# PAPYROLOGICAL TEXTS IN HONOR OF ROGER S. BAGNALL

Edited by

Rodney Ast, Hélène Cuvigny,

Todd M. Hickey, and Julia Lougovaya

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# 35. P.Cornell inv. 69 Revisited: A Collection of Geometrical Problems

Alexander Jones

In his *Exact Sciences in Antiquity* Neugebauer published a photograph of the front side of P.Cornell inv. 69 and referred in passing to one of its diagrams as an example of an instructional papyrus containing mathematical texts similar to those transmitted in the Heronian corpus, but it was only in 2003 that an edition and detailed study of the papyrus by Bülow-Jacobsen and Taisbak made this interesting text accessible.[1] Despite the broken state of preservation (no complete line of the text survives), the editors identified the kinds of problems being dealt with, namely determining the areas of rectilinear quadrilaterals, substantially reconstructed the problem we will designate Problem 2 and its method of solution, and pointed to a few other mathematical papyri containing comparable problems. They correctly observed that the problems, despite their superficial "real-world" appearance, are really artificial exercises whose solutions depend on the circumstance that their geometrical configurations are built out of perfect "Pythagorean" right triangles with whole-number sides; practical surveying mathematics surely never required the extraction of square roots. Building on their edition, Friberg succeeded in reconstructing the mathematical contents of Problem 1, and drew attention to the especially close parallels between P.Cornell inv. 69 and P.Ayer (= *P.Chic.* 3), situating these—and many other Greek and Demotic mathematical papyri—as part of a tradition descending ultimately from Old Babylonian mathematics.[2] The present reedition owes a great deal to both these publications.

Bülow-Jacobsen and Taisbak provided a technical description of the papyrus, from which I only need repeat that the mathematical text is written along the fibers in a round, "unpretentious" second-century hand, with frequent abbreviations, while the back preserves a fragmentary third-century document. The fragment's dimensions are approximately 13.5 cm (width) by 27.5 cm (height), including about 2.5 cm upper margin, so that it is likely that close to the full height of the roll is preserved. The provenance of the papyrus is unknown; it was probably one of the 138 papyri that Cornell purchased in 1921–1922.[3]

My reasons for thinking it worthwhile to offer a reedition of P.Cornell inv. 69 begin with an abbreviation that recurs frequently in it, and that appears (in one of its clearer instances, ii 8) as follows:

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[1] A. Bülow-Jacobsen and C. M. Taisbak, "P.Cornell inv. 69. Fragment of a Handbook in Geometry," in A. Piltz *et al.*, *For Particular Reasons: Studies in Honour of Jerker Blomqvist* (Lund 2003) 55–70 (referred to henceforth as *ed. pr.*). O. Neugebauer, *The Exact Sciences in Antiquity*, 1st ed. (Acta historica scientiarum naturalium et medicinalium 9) (Copenhagen 1951, also Princeton 1952) 172 and plate 12; 2nd ed. (Copenhagen—Providence 1957) 179 and plate 12. Pack$^2$ = MP$^3$ 2317 (not in Pack$^1$); LDAB 7092. The papyrus is now part of the papyrus collection of the University of Michigan, Ann Arbor. I wish to thank the Michigan papyrus collection, and specifically Arthur Verhoogt and Adam Hyatt, for providing the excellent photographs of the papyrus and giving permission to publish them. I am also indebted to Hélène Cuvigny and Rodney Ast for weeding out several errors in my transcription.

[2] J. Friberg, *Unexpected Links between Egyptian and Babylonian Mathematics* (Singapore 2005) 226–233. E. J. Goodspeed, "The Ayer Papyrus: A Mathematical Fragment," *AJP* 19 (1898) 25–39 with unnumbered plate between 24 and 25, and "The Ayer Papyrus," *American Mathematical Monthly* 10 (1903) 133–135 with unnumbered plate between 133 and 134. The Ayer Papyrus is in the collection of the Field Museum, Chicago.

[3] W. Westermann and C. J. Kraemer, Jr., *Greek Papyri in the Library of Cornell University* (New York 1926), iii; see also http://www.library.cornell.edu/colldev/mideast/cdhist.htm.

The editors describe this as a “cursive β (κ would also be possible) with a hook above,” and comment that it “must be an abbreviation for βᾴϊον, a measuring rod of 6 cubits.” However, a very similar-looking abbreviation

occurs in a first-century CE astronomical papyrus, *P.Oxy.Astr.* 4136, lines 6 and 18, in which its resolution as καταλείπεται is evident from a parallel instance in lines 15–16 where the word is spelled out.[4] This verb has the technical sense in Greek mathematical texts of “the result of the subtraction is,” and it generally follows a statement that some quantity is to be subtracted from another quantity. The same resolution fits wherever the abbreviation appears in P.Cornell inv. 69, and wherever it occurs we may infer that a subtraction has just been performed, even when the preceding text is lost—a significant help in reconstructing the missing text and its procedures.

In fact a conjectural restoration of a large part of the lost text is possible, taking advantage of the stylized, even formulaic language, mathematical logic, and the information in the extant text and diagrams. The exercise yields a clearer understanding of the text’s mathematical procedures and conventions, as well as helping to confirm some of the less secure readings of the papyrus.

## Conventions

Lines are numbered continuously in each column, with a single line number assigned to a diagram or to the space where a lost diagram is presumed.[5] Interlinear insertions are assigned the number of the preceding line followed by the letter “a.” The diagrams have been modelled on tracings of the originals, with no attempt to make the lengths of the lines proportionate to their dimensions according to the text. Restored lines or portions of lines are represented by broken lines.

The translation attempts to represent the terminology as literally as is practicable. The expressions for arithmetical operations and results are as follows:

| | *translation* | *meaning* |
|---|---|---|
| ποίει | make | multiply |
| παράβαλε | apply to | divide into |
| πρόσαγε | bring | add |
| ἔκβαλε | cast out | subtract |
| καταλείπεται | there is left behind | the difference is |
| πλευρά | side | square root |

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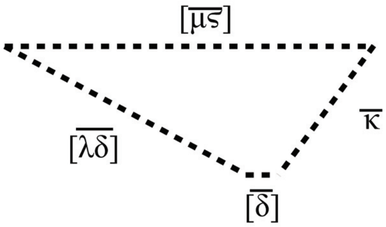
[4] A. Jones, *Astronomical Papyri from Oxyrhynchus* (Memoirs of the American Philosophical Society 233), 2 vols. in 1 (Philadelphia 1999) (= *P.Oxy.Astr.*), 2.14.

[5] It was not possible to retain the line numbering of *ed. pr.*

## Text and Translation

→
col. i



[ὡς δεῖ ποιῆσαι. ποίει τὰ λδ̅ τὰ ἐν τῶι νο(τίωι) ἐφ’ ἑατά·] (γίνεται) ,αρνϛ̅. καὶ τὰ κ̅
[τὰ ἐν τῶι βο(ρείωι) ἐφ’ ἑατά· (γίνεται) υ̅. ἀπὸ ,αρνϛ̅· κα(ταλείπεται) ψνϛ̅. ἔκβ]α̣λε τὰ δ̅ τὰ ἐν τῶι
[ἀπηλ(ιώτηι) ἀπὸ τῶν μϛ̅ τῶν ἐν τῶι λι(βί)· κα(ταλείπεται) μβ̅. παράβα]λε εἰ̣ς̣ ψνϛ· (γίνεται) ιη̅.
[πρόσαγε ταῖς μβ̅· (γίνεται) ξ̅. τὸ (ἥμισυ), λ̅, ὃ ἀπηλ(ιώτης) ἐστὶ τοῦ π]ρὸς νο(τίωι) τριγώνου.
[τὰ ιη̅ ἀπὸ τῶν λ̅· κα(ταλείπεται) ιβ̅, ὃ ἀπηλ(ιώτης) ἐστὶ τοῦ πρὸς βο(ρείωι) τριγώνου.] ὡς δε̣ῖ̣ ε̣ὑ̣ρε̣ῖ̣ν̣
[τὴν ὀρθὴν βάσιν. ποίει τὰ λδ̅ τὰ ἐν τῶι νο(τίωι) ἐφ’ ἑα]τ̣ά· [(γίνεται) ,α]ρ̅ν̅[ϛ̅.] κ̣[αὶ] τὰ λ̅
[τὰ ἐν τῶι ἀπηλ(ιώτηι) τοῦ πρὸς νο(τίωι) τριγώνου ἐφ’ ἑατά· (γίνεται)] ϡ̣. ἀπὸ ,α̣ρν̣ϛ̣· κα(ταλείπεται) σνϛ.
[πλευρά, ιϛ̅, ἣ ὀρθὴ βάσις ἐστίν. ὁμοίως τὰ κ̅ τ]ὰ̣ ἐν τ̣ῶ̣ι̣ ⟦ἀπηλ(ιώτηι) τοῦ⟧ ‵βο(ρείωι) ἐφ’ ἑατά· (γίνεται) υ̣̅.′
[‵καὶ τὰ ιβ̅ τὰ ἐν τῶι ἀπηλ(ιώτηι) τοῦ′]
[πρὸς βο(ρείου) τριγώνου ἐφ’ ἑατά· (γίνεται) ρμδ̅. ἀπὸ υ̅· κ(αταλείπεται) σνϛ̅. πλευρά,]ιϛ̅, ἣ ὀρθὴ βάσις
[ἐστίν. ]ομενων̣ νο() τριγω()
] ιϛ $\overset{\delta}{\underset{\delta}{\square}}$ιϛ, ἀρουρ() ξδ̅
] τὸ ἐ(μβαδόν), υ̅. τοσαύτ(ας)
]ναι τῶι ἀρουρισμ(ῶι)
] γ̣ένηται ἄλλο σχ̣[ῆ]-
[μα ]̣ ἀρο(ύρας) ψλϛ. τὸ ὑπόδ̣ι̣(γμα)

] *vac.*

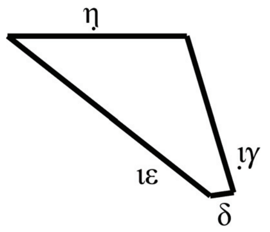
] νο() τρίγωνο̣ν̣
‵τετρά]γ̣ω̣νο̣ν̣ ἔχο̣ν̣ σ̣χ̣ῆ̣μ̣α̣ ‾‾′
τετρ]ά̣γ̣ω̣ν̣ον̣ ἔ̣χ̣ον
]̣̅ βο() ἐχομεν()
]̣̅. (γίνεται) ἐπὶ τὸ αὐτὸ
] ἀρουρῶ(ν) τλϛ̅
] ἀπὸ τῶν συν-

] *vac.*

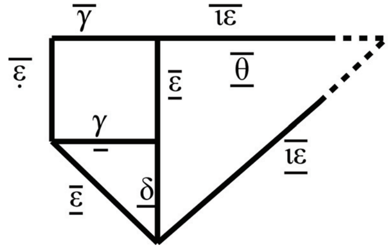
] .  . ṭọ διαφορ̣()
]   ṭίνα τὰ τμήμ̣(ατα);
]ν ι̅ .̅  ε .  . [
] . [
] . [
- - - - - - - - - - - -

col. ii

τραπεζοε̣[ιδὴς
. . . [  ] . [    ] . [   ] . [
ε . π̣ειπ[    ] . α̣ . α̣ṭ[
διαφόροụ [ἀρου]ρισμός τị[ς
σχήματος [τ]ọ̀ ụ̔πόδιγμ[α]



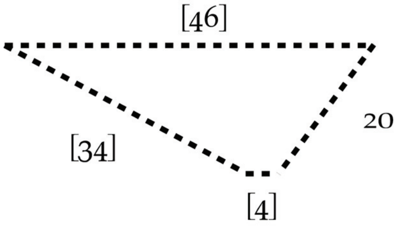
ὡς δεῖ ποιῆσαι. ποίει τὰ ιε τὰ ἐν τῶι νο(τίωι) ἐφ’ [ἑατά· (γίνεται) σκε̅. καὶ τὰ ιγ̅]
τὰ ἐν τῶι βο(ρείωι) ἐφ’ ἑατά· (γίνεται) ρξ̅θ̅. ἀπὸ σκε· κα(ταλείπεται) [νϛ̅. τὰ δ̅ τὰ ἐν]
τῶι λι(βὶ) ἀπὸ η̣̅ τῶν ἐν τῶι ἀπηλ(ιώτηι)· κα(ταλείπεται) δ. π̣[αράβαλε εἰς νϛ̅·]
(γίνεται) ιδ̅. πρόσαγε ταῖς δ̅· (γίνεται) ι̣η̣̅. τὸ (ἥμισυ) [θ̅, ὃ λί(ψ) ἐστι τοῦ πρὸς νο(τίωι)]
σφραγεῖδο(ς). ἀπὸ ιδ· κα(ταλείπεται) ε̣, ὃ ἀπηλ(ιώτης) ἐστὶ̣ [τοῦ πρὸς βορ(είωι) σφραγεῖδο(ς).]
ὡς δεῖ εὑρε̣ῖν τὴν ὀρθὴ̣ν βάσιν. [ποίει τὰ ι̅ε̅ τὰ ἐν τῶι]
νο(τίωι) ἐφ’ ἑατά. (γίνεται) σκ̅ε̅. καὶ τὰ θ̅ τὰ ἐν [τῶι λι(βὶ) τοῦ πρὸς νο(τίωι) τριγώνου]
ἐφ’ ἑατά· (γίνεται) πα̅. ἀπὸ σκε· κα(ταλείπεται) ρμδ. [πλ(ευρὰ) ιβ̅, ἣ ὀρθὴ βάσις ἐστίν.]
ὁμοίως καὶ τὰ ιγ̅ ṭὰ̣ ἐν ṭῶ̣ι βο̣(ρείωι) [ἐ]φ̣’ [ἑατά· (γίνεται) ρξθ̅. καὶ τὰ ε̅ τὰ]
ἐ̣ν τῶι ἀπηλ(ιώτηι) τοῦ πρὸς βο(ρείωι) τριγών[ου ἐφ’ ἑατά· (γίνεται) κε̅. ἀπὸ ρξθ̅·]
κα(ταλείπεται) ρμδ. πλ(ευρά), ιβ̅, ἣ ὀρθὴ βάσις ἐστίν.  . [
ἀναμέτρησις ὅλης τῆς σφραγε̣[ῖδος. τὸ συναμφότερον]
ṭῶ̣ν δύο τριγώνων τῶν πρὸς ν[ο(τίωι) καὶ πρὸς βο(ρείωι), πδ̅, ἀπὸ τῶν]
[ρ]νϛ· κα(ταλείπεται) οβ̅. τοσαύτας ἀρούρας ἔχε̣[ι
. . διαφόρωι̣ ἀρο(υρῶν) πδ̣̅· κα(ταλείπεται) ιβ̅  . . [
————
. ζ̅ μέθ̣οδο̣ς [
[σφραγὶ]ς̣ ἔ̣χουσα ιε $\frac{\overline{\iota\varepsilon}}{\varepsilon}$ ε συν . . . . . . [
τί]ς̣ ἡ̣ ὀρθὴ βάσις, τίνα τὰ ṭμ̣ή̣μ̣α̣[τα
] διαφόρου ἀρουρισμός̣ τις  . . [
σχήμ]ατος τὸ ὑπόδιγμα.



28 [ὡς δεῖ ποιῆσα]ι̣. πο[ί]ε̣[ι] τὰ ι̣ε̣̅ τὰ̣ ἐ̣ν τῶ̣ι [ἀπ]η̣[λ(ιώτηι) ἐφ’ ἑατά· (γίνεται) σκε̅. καὶ τὰ ε̅ τὰ ἐν τῶι]
[βο(ρείωι) ἐφ’ ἑα]τά· (γίνεται) κε̅. [σ]ύ̣ν̣θες· (γίνεται) σ̅[ν̅]. ὁ̣μ̣[ο]ίως κα̣[ὶ τὰ ιε̅ τὰ ἐν τῶι νο(τίωι)]
[ἐφ’ ἑατά· (γίνεται)] σ̣κ̣ε̣̅. καὶ τὰ ε̅ τ̣ὰ ἐν τῶι λι(βὶ) ἐ̣φ̣’ [ἑατά· (γίνεται) κε̅. σύνθες· (γίνεται) σν̅.]
]. . .μερος [ . ] . ι λ̅ τω[
32 ]. .[ . ] τοῦ πρὸς β̣ο(ρείωι) [τ]ριγώ̣ν̣[ου
]. . . . . ὡς δε̣ῖ εὑ̣ρε[ῖν τὴν ὀρθὴν βάσιν.]
[ποίει τὰ ιε̅ ἐν τ]ῷ̣ι̣ νο(τίωι) ἐφ’ ἑατά· (γίνεται) σκε. [καὶ τὰ ιβ̅ τὰ ἐν]
[τῶι τοῦ πρὸς ἀπ]ηλ̣(ιώτηι) [τ]ρ̣ι̣γώ̣νου ἐφ’ ἑα̣τά̣· (γίνεται) ρ[μδ̅. ἀπὸ σκε̅·]
36 [κα(ταλείπεται) πα̅. πλ(ευρά), θ̅, ἣ ὀρθὴ βάσ]ι̣ς̣ ἐ̣σ̣τ̣ί̣ν̣. . . . .[
] πρὸς ν̣ο̣(τίωι) τ̣ρ̣[ιγων
]. . .[
]. .[
40 ]. .[
- - - - - - - - - - - -

col. i 2 *et passim*, l. ἑαυτά, ˏαρνς̅κ̅α̅ι̅ *pap.* 11 τριγ$^{ω}$ 12 αρου$^{ρ}$ 13 τοσαυ$^{τ}$ 14 αρουρισ$^{μ}$ 16 αρ$^{ο}$ ϋποδι$^{γ}$ l. ὑπόδειγμα 21 εχομε$^{ν}$ 22 αρουρ$^{ω}$ 25 διαφο̣ρ¯
col. ii 5 ϋ̣ποδιγμ[α l. ὑπόδειγμα 10 ∟ 11 σφραγειδ$^{ο}$ l. σφραγῖδος (*bis*) 1 l. σφραγῖδος 21 αρ$^{ο}$ 26 ϋποδιγμα l. ὑπόδειγμα

col. i



[How one should do it. Make the 34 on the south by itself;] there results 1156. And the 20
[on the north by itself; there results 400. From 1156; there is left behind 756. Cas]t out the 4 on the
4 [east from the 46 on the west; there is left behind 42. Appl]y to 756; there results 18.
[Bring it to the 42; there results 60. Half, 30, which is the east of the] triangle to the south.
[The 18 from the 30; there is left behind 12, which is the east of the triangle to the north.]
How one should find
[the upright base. Make the 34 on the south by itse]lf; [there results 1]15[6.] A[nd] the 30
8 [on the east of the triangle to the south by itself; there results] 900. From 115[6]; there is left behind 256.

[Side, 16, which is the upright base. Similarly the 20] on the [east of the] ˋnorth by itself;
there results 400.ˊ
[ˋAnd the 12 on the east of theˊ]
[triangle to the north by itself; there results 144. From 400; there is left behind 256. Side,] 16, …
southerly triangle
] 16 $^{4}_{4}$ 16, 64 arouras
] the area, 400. So many
] … the determination of arouras
] there should arise another shape
] 736 arouras. The diagram

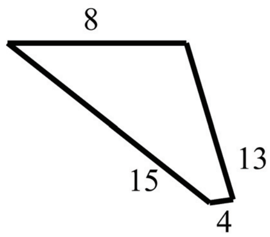
] *vac.*

] southerly triangle
]ˋhaving [quadra]ngular shapeˊ
] having [quadr]angular
] … north …
] … there results, by itself
] 336 arouras
] from the …

] *vac.*

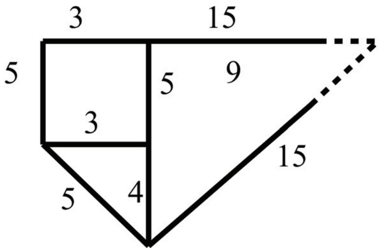
] … different
] What are the segments?
] … [
] … [
] … [
- - - - - - - - - - -

col. ii
trapezoid [
… [
… [
A different(?) determination of arouras. [
[The] example of the shape:



How one should do it. Make the 15 on the south by [itself; there results 225. And the 13] on the north by itself; there results 169. From 225; there is left behind [56. The 4 on] the west from the 8 on the east; there is left behind 4. A[pply to 56;]

there results 14. Bring it to the 4; there results 18. Half, [9, which is the west of the] plot
[to the south.]
From 14; there is left behind 5, which is the east of [the plot to the north.]
How one should find the upright base. [Make the 15 on the]
south by itself. There results 225. And the 9 on [the west of the triangle to the south]
by itself; there results 81. From 225; there is left behind 144. [Side, 12, which is the upright base.]
Similarly the 13 on the north by [itself; there results 169. And the 5]
on the east of the triangle to the north [by itself; there results 25. From 169;]
there is left behind 144. Side, 12, which is the upright base. … [
Surveying of the entire plot. [The sum]
of the two triangles to the s[outh and to the north, 84, from the]
[1]56; there is left behind 72. So many arouras does it have. [
… by(?) difference 84 arouras. There is left behind 12 … [
7(th?) method [
[Plo]t having $15\ \overset{15}{\underset{5}{\ }}\ 5$ … [
] What is the upright base? What are the segment[s?
] A different(?) determination of arouras. … [
The example of the [sha]pe:



[How one should do] it. Make the 15 on the e[ast by itself; there results 225. And the 5 on the]
[north by itse]lf; there results 25. Add; there results 2[50]. Similarly also [the 15 on the south]
by itself; there results 225. And the 5 on the west by [itself; there results 25. Add; there results 250.]
] … [
] of the triangle to the north [
] … How one should fi[nd the upright base.]
Make the 15 on] the south by itself; there results 225. A[nd the 12 on]
[the …] of the triangle [to the] east by itself; there results 1[44. From 225;]
[there is left behind 81. Side, 9, which is the upright ba]se. … [
] tr[iangle] to the south [
] … [
] … [
] … [
- - - - - - - - - - - -

## Abbreviations (see also apparatus)

*Directions*

| | |
|---|---|
| $\overset{o}{\beta}$ | βορρᾶς, βόρειος |
| $\overset{o}{\nu}$ | νότος, νότιος |
| $\alpha\pi\overset{\lambda}{\eta}$ | ἀπηλιώτης |
| $\underset{\iota}{\lambda}$ | λίψ |

*Mathematical*

| | |
|---|---|
| $\vert$ | γίνεται |
| $\overset{\llcorner}{\kappa}$ | καταλείπεται |
| $\breve{\varepsilon}$ | ἐμβαδόν |
| $\pi^{\lambda}$ | πλευρά |

## Commentary and Notes

### col. i 1–16: Problem 1

The problem concerns a trapezoidal field, i.e. a quadrilateral having one pair of opposite sides parallel, such that the two remaining sides are unequal and the internal angles that they make with the longer of the parallel sides are both acute; Heron calls such a figure τραπέζιον ὀξυγώνιον (*Metrica* 1.12). Linear dimensions are expressed throughout the papyrus as numbers without units, but since areas are explicitly stated to be arouras calculated on the assumption that 1 aroura equals 1 square linear unit, the linear units must implicitly be *schoenia*. The four sides of the field, though not orthogonal, are nominally designated by the cardinal directions. In the present instance, the east and west sides, respectively 4 and 46 *schoenia*, must be presumed to be parallel, while the north and south sides, respectively 20 and 34 *schoenia*, form acute interior angles with the longer of the parallel sides, so that they converge with *b* tending to the southeast and *d* to the northeast. The diagram in the papyrus, which followed the (lost) statement of the problem and precedes the method of solution, is broken away except for the numeral 20, which however suffices to show that it was oriented such that north was to the right. The restoration offered here, with west at the top, assumes that the diagram showed the field as seen from above, like a map; but see the introductions to Problems 2 and 3 below.

The object of Problem 1 is to find the area of the field, but this is accomplished in stages. The method can be set out in more general terms as follows.

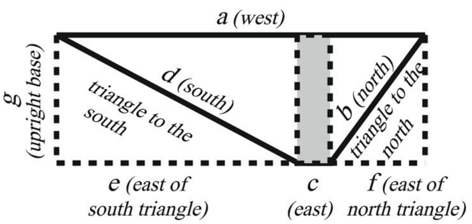


Fig. 1. Geometrical configuration of Problem 1

We imagine the completed rectangle consisting of the trapezoidal field and two neighboring right triangular plots on the north and south. The first step is to calculate the east sides of these triangles (*e* and *f*), which together with the trapezoid's east side, *c*, make up the east side of the rectangle. These may be found as:

(1) $$e = \frac{(a-c) + \frac{(d^2 - b^2)}{(a-c)}}{2}$$

(2) $$f = \frac{(a-c) - \frac{(d^2 - b^2)}{(a-c)}}{2} = e - \frac{(d^2 - b^2)}{(a-c)}$$

This algorithm works because, by the Pythagorean Theorem,[6]

(3) $$\begin{aligned} d^2 - b^2 &= (e^2 + g^2) - (f^2 + g^2) \\ &= e^2 - f^2 \\ &= (e+f)(e-f) \\ &= (a-c)(e-f) \end{aligned}$$

and we have the identities

(4) $$e = \frac{(e+f) + (e-f)}{2}$$

(5) $$f = \frac{(e+f) - (e-f)}{2}$$

Second, we find *g*, the “upright base,” by applying the Pythagorean Theorem to either of the right triangles:

(6) $$g = \sqrt{d^2 - e^2} = \sqrt{b^2 - f^2}$$

Now, to find the area of the trapezoid, one can proceed in either of two ways. One can calculate the areas of the two right triangles and subtract them from the area of the complete rectangle:

(7) $$A = ga - \left( \frac{ge}{2} + \frac{gf}{2} \right)$$

or, taking advantage of the fact that the trapezoid can be decomposed into two right triangles equal to the neighboring ones flanking a smaller rectangle (shaded in the figure above),

---

[6] Friberg prefers to designate the relation of the sides of a right triangle the “Diagonal Rule,” a more historically accurate if less recognizable name, since it was known already in Old Babylonian mathematics and outside of the “Euclidean” tradition of Greek geometry it is primarily an algorithm, not a proved theorem, applied to the diagonals of rectangles at least as often as to right triangles.

(8) $$A = gc + \left( \frac{ge}{2} + \frac{gf}{2} \right)$$

Despite the broken condition of this part of the text (lines 11–16) it is evident that the text gave both methods, first adding the combined area of the triangles (336 arouras) to the area of the small rectangle (64 arouras), and then subtracting the 336 arouras from the area of the complete rectangle (736 arouras), each time obtaining 400 arouras as the area of the trapezoid.

It is interesting to compare Heron's handling of this kind of quadrilateral in *Metrica* 1.12. In accordance with his normal approach in the *Metrica*, Heron first provides a geometrical "analysis" of the problem in which each step towards the solution is given a geometrical interpretation and justification, which he follows with a "synthesis" consisting of just the arithmetical operations on the given data, in the manner that one encounters in the mathematical papyri. Heron's algorithm for finding the area of the τραπέζιον ὀξυγώνιον begins with the calculation of $(a - c)$ and, from this, the "κάθετος" $g$. The procedure for calculating $g$ is not stated explicitly, but one is referred back to *Metrica* 1.5 for an algorithm that reflects a different line of reasoning from the one underlying the algorithm of P.Cornell inv. 69 and P.Ayer. Finally, Heron obtains the area of the figure as

(9) $$A = g\left( \frac{a + c}{2} \right)$$

which is of course much more direct though less intuitively obvious than the procedure of the papyri.

1 Although the numeral in the diagram representing the length of the field's north side is almost completely preserved, none of the corresponding line can be seen above or below it; this side must have been drawn as nearly vertical, or the numeral was written well to the right of it.

2–3 $d = 34$, $b = 20$, $d^2 = 1156$, $b^2 = 400$, $d^2 - b^2 = 756$.

2 (γίνεται): represented here and throughout by a simple vertical stroke, a much less common abbreviation in mathematical and astronomical papyri than the crossed gamma symbol. Resolved in *ed. pr.* as (γίνονται), but the singular is often attested in such contexts.

The elevated horizontal stroke intended to mark ͵αρνϛ̄ as a numeral is sloppily extended over the following letters. In general, the numerals in the papyrus are typically marked by a sinuous or straight horizontal stroke either over or above and to the right of the last letter, but the stroke sometimes begins earlier or is entirely omitted.

3–4 $a = 46$, $c = 4$, $a - c = 42$, $(d^2 - b^2)/(a - c) = 18$.

5 $(a - c) + 18 = 60$, $e = 60/2 = 30$.

6 An independent computation of $f$ would parallel the computation of $e$: $(a - c) - 18 = 24$,
$f = 24/2 = 12$. Instead the text uses the fact that $18 = (e - f)$, so that $f = e - 18 = 12$.
ὡ̣ς δεῖ̣ ε̣ὑ̣ρεῖ̣ν̣: ως δὲ ῡ̣ ,αρνϛ *ed. pr.*

7–9 $d^2 = 1156$ (repeated from line 2), $e^2 = 900$, $g^2 = d^2 - e^2 = 256$, $g = 16$.

8 σνϛ: no horizontal stroke over the numerals as reported in *ed. pr.*

9–9a The scribe accidentally skipped from one ἐν τῶι to the next. The words ἀπηλ(ιώτηι) τοῦ, which prematurely follow the first ἐν τῶι in line 9, have been circled, and the first words of the correct continuation are written in the intercolumnium to the right, in a hand of different appearance though ἑαυτὰ is spelled ἑατὰ as in the main text. The remainder of the skipped text was presumably inserted interlinearly.

9a–10 $b^2 = 400$ (cf. line 3), $f^2 = 144$, $g^2 = b^2 - f^2 = 256$, $g = 16$.

10 ὀρθὴ βάσις: I have not found this exact expression for the altitude of the figure in other mathematical texts. The counterpart expression in P.Ayer is κάθετος.

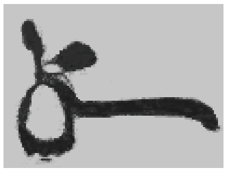
11 The statement of the areas of the two right triangles and their sum ought to have been in this line. The calculation would have been:

south triangle = (*eg*)/2 = 240 arouras,
north triangle = (*fg*)/2 = 96 arouras,
sum of triangles = 336 arouras.

Perhaps one should restore [συναγ]ο̣μ̣ένων, “added together,” but the syntax appears awkward.

12 This is part of the second method of computing the area of the field (8). The four sides of the small rectangle with sides *c* and *g* are indicated by writing them in a cruciform arrangement around what seems to be a small square, a notation I have not seen in other papyri. Cf. ii 23. *cg* = 64; thus the area of this rectangle is 64 arouras.

13 ἐ(μβαδὸν): β̣ά̣(ϊα) *ed. pr.*, i.e. reading the abbreviation interpreted here as καταλείπεται. If correct, this would make the area 400 arouras the result of a subtraction, as in the first method of computation (7), whereas line 12 is evidently using the second method. The symbol appears to me to be an epsilon, not a kappa, with a superscribed stroke.



Hence the operation in this line was probably:

area of field = 336 arouras + 64 arouras = 400 arouras.

16 736 arouras is *ag*, the area of the complete rectangle composed of the field and its two neighboring triangles. The area of the field was likely restated in a lost short line immediately following, as the result of subtracting the areas of the triangles from 736:

area of field = 736 arouras – 336 arouras = 400 arouras.

### col. i 17–27

The remainder of column i is too broken to make much sense of. After a space that presumably accommodated a lost diagram, lines 18–23 apparently discuss a configuration similar to that of Problem 1, if it is not in fact the same configuration, since an area of 336 arouras is mentioned. Then there is a smaller space, perhaps corresponding to a section break rather than a diagram, and three lines of text (25–27) that are too fragmentary to interpret.

18a–19 The restoration [τετρ]άγωνον of *ed. pr.* in 19 (“quadrangle/quadrangular,” or more specifically, “square”) seems quite plausible, though it is not clear just what is going on in the lines following the presumed diagram in 17. The interlinear text appears to duplicate this text, perhaps restoring a passage accidentally omitted through eyeskip. The intention of the elevated horizontal stroke at the end of 18a is not clear.

22 336 arouras was the area of the two right triangles in Problem 1. The present set of lines presumably referred to a different diagram, but the geometrical configuration must have been similar to that of Problem 1 for this area to arise again.

23 A paragraphos appears below this line.

26 Cf. ii 24. τμήματα might mean geometrically defined subdivisions of a field.

27 *Ed. pr.* reads ]ν̣ι π̣ερι ̣ for this line, but the stroke or strokes read there as π form an inverted U high above the line, more likely the top of a numeral letter (α or δ?) and the descending curved superstroke used elsewhere as a mark of a numeral or abbreviation.

**col. ii 1–21: Problem 2**

This is similar to problem 1, except that one of the oblique sides of the field makes an obtuse interior angle with the longer of the two parallel sides; Heron (*Metrica* 1.13) calls this figure τραπέζιον ἀμβλυγώνιον. The drawing is again oriented with north to the right, but it is curiously mirror-reversed in the vertical dimension so that east is at the top, in the direction counterclockwise from north, as if the field was seen from below. A generalized (and conventionally oriented) version of the diagram is as follows:

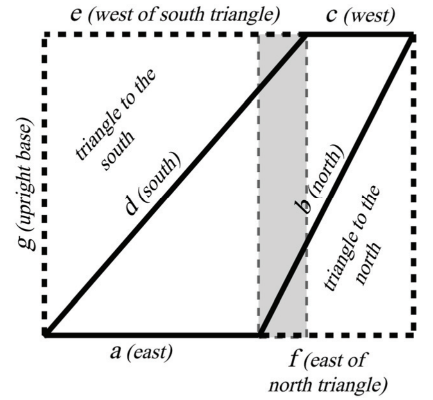


Fig. 2. Geometrical configuration of Problem 2

The procedure for finding the field's area is practically identical to that of Problem 1; with the diagram labelled as above, (3) through (7) remain valid, except that the last step of (3) is different:

(3a)
$$\begin{aligned} d^2 - b^2 &= (e^2 + g^2) - (f^2 + g^2) \\ &= e^2 - f^2 \\ &= (e+f)(e-f) \\ &= (e+f)(a-c) \end{aligned}$$

and thus while the algorithm (1) works with this configuration, the counterpart of (2) is:

(2a)
$$f = \frac{\frac{(d^2 - b^2)}{(a-c)} - (a-c)}{2} = \frac{(d^2 - b^2)}{(a-c)} - e$$

The alternative final derivation of the field's area (8) is no longer applicable without adjustment. The field cannot be decomposed into two right triangles equal to the two neighboring triangles *plus* a small rectangle, but the field's area can be calculated as the sum of the areas of the two triangles *minus* the area of a small rectangle:

(10)
$$A = \left(\frac{ge}{2} + \frac{gf}{2}\right) - g(e-a)$$

It is unclear whether the text employed this second method. Before using it, one has to know that no parts of the parallel sides of the field face directly opposite each other, which, if not a given fact at the outset, becomes evident when $e$ is found to be greater than $a$.

Heron's treatment of this figure in *Metrica* 1.13 follows the same lines as for the τραπέζιον ὀξυγώνιον in 1.12.

1 The line is written in ekthesis, probably indicating the beginning of the statement of a new problem.

4 διαφόρου ἀρουρισμός (cf. also ii 21 and 25): the sense of διαφόρου/διαφόρωι and how it connects with ἀρουρισμός is unclear.

5 σχήματος ὑ̣π̣όδιγμ[α] *ed. pr.*, but the gap in the papyrus here is much too wide for the trace left of υ to belong to the final letter of σχήματος. ὑπόδιγμα presumably refers to the actual diagram, and σχῆμα to the configuration that it portrays.

6 Well to the right of the bottom of the diagram (roughly on a level with the ιε), and close to the edge of the papyrus, is a vertical stroke, about the height of an iota. It is not clear whether this was text, and if so, whether it was related to the diagram.

7 The line is written in ekthesis.

7–8 $d = 15$, $b = 13$, $d^2 = 225$, $b^2 = 169$, $d^2 - b^2 = 56$.

8–10 $a = 8$, $c = 4$, $a - c = 4$, $(d^2 - b^2)/(a - c) = 14$.

9 η̣̅: ι̣̅γ̅ *ed. pr.*

10–11 $14 + (a - c) = 18$, $e = 18/2 = 9$, $f = 14 - (a - c) = 5$.

10 τὸ (ἥμισυ): ο̣ὖ̣ τὸ (ἥμ̣ι̣σ̣υ̣) *ed. pr.*, but I can see no trace of ο̣ὖ̣ or indeed space for it.

12–14 $d^2 = 225$ (repeated from line 7), $e^2 = 81$, $g^2 = d^2 - e^2 = 144$, $g = 12$.

15–17 $b^2 = 169$ (repeated from 7–8), $f^2 = 25$, $g^2 = b^2 - f^2 = 144$, $g = 12$.

18–20 The calculations of the areas of the triangles and the complete rectangle are not given explicitly:

south triangle = $(eg)/2$ = 54 arouras,
north triangle = $(fg)/2$ = 30 arouras,
sum of triangles = 84 arouras;
area of complete rectangle = $(a + f)g$ = 156 arouras. Hence
area of field = 156 arouras – 84 arouras = 72 arouras.

21 ] διαφόρου ἀρουρισμοῦ ν . . . [ *ed. pr.* The figure 84 arouras, which is the sum of the areas of the two right neighboring right triangles, is stated in the genitive case, and then 12, evidently arouras, is given as the result of a subtraction. Thus it seems that the area of the field, 72 arouras, has been subtracted from the area of the two triangles. According to (9) this yields the area of the small shaded rectangle in the diagram above, but it is not clear why one would want to find the area of the small rectangle *from* the field's area rather than the other way around.

## col. ii 22–40: Problem 3

This diagram for this problem shows a kite-shaped quadrilateral having two pairs of equal adjacent sides and two right angles; moreover, the quadrilateral is decomposed into two right-angled triangles and a rectangle. A generalized version of the configuration is as follows, with $a = b$ and $c = d$:

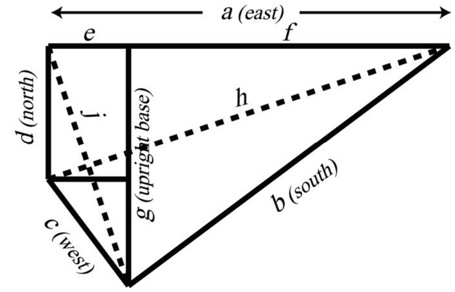


Fig. 3. Geometrical configuration of Problem 3

Determining the area of a quadrilateral is trivial if it is given that two opposite angles are right (whether or not it also has pairs of equal adjacent sides as in the present problem), since the figure can be decomposed into two right-angled triangles by its diagonal *h*, and the combined area of these triangles is obviously equal to a rectangle with sides *a* and *d*, i.e. *ad*. Despite the poor state of preservation of the text, we can see that the treatment of the figure was considerably more complicated, involving finding its "upright base," i.e. *g*, which is the maximum dimension of the field perpendicular to side *a*, and probably also the areas of the triangular and rectangular subdivisions that are delineated in the diagram.

It therefore seems likely that the problem only asserted that one of the angles (say the one at the northeast corner) was right. The first step in solving this more general problem would be to determine whether the opposite angle is right, by checking whether $b^2 + c^2 = a^2 + d^2$, in which case the area is simply the sum of the areas of the two right triangles composing the quadrilateral. If the two sums of squares turn out not to be equal, one would have to employ a more complicated algorithm, such as the one Heron presents in *Metrica* 1.14. Problem 3 of the papyrus thus was probably followed by a problem dealing with a similar figure but with dimensions such that the angle opposite the given right angle was not right.

The "upright base" *g* is apparently found in lines ii 33–36 by means of the Pythagorean Theorem as the square root of the difference between the squares of *b* and *f*, so that by this point *f* and *e* have been determined.[7] How this was done is not clear. In lines ii 28–30, the Pythagorean Theorem has been applied to find the square of the diagonal *h* (as the sum of $a^2$ and $d^2$ as well as of $b^2$ and $c^2$). One possible reconstruction of the intervening stages is that the other diagonal *j* was then found by some relation such as

$$(11) \qquad h^2 j^2 = 4a^2 c^2$$

which follows from Ptolemy's Theorem since a quadrilateral with two opposite right angles must be cyclic. Then a procedure similar to that used in Problems 1 and 2 can be used to find *e* and *f*:

$$(12) \qquad e = \frac{a - \frac{\left(h^2 - j^2\right)}{a}}{2}$$

---

[7] Heron, *Metrica* 1.15, also gives an algorithm for finding this length (which he again calls κάθετος), but his procedure was certainly not the same as the one of the papyrus since his final result is obtained as the sum of two component line segments. The diagram for *Metrica* 1.15 (*Seragliensis* GI 1, f. 75v) has the same lines dividing the quadrilateral into two right triangles and a rectangle as the diagram of our papyrus, though their function is to determine certain points and line segments involved in deriving the κάθετος, not to partition the quadrilateral into constituent areas.

(13) $$f = \frac{a + \frac{\left(h^2 - j^2\right)}{a}}{2}$$

If this approach was used, it must have been expressed very compendiously so as to fit in two text lines.

22 The horizontal stroke may be a paragraphos marking the beginning of the new problem. Before the ζ is a large speck of ink, perhaps just an accidental block. If this line is a heading, as it appears to be, it suggests an attempt at a systematic structure for the manuscript as a whole. A further indication that there existed a conventional order for presenting the various quadrilaterals is that in Heron's *Metrica* the quadrilateral with one right angle but no parallel sides is treated in 1.14, immediately after the two kinds of τραπέζια corresponding to Problems 1 and 2 in the papyrus.

23 Cf. i 12. Here there is no small square surrounded by the numerals.

28–29 $a^2$ = 225, $d^2$ = 25, hence $h^2$ = 250. This is not a perfect square, and presumably whatever operations followed did not require taking its square root.

29–30 $b^2$ = 225, $c^2$ = 25, hence again $h^2$ = 250.

33–36 $b^2$ = 225 (repeated from line 29), $f^2$ = 144, hence $g^2$ = 81, $g$ = 9.

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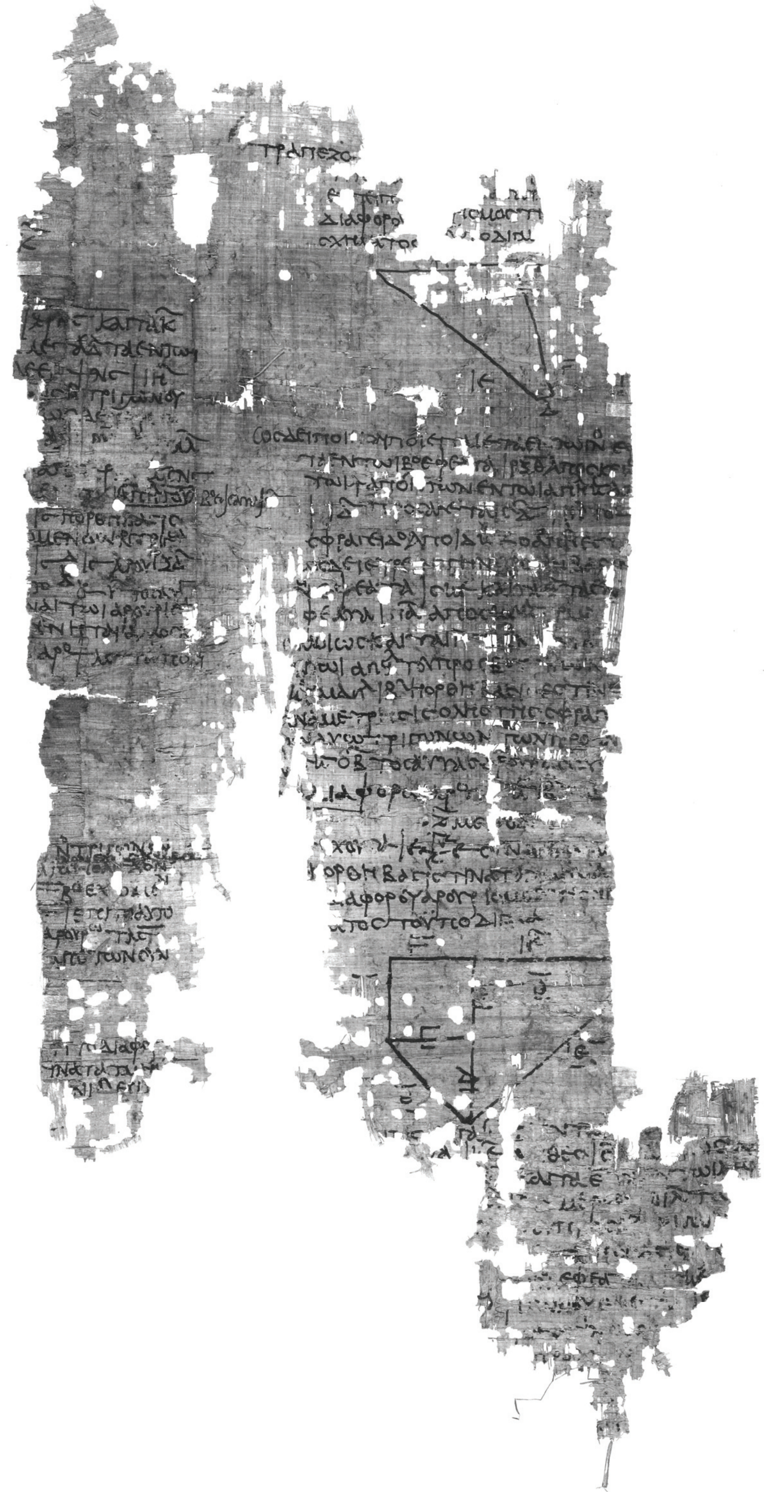
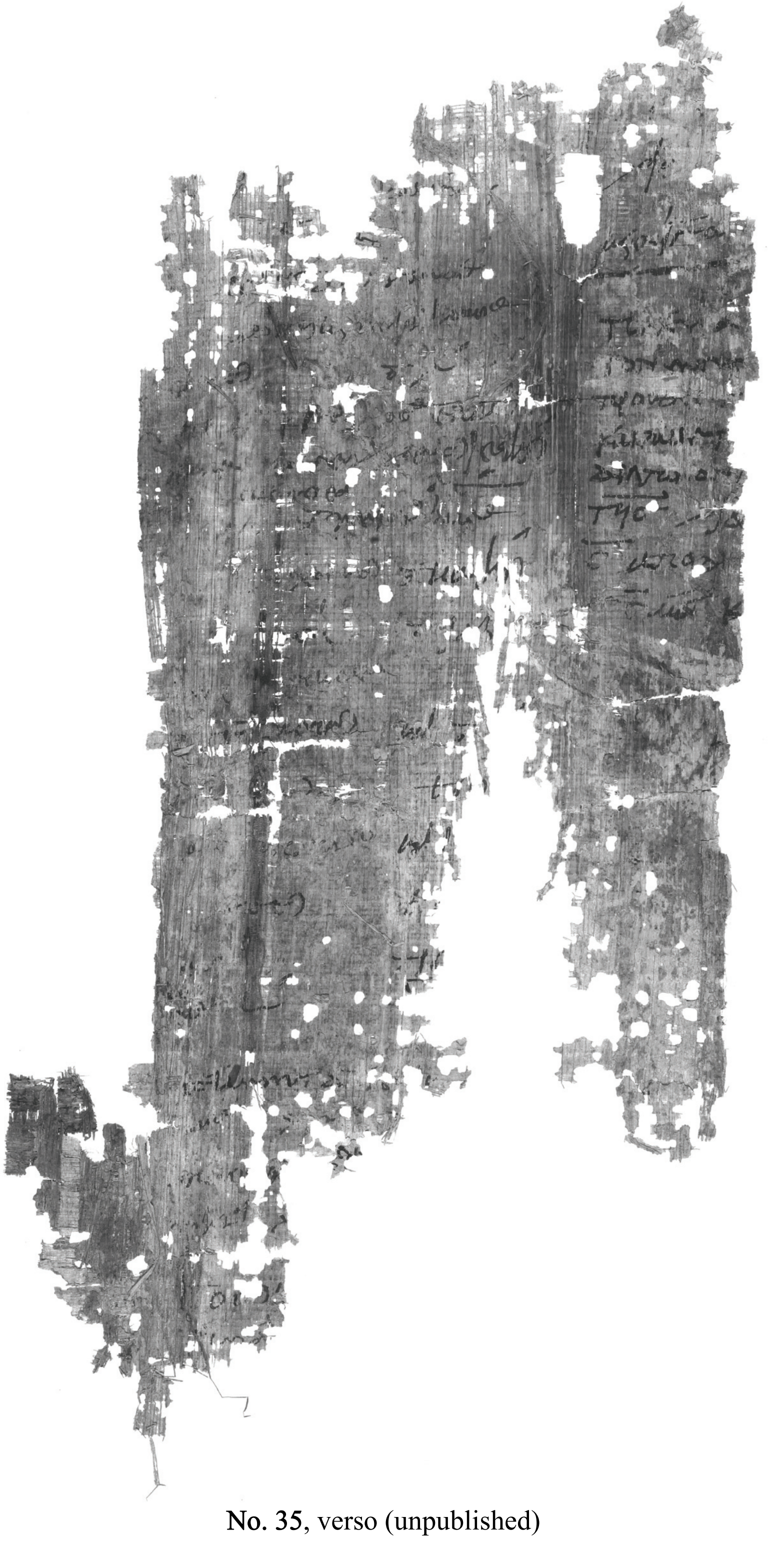


No. 35, recto



No. 35, verso (unpublished)