ἀνωμαλίας δὲ ἀποκαταστάσεις σξθ, οὐκέτι μέντοι καὶ τὴν κατὰ πλάτος ἀπηρτισμένην ἀποκατάστασιν.

§4 τὴν δὲ τοιαύτην περίοδον εὑρῆσθαι μὲν ὑπὸ Κηδῆνα λέγεται· φαίνονται δὲ πόλλοι αὐτῇ κεχρημένοι, καὶ ὁ Πτολεμαῖος, ἀλλὰ μετὰ διορθώσεως.

§5 ἤδη μέντοι ὁ Ἵππαρχος, προκατειλημμένου τοῦ τῆς ἀνωμαλίας ἀποκαταστατικοῦ χρόνου, παραθέμενος διαστάσεις μηνῶν κατὰ πάντα ἐκλείψεις ὁμοίας καὶ ἴσας καὶ τοῖς μεγέθεσι καὶ τοῖς χρόνοις ἐχούσας, ἐν αἷς οὐδὲν διάφορον ἐγίνετο παρὰ τὴν ἀνωμαλίαν, ὡς διὰ τούτου καὶ τῆς κατὰ πλάτος παρόδου δεικνυμένης τῆς ἀποκαταστάσεως, ἐν μησὶ μὲν ͵ευνη, πλατικαῖς δὲ περιόδοις ͵εϡκγ, τὴν τοιαύτην διάστασιν ἐξέθετο.

§6 κατὰ δὴ ταύτην μάλιστα τὴν ὑπόθεσιν αἵ τε σεληνιακαὶ καὶ ἡλιακαὶ ἐκλείψεις

1 ὑπὸ Heiberg: ἀπὸ A || 3 περιδρομαῖς scripsi: περιοδικοῖς A: περιόδοις Cumont | ͵δχια ἴσας Cumont: ͵δχμ εἰς ἃς A || 9 (πολυπραγμον)οίη Cumont (B): –είη A | τὸν... ἀριθμόν e τῶν... ἀριθμῶν corr. A || 12 αὐτῶν ιζ scripsi: ἀπὸ τῶν ἑπτακαιδέκατον A: ἀπὸ τῶν ιζ′ Cumont || 14 ἀπηρτ(ισμένην) Cumont (B): ἀπαρτ– A || 19 ἐκλείψεις Cumont (B): ἔκλειψιν A || 21 παρὰ Cumont: περὶ A || 22 παρόδου Cumont: περιόδου A || 23 περιόδοις e περιόδους corr. A

§1 ... the foregoing intervals, a restitution from one eclipse always to another similar eclipse is demonstrated by Hipparchus, [always] containing the same number of months, and [always] taking up the same number of [longitudinal] revolutions, 4611, plus the same number of degrees, $352\frac{1}{2}$, in accordance with [the moon's] syzygies
§2 with the sun. But the repetition of eclipses turns out to preserve equalities only with respect to intervals of time and longitudinal revolutions, not with respect to magnitudes
§3 and similarities in [the direction of] obscuration. In general, however, if one does not concern oneself with the number from eclipse to eclipse, but rather the number from simple syzygy to the like, one would find the time of restitution in months and anomaly by taking their common measure, $\frac{1}{17}$, which comes to 251 months and 269 restitutions of anomaly; but [one would find] that the latitudinal restitution
§4 is no longer completed too. This period is said to have been discovered by Kedenas; and many prove to have used it, as
§5 has Ptolemy, albeit with a correction. But already Hipparchus, after determining the time of restitution in anomaly, had compared intervals of months having [at each end] eclipses that were absolutely alike and equal in magnitude and duration, and in which there was no discrepancy in anomaly, so that the restitution of latitudinal motion was thereby demonstrated, in 5458 months and 5923 latitudinal revolutions; and he published this interval.
§6 Lunar and solar eclipses worked out according to this hypothesis are indeed found to be in best agreement with the

πραγματευόμεναι ˈσύμφωνοι τοῖς φαινομένοις εὑρίσκονται, A 32v μενόντων τῶν κατὰ τὰς ἐκκεντρότητας θεωρημάτων. περιέχει δὲ ἡ τῶν ͵ευνη μηνῶν περίοδος ἐκλείψεις §7 πενταμηνιαίας μὲν ρκβ, ἑξαμηνιαίας ⟨δὲ⟩ ωη. πεντάκις μὲν γὰρ τὰ ρκβ γίνεται χι, ἑξάκις δὲ τὰ ωη, ͵δωμη, συντεθέντα δὲ ποίει τοὺς ͵ευνη μῆνας τοὺς τῆς περιόδου.

οἱ μὲν οὖν τρόποι οἷς οἱ παλαιότεροι ἐχρήσαντο καὶ ὁ C 127 §8 Ἵππαρχος ἦσαν τοιοῦτοι· ἃ δ' ἐπέστησε πρὸς αὐτὰ ὁ Πτολεμαῖος ὕστερον ῥηθήσεται. ζητήσαις ⟨δ'⟩ ἂν εἰκότως ἐν §9 ταῖς εἰρημέναις ἀποκαταστάσεσι τῶν περιόδων διότι αἱ πλατικαὶ ἀποκαταστάσεις πασῶν εἰσι πλείους, μεθ' ἃς αἱ τῶν περιδρομῶν τῶν μηκικῶν, ἐλάττους μὲν τῶν πλατικῶν, πλείους [μείζους] δὲ τῶν ἀνωμαλιῶν αὐτῆς. πάλιν δὲ αἱ τῶν ἀνωμαλιῶν περίοδοι ἐλάττους οὖσαι τῶν περιδρομῶν [μείζους] πλείους εἰσὶ τοῦ τῶν μηνῶν ἀριθμοῦ. οἷον ἵν' ἔτι §10 σαφέστερον γένηται παραδείγματι, ἡ ὑπόθεσις ἔστω ἡ ὑπὸ τῶν ἀρχαίων παραληφθεῖσα περίοδος ἣν ἐλέγομεν μηνῶν εἶναι σκγ. ἀλλ' ἐν τούτοις τοῖς μησὶν οὖσι σκγ, αἱ πλατικαὶ §11 περίοδοι εὑρίσκονται σμβ, αἱ δὲ τῶν μηκικῶν περιδρομαὶ σμα, αἱ δὲ τῆς ἀνωμαλίας σλθ, μῆνες δέ, ὡς εἴρηται, σκγ. δῆλον οὖν ὅτι πασῶν μὲν ἐλάχιστος ἀριθμὸς ὁ τῶν μηνῶν §12 ὑπάρχων σκγ, πασῶν δὲ τῶν περιόδων μείζων ἀριθμὸς ὁ τῶν πλατικῶν ὑπάρχων σμβ, μέσοι δὲ ὅ τε τῶν μηκικῶν περιδρομῶν οὐσῶν σμα, καὶ ὁ τῆς ἀνωμαλίας, εἰσὶ γὰρ σλθ. καὶ δῆλον ὅτι ὀλιγώτερα μὲν τὰ σμα τῶν σμβ, τὰ δὲ σλθ §13

4 δὲ addidi || 5 ωη, ͵δωμη Cumont (B): ωι ͵δωνη A || 9 δ' add. Heiberg || 13 μείζους del. Heiberg || 15 μείζους del. Heiberg | ἔτι Heiberg: ἐπὶ A || 16 παραδείγματι scripsi: παραδείγμα A: παράδειγμα Cumont (B) || 20 σλθ scripsi: σλε A || 21 πασῶν: πάντων Cumont || 23 ὅ Cumont: οἵ A || 24 ὁ Heiberg: αἱ A | σλθ scripsi: σλε A || 25 σλθ scripsi: σλε A

phenomena, given the same theory for the eccentricities.
§7 The period of 5458 months contains 122 eclipses at five-month intervals, and 808 at six-month intervals; for five times 122 is 610, and six times 808 is 4848, and together they make the 5458 months of the period.

§8 Such were the methods that the more ancient [astronomers] and Hipparchus used; what Ptolemy added to
§9 these things, will be stated later. You might reasonably ask why, among the foregoing periodic restitutions, the restitutions in latitude are more numerous than all the others, and after them the longitudinal revolutions, which are fewer than the latitudinal ones but more numerous than those of [the moon's] anomaly; and again the periods in anomaly, though fewer than the [longitudinal] revolutions,
§10 are more numerous than the number of months. To make this still clearer by an example, let the hypothesis be the period adopted by the ancients, which we said was 223
§11 months. In these 223 months the periods in latitude are found to be 242, the longitudinal revolutions 241, those of
§12 anomaly 239, and, as was said, the months 223. Hence it is evident that the smallest number of all is that of the months, being 223, and the greatest number of all the periods is that of latitudinal [periods], which is 242, and in between are the number of longitudinal revolutions, 241, and that of
§13 anomaly, namely 239. And obviously 241 is less than 242, and 239 less than 241, and the months, 223, are fewer than

ὀλιγώτερα τῶν $\overline{σμα}$, τούτων δὲ τὰ μὲν τῶν μηνῶν ὄντα $\overline{σκγ}$
ὀλιγώτερα πάντων τῶν ἀριθμῶν. αἴτιον δὲ τούτου ὅτι ἡ μὲν §14
πλατικὴ ἀποκατάστασις γίνεται δι' ἡμέρων $\overline{κζ}$ ε' ἔγγιστα, ἡ
δὲ μηκικὴ διὰ $\overline{κζ}$ γ', ἡ δὲ τῆς ἀνωμαλίας διὰ $\overline{κζ}$ L', ἡ δὲ ἀπὸ
συνόδου πρὸς ἥλιον ἐπικατάληψις διὰ $\overline{κθ}$ L' λ' ἔγγιστα. ἐπεὶ §15
γάρ, ὥσπερ ἔφαμεν, ὁ ἡλιακὸς κύκλος ὁ αὐτὸς δυνάμει ἐστὶ
τῷ διὰ μέσων τῶν ζῳδίων κύκλῳ, ἐπείπερ ἐκβαλλόμενος διὰ
τοῦ ἰδίου ἐπιπέδου ἐκείνῳ συμπίπτει, ὥσπερ δὲ οὗτος
ἐγκέκλιται πρὸς τὸν ἰσημερινόν, καὶ ἔστιν αὐτῶν δύο σημεῖα
τὰ ἰσημερινὰ κοιναὶ τομαί, οὕτω καὶ ὁ τῆς σελήνης κύκλος,
καθ' οὗ ἡ πορεία αὐτῆς γίνεται, ἐγκέκλιται πρὸς τὸν ἡλιακόν,
ἐκβαλλομένου οὖν πάλιν τοῦ σεληνιακοῦ ἐπιπέδου, τομαὶ
γίνονται ἀμφοτέρων τῶν κύκλων περί τινα σημεῖα δύο, ἃ
κυρίως αὐτῶν κέκληνται σύνδεσμοι, ἀλλ' ἀφ' οὗ μὲν φέρεται
ἡ σελήνη πρὸς τὰ βόρεια, ἐπείπερ ταῦτα ἀνώτερά ἐστιν ὡς
πρὸς ἡμᾶς, Ἀναβιβάζων κέκληται, ἀφ' οὗ ⟨δὲ⟩ πρὸς τὰ νότια
διὰ τὸ ταῦθ' ἡμῖν εἶναι ταπεινότερα, Καταβιβάζων. τὸ δὲ §16
ὅλον σεληνιακὸν ἐπίπεδον οὐ, καθάπερ τὸ ἡλιακόν, ἐπὶ ταὐτὸ
μένει, ἀλλὰ παραφέρεται τεταγμένως ἐπὶ τὰ προηγούμενα
τῶν δωδεκατημορίων ὡς ἀπὸ Κριοῦ ἐπὶ Ἰχθύας ἡμερήσια
ἔγγιστα λεπτὰ τρία. φανερὸν δὲ τοῦτο ἐκ τῶν ἐκλείψεων, §17
αἵπερ περὶ τοὺς συνδέσμους τούτους γινόμεναι ἀεὶ φαίνονται
κατὰ τοὺς ἑξῆς χρόνους ἐν τοῖς προηγουμένοις ζῳδίοις· εἰ C 128
γὰρ ἔμενε τὸ σεληνιακὸν ἐπίπεδον, καθάπερ τὸ ἡλιακόν, καὶ
οἱ σύνδεσμοι, κατὰ τῶν αὐτῶν ἂν τόπων αἱ ἐκλείψεις
ἐγίνοντο. ἡ μὲν οὖν πλατικὴ αὐτῆς κίνησις ἐπὶ τοῦ ἰδίου §18
αὐτῆς τοῦ ἐγκεκλιμένου λαμβάνεται κύκλου, καὶ εὑρίσκεται

5 ἐπικατάληψις Cumont: ἐπικαταλήψεις A | λ' scripsi: λα' A || 16 δὲ add. Cumont | νότια corr. e νοτεια A || 22 αἵπερ scripsi: εἴπερ A | φαίνονται Cumont (B): φαίνεται A || 25 ἂν scripsi: ἀεὶ A: ἂν ἀεὶ coni. Heiberg

§14 all the other numbers. The reason for this is that the latitudinal restitution takes place in approximately $27\frac{1}{5}$ days, [the restitution] in longitude in $27\frac{1}{3}$ days, [the restitution] in anomaly in $27\frac{1}{2}$ days, and the [moon's] catching up with the sun after conjunction in approximately
§15 $29\frac{1}{2}+\frac{1}{30}$ days. For since, as we have said, the sun's circle is effectively the same as the ecliptic, because when projected through its own plane it meets [the ecliptic], and just as [the ecliptic] is inclined with respect to the equator, and [the ecliptic and equator] have two equinoctial points as intersections, so too the moon's circle, along which its progress takes place, is inclined with respect to the sun's [circle]: therefore if the lunar plane too is projected, the intersections of the two circles [i.e. the moon's and the ecliptic] are at some two points, which are specially named their 'nodes', and the one from which the moon travels northward, since this direction is higher with respect to us, is named the 'ascending node', while the one from which it travels southward, because this direction is lower with
§16 respect to us, is named the 'descending' node. The entire plane of the moon does not remain stationary like the sun's, but shifts uniformly towards the leading parts of the zodiacal signs, as from Aries to Pisces, approximately 3
§17 minutes [of arc] each day. This is evident from the eclipses, which, taking place near these nodes, are observed at sequential times in [progressively] leading signs; for if the lunar plane and the nodes stayed still like the solar plane, the eclipses would occur always in the same places.

§18 Now the [moon's] latitudinal motion is reckoned on [the moon's] own inclined circle, and its greatest deviation

ἡ μεγίστη αὐτῆς ἐπὶ τῆς ἐγκλίσεως παραχώρησις ἐφ'
ἑκάτερα ¦μοιρῶν ε̅· ἡ δὲ μηκικὴ ὡς πρὸς τὸν διὰ μέσων A 33
[διαφόρει], οὐδεμίαν γὰρ διαφορὰν αἰσθήτην ποιεῖ τῇ
φαινομένῃ μηκικῇ κινήσει.
⟨ἡ δὲ σελήνη⟩ φαίνεται καθ' ἕκαστον μῆνα καὶ ἐλάχιστα §19
καὶ μέσα καὶ μέγιστα κινουμένη, ἄλλοτε ἀπὸ ἄλλων ἀρχομένη
καὶ μὴ ἀκριβῶς ἐπὶ τὰ αὐτὰ λήγουσα. δι' ἃς δὲ αἰτίας ταῦτα §20
γίνεται, ἐν τοῖς περὶ τῶν ἀνωμαλιῶν αὐτῆς λέγεται.
φαίνεται δὲ ἐκ τῶν προειρημένων ἡ προειρημένη ἀνωμαλία §21
μὴ συναποκαθισταμένη αὐτῆς τῇ μηκικῇ κινήσει, ἀλλὰ
πλεονάζουσα μοίραις β̅ καὶ λεπτοῖς μ̅ς̅, ὡς τετήρηται. ἐὰν §22
τοίνυν ὑποθώμεθα τὴν σελήνην ἐπὶ τῆς τοῦ Κριοῦ ἀρχῆς καθ'
ἑνὸς τῶν συνδέσμων, σύνοδον μεθ' ἡλίου πεποιημένην, καὶ τὰ
ἐλάχιστα ἀρχομένην δρομήματα ποιεῖσθαι, κινήσεως
γινομένης μετὰ ταῦτα ἐν μημιαίῳ χρόνῳ, ἡ σελήνη περιίουσα
τὸν ἴδιον αὐτῆς κύκλον, πρότερον ἐπὶ τὸν Ἀναβιβάζοντα
ἥξει διὰ τὸ αὐτόν, παραφερομένου τοῦ ἐπιπέδου εἰς τὰ
προηγούμενα, γίνεσθαι ἐν τῷ τοσούτῳ χρόνῳ περὶ μοίρας
Ἰχθύων ἔγγιστα κ̅η̅ ∟', εἶθ' οὕτω πάλιν ἐπὶ τὴν τοῦ Κριοῦ
ἀρχὴν ἐλεύσεται, καὶ μετὰ ταῦτα ἐπὶ τὴν ἀνωμαλίαν
προσλαβοῦσα μοίρας β̅ μ̅ς̅ ἀποκατασταθήσεται, τουτέστιν,
ἐπὶ τῷ ἄρχεσθαι ἀπὸ τῶν ἐλαττόνων δρομημάτων ἐπὶ τὰ
μέσα, καθάπερ πρότερον κινεῖσθαι. ἐπὶ δὲ τούτοις λοιπὸν τὸν §23
ἥλιον ἐπικαταλαβοῦσα, δηλονότι καὶ αὐτὸν ἐπικινηθέντα,
τὴν μηνιαίαν ἀποπληρώσει κίνησιν. εἴρηται γὰρ ὡς ἡ μὲν §24
πλατικὴ ἀποκατάστασις γίνεται δι' ἡμέρων κ̅ζ̅ ε' ἔγγιστα, ἡ

1 παραχώρησις Cumont (B): περιχωρήσεις A ‖ 2 μοιρῶν Cumont: μοίρας A ‖ 3 διαφόρει seclusi ‖ 5 ἡ δὲ σελήνη addidi exempli gratia: lacunam ind. Heiberg ‖ 7 λήγουσα Cumont: λέγουσα A ‖ 12 καθ' Cumont: καὶ A ‖ 14 δρομήματα Cumont (B): δρομήματο A ‖ 15 γενομένης Cumont (B) | περιίουσα Cumont: περιοῦσα A ‖ 17 τὸ scripsi: τὸν A ‖ 25 ἀποπληρῶσαι Cumont (B)

in inclination is 5° in either direction; but the [motion] in longitude [is reckoned] as if with reference to the [plane of] the ecliptic, since this makes no perceptible difference with respect to the apparent longitudinal motion.

§19 The moon is seen to make its least, mean, and greatest motion during each month, starting from a different [motion] each time, and not reattaining the same [motions]
§20 exactly. The reasons for this are stated in the [section] on [the
§21 moon's] anomalies. But from what has been said above, the stated anomaly is evidently not restored at the same time as [the moon's] longitudinal motion, but is in excess by 2° 46',
§22 as has been observed. Now if we suppose that [at a certain moment] the moon was [simultaneously] at the beginning of Aries and at one of the nodes and making its conjunction with the sun and starting its least courses [i.e. moving most slowly], and that thereafter a month elapses during which there is motion [in the solar and lunar models], the moon in its revolution about its own circle will first reach the ascending node, because [the node], on account of the plane's shifting in the leading direction, is then approximately at Pisces $28\frac{1}{2}$°. Then [the moon] will come back to the beginning of Aries, and after that it will be restored in anomaly, having taken up an additional 2° 46'; that is, assuming that it started from the least courses towards the mean, [it will be restored] to moving as before
§23 [i.e. most slowly]. Next after these things, [the moon], catching up with the sun (which itself of course will have made an additional motion), will complete its monthly
§24 motion. For it was said [above] that the latitudinal restitution takes place in approximately $27\frac{1}{5}$ days, the

δὲ μηκικὴ διὰ $\overline{\text{κζ}}$ γʹ, ἡ δὲ τῆς ἀνωμαλίας διὰ $\overline{\text{κζ}}$ ∟ʹ, ἡ δὲ ἀπὸ συνόδου πρὸς ἥλιον ἐπικατάληψις διὰ $\overline{\text{κθ}}$ ∟ʹ λʹ ἔγγιστα.
διόπερ, ὡς εἴρηται, χρόνον περιέχοντα τὰς προειρημένας §25
αὐτῆς πάσας ἀποκαταστάσεις κοινῶς διαφόροις ἀριθμοῖς εὑρόντες, περίοδον τοῦτον κεκλήκασιν, ἐν ᾗ πλείων μέν, ὡς ἂν ταχυτέρων οὐσῶν, ⟨ὁ⟩ τῶν πλατικῶν ἀποκαταστάσεών ἐστιν ἀριθμός, ἐλάττων δὲ τούτου ὁ τῶν περιδρομῶν τῶν μηκικῶν αὐτῆς, τούτου δὲ πάλιν ἐλάττων ὡς ἂν διὰ πλείονος
γιγνομένων ὁ τῶν ἀνωμαλιῶν. ἐμπεριεχονται δὲ ὡς μείζους C 129 §26
πασῶν τῶν προειρημένων ἐλάττονι αὐτῶν ἀριθμῷ οἱ μηνιαῖοι χρόνοι.

γράφει δὲ Ἀρτεμίδωρος περὶ τῶν κατὰ Πτολεμαῖον §27
ψηφοφοριῶν ταῦτα. «τὴν σελήνην ὁ Πτολεμαῖος ἐν τοῖς $\overline{\text{͵ευνη}}$ §28
μησὶν ὑποτίθεται κατὰ πλάτος κύκλους διανύειν $\overline{\text{͵εϡκγ}}$ κατὰ τὰ αὐτὰ τῷ Ἱππάρχῳ, ἐπιλαμβάνειν τε κατὰ μῆκος μεθ' ὅλους κύκλους μοίρας $\overline{\text{ργ}}$ $\overline{\text{με}}$. αἱ δὲ κατὰ βάθος πλεῖσται προσθαφαιρέσεις, ἐπὶ μὲν τοῦ μεγίστου ἀποστήματος ὄντος τοῦ κέντρου τοῦ ἐπικύκλου, ἡ διαφορὰ γίνεται μοιρῶν $\overline{\text{ε}}$ $\overline{\text{α}}$,
ἐπὶ δὲ τοῦ ἐλαχίστου, $\overline{\text{ζ}}$ $\overline{\text{λθ}}$. καινοτομεῖν δὲ δοκεῖ διά τινων §29
φαινομένων δι' ὀργάνου τετηρημένων μηδὲ ποδιαίαν ἔχοντος τὴν διάμετρον, ἐπὶ τῆς σελήνης ὅτι τὸ ἀπόγειον τοῦ

3 ὡς Cumont (B): ἓν A ‖ 4 κοινῶς scripsi: καὶ ἡ ἐν A | διαφόροις Cumont (B): διαφόραν A ‖ 5 περιοδικὸν Heiberg | πλείων: –ω– incertum A ‖ 6 ὁ add. Heiberg ‖ 7 ἐλάττων Cumont (B): ἔλαττον A ‖ 9 ἐμπεριέχονται – χρόνοι scripsi: ἐμπεριέχον῎ δὲ ὡς μείζους πάσας τὰς προειρημένας ἐλάττονές αὐτοὶ ἀριθμοὶ ὡς μηνιαίῳ χρόνῳ A: ἐμπεριέχονται δὲ εἰς μείζους (et inde ut A) Cumont ‖ 12 τῶν Cumont (B): τὸν A ‖ 14 $\overline{\text{͵εϡκγ}}$ Cumont (B): $\overline{\text{͵επκγ}}$ A ‖ 15 μεθ' ὅλους Heiberg: μεθόδους A ‖ 16 $\overline{\text{ργ}}$ scripsi: $\overline{\text{λγ}}$ A | πλεῖσται προσθαφαιρέσεις Cumont (B): πλεῖστα προσθαφαιρεσις A ‖ 17 ὄντος Rome: ἐντὸς A ‖ 18 $\overline{\text{α}}$ Rome: $\overline{\text{ο}}$ A ‖ 19 $\overline{\text{λθ}}$ Cumont (B): λ῀ θ῀ A | καινοτομεῖν Cumont: κενοτομεῖν A ‖ 20 post φαινομένων add. ἃς Rome | τετηρημένων scripsi: τετήρηκεν A

longitudinal [restitution] in $27\frac{1}{3}$ days, that in anomaly in $27\frac{1}{2}$ days, and the catching up with the sun after a conjunction in approximately $29\frac{1}{2}+\frac{1}{30}$ days.

§25 Hence, finding as was said, a time encompassing in common all the foregoing restitutions of [the moon] in different numbers, [the ancient astronomers] called it a 'period': the number of latitudinal restitutions in it is greatest because they are the fastest; less than them is the [number] of longitudinal revolutions of [the moon], and still less than this is [the number] of anomalistic [restitutions]
§26 because they take place in a longer time. The monthly intervals, being greater [in length] than all the foregoing [kinds of month], are contained [by the period] in a fewer number than they are.

§27 Artemidorus writes the following about the compu-
§28 tations according to Ptolemy: 'Ptolemy assumes that the moon completes 5923 cycles in latitude in 5458 months, in agreement with Hipparchus, and that it takes up in addition to whole circles 103° 45' in longitude; the maximum equations in depth [are as follows]: when the center of the epicycle is at the greatest distance, the difference [i.e. maximum equation] is 5° 1'; when at least distance, 7° 39'.
§29 [Ptolemy] seems to innovate on account of certain phenomena observed by means of an instrument of less than a foot in diameter: for the moon, that the epicycle's apogee

ἐπικύκλου μὴ πάντοτε νεύει ἐπὶ τὸ κέντρον τῆς γῆς, ἀλλ' ἐπὶ
τὸ ⟨τῆς⟩ δι' ἀμφοτέρων τῶν κέντρων σημεῖον τὸ τὴν ἴσην τῇ
μεταξὺ τῶν κέντρων κατὰ τὰ περίγεια τοῦ ἐκκέντρου αὐτῆς
κύκλου διάστασιν περιέχον. καὶ διὰ ταῦτα τὴν τῆς ἀνω- §30
μαλίας ἀποκατάστασιν ποιεῖται. διορθούμενος δὲ τὰς τῆς §31
σελήνης κατὰ τὸν Ἵππαρχον ὁμαλὰς κινήσεις, ἃς ἐκεῖνος τῇ
διὰ τῶν ,ευνη μηνῶν ὑποθέσει ἐκτίθεται, ὅμως ἐν τῇ τῶν
συζυγιῶν πραγματείᾳ αὐτῇ τῇ διὰ τῶν ,ευνη μηνῶν γινο-
μένῃ κατὰ μῆκος ἐμμήνῳ προκοπῇ κέχρηται. ἐξ οὗ δῆλον ὅτι §32
ὁ τῆς |συζυγίας χρόνος οὐ περιέξει τὸν αὐτὸν τόπον τῷ ἐκ A 33v
τῆς Συντάξεως. συμβέβηκεν οὖν αὐτὸν ἄλλα μὲν ἀπο- §33
δεικνύναι διὰ τῶν αὐτοῦ τηρήσεων, οἷς οὐκ ἠκολούθησεν,
ἄλλα δὲ ὑποτίθεσθαι.»

ὑπολείπεται ⟨δ'⟩ ἐπὶ τῆς σελήνης ιζ ἑξηκοστὰ εἰς τὰ §34
προηγούμενα ἐν τοῖς Προχείροις τῶν κατὰ τὴν Σύνταξιν.
ἔχουσι δ' αὐταῖς ἐφεξῆς ⟨αἱ ἐκ⟩ τῶν κανόνων οὕτως. αἱ μὲν §35 §36
ἐκ τῆς Συντάξεως, καθ' ἃς ἐνειστήκει εἰς τὸν ἐπιζητούμενον
χρόνον τὰ ἀπὸ Ναβονασάρου ἔτη ϡνθ, ἥλιος μὲν Ταύρῳ
μοίραις β ζ ἀκριβῶς, ἐγεγόνει δὲ μέσως ο μη· τὸ δὲ παρὰ τὴν
ἐκκεντρότητα διάφορον μοίρας α ιθ. ἡ δὲ σελήνη ὁμάλως §37
μὲν Σκορπίῳ μοίραις κς ις· ἀπεῖχε δὲ τὸ μὲν κέντρον τοῦ
ἐπικύκλου τοῦ ἀπογείου τοῦ ἐκκέντρου μοίρας ν νη. ἀφ' §38
οὗ ἀκριβῶς αὐτὸ τὸ κέντρον ἀπεῖχε τοῦ ἀπογείου τοῦ
ἐπικύκλου τκδ ις, δι' ὧν καὶ ἡ διαφορὰ ἐγίνετο μοίρας β νς.
ἡ δὲ ἀκριβὴς αὐτῆς πάροδος Σκορπίῳ μοίραις κθ κβ· C 130 §39
διειστήκει δὲ ἀκριβῶς καὶ τοῦ βορείου πέρατος μοίρας κη μς.

2 τῆς add. Rome | τῇ Rome: τῶν A || 4 περιέχον Cumont: περιέχοντι A || 5 τὰς Cumont (B): τὰ A || 12 οἷς scripsi: αἷς A || 14 δ' addidi: οὖν add. Heiberg || 16 αὐταῖς scripsi: αὐτῷ A | αἱ ἐκ addidi || 18 ϡνθ scripsi: ϡνη A || 19 παρὰ Rome: περὶ A || 23 τοῦ^2 bis A: corr. Cumont || 26 διειστήκει Heiberg: διαστήκει A

does not point always to the earth's center, but to that point on the [line] between the two centers that has a distance [from the earth's center] in the direction of the perigee of the [moon's] eccentric circle equal to the [line] between the two
§30 centers. By means of these [hypotheses] he makes the
§31 restitution of the anomaly. But although he corrected the mean motions of the moon according to Hipparchus, which [Hipparchus] sets out by the hypothesis of the 5458 month [period], nonetheless in the operation of the syzygies he has used this very monthly advance in longitude that arises from
§32 the 5458 month [period]. It is evident from this that the time of syzygy will not encompass the same place as that
§33 [derived] from the *Syntaxis* [i.e. *Almagest*]. Thus it has come about that there are some things that [Ptolemy] demonstrates through his own observations, but has not followed, assuming other things instead.'

§34 There is a shortfall [in longitude] of 17 minutes in the direction of the leading parts in the case of the moon in the *Handy Tables* with respect to the [longitudes] in the
§35 *Syntaxis*. The [computations] from the tables, following
§36 [those of the *Syntaxis*] are as follows. Those from the *Syntaxis*, according to which 959 years have been completed from Nabonassar to the time in question: the sun at Taurus 2° 7' in true position, and its mean position was [Taurus] 0° 48', and the difference due to eccentricity was
§37 1° 19'. The moon was at Scorpio 26° 16' in mean motion, and the center of [its] epicycle was 50° 58' from the eccenter's
§38 apogee. Hence the center itself was 324° 16' in true motion from the epicycle's apogee, on account of which the difference [i.e. equation] was 2° 56'. [The moon's] true
§39 position was Scorpio 29° 12', and it was 28° 46' from the northern limit in true motion.

αἱ δὲ ἐκ τῶν Προχείρων σχεδὸν μὲν αἱ αὐταὶ γίνονται — §40
ἑξηκοστοῖς γὰρ τὸ πλεῖστον παρ' αὐτοὺς τοὺς ἐπιλογιζο-
μένους διοίσουσι $\overline{\text{β}}$ — ὁμοίως δὲ καὶ ταύτας ἐκτίθεται. ὁ μὲν §41
ἥλιος ἀπέχει τοῦ ἀπογείου μοίρας $\overline{\text{τκε}}$ $\overline{\text{κ}}$, αἷς ἐπιβάλλει μοῖρα
$\overline{\text{α}}$ καὶ ἑξηκοστὰ $\overline{\text{ιθ}}$· καὶ γίνονται ἐπὶ τὸ αὐτὸ μοῖραι $\overline{\text{τκς}}$ $\overline{\text{λθ}}$.
αὗται δὲ ἐκβληθεῖσαι ἀπὸ Διδύμων $\overline{\text{ε}}$ μοιρῶν καὶ ἑξηκοστῶν §42
$\overline{\text{λ}}$ καταλήγουσιν εἰς Ταύρου μοίρας $\overline{\text{β}}$ καὶ ἑξηκοστὰ $\overline{\text{θ}}$. τῆς §43
δὲ σελήνης τὸ μὲν ἀπόγειον [ὂν] τοῦ ἐκκέντρου κύκλου
ἀπεῖχε τῆς τοῦ Κριοῦ ἀρχῆς μοίρας $\overline{\text{ροδ}}$ $\overline{\text{κβ}}$, τὸ δὲ κέντρον
τοῦ ἐπικύκλου τοῦ ἀπογείου τοῦ ἐκκέντρου μοίρας $\overline{\text{ν}}$ $\overline{\text{κβ}}$, τὸ
δὲ κέντρον αὐτῆς τοῦ ἀκριβοῦς ἀπογείου τοῦ ἐπικύκλου
μοίρας $\overline{\text{τκγ}}$ $\overline{\text{νε}}$. καὶ διὰ ταῦτα προσετίθει μοίρας $\overline{\text{β}}$ $\overline{\text{νς}}$. αὗται §44 §45
δὲ προστεθεῖσαι ταῖς $\overline{\text{ν}}$ $\overline{\text{κβ}}$ γίνονται $\overline{\text{νγ}}$ $\overline{\text{ιη}}$, ἀφ' ὧν ἐὰν
ἀφέλωμεν τὰς $\overline{\text{ροδ}}$ $\overline{\text{κβ}}$, καὶ τὰς λοιπὰς $\overline{\text{σλη}}$ $\overline{\text{νς}}$ ἐκβάλλωμεν
ἀπὸ τῆς ἀρχῆς τοῦ Κριοῦ, ἕξομεν τὴν ἀκριβῆ τῆς σελήνης
ἐποχήν, Σκορπίῳ μοίρας $\overline{\text{κη}}$ $\overline{\text{νς}}$. διενήνοχεν ἄρα τῆς ἐκ τῆς §46
Συντάξεως ἑξηκοστοῖς $\overline{\text{ιζ}}$· ταῦτα δὲ γίνεται παρὰ τὴν τῶν
νυχθημέρων ἀνωμαλίαν [διαφόρου].

περὶ μὲν οὖν τῶν ἐκ τῆς Συντάξεως λαμβανομένων τῆς §47
σελήνης παροδῶν τὰ νῦν ἀφείσθω· περὶ δὲ τῶν ἐκ τῶν
Προχείρων Κανόνων λεγέσθω. $\overline{\text{ε}}$ δὴ χρόνοι ὁμοίως ἐπὶ τοῦ §48
ἡλίου καὶ ἐνταῦθα κεῖνται, εἰκοσαπενταετηρίδων μὲν
πρῶτος, δεύτερος δὲ ἁπλῶν ἐτῶν, τρίτος μηνῶν αἰγυπτίων,
τέταρτος ἡμερήσιος δρόμος, πέμπτος ὁ καθ' ὥραν.
λαμβάνεται δὲ ὥρα ὁμοίως μὲν ἡ ἀπὸ μεσημβρίας §49

2 ἑξηκοστοῖς ex ἑξεικοστοις corr. A ‖ 5 $\overline{\text{ιθ}}$ coni. Cumont: $\overline{\text{θ}}$ A | $\overline{\text{τκς}}$ Rome: $\overline{\text{τκθ}}$ A ‖ 7 $\overline{\text{λ}}$ Rome: τετάρτων A: τεσσάρων in marg. A ‖ 8 ὂν del. Rome ‖ 13 προστεθεῖσαι Toomer: προστιθεῖσαι A ‖ 17 $\overline{\text{ιζ}}$ Heiberg: $\overline{\text{σιζ}}$ A (immo $\overline{\text{ις}}$!) | παρὰ coni. Cumont: περὶ A ‖ 18 διαφόρου deleui: διάφορον Cumont ‖ 20 ἀφείσθω Toomer: ἀφίσθω A: ἀφέσθω Cumont (B) ‖ 21 $\overline{\text{ε}}$ δὴ: οἱ δὲ Cumont (B) ‖ 23 πρῶτος scripsi: πρῶτα A ‖ 24 πέμπτος: ε' A

§40 The [computations] from the *Handy Tables* are nearly
the same [for the sun]—for at most they differ by two
minutes from those computed [above]—and he [?] sets
§41 these out too in the same way. The sun is 325° 20' from the
apogee; 1° 19' corresponds to this [in the table of solar
§42 anomaly], and the sum is 326° 39'. When these are counted
§43 off from Gemini 5° 30', they reach Taurus 2° 9'. The apogee
of the moon's eccentric circle was 174° 22' from the
beginning of Aries, the center of the epicycle was 50° 22'
from the eccenter's apogee, its center was 323° 55' from the
§44 epicycle's true apogee. And consequently he [?] added on
§45 2° 56'. These, added to 50° 22', make 53° 18'; and if we
subtract 174° 22' from these and count off the remainder,
238° 56', from the beginning of Aries, we will get the moon's
§46 true position, Scorpio 28° 56'. It differed therefore by 17
minutes from the [figure] from the *Syntaxis*; this happens on
account of the variation in the solar days [i.e. the equation
of time].

§47 Let us dismiss for now the subject of the moon's
motions as taken from the *Syntaxis*; and now let us speak of
§48 those from the *Handy Tables*. Five time intervals are laid out
similarly here as for the sun, first 25-year intervals,
secondly single years, thirdly Egyptian months, fourthly
§49 daily course, fifthly hourly [course]. The hour is taken in the
same way, as reckoned from noon, but the seasonal hour is

ἀριθμουμένη, μεταβληθεῖσα δὲ ἡ καιρικὴ εἰς τὴν δι'
'Αλεξανδρείας ἰσημερινήν, ὥστε διαφέρει ἡ ψηφοφορία κατὰ
τὰς ὥρας. ὅσας γὰρ ἂν ποιήσῃ ἰσημερινὰς ὥρας ἀπὸ §50
μεσημβρίας, αὐτὰς εἴσαγε. ἔτι τὰς εἰκοσαπενταετηρίδας τὰς §51
αὐτὰς καὶ τὰ ἁπλᾶ ἔτη καὶ τὸν μῆνα τὸν αὐτὸν λήψῃ ὅνπερ
καὶ ἐπὶ τοῦ ἡλίου. διὰ ⟨δὲ⟩ τὰς ἰσημερινὰς ὥρας παραλλάξεις §52
τινάς γε μὴν λάβοι καὶ ἡ ἡμέρα.

οὐχ ὥσπερ ὁ ἥλιος μίαν [ἂν] ἀπογραφὴν ἔχει ἑκάστῳ §53
χρόνῳ μοιρῶν τε καὶ λεπτῶν, οὕτω καὶ ἐπὶ τῆς σελήνης διὰ
μίας προσθέσεως τῶν μοιρῶν τὸ μῆκος εὑρίσκεται τὸ
ἀκριβές, ἀλλὰ πλείους [τε] εἰσὶ σελίδες παρακείμεναι
ἀριθμῶν ⟨ταῖς⟩ τε εἰκοσαπενταετηρίσι καὶ τοῖς ἁπλοῖς ἔτεσι
καὶ τοῖς μησίν, ὁμοίως καὶ ταῖς ἡμέραις καὶ ταῖς ὥραις· διὰ C 131
τριῶν γὰρ σελίδων [καὶ] τῶν τούτοις παρακειμένων μοιρῶν
τε καὶ λεπτῶν ψηφίζεται τὸ μῆκος.

συμβέβηκε δὲ καὶ ἄλλη διαφορὰ τῇ τῆς σελήνης §54
ψηφοφορίᾳ πρὸς τὴν τοῦ ἡλίου. ἐπὶ μὲν γὰρ τοῦ ἡλίου §55
πάντας τοὺς εὑρισκομένους τῆς ὁμαλῆς κινήσεως ἀριθμοὺς
εἰσήγομεν εἰς τὸν |τῆς ἀνωμαλίας κανόνα· ἐπὶ δὲ τῆς σελήνης, A 34
οὐκέτι πάντας τοὺς μετὰ κύκλον ἢ κύκλους παραλειπομένους
ἀριθμοὺς εἰς τὸν τῆς ἀνωμαλίας κανόνα εἰσφέρομεν, ἀλλὰ
μόνον ἐκ δυεῖν ὡς ἐπιδείξομεν. ἔχει γὰρ οὕτως· ἐπὶ τοῦ ἡλίου §56
μῆκος μόνον ἐψηφίζομεν, ἐπὶ δὲ τῆς σελήνης τρεῖς εἰσι χῶραι
αἱ ψηφιζόμεναι ἐφεξῆς κείμεναι τῇ τοῦ μήκους ἡλίου χώρᾳ.
καὶ ἡ μὲν μετὰ τὴν σελίδα τοῦ μήκους τοῦ ἡλίου ἐστὶ σέλις §57
περιέχουσα τοὺς τοῦ ἀπογείου τῆς σελήνης ἀριθμοὺς ἥτις
ἐπιγραφὴν ἔχει «ἀπογείου ἐκκέντρου»· ταύτῃ δὲ ἐφεξῆς

4 ἔτι scripsi: ἐπὶ A | εἰκοσα(πενταετηρίδας) Cumont (B): εἰκοσι– A ‖ 6 δὲ addidi ‖ 8 ἂν deleui: om. Cumont ‖ 9 οὕτω Cumont (B): ὅστω A ‖ 10 προσθέσεως scripsi: περιθέσεως A ‖ 11 τε deleui: δέ coni. Cumont ‖ 12 ταῖς add. Heiberg ‖ 14 καὶ deleui ‖ 19 εἰσάγομεν Cumont ‖ 26 τοὺς Cumont (B): ὡς A

converted to the equinoctial hour [for the meridian] through Alexandria, so that the computation is different in respect to
§50 the hours. For [you] enter as many equinoctial hours as you
§51 obtain after noon. You still take the same [number of] 25-year intervals and single years and the same month as for the
§52 sun. But because of the equinoctial hours, the day could be subject to some shiftings.

§53 Unlike the sun, which has one thing to record in degrees and minutes for each [component of] time, in the moon's case the true longitude is not found by a single addition of degrees, but rather there are several columns of numbers adjacent to the 25-year intervals and single years and months, and likewise to the days and hours; for the longitude is computed through three columns of degrees and minutes adjacent to these [time intervals].

§54 There is another difference between the moon's
§55 computation and the sun's. For the sun we entered in the table of anomaly all the numbers found for the mean motion; but for the moon we do not enter in the table of anomaly all the numbers remaining after a circle or circles [i.e. multiples
§56 of 360°], but only in pairs, as we shall show. For it is as
§57 follows. For the sun we computed only the [mean] longitude, but for the moon there are three places computed, which are situated right next to the place [i.e. column] of the sun's longitude. The one after the column of the sun's longitude is a column containing the numbers [of degrees] of the apogee of the moon, which has the title 'Of the eccenter's apogee'; next to this lies the column containing

κεῖται ἡ περιέχουσα τοὺς τοῦ μήκους τῆς σελήνης ἀριθμοὺς σέλις· ἐπιγέγραπται δὲ «κέντρου ἐπικύκλου», ἐν δέ τισιν ἀντιγράφοις οὕτως ἐπιγέγραπται, «μῆκος κέντρου ἐπικύκλου». ταύτῃ δὲ ἐφεξῆς σέλις κεῖται ἡ τοῦ βάθους §58 αὐτῆς, ἐπιγραφὴν ἔχουσα «κέντρου σελήνης», ἐν δέ τισι «βάθος κέντρου σελήνης». καὶ αὗται μέν εἰσιν αἱ τρεῖς χῶραι §59 ἃς χρὴ ἀριθμεῖν, τουτέστι τοὺς κατ' αὐτὰς ἀριθμοὺς ψηφίζειν, τὸν μέλλοντα εὑρήσειν πόσον κατὰ μῆκος ἡ σελήνη κεκίνηται. τὸ γὰρ «βόρειον πέρας», τουτέστι τὸ πλάτος, τὸ ἐν τῷ ἐφεξῆς σελιδίῳ ἀναγεγραμμένον, ἢ τὸ σελίδιον τὸ «Λέοντος καρδίας» ἐπιγεγραμμένον, ὃ ἔν τισιν ἀντιγράφοις ἐφεξῆς κεῖται, εἰς τὸ μῆκος οὔκ ἐστι χρήσιμα, ἀλλὰ εἰς τὴν τῆς ἐκλείψεως, ὡς ἑξῆς ἐροῦμεν. παράκειται δὲ τὰ τρία §60 ταῦτα σελίδια, τό τε ἀπογείου καὶ τὸ ἐπικύκλου καὶ τὸ τοῦ βάθους, πᾶσι τοῖς ὁμάλοις ἀριθμοῖς, τουτέστι καὶ ταῖς εἰκοσαπενταετηρίσι καὶ τοῖς ἁπλοῖς ἔτεσι καὶ τοῖς λοιποῖς τῶν πέντε χρόνων.

ζητῆσαι δὲ ἄξιον πῶς, τεσσάρων διαφορῶν ἐπινοουμένων §61 ἐν τῇ σεληνιακῇ κινήσει, μήκους, βάθους, πλάτους, μηνιαίας ἀποκαταστάσεως (ἣ κατὰ σχέσιν θεωρεῖται τῆς πρὸς τὸν ἥλιον ἀποκαταστάσεως), ὅταν λέγωσι «ζῳδιακοῦ περιδρομὴν» καὶ «ἀνωμαλίας ἀποκατάστασιν», τήν τε «ἀπὸ συνόδου πρὸς ἥλιον ἐπικατάληψιν», τί τούτων τίνι ἐφαρμόζει τῶν προρηθέντων, ὅταν ᾖ πάλιν ἐν τῇ πραγματείᾳ ψηφοφορία, ⟨καὶ⟩ τίθωνται «μήκους», «ἀνωμαλίας», «πλάτους», «ἀποχῆς», πάλιν τί τούτων ταὐτόν ἐστι τοῖς

3 κέντρου ἐπικύκλου: κύκλου ἐπικέντρου A: γρ(άφεται) κέντρου ἐπικύκλου in marg. A || 11 ἐπιγεγραμμένον Heiberg: ἀπογεγραμμένον A || 13 ἐροῦμεν Cumont (B): εὑροῦμεν A | τρία: $\overline{\gamma}$ A || 15 ταῖς Cumont: τοῖς A || 18 τεσσάρων: $\overline{\delta}$ || 23 τί: τε Cumont (B) | τίνι scripsi: τινὰ A: τίνα Heiberg || 25 καὶ addidi || 26 τί: τε Cumont (B)

the numbers of the moon's longitude, which is entitled 'Of the epicycle's center' (but in some copies it is entitled thus:
§58 'Longitude of the epicycle's center'). Next to this lies the column of [the moon's] depth, with the title 'Of the moon's center' (or in some copies, 'Depth of the moon's center').
§59 These are the three places that one should count up, that is, one should compute the numbers in them, if one intends to find how much the moon has moved in longitude; for the 'Northern limit', that is the latitude, which is recorded in the next column, or the column entitled 'Heart of the lion', which lies next in some copies, is not useful for the longitude, but rather for the [computation] of eclipses, as we
§60 shall explain next. These three columns, those of the apogee and the epicycle and the depth, lie next to all the numbers of mean motion, that is the 25-year intervals and single years and the rest of the five time intervals.

§61 Given that four different things are contemplated in the lunar motion, namely longitude, depth, latitude, and monthly restitution (which is reckoned relatively from the return to the same position with respect to the sun), it is fair to ask how, when people speak of 'revolution of the zodiac' and 'restitution of anomaly' and of 'catching up with the sun after conjunction'—which of these fits which of the [expressions] stated above; and again, when there is a computation in the treatise and [the headings] are set down 'Of longitude', 'Of anomaly', 'Of latitude', and 'Of elongation', which of these is the same as [which of] the

εἰρημένοις. ὅτι μὲν γὰρ τὸ πλάτος ταὐτὸν τῷ ἐν τοῖς §62
Προχείροις ἐπιγραφομένῳ «βορείου πέρατος», δῆλον· ⟨ἡ⟩
μὲν γὰρ πρὸς βορρᾶν ἢ νότον κατάβασις ἢ ἀνάβασις
ἀφορίζουσι τὸ πλάτος τῆς κινήσεως. πάλιν δὲ ὅταν ἐν τοῖς §63
Προχείροις αἱ ψηφοφορίαι γίνωνται τοῦ τε ἀπογείου C 132
ἐκκέντρου, ὃ καὶ ἐπιγράφεται «ζῳδιακοῦ ἀπογείου
ἐκκέντρου», καὶ πάλιν ἄλλου ὃ ἐπιγράφεται «⟨κέντρου⟩
σελήνης», ἢ ὥς τινες, «βάθος κέντρου σελήνης», γένοιτο δ'
ἂν, οἶμαι, τούτων φανερὰ ἡ συνωνυμία, ἐάνπερ αὐτὰ
ἀφορίσωμεν καθ' ἕκαστον.

λέγει δὲ ὁ Ἀπολλινάριος περὶ αὐτῶν οὕτως. «μὴν μέν §64 §65
ἐστιν ὁ χρόνος ἐκ συνθέτου κινήσεως ἡλίου καὶ σελήνης.
πλάτους δὲ ἀποκατάστασις λέγεται χρόνος ἀφ' οὗ ἂν τὸ §66
σεληνιακὸν τῷ διὰ μέσων ἐφαρμόσαν κέντρον καὶ περι-
ενεχθὲν τοῖς τοῦ πλάτους τέρμασιν εἰς τὸ τοῦ διὰ μέσων
ἐπίπεδον ἀποκαταστῇ. βάθους δ' ἀποκατάστασις λέγεται §67
χρόνος ἀφ' οὗ ἂν τῆς τοῦ ἀστέρος σφαίρας τὸ ἀπογειότατον
τῆς ἐπιφανείας μέρος ἀπὸ τοῦ ἀπογειοτάτου τῆς ἑαυτοῦ
κινήσεως γενόμενον ἐπὶ τὸν ἀπογειότατον αὐτοῦ πάλιν
ἀποκαταστῇ τόπον. μήκους δὲ ἀποκατάστασις λέγεται §68
χρόνος ὁπόταν οἱουδηποτοῦν ἀστέρος τὸ κέντρον ὁρμῆσαν
ἀπό τινος ἐπιπέδου τῶν διὰ ⟨τῶν⟩ τοῦ ζῳδιακοῦ πόλων
γραφομένων κύκλων καὶ περιοδεῦσαν τὸ ζῳδιακόν, εἰς ταὐτὸ
πάλιν ἐπίπεδον ἀποκαταστῇ, τοῦτο ἀφ' οὗπερ ἤρξατο

2 ἡ add. Cumont ‖ 4 δὲ coni. Cumont: τε A ‖ 5 γίνωνται Cumont (B): γίνονται A ‖ 6 ἐκκέντρου scripsi: κέντρου A ‖ 7 ἐκκέντρου scripsi: κέντρον A | ἐπι-γράφεται Cumont: ἐπιγράφει A | κέντρου add. Cumont ‖ 11 Ἀπολλινάριος Cumont: ἀπολινάριος A ‖ 21 οἱουδηποτοῦν Cumont (B): οὐδήποτουν A | ὁρμῆσαν Heiberg: ὁρμήσαντ(ος?) A ‖ 22 τῶν add. Heiberg ‖ 23 γραφομένων κύκλων coni. Heiberg: γραφομένου κύκλου A | ζῳδιακὸν scripsi: ζῴδιον A

§62 stated [expressions]. For it is clear that the latitude is the same as what is entitled 'Northern limit' in the *Handy Tables*, because the descent or ascent northward or
§63 southward delimits the latitude of the motion. Again, when in the *Handy Tables* there is computation of the eccenter's apogee, which is also entitled 'Of the zodiacal apogee of the eccenter', and also of another, which is entitled 'Of the moon's center' (or as some [copies have it] 'Depth of the moon's center'), the synonymity of these should, I think, be obvious, at least if we define them individually.

§64-5 Apollinarius says the following about them: '"Month" is the interval [resulting] from the combined motion of the
§66 sun and moon. "Restitution of latitude" is the name of the interval from when the lunar center coincides with the ecliptic to when it has revolved through the latitudinal limits
§67 and is returned to the plane of the ecliptic. "Restitution of depth" is the name of the interval in which the exact apogee of the surface of the star's [i.e. moon's or planet's] sphere, starting from the exact apogee of its own motion, is returned
§68 again to its exact apogee. "Restitution of longitude" is the name of the period in which the center of any star, having set out from some plane of [one of] the circles drawn through the poles of the zodiac [i.e. ecliptic], and having revolved around the zodiac, is returned to that same plane from which

φέρεσθαι. ἄλλως δ' ἀποκατάστασις μέση λέγεται ἣ καὶ μὴ §69
οὕτως ἔχουσα.

«κατελήφθη μὲν δὴ ὁ μηνιαῖος χρόνος μικτὸς ὢν ἐκ §70
κινήσεως ἡλίου καὶ σελήνης, ἀρξαμένη γὰρ ἀπὸ ⟨τῆς⟩ πρὸς
ἥλιον συνόδου ἡ σελήνη καὶ περιέλθουσα τὸν ἑαυτῆς κύκλον
ἐπιλαμβάνει τοσαύτην περιφέρειαν ὅσην ὁ ἥλιος ἐν τῷ
μεταξὺ μέχρι τῆς ἐπικαταλήψεως κεκίνηται χρόνῳ. [πῶς δὲ
ἀστὴρ ἐλάχιστα καὶ μέγιστα κινεῖται.] τῆς δὲ ἀνωμαλίας §71
αἴτιόν ἐστι τὸ βάθος. οὐκοῦν τοῦ μηνιαίου χρόνου καταληφ- §72
θέντος ἀνάγκη μῆκός τε καὶ βάθος κατειλῆφθαι. τὰ δὲ §73
ζητούμενά ἐστι βάθους περίοδος, πλάτους περίοδος, μήκους
περίοδος, μηνῶν πλήθους περίοδος. τοῦ βάθους μέρη τὸ §74
ἀπόγειον καὶ τὸ πρόσγειον καὶ τὸ μέσον. ἡ σελήνη ἀπὸ τοῦ §75
μέσου αὐτῆς ἀποστήματος ἐπὶ τὰ πρόσγεια χωροῦσα,
τοὐντεῦθεν δὲ ἐπὶ τὸ μέσον ἀνατρέχουσα ἀπόστημα, τὸν
μηνιαῖον χρόνον ὅσον ἐφ' ἑαυτῇ μειοῖ τάχιον τὸν ἥλιον
περικαταλαμβάνουσα διὰ τὸ τῷ μέσῳ μήκει προστιθέναι—
προστιθεῖσα δὲ τῷ μήκει τοῦτο, καὶ τῷ πλάτει προστίθησι—
τοὔμπαλιν δὲ τοῦ μέσου ἐπὶ τὸ ἀπόγειον καὶ ἀπὸ τοῦ
ἀπογείου ἐπὶ τὸ μέσον φερομένη, αὔξει τὸν μηνιαῖον χρόνον,
βράδιον τὸν ἥλιον περικαταλαμβάνουσα, τῷ δὲ ἴσῳ καὶ τὸ C 133
πλάτος |μειοῦσα ⟨καὶ τὸ μῆκος⟩ [συναύξει τὸν μηνιαῖον A 34v
χρόνον]. ἐξισοῖτο δ' ἂν εἰ ὑποστησαίμεθα τὴν φορὰν τοῦ §76
πλάτους εἰς ⟨ἴσον⟩ ἀριθμὸν μοιρῶν τῷ μήκει, λέγω δὲ $\overline{\text{τξ}}$.

1 ἣ scripsi: ἡ A ‖ 4 τῆς add. Cumont ‖ 7 πῶς – κινεῖται deleui ‖ 12 τοῦ βάθους bis A: corr. Cumont (B) ‖ 13 ἀπόγειον scripsi: ὑπόγειον A ‖ 14 αὐτῆς scripsi: ἀπὸ τοῦ A: delendum coni. Cumont ‖ 18 προστιθεῖσα Cumont: προστεθεῖσα A | τοῦτο scripsi: τούτων A: τὸ ἴσον coni. Heiberg ‖ 22 καὶ τὸ μῆκος addidi | συναύξει... χρόνον deleui ‖ 24 ἴσον addidi

§69 it began revolving. Besides, a restitution is called [either] "mean" or not [mean].

§70 'Now the length of a month has been determined as being a composite of the motion of the sun and moon; for the moon, after starting from its conjunction with the sun and revolving around its own circle, takes up additionally as much arc as the sun has traversed during the intervening
§71 time until the catching up. The reason for the variation [in
§72 its length] is the depth. If therefore the length of the month has been determined, both [the motions in] longitude and
§73 depth necessarily must have been determined. What one is looking for is a period of depth, a period of latitude, a period
§74 of longitude, and a period of a number of months. The parts of the depth are the "apogee" and the "perigee" and the
§75 "middle". When the moon moves from its middle distance towards the perigee, and from there ascends to the middle distance, it diminishes the length of the month on its own account by catching up with the sun more quickly because it adds on to the mean [motion in] longitude (when it adds this to the [motion in] longitude, it adds also to the [motion in] latitude); contrariwise, as it moves from the middle to the apogee and from the apogee to the middle, it increases the length of the month by the same amount by catching up with the sun more slowly; and it diminishes the [motions in]
§76 latitude and longitude by an equal amount. It would be equal, [that is], if we established the motion in latitude for the same number of degrees as in longitude, namely 360.

τῶνδε γενομένων, διαφορὰν γίνεται ⟨οὐ⟩ μόνον ἐν ἐλαχίστῳ §77
διαστήματι τὸν ἐκλογισμὸν ποιουμένων, ἀλλὰ καὶ ἐν μείζονι.
ὅθεν δεήσει τὴν περὶ τὸν ἥλιον καὶ τὴν σελήνην γινομένην §78
διαφορὰν τῆς αὐξήσεως τῶν μηνῶν ἢ καὶ μειώσεως
ἐκλογισαμένους διαστεῖλαι.

«ἔστι μὲν οὖν ὀξυτάτη ἡ πρὸς τὰ πρόσγεια καὶ ἀπόγεια §79
παραχώρησις τῶν κύκλων, περὶ δὲ τὰ μέγιστα καὶ ἐλάχιστα
δρομήματα τῆς σελήνης, καὶ τὸ πλάτος τὴν ἰδίαν
ἀπολαμβάνει μέσον· ἀπὸ γὰρ τοῦ βάθους τὸ πλάτος αὔξεται
καὶ μειοῦται. Χαλδαῖοι δὲ ᾤοντο, τὰ μέσα κινουμένης τῆς §80
σελήνης, ἀμείωτόν τε καὶ ἀπρόσθετον τὸ πλάτος εἶναι.
εὕρηται δὲ περὶ τὰς ἐλαχίστας καὶ μεγίστας κινήσεις πλείστη §81
πρόσθεσις γινομένη ἢ ἀφαίρεσις, ὥστε διαφοράν τινα καὶ
τοῦ πλάτος περὶ τὸ τῆς σελήνης πρόσγειον ἢ ἀπόγειον εἶναι
(κατὰ γὰρ τὸ προσγειότατον μέρος ἢ καὶ ἀπογειότατον, τὸ
ὁμαλὸν ἵσταται πλάτος). ἄλλη ⟨μὲν⟩ γὰρ ἡ κατὰ τὸ §82
ἀπόγειον, ἄλλη δὲ ἡ κατὰ τὸ προσγειότατον τοῦ ὁμαλοῦ
πλάτους γίνεται σχέσις. καὶ οὐ μόνον δὲ περὶ τὸ ἀπογει- §83
ότατον ἢ προσγειότατον τοῦτο γίνεται, ἀλλὰ κατὰ πᾶν
μέρος τοῦ βάθους τὸ πλάτος ἐναλλάσσεται, ὥστε καὶ κατὰ
τὴν μέσην ἀπόστασιν τῆς σελήνης τυγχανούσης, ἄλλη μὲν

1 τῶνδε γενομένων διαφορὰν scripsi: ὧδε γὰρ γενομένου διάφορον A | γίνεται scripsi: γένηται A: γένοιτο coni. Cumont | οὐ add. Cumont || 2 ἐκλογισμὸν Cumont (B): ἐκλογισμῶν A | ἐν Cumont: ἒ A | μείζονι Cumont (B): μείζωνι A || 3 τὴν σελήνην γινομένην Cumont: ἡ σελήνη γινομένη A || 5 ἐκλογισαμένους Cumont (B): ἐκλογισαμένου A || 11 εἶναι scripsi: ἵνα A: post ἵνα lacunam ind. Heiberg || 12 εὕρηται Heiberg: εὕροιτε A || 13 διαφοράν τινα καὶ scripsi: διαφορὰ γίνεται A || 14 περὶ scripsi: παρὰ A | τὴν σελήνην Cumont (B corr.) | ἀπόγειον scripsi: ὑπόγειον A || 16 μὲν add. Cumont || 17 an ἀπογειότατον? || 18 δὲ Cumont (B): σε A || 20 ἐναλλάσσεται Cumont (B): ἐναλάσεται A || 21 ἀπόστασιν scripsi: ἀποκατάστασιν A

§77 This being so, a difference arises not only when we are making the selection [of eclipses] at an extremely short
§78 interval, but also at a greater [interval]. Hence it will be necessary to define the difference, whether increase or decrease, in [the length of] the months that occurs because of the sun and the moon, when one has made the selection.

§79 'Now the relative motion of the circles [i.e. the eccenter and epicycle] is most pronounced at the perigee and apogee, while it is near the moon's greatest and least courses that the [motion in] latitude assumes its own mean; for the [motion in] latitude increases and decreases in consequence of the
§80 depth. The Chaldeans, however, believed that, with the moon moving at its middle [distances], the latitude is not
§81 subject to increase or decrease. But the greatest increment or decrement has been found to occur near the least and greatest motions, so that there is some difference [from mean motion] in the [motion in] latitude too about the moon's perigee or apogee (for the mean [motion in] latitude
§82 is in effect at the exact perigee or apogee). This is because the situation of the mean [motion in] latitude at the apogee
§83 is different from its situation at the exact perigee. And not only does this occur at the exact apogee or exact perigee, but the [motion in] latitude changes at every part of the [motion in] depth, so that also when the moon is at its mean distance,

γίνεται σχέσις τοῦ πλάτους καταβαινούσης τῷ βάθει, ἄλλη
δὲ ἀναβαινούσης. οὐ μὴν ἀλλὰ καὶ καθ' ἕκαστον ζῴδιον §84
ἐναλλαγή τις θεωρεῖται τοῦ πλάτους, ἐναλλασσομένου
ἄλλοτε ἄλλως τοῦ βάθους πρὸς τὸ πλάτος. [μόνον δ' ἂν] §85
μάλιστα δ' ἂν τὸ πλάτος καταληφθείη εἴπερ ἐπὶ τῶν αὐτῶν
σχέσεων τυχόντων καὶ τῶν αὐτῶν οὐ μόνον ζῳδίων ἀλλὰ καὶ
μοιρῶν ἡλίου καὶ σελήνης αἱ τηρήσεις γίνονται. τοῦτο δ' §86
ἐστὶν ἀδύνατον διὰ τὸ ἐν μυριάσιν ἐτῶν πάνυ πολλαῖς εἰκὸς
γενέσθαι τὸ τοιοῦτον.»

τούτων οὕτω δεδειγμένων ἐπίστησον ὡς τὸ μὲν «μῆκος» §87
παρίστησιν ἡ ζῳδιακὴ περιδρομὴ καὶ τὸ ἐπιγραφόμενον
«κέντρου ἐπικύκλου» σελίδιον, διὸ καὶ ἐπιγράφουσί τινες
αὐτὸ «μῆκος κέντρου ἐπικύκλου». τὸ δὲ βάθος ἐν μὲν τῇ §88
πραγματείᾳ «ἀνωμαλίας» ἐπέγραψεν [ὃ γίνεται ὅταν ἀπὸ
μεγίστου κινήματος ἐπὶ μέγιστον κίνημα παραγένηται], ἐν δὲ
τοῖς Προχείροις ἐπέγραψε «κέντρου σελήνης». τὸ δὲ πλάτος C 134 §89
τὸ «τοῦ βορείου πέρατος» σημαίνει ἐπίγραμμα· ἡ δ' «ἀποχὴ»
τὴν ἀπὸ συνόδου πρὸς ἥλιον ἐπικατάληψιν, ὅσον ἀπέχει καθ'
ἕκαστον χρόνον.

ταῦτα μὲν οὖν περὶ τῆς ὀνομασίας ἀπαιτεῖ τὴν ἀληθῆ §90
ἱστορίαν. ἡμεῖς δὲ τοὺς τῆς πραγματείας ἀφέντας ἀριθμούς, §91
περὶ τῶν ἐν τοῖς Προχείροις λέγωμεν κανόσιν.

4 μόνον δ' ἂν deleui ‖ 5 δ' ἂν del. Cumont | ἐπὶ Heiberg: εἰπεῖν A ‖ 8 ἀδύνατον Cumont: ὃ δυνατὸν A ‖ 12 κέντρου scripsi: κέντρον A ‖ 14 ὃ – παραγένηται deleui (ad alium locum perditum pertinere uidetur) | ἀπὸ μεγίστου κινήματος scripsi: ἐπὶ μεγίστου κινήματος A: del. Cumont ‖ 20 ἡ ἀληθὲς ἱστορία coni. Cumont

the situation of the [motion in] latitude is different when [the moon] is descending in depth from when it is ascending.
§84 But moreover in each zodiacal sign some variation is seen in the [motion in] latitude, as the [motion in] depth varies differently at all times with respect to the [motion in]
§85 latitude. The [motion in] latitude would best be determined if the observations occur when the sun and moon are at the same situations and not merely in the same signs, but even
§86 [the same] degrees. But this is impossible because such a thing probably takes place [only] in many tens of thousands of years.'

§87 Now that these things have been shown, know that the zodiacal revolution furnishes the 'Longitude' [in the *Almagest*] and the column entitled 'Center of the epicycle' [in the *Handy Tables*], hence some give it the heading
§88 'Longitude of the center of the epicycle'. [Ptolemy] gave the depth the title 'Of anomaly' in the treatise, but in the
§89 *Handy Tables* he gave it the title 'Of the moon's center'. The title 'Of the northern limit' indicates the [motion in] latitude, and 'Elongation' [indicates] how far away at each time is the catching up with the sun after conjunction.

§90 These things concerning the nomenclature call for a
§91 true account. We shall dismiss the [subject of] the numbers in the treatise, and speak [now] of those in the *Handy Tables*.

III. COMMENTARY

References to the *Almagest* are by book and chapter (e.g., IV, 6), with page references to the first volume of Heiberg's edition (e.g., Heiberg, 302) and to Toomer's translation (e.g., Toomer, 191).

§1. In the lost part preceding the extant fragment, our author had already written about the tables for the sun's motion in Ptolemy's *Handy Tables* (cf. §§47-60). His next topic (we may conjecture) was to explain the lunar tables in the same work. The plan, so far as one can discern it through the muddle of digressions, seems to have been, first to say something about the theoretical derivation of the tables, then to describe how to use them. At the point where our fragment commences, he seems to have progressed from the lunar mean motion table in the *Handy Tables*, by way of the corresponding table in the *Almagest* which was its source, to the period relations from which Ptolemy initially derived his lunar mean motions (see Chapter I, section 3d). §§1-5 is a close (if disordered) paraphrase of Ptolemy's chapter on the lunar periods, *Almagest* IV, 2.[1] The 223-month eclipse period of the "even more ancient" astronomers (1) has already been mentioned (cf. §10); now our author turns to Hipparchus' period of 4267 months (3).
—"in accordance with [the moon's] syzygies with the sun": that is, the excess of $352\frac{1}{2}°$ over 4611 revolutions in period relation (3) was calculated by Hipparchus on the basis of his already established solar theory, since the period was bounded by oppositions (i.e., lunar eclipses). The $7\frac{1}{2}°$ shortfall from an integer number of revolutions seems in fact to have been a rounding of a more exact figure, probably 7° 46′, to the nearest quarter of a zodiacal sign.[2]

§2. Hipparchus' 4267-month eclipse period (3) brings the moon from near one node to an almost diametrically opposite point near the other; hence if the moon is eclipsed from its north side at the period's beginning, it will be eclipsed from the south at the end, and *vice versa*. Since moreover the moon is not at the same distance from the nodes at each eclipse, the eclipse durations and magnitudes will be different. For this reason, the 4267-month period was suitable for establishing the length of the anomalistic month, but not the draconitic month, which Hipparchus derived from period relation (4).

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[1] §1 ≈ Heiberg, 271 lines 15–19; §2 ≈ Heiberg, 272 lines 6–10; §3 ≈ Heiberg, 271 line 20–272 line 6; §5 ≈ Heiberg, 272 lines 12–20. Translation of the entire passage: Toomer, 176.

[2] Cf. Neugebauer, *HAMA*, 312, where, however, the assertion that our fragment ascribes a shortfall of 8° to Hipparchus derives from a typographical error in the *CCAG* edition. See also Toomer's note, 176 note 10.

§4. "Kedenas" is in all probability to be equated with a Kidinnu whose name figures in the colophons of some Seleucid cuneiform tablets from Babylon.[3] These texts, all of them lunar ephemerides, do not inform us of what his contributions were, nor indeed when he lived. In Greek sources, however, he is credited with three specific elements: the 251-month lunar anomalistic period (2) in our fragment, a maximum elongation of Mercury from the sun of 22° in Pliny's *Natural History* (II 6,39), and tables for computing lunar longitudes in Vettius Valens' astrological *Anthologies* (IX, 11). Neugebauer doubts whether the parameter for Mercury had an authentic Babylonian origin, but the lunar tables do seem to have descended (with some Greek modification) from a Babylonian scheme based on the approximate equation,

(5) 9 anom. m. = 248 days.[4]

It is possible that the various Greek *testimonia* on Kedenas derived from a single Hellenistic source transmitting Babylonian data, perhaps the authority on the "Chaldeans" from which Geminus (*Isagoge* chapter 18) cites Babylonian lunar parameters connected with relation (5). This transmission cannot be later than Hipparchus, i.e., the middle of the second century B.C., since he knew (5).[5] Van der Waerden has attempted to recover details of this source, on the assumption that all ancient astronomical and astrological references to "Chaldeans" descend from it.[6]

—"many prove to have used it": Beyond this assertion, we have no information about what astronomers, other than Hipparchus and Ptolemy, used the 251-month anomalistic period (2). For Ptolemy's correction, see Chapter I, section 4.

§6. This sentence surely does not mean to say that the value for the mean motion in latitude derived from (4) is more satisfactory than Ptolemy's slightly corrected value. Our author may, however, have been misled by Artemidorus into believing that Ptolemy's correction was a consequence of his modification of the lunar model (cf.§29, and Chapter I, section 4); or he may merely be comparing (4) with the inferior relation (1).

§7. This passage and related texts concerning eclipse intervals were discussed by Rome in his first article on the fragment.[7] Lunar eclipses, as was already known to the Babylonian astronomers, occur only at intervals of five or six synodic months, or sums of the two. Our author seems to believe, incorrectly, that a lunar eclipse will take place *every* five or six months when the sun is nearest a lunar node at conjunction, an assumption that leads to the figures in the text. The sun passes through one or the other node 930 times in 5458 synodic months (i.e., 5923 - 5458 times for each node); hence if f and s are

[3] Neugebauer, *HAMA*, 611–12.
[4] Jones [1983], especially 14–33. See also Chapter I, section 5.
[5] Jones [1983], 23–27.
[6] Van der Waerden [1972].
[7] Rome [1931,1]. Also Neugebauer, *HAMA*, 321–22.

the numbers of five-month and six-month intervals between ecliptic oppositions during 5458 months, then

$$f + s = 930,$$
$$5f + 6s = 5458,$$

so that $f = 122$ and $s = 808$.

This passage is not the only evidence for use of the 5458-month period as a recurring cycle of lunar eclipse possibilities. Plutarch (*De facie in orbe lunae* 20, 933E) mentions that intervals of 465 synodic months contain 404 six-month eclipse intervals and 61 five-month intervals; these are simply the figures in our fragment divided by two. It is possible that this application of period relation (4) goes back to Hipparchus, who is known to have studied eclipse intervals for both solar and lunar eclipses.[8]

§8. "what Ptolemy added . . . will be stated later": apparently in the passage beginning at §27.

§11. Three times in §§11-13 Cumont retains the manuscript's wrong figure 235 for the number of anomalistic months, which is certainly just a scribe's persistent misreading of the correct figure 239 (cf. *Almagest* IV, 2, Heiberg, 270; Toomer, 175).

§14. Manuscript A gives the length of the synodic month as $29\frac{1}{2} + \frac{1}{31}$ days here, but $29\frac{1}{2} + \frac{1}{30}$ in §24. The first figure approximates no attested or plausible value for the length of the month. Ptolemy's value, a parameter transmitted through Hipparchus from Babylonian astronomy, is 29;31,50,8,20 days in sexagesimal notation; rounded to one sexagesimal place, this becomes 29;32 (i.e. $29\frac{1}{2} + \frac{1}{30}$). Since the two sentences must originally have given the same numbers, I have emended §14 to agree with §24.

§15. "to us": i.e., to observers in the northern hemisphere.

§17. "leading signs": a standard expression meaning signs leading in the order of their rising; hence, with lower longitudes.

§18. "latitudinal motion": i.e., argument of latitude.
—Ptolemy does indeed always treat lunar longitudes as if the moon traveled in the plane of the ecliptic, ignoring the very slight longitudinal deviations (never more than 6 minutes) that result from disregarding the inclination of the moon's plane. This simplification is mentioned in *Almagest* IV, 6 (Heiberg, 302; Toomer, 191), and justified geometrically in VI, 7 (Heiberg, 503-506: Toomer, 296-98).

§20. "the [section] on [the moon's] anomalies": It is not clear whether our author is referring to relevant parts of the Almagest (e.g., IV 5-6) or to a lost part of his own commentary in which the geometrical model of the lunar motion was discussed.

[8] Neugebauer, *HAMA*, 321–22.

§21. "but is in excess by 2° 46′": i.e., the mean longitudinal motion in one anomalistic month is said to be 362° 46′. This is inaccurate. From relation (3), for example, one obtains a motion of about 363° 4′ per anomalistic month. Our author has evidently multiplied a crude mean daily motion of 13° 10′ (instead of 13° 10′ 34′ . . .) by a rough value for the length of the anomalistic month, say 27 $\frac{1}{2}$ + $\frac{1}{19}$ days, rounding the result to the nearest minute.

§22. For the purposes of illustration, it is assumed that the moon, sun, and a node all coincide at Aries 0°, and that the moon is at apogee (i.e., its least apparent speed). The order in which the moon afterward reattains the node, Aries 0°, its apogee, and the sun obviously is a consequence of the relative values of the four lunar mean motions (in latitude, longitude, anomaly, and elongation from the mean sun). The node will be at about Pisces 28° 34′ when the moon reaches it again at the end of one draconitic month.

The hypothetical situation used here resembles one that Ptolemy uses in *Almagest* V, 2 (Heiberg, 357-60; Toomer, 221-22) to illustrate his eccentric-epicyclic lunar model.

§25. "period": Our fragment gives "περίοδος" instead of the *Almagest*'s "περιοδικός," i.e., "periodic" (Heiberg, 270 line 10). This may be the commentator's slip. Cumont (following Heiberg) emends it as a copyist's error, but has not noticed that the gender of the following relative pronoun must then be changed.

§27. For general discussion of the quotation from Artemidorus, see Chapter I, section 4.

§28. The 5458-month latitudinal period (4) does not contain even nearly an integer number of anomalistic months, so that the moon's longitudinal motion during this period is not constant. It therefore makes no sense to assign to this period an excess in longitudinal motion over whole revolutions, nor does Ptolemy do so in the *Almagest*. If Artemidorus' figure is to have any meaning, it must refer to mean motion; but the reading in manuscript A, 33°45′, cannot be correct. Using Ptolemy's value for the mean motion in longitude (13;10,34,58, . . . ° per day), one would find that the moon travels approximately 5899·360° + 102° 42′ 22″ in 5458 mean synodic months of 29;31,50,8,20 days. A more plausible emendation of Artemidorus' number, 103° 45′, follows from assuming a rounded value, 13° 10′ 35″ for the mean daily motion:

$$\begin{aligned} 13;10,35°/\text{d} \cdot 29;31,50,8,20\text{d} &= 389;6,23,43,58, \ldots °/\text{syn. m.} \\ &\approx 389;6,23,43°/\text{syn. m.} \\ 389;6,23,43° \cdot 5458 \text{ syn. m.} &= 5899 \cdot 360° + 103;45,25,34° \\ &\approx 5899 \cdot 360° + 103;45° \end{aligned}$$

It is hard to see, however, why Artemidorus would choose to misrepresent Ptolemy's mean motions in this way.

The manuscript has 5° 0′ for the maximum equation at the moon's greatest distance, but Rome's correction to 5° 1′ seems necessary (cf. for example *Almagest* V, 7, Heiberg, 384; Toomer, 235). Ptolemy's theoretical maximum equation at least distance (*Almagest* V, 3, Heiberg, 362-65; Toomer, 223-25) is $7\frac{2}{3}°$ (= 7° 40′), but the greatest value derivable from his table (V, 8) is 7° 39′.

§29. The instrument mentioned here is presumably the *astrolabon* or armillary sphere described in *Almagest* V, 1 (Heiberg, 351-54; Toomer, 217-19), where Ptolemy does not specify its dimensions. Ptolemy is, however, known to have written a work specifically devoted to the description of a more elaborate armillary sphere (with nine rings instead of the *astrolabon*'s seven) called the *meteoroskopeion*.[9] From a quotation by Pappus we learn that Ptolemy specified that the largest ring of this instrument was to be "not less than twelve digits," i.e., $\frac{3}{4}$ foot.[10] Artemidorus may therefore have transferred this dimension to the simpler instrument of the *Almagest*, and reasonably interpreted "not less than" as "not much more than." Alternatively, he could have had some other source of information about the instrument, or even (considering his early date) seen it himself.

It is not clear whether Artemidorus mentions the smallness of the instrument in order to cast doubt on its accuracy. In fact Ptolemy refined his lunar model on the basis of observations by Hipparchus as well as his own, and the nature of Hipparchus' instruments is open to conjecture.

§33. This sentence may be our author's summing up of Artemidorus' argument. For my belief that what follows (§§34ff) is not by Artemidorus, see Chapter I, section 2.

§34. Ptolemy's *Almagest* and *Handy Tables* use different epochs from which their mean motions are counted, the era Nabonassar (1 Nabonassar, Thoth 1 = 26 February, 747 B.C.) and the era Philip (1 Philip, Thoth 1 = 12 November, 324 B.C.); in both sets of tables times are reckoned from noon. However, the intervals between consecutive noons, that is between successive meridian crossings of the sun, are not always exactly 24 equinoctial hours, because the sun's actual anomalistic motion along the ecliptic during the elapsed day is not constant, and because the ecliptic itself is inclined with respect to the uniformly revolving celestial equator. The correction that must be made to a given time in order to convert it to mean nychthemera (i.e., days of exactly 24 equinoctial hours) reckoned from the epoch date is called the "equation of time," and is a periodic function dependent on the sun's longitude at both the given date and the epoch date.[11] The equation of time is never greater than about 32 minutes, which is however enough to make a perceptible difference in the longitudes of the quickly moving moon. In converting his mean motion tables from the *Almagest*'s epoch to the *Handy Tables*', Ptolemy compensated for this effect, so that if lunar longitudes are computed for the same date by the two sets of tables without correcting the given date for the equation of time in each case, the results will differ by roughly 17 minutes (some

[9] Rome [1927].
[10] Rome, *CA* vol. 1, 6
[11] Neugebauer, *HAMA*, 61-68, 984-86.

variation is caused by the moon's anomaly and by rounding errors). Our author, in §46, correctly ascribes the discrepancy to the equation of time ("the variation in the solar days") but without further elaboration. The correct explanation is also given at somewhat greater length by Theon in his *Greater Commentary* to the *Handy Tables*.[12]

§36. The date of the example is given in the manuscript as 958 years elapsed since 1 Nabonassar (i.e., 959 Nabonassar), while the month, day, and hour are not given. Rome has, however, shown that the mean motions cited in the text pertain to 960 Nabonassar (= 536 Philip), Payni 28/29 at midnight[13]; the years since epoch should therefore have been given as 959, for which 958 could be the author's or a copyist's mistake. I have given the author the benefit of the doubt, and emended the text. Perhaps the details of the day and time were omitted because the same date had been used earlier in the lost part of the commentary. The verbs in the third person in §40 and §44 (if they are not textual corruptions) might suggest that the author is writing down his teacher's oral working out of the problem or copying from a written source (Artemidorus?); alternatively, they may merely mean "Ptolemy," referring to his general rules for using his tables. In any case, only a few intermediate stages of the computations are given in the text. A complete recomputation is given below (for notations, see Chapter I, section 3f).[14]

FROM THE ALMAGEST, FOR 960 NABONASSAR, PAYNI 28/29, MIDNIGHT:
Mean Motions

	sun: $\bar{\lambda}$	moon: $\bar{\lambda}$	$\bar{\alpha}$	$\bar{\omega}$	$\bar{\eta}$
810 y	163; 4,12	37;24, 7	222;10,57	217;37,22	234;19,55
144 y	324;49,25	270;38,57	175;29,57	174;41,19	305;39,32
5 y	358;47, 4	286;53,51	83;35,37	23;33,56	288; 6,47
270 d	266; 7,17	317;37,24	287;32,43	331;55,29	51;30, 6
27 d	26;36,44	355;45,44	352;45,16	357;11,33	329; 9, 1
12 h	0;29,34	6;35,17	6;31,57	6;36,53	6; 5,43
epoch	330;45	41;22	268;49	354;15	70;37
	30;49°	236;17°	316;55°	25;52°	205;28°
(text)	(30;48°)	(236;16°)	(omitted)	(omitted)	(205;29°)

N.B. The text in fact gives, not $\bar{\eta}$, but $2\bar{\eta}$ (= 50;58°).

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[12] Theon, *GC* 192.

[13] Rome [1931,2]. Neugebauer, *HAMA*, 949, mistakenly asserts that the solar longitude was computed for 958 Nabonassar, Payni 28 (= 25 April, A.D. 211). The longitudes are of course nearly the same for the same day in both years.

[14] For the *Handy Tables* I have used the manuscript *Vat. gr.* 1291. Rome ([1931,2], 109–12) gives the results but not all details of these computations; his value for the argument of latitude from the *Almagest* is too great by one degree. Three corrections have to be made to the numbers transmitted in manuscript A of our fragment, all in the computation according to the *Handy Tables*: 1° 19′ for 1° 9′ as the sun's equation, 326° 39′ as the sun's true longitude, and Gemini 5° 30′ for 5° 4′ as the sun's apogee. Other errors in the *CCAG* text, corrected by Rome, turn out to be Cumont's misreadings.

Calculation of True Positions

sun: elongation from apogee = 30;48° - 65;30° = 325;18°

equation corresponding to 325;18° = 1;18,24° (text: 1;19°)

true longitude = Taurus 0;48° + 1;19°
= Taurus 2;7° (text: Taurus 2;7°)

moon: c_1(50;58°) = 7;22,44

c_4(50;58°) = 0;9,19

α = 316;55 + 7;23° = 324;18°

c_2(324;18°) = 2;42,45

c_3(324;18°) = 1;22,21

c = 2;42,45° + $c_3 \cdot c_4$ = 2;55,32° (text: 2;56°)

λ = Scorpio 26;16° + 2;56°
= Scorpio 29;12° (text: Scorpio 29;12°)

ω = 25;52° + 2;56° = 28;48° (text: 28;46°)

FROM THE HANDY TABLES, FOR 536 PHILIP, PAYNI 28/29, MIDNIGHT:
Mean Motions

	sun: $\bar{\lambda}$	moon: $2\bar{\eta}$-$\bar{\lambda}$	$2\bar{\eta}$	$\bar{\alpha}$
526 Philip	34;32	222;11	284;26	222;37
10 y	357;34	218;39	72;27	167;11
Payni	266; 7	145;23	103; 0	287;33
28th day	26;37	302;32	298;18	352;45
12 h	0;30	5;36	12;11	6;32
	325;20°	174;21°	50;22°	316;38°
(text)	(325;20°)	(174;22°)	(50;22°)	(omitted)

Calculation of True Positions

sun: equation corresponding to 325;20° = 1;18,20° (text: 1;19°)

elongation from apogee = 325;20° + 1;19°
= 326;39° (text: 326;39°)

true longitude = 326;39° + Gemini 5;30°
= Taurus 2;9° (text: Taurus 2;9°)

moon: c_1(50;22°) = 7;17,56

c_4(50;22°) = 0;9

α = 316;38 + 7;18° = 323;56° (text: 323;55°)

c_2(323;55°) = 2;44

c_3(323;55°) = 1;23

c = 2;56,27° (text: 2;56°)

λ = 2;56° + 50;22° - 174;22°
= Scorpio 28;56° (text: Scorpio 28;56°)

N.B. For clarity the notations c_1, c_2, c_3, c_4 here refer to the same functions as for the *Almagest*, although they are tabulated in the order c_1, c_4, c_2, c_3 in the *Handy Tables*.

§40. The stated maximum difference of 2 minutes refers, of course, only to the solar longitudes. Neugebauer faults our author for ascribing the discrepancy in the solar longitudes to the equation of time (it results in fact from rounding errors); it seems to me that the text makes no such claim.[15]

§42. Manuscript A gives the difference as 217 minutes, but the first digit is probably a dittography of the end of the preceding word. The difference in the author's example is of course only 16 minutes.

§49. The time of day for which an astronomical computation was to be made would normally have been given in seasonal hours of day or night (equal to one twelfth of the interval between sunrise and sunset, or between sunset and sunrise). Ptolemy's mean motion tables, however, use uniform equinoctial hours counted from noon at the meridian of Alexandria. To obtain the most accurate results one must therefore convert a given time to seasonal hours, and then adjust it for the difference in longitude between one's location and Alexandria, and for the equation of time (see the note to §34 above). But the conversion of seasonal to equinoctial hours requires knowledge of the sun's current longitude in the first place. In his introduction to the *Handy Tables* Ptolemy therefore says to compute a first approximation of solar longitudes using the *seasonal* hours, counted from the preceding noon, and not even corrected for the difference in longitude from Alexandria.[16] When computing final results, especially for the moon's position, one must take all the corrections into account or perceptible errors may result. Both the correction for longitude and (in rare instances) the equation of time can cause the date entered in the tables to be the day before or after the current day at the observer's location.

§55. In the extant fragment our author never gets around to explaining the use of the lunar anomaly table in the *Handy Tables*; perhaps he was turning to this topic at the point where the text is cut off in §93.

§56. The passage beginning at this point is the earliest evidence after Ptolemy's own introduction for the arrangement of tables in early copies of the *Handy Tables*. The prevailing opinion of historians has for some time been that the version of the *Handy Tables* presented in surviving manuscripts is a fourth-century revision by Theon. A. Tihon has shown, however, that there is no evidence anywhere in Theon's voluminous commentaries on Ptolemy that supports the hypothesis of a "Theonine recension."[17] The testimony of our fragment, although it describes only a small part of the *Handy Tables* (the solar and lunar mean motion tables), yields interesting new information pertinent to the textual history of the *Handy Tables*. It not only shows that there existed

[15] Neugebauer, *HAMA*, 949.
[16] Ptolemy, *OAO*, 160–61.
[17] Tihon [1985]. Doubts were already raised by Neugebauer, *HAMA*, 968.

significant variants in the arrangement of the tables in copies of the *Handy Tables* already in the third century, but one minor divergence between copies described in the fragment actually survives in the manuscript tradition of the tables. This gives reason to doubt whether this tradition can descend from an archetype much later than Ptolemy himself.

In our manuscripts of the *Handy Tables* the mean motion tables for the sun and moon are combined, so that the single column for the sun's mean motion is followed by four columns for the moon's mean motions.[18] The anomaly tables are similarly unified. This arrangement has the obvious advantages of saving space and the user's time. Ptolemy's own introduction to the *Handy Tables* does not make it clear whether he combined either the mean motion or the anomaly tables. Theon writes that there were copies of the tables in which the anomaly tables were separate, as in the *Almagest*, as well as copies with the unified anomaly table; but he does not seem to have found the mean motion tables in any other form than the one we possess.[19] The author of our fragment seems to have known only the combined format of the mean motion tables. He even mentions (§59) certain copies in which at least one more column, the precessional motion of the reference star Regulus ("the heart of the Lion"), followed the column for the moon's northern limit; in our manuscripts this column is given alongside the planetary mean motions, with which it is more closely associated in application. Obviously such variations were dictated by the dimensions of the copyist's pages, and how reluctant he was to waste space. In this connection it is worth noting that papyrus fragments of astronomical tables in codex format, dating as early as the second century, have been discovered[20]; if the *Handy Tables* were published in roll format, there would have been no physical limit to the number of parallel columns, although a table combining (say) the mean motions of *all* the heavenly bodies would have been inconvenient to use.

Our author gives two forms for the titles of three of the four columns of lunar mean motions (see also the note to §61 below). The longer forms, in which the words μῆκος ("longitude"), βάθος ("depth"), and πλάτος ("latitude") are added at the beginning, occur in one of our oldest copies of the *Handy Tables*, the ninth-century *Leid. B.P.G.* 78 (ff. 91-93^{v}).[21] In the contemporary *Vat. gr.* 1291 (ff. 38-40^{v}) the titles have the short forms. Since βάθος is not a Ptolemaic term (he would have written ἀνωμαλία), the additional words probably are early glosses.

§59. The column for Regulus surely followed in these copies the column for the moon's northern limit, which could hardly have been omitted. Regulus has nothing to do with eclipses, but serves as a reference star for the preces-

[18] Until all manuscripts of the *Handy Tables* have been examined, such generalizations as this must be considered tentative.

[19] Theon, *PC*, 222–23.

[20] Neugebauer [1958] and *HAMA*, 1056.

[21] There are trivial errors in the column headings in this manuscript, but these have no bearing on my argument.

sional motion of the fixed stars and the planet's apogees. The discussion of eclipses must have followed well after the end of our fragment.

§61. In the lunar mean motion table of the *Almagest* (IV, 4) Ptolemy tabulates the increments of the four lunar mean motions "of longitude" ($\bar{\lambda}$), "of anomaly" ($\bar{\alpha}$), "of latitude" ($\bar{\omega}$), and "of elongation" ($\bar{\eta}$), which have an obvious significance in a pre-Ptolemaic simple epicyclic lunar model (see Chapter I, section 3c) as well as in Ptolemy's eccenter-and-epicycle model (sections 3e-f). The periods of these mean motions are of course the longitudinal revolution ("revolution of the zodiac"), anomalistic month ("restitution of anomaly"), draconitic month, and synodic month ("catching up with the sun after conjunction"). In the *Handy Tables* Ptolemy tabulates $2\bar{\eta}$-$\bar{\lambda}$ (with the heading "of the eccenter's apogee") instead of $\bar{\lambda}$, and $2\bar{\eta}$ (headed "of the epicycle's center," i.e., reckoned from the apogee of the eccenter) instead of $\bar{\eta}$. These quantities have a direct geometrical significance only in Ptolemy's model. He moreover retains $\bar{\alpha}$ (headed "of the moon's center"), but instead of $\bar{\omega}$ he now gives $\bar{\omega}$-$\bar{\alpha}$ (headed "of the northern limit")

Our author does not succeed in answering his own question about the relationship between these various mean motions, beyond saying that they should be obvious. Nor does his appeal to the authority of Apollinarius in §§64ff help much, since Apollinarius could not possibly foresee Ptolemy's model, and writes (so far as we can tell) in terms of a simple epicyclic (or possibly a simple eccentric) lunar model.

—From this point our author starts calling the *Almagest* the *pragmateia* ("the treatise") instead of its actual title *Syntaxis*, i.e., "Compilation." Ptolemy refers to the *Almagest* as *pragmateia* in its very last sentence (XIII, 11, Heiberg II 608, Toomer 647): "So at this point our present treatise can be terminated at an appropriate place and at the right length."

§64. On Apollinarius, the author of the following quotation, see Chapter I, section 5. Throughout the quotation Apollinarius signifies by the terms μῆκος ("longitude") and πλάτος ("latitude") the moon's motion or position in longitude and (argument of) latitude (i.e., λ and ω in the notations defined in Chapter I, section 3f); πλάτος never means what we usually call latitude, the actual deviation of the moon from the ecliptic. The term βάθος ("depth") refers to the anomalistic component of the moon's motion; it seems to have alluded originally to the moon's moving nearer to and farther from the earth, although Apollinarius is concerned mostly with the way that the anomaly interferes with the longitudinal and latitudinal motions.

§67. This definition of the "restitution of depth" (i.e., period of anomaly) seems to fit an epicycle-and-eccenter model, such as one might expect for one of the five planets, but not for the moon before Ptolemy. The "star's sphere" corresponds to the epicycle, and since the epicycle's motion itself has an apogee, it must be borne on an eccenter. Presumably (but this is not quite clear as Apollinarius expresses it) the "restitution of depth" must simultaneously bring the planet back to the epicycle's apogee and the epicycle back to the eccenter's apogee from these initial positions. Apollinarius may be quoting a general definition of "restitution of depth," originally expressed in terms of eccenter-and-epicycle models for the five planets; but it is also barely conceivable that Apollinarius contemplated such a model for the moon.[22] Be that as it may, Apollinarius' intricate discussion of the interrelation of the lunar motions and their periods seems to allow for only a single component of lunar anomaly (though see the note to §84 below), and only a single anomaly was accounted for in the Apollinarian lunar tables (see Chapter I, section 5). For clarity we shall assume in the following notes that Apollinarius employed a simple epicyclic model for the moon. In fact there is nothing in the quotation that establishes whether he preferred an epicyclic model or the geometrically equivalent eccentric model. We know from Ptolemy that Hipparchus had worked with both kinds of model at various times (*Almagest* IV, 11, Heiberg, 338: Toomer, 211), and this ambivalence may have persisted up to Ptolemy's time.

§70. At the end of this sentence there follows an interpolated title, "On how a planet makes its least and greatest motion." This undoubtedly was a reader's addition, and stood in the margin of an ancestor of manuscript A.

§74. Apollinarius uses the terms "apogee" and "perigee" (πρόσγειον, not the normal Ptolemaic term περίγειον) to signify the sections of the epicycle about the points farthest from and nearest to the earth. For these points themselves he uses superlatives, which I translate as "exact apogee" and "exact perigee."

§75. More simply put, the moon's motion (whether with respect to the ecliptic, the sun, or the nodes) is fastest at perigee and slowest at apogee. Since the moon takes longer than exactly one anomalistic month to repeat a conjunction or opposition with the sun, this excess of time over one anomalistic month obviously is inversely dependent on the moon's speed at the beginning or end of the interval in question. The same phenomenon may be considered in another way: at a fixed time shortly after an anomalistic month has elapsed, the moon's progress in longitude and in argument of latitude since the beginning of the anomalistic month will be greatest if the anomalistic month begins and ends with the moon at perigee, and least if the moon begins and ends at apogee.

In manuscript A the last phrase of this sentence reads, "and it simultaneously increases the length of the month while diminishing the [motion in] latitude by an equal amount." This is clearly nonsense, since an increase in time cannot be equal to a decrease in latitudinal motion, which is an arc. The parallel clause a few lines above equates the increases in latitudinal and longitudinal motion during the faster part of the anomalistic month; here we expect a corresponding statement, that the decreases in these motions in the slower part are equal. The words referring to the motion in longitude must

[22] Ptolemy (*Almagest* IV, 5, Heiberg, 294: Toomer, 180–81) falls just short of saying that no one before him tried to account for a second component of lunar anomaly. He does clearly state that he was the first to discover just how the second component depended on the moon's elongation from the sun.

have dropped out, and a phrase from earlier in the sentence has been unintelligently copied in their place.

§76. In the pre-Ptolemaic tables for lunar motion (see Chapter I, section 5) the argument of latitude was measured in 15° units called βαθμοί ("steps"). The anomalistic component of the argument of latitude, in such units, will of course be one fifteenth of what it would be if it were expressed in degrees (as in Ptolemy's tables).

§77. The remainder of the quotation from Apollinarius (has a connecting passage dropped out somewhere?) concerns specifically the difficulty of establishing a period relation for the moon's latitudinal motion (i.e., an equation between whole numbers of draconitic and synodic months) by comparison of observed lunar eclipses. The procedure that Apollinarius evidently has in mind was used by Hipparchus to confirm the Babylonian period relation (4):

5458 syn. m. = 5923 drac. m.

According to Ptolemy (*Almagest* IV, 2, Heiberg, 272: Toomer, 176), Hipparchus found a pair of lunar eclipses that were observed to be identical in duration and magnitude, and occurring at times when the moon's true longitude (λ) nearly coincided with its mean longitude (i.e., the longitude of the center of its epicycle, $\bar{\lambda}$). Ptolemy later (*Almagest* VI, 9, Heiberg, 525-27: Toomer, 309-10) gives a more detailed criticism, in the course of which he identifies the two eclipses. The first was an eclipse observed in Babylon at midnight, March 8/9, 720 B.C. (the observation is quoted earlier in the *Almagest*, IV, 6, Heiberg, 303: Toomer, 191-92). The situation of the moon at the time of this eclipse, based on a simple epicyclic model and Ptolemy's mean motions, is shown in Figure 5.

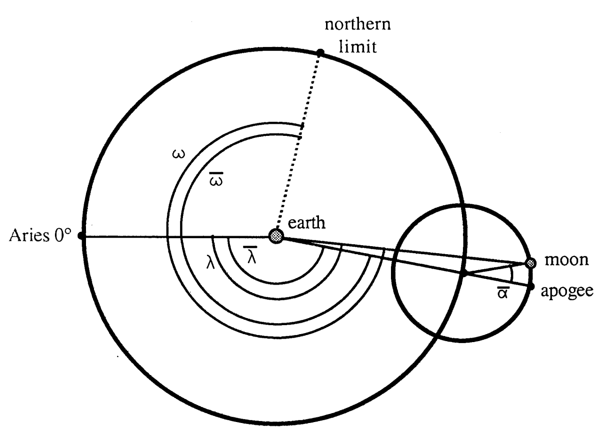

Figure 5. Configuration of lunar model at eclipse of March 8/9, 720 B.C.
Parameters according to the *Almagest*:

$\bar{\alpha}$ = 12° 24′ c = $\lambda - \bar{\lambda} = \omega - \bar{\omega} = -59'$
$\bar{\lambda}$ = 164° 45′ λ = 163° 46′
$\bar{\omega}$ = 280° 34′ ω = 279° 35′

Hipparchus himself observed his second eclipse at Rhodes, about two hours before midnight on 27 January, 141 B.C. (details quoted in *Almagest* VI, 5, Heiberg, 477-78: Toomer, 284). The configuration of the lunar model is shown in Figure 6.

According to Ptolemy (VI, 9), Hipparchus assumed that since the two eclipses were reported to have had identical durations and magnitudes (1 equinoctial hour and 3 digits from the south) the moon must therefore have been at exactly the same distance from the ascending node at both eclipses. The numbers of draconitic and synodic months most nearly corresponding to the interval between the eclipses could be derived from period relations such as (5) that Hipparchus already knew were roughly correct. Taking the moon to have been at its exact apogee and perigee at the two eclipses, Hipparchus could conclude that *exactly* a whole number of mean draconitic months had elapsed in the interval, as well as exactly a whole number of true synodic months. A very small correction (less than half an hour) would account for the difference between the interval and a whole number of *mean* synodic months that results from the different solar anomalies at the two dates; no correction for the lunar anomaly would have been necessary because of the moon's special situations at apogee and perigee. The result was therefore an empirical relation between mean synodic and draconitic months by which Hipparchus could check the accuracy of relation (5).

In the same passage (VI, 9), Ptolemy exposes two defects in Hipparchus' argument. Since the moon is at its greatest distance from the earth in the first eclipse, and at its least distance in the second, the size of the earth's shadow will have been distinctly greater at the first than at the second, so that to be

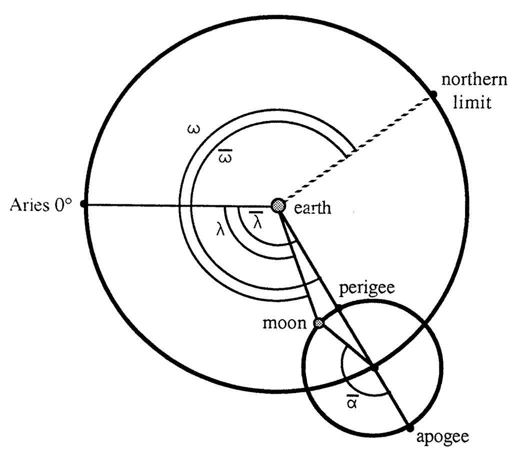

Figure 6. Configuration of lunar model at eclipse of January 27/28, 141 B.C.
Parameters according to the *Almagest*:

$\bar{\alpha} = 178°\ 46'$ $\quad c = \lambda - \bar{\lambda} = \omega - \bar{\omega} = -8'$
$\bar{\lambda} = 125°\ 16'$ $\quad \lambda = 125°\ 8'$
$\bar{\omega} = 280°\ 36'$ $\quad \omega = 280°\ 28'$

eclipsed by the same amount the moon must have been farther from the node in the first eclipse. Contrariwise, the moon was not exactly at apogee or perigee during the eclipses; at the first, the mean longitude of the moon was about a degree nearer the node than the true, while at the second the mean moon was about eight minutes nearer the node than the true. As Ptolemy points out, these two effects tend almost to cancel each other, although he can only conjecture that Hipparchus might have been conscious of the fact. Apollinarius' discussion is devoted entirely to the second effect.

§79. The sense of §§79-82 is that the parts of the anomalistic month when the moon passes through the apogee and perigee are the times when the moon's true rate of motion is most different from its mean motion. Obviously this is true whether the positions are reckoned from Aries 0° (i.e., the "motion in longitude") or from the northern limit (i.e., the "motion in latitude"). A slight deviation of the moon from the exact apogee or perigee will therefore bring about a greater variation in the moon's longitude or argument of latitude (with respect to their mean values) than an equal deviation elsewhere in the moon's revolution.

§80. Apollinarius seems to mean that the "Chaldeans" (i.e., Babylonian astronomers) did not incorporate an anomalistic fluctuation in the moon's latitudinal motion. From what we know of Babylonian lunar theory, this claim appears to be correct.

§82. This is not clear. Perhaps Apollinarius actually wrote, "This is because the situation of the mean [motion in] latitude at the apogee is different from its situation at the exact *apogee*."

§83. Apollinarius might have it in mind that the interval of 7160 synodic months between Hipparchus' two eclipses is almost exactly half an anomalistic month over a whole number of anomalistic months, so that if it begins when the moon is at apogee, it will end when the moon is at perigee. If the same interval is taken starting when the moon is near mean distance and moving toward the earth ("descending in depth"), it will end with the moon near mean distance but now moving away from the earth ("ascending"). These situations produce the maximum difference between the true and mean positions in longitude (and latitude), with the moon lagging behind its mean position in the first case, and leading it in the second.

§84. The reference to zodiacal signs is a bit unexpected. There is no component in the Hipparchian lunar theory (or Ptolemy's for that matter) that depends on absolute longitude. Probably Apollinarius means only that the effect of the anomaly on the latitudinal motion is constantly changing as the moon progresses from sign to sign.

§85. In selecting eclipses to test a period of latitudinal motion it would obviously be convenient to have not only the moon in the same configuration at both times, but also the sun, so that the interval between the eclipses will be exactly a whole number of *mean* synodic months. This would require eclipses occurring at the same longitudes. As we have seen above (note to §77), however, a difference in solar anomaly between the two eclipses can easily be accounted for by introducing a small correction in the number of synodic months in the interval.[23]

§88. In the middle of this sentence (just before "but in the *Handy Tables*") manuscript A has the interpolated phrase, "which occurs when it [i.e., the moon] goes from greatest motion to greatest motion." This makes no sense in the present context; it may have been mistakenly inserted by a copyist from a marginal note whose original purpose is no longer recoverable.

[23] This is in contrast to Hipparchus' method of determining a period of *anomalistic* motion from *two* pairs of eclipse observations (Toomer [1980]), where the need to have exactly equal intervals between each pair compelled him to find an eclipse period containing nearly a whole number of solar years.

INDEX