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PERIPATETIC AND EUCLIDEAN THEORIES OF THE VISUAL RAY

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SUMMARY — The visual ray of Euclid's *Optica* is endowed with properties that reveal the concept to be an abstraction of a specific physical account of vision. The evolution of a physical theory of vision compatible with the Euclidean model can be traced in Peripatetic writings of the late fourth and third centuries B.C.

Euclid's *Optica* stands at the head of a tradition of geometrical optics, the fundamental assumption of which was that sight is effected by means of rectilinear visual rays (ὄψεις or ἀκτῖνες), radiating from the eye. Looking beyond the overtly mathematical style of argumentation that characterizes the work, recent studies have concentrated on the kinds of propositions that Euclid undertakes to prove, arguing (I think rightly) that the *Optica* is essentially a study in visual perception and the relation between objects and their subjective appearances.[1] By contrast, the origins and motivation of Euclid's concept of the visual ray have received scarce attention.

As early as the second century of our era Alexander of Aphrodisias associated the hypotheses of Euclidean optics with «the mathematicians». Expanding on Aristotle (*De sensu*, 438a25), Alexander distinguishes these people, who believe that sight is effected by means of a cone of

* I am grateful to the participants of the 1990 Dibner Institute Conference, my colleagues at the University of Toronto, and the referees of this journal, for many suggestions and criticisms.

[1] K. ANDERSEN [1987], G. SIMON [1988].

visual rays emanating from the eye to the object, from those who follow Plato in believing that light from the eye coalesces with light from the object to form a single body of light imparting vision to the eye.[2] Alexander's assault on the mathematicians' position begins by arguing that, according to physical principles, the emission from the eye that they posit would have to be corporeal in order to move as they believe it does; hence Alexander thought that the mathematicians were not already committed to a corporeal interpretation, or perhaps to any physical interpretation, of their visual rays.

This 'instrumentalist' reading of geometrical optics is expressed still more explicitly by a short anonymous text that modern scholarship assigns, on rather shaky evidence, to Geminus (c. 50 A.D.).[3] According to this writer,

> Optics neither treats physical nature [φυσιολογεῖ] nor seeks to know whether certain emanations travel to the boundaries of bodies with rays [ἀκτίνων] flowing from the eyes [ὄψεων], or whether images emanate from the objects of perception and enter the eyes after travelling in a straight line, or whether the intervening air is stretched out or moved along with the luminous *pneuma* of the eye. It only investigates whether in each hypothesis the rectilinearity of the motion or extension and the convergence [of the rays] into an angle where they come together is preserved whenever there is an eye's beholding of greater or lesser things.

And in truth one can read parts of Euclid's *Optica* without gaining a clear notion of what kind of physical existence, if any, Euclid assumed for his visual rays. This is true in particular of those propositions that concern what one might loosely call problems of perspective, that is, deductions of the appearance of objects based purely on the directions of lines of sight from eye to object; and modern commentators who have regarded 'perspective' as the chief concern of Euclidean optics have also tended to impute a high degree of mathematical abstraction to Euclid's visual rays.[4] According to this reading, the *Optica* was not so much an attempt to apply geometrical argumentation to certain elements abstracted from a physical theory of visual perception as an *alternative*

[2] *Commentaria in Aristotelem Graeca*, Berlin, 1882 (hereinafter referred to as 'CAG'), III, pt. 1, pp. 27-28.

[3] R. Schöne [1897], p. 24.

[4] E.g. C. D. Brownson [1981], esp. p. 169 and note 10; O. Pedersen [1993], pp. 112-113. Examples could be multiplied.

to existing physical theories. But will this interpretation stand up to closer scrutiny of the *Optica* and contemporary writings on related aspects of natural philosophy?

The Two Texts of Euclid's *Optica*

In the first place one ought to ask which version of the *Optica* we mean, since in 1879 Heiberg found the first of several manuscript copies of a text that differs substantially from the *Optica* as it was hitherto known to scholarship. Heiberg came swiftly to the conclusion that the newly discovered version was the 'genuine' text of the *Optica*, whereas the better known version represented a systematic revision made in the fourth century of our era by Theon of Alexandria.[5] Heiberg's hypotheses have since passed for established fact, with the consequence that the so-called 'Theonine recension' is almost never read.[6]

In reviewing Heiberg's line of argument, I will adopt a conveniently neutral nomenclature suggested by Knorr. I refer to the text that Heiberg ascribed to Theon, which is found in numerous manuscripts of the so-called 'Little Astronomy' (a corpus of treatises relevant to elementary astronomy, e.g. spherics), as version **B**. Heiberg's 'genuine' text, known from a half dozen Greek manuscripts and a twelfth-century Latin translation, I call version **A**.[3] Heiberg's first contention, then, was that version **A**, although not free from errors and interpolations, is the essentially faithful transmitter of Euclid's book, and that **B** is a revision of **A**. The argument rests on the claim that the proofs in **A** are in general more complete and clear, and Heiberg cites eight specific passages where **A** has a phrase that is lacking or less precisely expressed in **B**, and that Heiberg regards as important for the sense.[8] Heiberg assumes that these instances prove that **B** is a careless paraphrase of **A**; but in most

[5] J. L. Heiberg [1882], pp. 90-153.

[6] So far as I know, no doubts were ever raised about Heiberg's assessment of the two versions of the *Optica* between 1882 and 1990, when I discussed some parts of the present investigation at the Dibner Institute conference. More recently W. R. Knorr has come by different reasoning to conclusions similar to my own: see W. R. Knorr [1991], pp. 194-195 and note 7, and his article in this volume.

[7] The proposition numbers of the two versions will be cited following Heiberg's later Teubner edition, which differs in some respects from the numeration employed in J. L. Heiberg [1882].

[8] J. L. Heiberg [1882], pp. 129-130.

or all the cases one could maintain with at least equal plausibility that the fuller text in **A** is an editor's attempt to clarify perceived ambiguities in **B**. A telling instance is proposition 30, concerning the appearance of a cone seen by one eye, where **A** alone makes it explicit that the cone is a right cone. As Heiberg himself notes, the definition of a cone in *Elementa*, XI def. 18 only admits of right cones, so that the qualification of the cone in *Optica*, 30 likely was interpolated in **A** at a later date, to take account of the more general definition of the cone popularized by Apollonius's *Conica*.

According to Heiberg, the greater clarity of **A**, demonstrated by these eight examples, not only proved that **A** was the more sincere text, but also freed the *Optica* from the charge that its proofs were too filled with errors and irregularities to be by the same author as the *Elementa*. One might well ask whether Euclid should be expected to have applied the same style and standards of demonstration in two very disparate subject fields that had moreover developed to quite different degrees of maturity by the early third century B.C. But the same arguments, good or bad, that impugn the authenticity of version **B** can also be levelled against **A**. Not only do both versions A and B share numerous strange and 'illogical' features (e.g. the very odd proposition **A** 57 = **B** 56), but we shall see presently that there are passages of greater importance than any cited by Heiberg where **A** presents a more senseless text than **B**.

Having identified **B** as the revised text, Heiberg next attributed the revisions to Theon of Alexandria on the following grounds.[9] Version **B** is preceded by an anonymous preface summarizing part of a lecture (or, less likely, a written text). The lecturer, also unnamed, justifies the hypotheses of the *Optica* on the basis of everyday phenomena, and vigorously attacks the Epicurean theory that vision is effected by means of images (εἴδωλα) given off by the object. In a single late manuscript (*Par. gr.* 2468), the copyist Angelus Vergetius (mid sixteenth century) wrote that «the preface is from the instruction of Theon». But as Heiberg admits, this note, lacking further support, was probably a mere guess, and the further assumption that the lecturer was also responsible for the state of the *Optica*'s text is pure speculation. Heiberg then attempts to narrow down the date of the preface. The Christian writer Nemesius (c. 400) apparently knew it; but its style, according to Heiberg, is late, hence possible for Theon – though he does not give specific grounds for this judgement or show why the preface cannot have been written, say, in the third or second century rather than the fourth. And, Heiberg goes on, we know that Theon was both a teacher and an editor of other writings (but there were many other teachers of mathematics in late antiquity, and Theon's only two authenticated 'editions', of Euclid's *Elementa* and *Data*, show no such extensive deviations as we find between the two versions of the *Optica*).[10]

On these weak arguments hang all Heiberg's hypotheses about texts **A** and **B**. Obviously the true relationship between the two versions will only emerge in all its complexities from a thorough collation of the entire work.[11] For the present it will suffice, however, to give grounds for regarding **A** as the more authoritative text for the greater part of the *Optica*, including passages that are pertinent to Euclid's conception of the visual ray.

One of the most conspicuous ways in which **A** and **B** diverge is in the lettering of the points, lines, and angles in the propositions and figures. The normal procedure in Greek mathematics, at least at later periods, is to follow the sequence of the alphabet. Throughout **B**, however, the lettering follows an irregular pattern, usually giving priority to the letters ΒΓΔΖΚΛ and tending to avoid the vowels ΑΕ. Most of the propositions in **A**, whether or not they use essentially the same figures as **B**, follow the customary alphabetic order; but in several propositions the lettering in **B** is the same as in **A**.[12] It is hardly plausible (and Heiberg made no such claim) that these nonstandard letterings entered the **B** text by contamination from **A**, especially since some of the propositions in question (e.g. **A** 50-51 = **B** 49-50) are otherwise significantly different in **A** and **B**. Rather, they are the relics of the original author's idiosyncratic lettering, which an editor has elsewhere replaced in **A** with the familiar alphabetic sequence while otherwise revising or rewriting the text.[13]

[9] J. L. Heiberg [1882], pp. 139-147.

[10] In the prolegomena of his 1895 Teubner edition of the *Optica* (p. xxx) Heiberg conceded that the citations of the *Optica* in Theon's commentary to Ptolemy's *Almagest* follow the **B** text; but by then his preference for **A** was unshakable.

[11] I hope to do this elsewhere.

[12] The propositions with the same lettering include 1-3, 9, 22, **A** 45 = **B** 46, 47-48, **A** 50-54 = **B** 49-53, **A** 57 = **B** 56. Some trivial variants, e.g. Α for Λ, are probably due to scribal errors.

[13] I do not know any parallel to the lettering scheme of **B** in the Euclidean corpus or elsewhere in Greek mathematics.

Another method of identifying which of two obviously related mathematical texts is derived from the other is to find instances where the divergence can only be explained as the result of editorial misunderstanding and conjecture, sometimes inspired by a defective text.[14] Of the two illustrations that I will give here, the first is of fundamental importance: Euclid's definition (def. 1) of the visual ray itself. In order to show the nature and cause of the variant, I quote the Greek text of the two versions:

A	B
ά. ὑποκείσθω τὰς ἀπὸ τοῦ ὄμματος ἐξαγομένας εὐθείας γραμμὰς φέρεσθαι διάστημα μεγεθῶν μεγάλων	ά. ὑποκείσθω τὰς ἀπὸ τοῦ ὄμματος ὄψει̧ κατὰ εὐθείας γραμμὰς φέρεσθαι διάστημά τι ποιούσας ἀπ' ἀλλήλων.
(1) Let it be postulated that the straight lines drawn out from the eye travel an interval of great magnitudes;	(1) Let it be postulated that the visual rays from the eye travel along straight lines, while making some interval one from another;

Both versions

(2) and that the figure contained by the visual rays is a cone with its vertex at the eye and its base at the limits of the object of vision;

(3) and that those things are seen upon which the visual rays fall, and those are not seen upon which the visual rays do not fall.

Comparison of the Greek reveals what is less apparent in translation, that the two versions alternate between literally identical and entirely divergent (here underlined) words or parts of words, a strange situation that strongly suggests that at least one version is an attempt to restore by conjecture a mutilated text. The reader presented with **A** is told simply that the «straight lines» that are drawn outward from the eye travel «an interval of great magnitudes», a rather awkward expression that presumably means 'very far'. It seems odd that the text does not speak of ὄψεις or ἀκτῖνες the terms for visual rays employed throughout the rest of the book, but of straight lines. Such language

[14] I have illustrated this method in analysing the mutual borrowings of several writers on mirror optics; see A. JONES [1987].

might seem to support a reading of the *Optica* as an abstract mathematical treatment of visual phenomena that did not presuppose a well-defined underlying physical interpretation of what the visual rays are supposed to be (e.g. bodies, boundaries of bodies, or properties of the medium between eye and object). And the «interval of great magnitudes», although of course implicit in the fact that we can see distant objects such as the stars, has no demonstrative value in the body of the *Optica*, where all distances are relative anyway.

The first definition in **B**, on the other hand, gives us two properties of the visual ray (here so named) that not only are usable in geometrical argument but are actually hypotheses essential to the propositions of the *Optica*: visual rays travel *along* straight lines (so that straight lines can be used to represent them in the proofs), and they have «intervals» between them. In this context «interval» (διάστημα) obviously has its usual meaning of 'gap' rather than the trivial sense of 'geometrical interval', i.e. angle. The assumption that there are spaces between the visual rays is fundamental to many propositions of the *Optica*; in the very first proposition (both versions) it is invoked in words closely paralleling **B**'s def. 1 (ἐπεὶ ἐν διαστήματι φέρονται αἱ [...] ὄψεις). The reader of **A** is brought to a halt, wondering why the rays, which he was led to believe were the straight lines drawn from the eye, do not possess the continuity properties of straight lines in space. **B** thus manifestly preserves the appropriate definition of the behaviour of the visual ray.

As a second example of misunderstanding on the part of the editor of **A**, I quote two propositions from the section concerning appearances of motion:

A	B
(Prop. 54, cf. Fig. 1) Of [objects] travelling with equal speed, those that are farther away appear to travel more slowly.	(Prop. 53, cf. Fig. 1) Of [objects] travelling with equal speed, those that are farther away appear to travel more slowly.
For let B and K travel with equal speed, and from the eye, A, let rays AG, AD, and AZ be drawn. Then B has greater rays drawn from the eye than has K. Hence [B] will cross a greater interval and, having traversed ray AZ later, it will appear to travel more slowly.	For let B and K travel with equal speed in the direction of Z, and from the eye, A, let rays AG, AD, and AZ be drawn. Then K has smaller rays drawn from the eye, A, than has B. Hence [K] will cross a smaller interval and, having traversed ray AZ first, it will appear to travel faster.

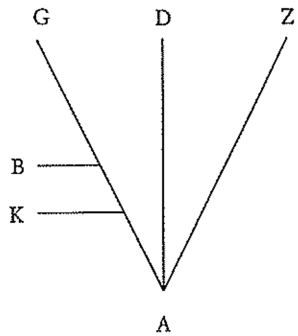

Fig. 1

(Prop. 55, cf. Fig. 2)
With the eye standing still and the rays moving sideways, the further away of the object of vision will appear to be left behind.

For let A and G be the objects of vision, and let them be upon straight lines AB and GD, and let E be the eye, and let rays EG, ED, EA, EB fall from it upon [the objects of vision]. I say that the [object] at A will appear to be left behind. Let ED be produced to where it meets AB, and let it be EB. Then since angle GEB is greater than angle AEB, interval GD will therefore seem to be greater than AB. Thus with the eye standing still at E, the rays, as they move sideways in the direction of A and G, will cross A faster than G. Hence AB will appear to be left behind.

(Prop. 54, cf. Fig. 3)
With the eye moving sideways, the further away of the objects of vision will appear to be left behind.

For let B be the eye, and let rays BG, BD, and BZ be drawn from it, and let K and L be objects of vision. Then as the eye moves sideways in the direction of G, the rays will move across K faster than L. Hence K will appear to be left behind, and L [will appear] to move in the opposite direction, that is in the direction of Z.

The proof of the first of this pair of propositions is clear and satisfactory in both versions: the fan of rays through which the nearer object must pass is shorter (and hence narrower), so that it is seen by all the rays in less time than the further object. For the second

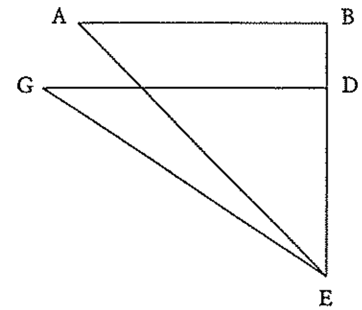

Fig. 2

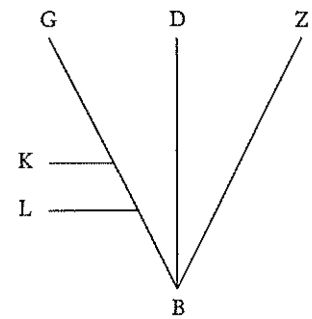

Fig. 3

proposition we should expect an analogous proof, and version **B** provides it, but with a trivial error: in the statement «the rays will move across K faster than L» the names of K and L should be exchanged. The editor of version **A** rewrote the proposition because he was misled by this mistake into thinking that the fan of rays was supposed to be *revolving* about a stationary eye (in which case it would indeed traverse the further object at a greater velocity, although of course still in the same time). It did not occur to him that the phenomenon he was now setting out to prove is spurious.

Euclid's use of the fan of rays in these propositions is consistent with the implication of the authentic first definition in **B**: his visual rays are not geometrical straight lines, as the mutilated version of def.

1 in A implied, but travel along straight lines. The straight lines of the figures and proofs, in other words, are to be understood as abstractions of some kind of physical entity, perhaps an actual body emitted from the eye. Although the *Optica* nowhere tells us just what the rays are, we can at least list various properties of the rays invoked in the definitions and propositions. According to the first two definitions they originate from the eye; they travel along straight lines; there are gaps between them; when we see an object, the visual rays form a «cone» with the eye as vertex and the object as base; and vision of an object occurs if and only if a visual ray falls upon the object. Nevertheless (prop. 1) we may believe that we see the whole surface of a continuous object even though the rays fall only on discrete points of it; this is because the visual rays are moving quickly from side to side (ταχὺ παραφερομένων). This sideways movement is apparently only consequent on turning the eye or head, however, since the later propositions concerning perception of transverse motion quoted above assume that the fan of visual rays remains stationary relative to the eye while objects cross through it. The gaps between rays suffice to explain why more distant objects are seen less clearly or not seen at all (prop. 2-3), so that a single visual ray perhaps is able to 'see' equally well at any distance. The 'perspective' propositions, at any rate, presume that the ray does not convey to the eye any information about an object's distance.

We need not assume that all the properties that Euclid would have attributed to the visual ray must make an appearance in the abstracted treatment of the *Optica*: the conspicuous omission of any reference to colour, for example, means no more than that geometrical analysis seemed to have nothing to offer this aspect of the theory of appearances.

Visual Rays in the Aristotelean Corpus

It will be my purpose in the remainder of this paper to show that not merely the general concept of a visual ray emitted from the eye, but also a kind of visual ray compatible with Euclid's postulates, can be traced in the Peripatetic natural philosophy of the generations preceding and contemporary with Euclid, that is, the late fourth and third centuries B.C. These ideas may be traced further back to Presocratic roots (in particular, Empedocles and the *Timaeus*), but the relevant fragments and *testimonia* are perilously few and disputable.[15] We have some evidence that even before Empedocles and Plato the Pythagoreans suggested an optical emanation (Aetius, IV, 14, 3). The Platonic doctrine in the *Timaeus* gave an intelligible function to the visual ray as a rigid «stick» by which one can «feel» distant objects;[16] and in Peripatetic texts, the «stick» came to be attenuated to a narrow rectilinear rod that could be treated as a straight line.

Around the first decades of the third century, Strato of Lampsacus advocated a Peripatetic conception of matter and physical change that incorporated three concepts immediately relevant to physical optics. First, material bodies are composed of imperceptibly small particles (not indivisible atoms) separated by comparably small voids. Secondly, a consequence of these tiny voids is the presence of tiny channels or 'pores' through bodies. And thirdly, sight and light are corporeal entities radiated respectively through pores at the surface of the eye and the luminous body like straight narrow rods. If Euclid's visual rays were not simply mathematical models operating at a phenomenological level but abstractions of some physical theory of vision, the physical hypotheses that provide the most satisfactory motivation for their specific properties are Strato's.

Although the fusion of these principles into a reasonably consistent physics seems to have been Strato's work, the concepts of void, microscopic porosity, and sight by means of corporeal extromission from the eye were already familiar to Aristotle. Aristotle's arguments against the existence of void, whether external or internal to bodies, are too well known to need repetition here (*Physica*, IV, 213b14-22, 214a32-b11, 216b22-217b20). Porosity as a means of physical change is rejected in *De generatione et corruptione* (324b25-326b28), where Aristotle specifically addresses Empedocles's belief that bodies «possess

[15] Cf. J. I. Beare [1906], pp. 9-92.

[16] It is unclear how or where Plato intends this visual ray to interact with the colours of bodies (described as «a flame flowing from all bodies, having parts fitted [σύμμετρα] to sight [ὄψει] for sensation», *Timaeus*, 67C).

pores that cannot be seen because they are so small, but are closely spaced and lined up in rows» (324b30), and that these pores were the reason for transparency, miscibility, and qualitative change in bodies. Aristotle points out that the pores would have to be void to serve any such purpose (which contradicts Empedocles's assumption of continuous plenum), and moreover, accepting this corollary, that there is no good reason to postulate tiny voids and yet deny the existence of bigger ones.

Aristotle takes up the nature of sight in *De anima*, II (418a26-419a25) within the context of sensation as a function of the soul. Sight primarily receives the colours of objects by a process in which colour is a quality taken on from the objects by a medium whose transparency (defined as the potentiality to take on colour in this way) is actualized by the presence of light. Other formal qualities of a body (the «common sensibles», including shape, size, movement and rest, and number) are perceived only through colour. In this passage Aristotle gives only a positive account without criticizing other theories, and stays firmly on the level of qualitative description; there is not the slightest hint that deeper physical processes might be effecting the transmission of colour. Aristotle's account gives no more active role to the eye than the ear has in hearing: the conceptual and causal direction of vision is from object to eye. It is emphatically not an 'intromission' theory of light rays emanating from a luminous body and reflecting off a coloured body to the eye. In the chapters that conclude the *De anima* as we have it (435a2-10), Aristotle again insists that vision is an alteration of the air, affected by the shape and colour of a body and passing these on to the eye; the process is said to resemble the disturbance caused in water when an object is dipped in it.[17] Here he explicitly opposes and prefers this explanation to extromission: «hence concerning reflection too, it is better [to say] that the air is affected by the shape and colour[...] than that the visual ray [τὴν ὄψιν] goes out and is inflected» (435b5).[18] On the other hand, *De generatione animalium*, 781a1-13 describes an instance where, he maintains, the visual ray theory and his own explain a biological aspect of vision equally well.

Aristotle contrasts his visual theory more fully with the opinions of other philosophers in the *De sensu*. Of the two rival explanations of vision, Aristotle has little to say against the theory (ascribed to Empedocles) that the eye perceives material emanations from objects (438a4). He is principally intent on refuting the notion, also found in Empedocles, that the eye sees by issuing a visual ray of light, or the more elaborate theory in the *Timaeus* (45b) that quasi-light from fire in the eye «coalesces» with light from a luminous body to form a rigid visual ray (438a25-b2).

From these passages it is clear that Aristotle regarded his causal explanation of vision (from object through medium to eye by means of qualitative change) as different from and incompatible with theories of corporeal emanation from the eye, although in certain circumstances he believed that they could both adequately account for the phenomena. More problematical is the relationship between his theory and geometrical optics. That there existed a geometrical optics when Aristotle wrote is certain from various brief allusions, and especially a notorious passage (*Physica*, II, 194a7-12) where he writes that optics «has a certain contrary approach to geometry; for geometry investigates physical line, but not *qua* physical, whereas optics [investigates] mathematical line, but not *qua* mathematical but *qua* physical». I take this to mean that geometry studies abstractions of «physical lines», i.e. the limits of bodies, but optics studies what are supposed to be physical entities (say, corporeal visual rays) by means of geometrical abstractions. Thus Aristotle knew of a geometrical optics that was conceived of as an abstraction of some physical theory. But it does not follow either that this geometrical optics was substantially the same as the Euclidean optics of a generation later, or that Aristotle endorsed the physical presuppositions of the opticians.[19] In fact it is very difficult to conceive how Aristotle would have reconciled the

17 These last chapters of *De anima* are probably displaced from their original context, between II 4 and 5: see D. S. HUTCHINSON [1987].

18 Although this passage speaks of reflection, it is not an explanation of reflection, but rather of all vision through a medium.

19 According to Vitruvius, VII, preface, perspective techniques in scenography, based on the geometry of lines of sight, were introduced in Athens by Agatharchus for the production of one of Aeschylus's plays. But even Euclid's *Optica* has no treatment of perspective projection, so that Vitruvius's remarks are almost certainly an anachronism (K. ANDERSEN [1987]).

'abstractions' fundamental to rectilinear optics with the theory of the *De anima*.

Strangest of all, the visual ray is invoked, not just as a mathematical tool of analysis but as a means of physical explanation, in two other major works of the Aristotelean corpus: the *Meteorologica* and the *De caelo*. In particular, the long discussion of halos and rainbows that occupies most of *Meteorologica*, III accounts for these phenomena consistently in terms of visual rays reflected to the sun by atmospheric air or water. Nowhere does Aristotle allude to the theory of the *De anima*; indeed, he refers to the conclusions of visual ray optics as a matter for *belief*: «One must accept from the demonstrations concerning the visual ray the belief that the visual ray [ὄψις] is reflected» (372a30-32). The conflict of this explanation with Aristotle's theory of vision was apparent to Alexander of Aphrodisias:

> [Aristotle] does not approve the doctrine that says that we see by an emanation of rays, as the mathematicians [οἱ ἀπὸ τῶν μαθημάτων] allege, a consequence of which doctrine is the [doctrine] that we see all things that are seen through an image [δι' ἐμφάσεως] by a reflection of these same rays. For it is not possible for any rays emanating from our sight and falling upon the object of vision to be the reasons for our seeing, nor for these same [rays], when reflected from mirrors and all such things[20] to the object of vision, again to furnish the reason for our seeing them, where the reflection of the rays takes place at equal angles. For the fact that these things are impossibilities has been shown in what he [Aristotle] has also said concerning how we see in his *De anima*. For his opinion is that the things that are seen rectilinearly are seen not by rays, but because the transparent [medium] between the object of vision and the sight is affected by the seen colour and transfers this affection to the sight, which itself is transparent (for this is what it is to be transparent); and the things that seem to be seen by reflection are seen because the image of the object of vision is first made by the intervening transparent [medium] in the mirrors and all reflective things, and then it comes from this to the sight by means of the transparent [medium] between the mirror and the sight. But it makes no great difference in the context whether one says that the sight is reflected from the mirror at equal angles, and whenever the object of vision happens to be under [i.e. in line with] such a reflection, [the sight] falls on it and sees it, or [to say] that the object of vision itself, by the particular kind of position that it has with respect to the mirror, is imaged in it through the affection of the intervening transparent [medium], since the [mirror] is such that it is not only able to be affected by the colour so as to transfer its quality to another transparency, but it is also able to maintain the image because of its smoothness and brilliance, so that the transparent [medium] between it and the sight is affected by it as by a coloured thing. Hence [Aristotle] employs the doctrine of rays as being familiar and to the liking of the mathematicians.[21]

20 Reading καὶ πάντων τῶν [διὰ] τοιούτων [ὁρωνένων].

Alexander anticipates many modern commentators in asserting that the causal direction of vision is irrelevant to the explanations of the meteorological phenomena, and that Aristotle here uses extromitted visual rays merely because there was already in existence a well-established vocabulary of visual ray optics.[22] If this were so, then it should be possible to convert the arguments of *Meteorologica*, III, without impairment into legitimate Aristotelean physics, firstly by turning around the causal direction of the colour-bearing rays, and secondly by explaining what visual rays mean in terms of colour changes in the medium. There is no evidence that Aristotle ever attempted the second, more fundamental, task. And the reversal of direction of the rays, easy when we think of them as abstracted geometrical objects, becomes difficult or impossible when Aristotle endows them with other properties that depend on the seer. Thus in *Meteorologica*, III, he makes extensive use of the principle that «the visual ray when extended becomes weaker and less» (374b11) to account for the colours and brightness of the rainbow's arcs; and the initial strength of the visual ray before extension varies from one person to another:

> Because of weakness of the visual ray, [air] often causes a reflection [of the ray] even without condensation, as was once the affliction of someone who saw weakly and not sharply: for as he walked he always saw an image

21 ALEXANDER APHRODISIENSIS, *In Aristotelis Meteorologicorum libros commentaria*, edidit M. Hayduck, Berolini, 1899, in CAG III pt. 2, p. 141.

22 Cf. in this context C. BOYER [1959], p. 50: «Nevertheless, in some of the Aristotelean treatises the language of visual rays continued to be used, possibly in deference to views current at the time. It should be noted that the Aristotelian geometry of the rainbow is independent of the doctrine of vision in terms of which it is expressed». Similar views are expressed by H. B. GOTTSCHALK [1964], pp. 79-80, and G. SIMON [1988], pp. 48-51, although both are more sensitive to the difficulties involved.

before him, looking facing him. He had this experience because the visual ray was reflected [back] to him; for it was so weak and delicate because of its feebleness that even the neighbouring air became a mirror, and [the ray] could not push it away. (*Meteorologica*, III, 373b2)

This example is explicit and decisive, since a reflection that is made possible only by personal weakness of sight absolutely demands extromitted rays.[23]

What is one to make of this extensive, and unapologetic, use of visual rays in *Meteorologica*, III? The choices, so far as I see, are (a) that Aristotle thought that the visual ray theory was consistent with the theory of the *De anima*, (b) that he thought the theories were not everywhere compatible but were interchangeable in the present context, (c) that he held different opinions at various stages of his career, or (d) that *Meteorologica*, III, is a work of composite authorship or even spurious. I have already explained why I cannot believe either (a) or (b). Others have suggested that *Meteorologica*, III, represents an early, repudiated stage of Aristotle's thought, although the internal evidence (such as it is) suggests a date of composition in the 330s.[24] The suggestion that other people (e.g. Theophrastus) had a hand in writing parts of the *Meteorologica*, although repugnant to many scholars, is not new: a long standing debate centres on the authenticity of Book IV, although no one seems to have questioned I-III.[25]

But if someone other than Aristotle was responsible for this part of the *Meteorologica*, we might have to concede that he also added the reference to the visual ray in the *De caelo*, expanding on Aristotle's explanation of the apparent wobble of the sun when rising and setting; the closing phrase, expressing the impossibility of telling whether it is the object or just the visual ray that moves, is habitual with the

[23] Alexander of Aphrodisias tries to explain the situation of the man who always saw his own image as an illusion caused by a medical condition (op. cit., CAG, III, pt. 2, pp. 147-148); but this amounts to correcting Aristotle, not interpreting him. Alexander also, rather implausibly, identifies the unfortunate man with the hallucinator Antipheron of Oreus (mentioned by ARISTOTELES, *De Memoria*, 451a9).

[24] G. SIMON [1988], p. 49 and p. 56 note 48. On the date of the *Meteorologica* see S. M. COHEN, P. BURKE [1990].

[25] H. B. GOTTSCHALK [1961] gives a clear and accurate argument against Aristotelean authorship of *Meteorologica*, IV, and reasons to associate it with Theophrastus (on which, see also D. E. EICHHOLZ [1965], pp. 18-19 n. 1).

author of *Meteorologica*, III, but also, interestingly, with Theophrastus and certain of the Aristotelean *Problemata*:[26]

The visual ray, being extended over a great distance, wavers because of its weakness. This is perhaps the reason why the fixed stars seem to twinkle, while the planets do not twinkle. For the planets are near, so that the visual ray reaches them in its strength, but when it comes to the stationary [stars] it is shaken because of the distance and because it is extended too far. Its wobbling makes the motion seem to belong to the star; for it makes no difference [in appearance] whether the visual ray or the object of vision moves. (*De caelo*, 290a17)

Meteorologica, III, does not profess to give a complete account of optics (372b9, 374b15), and it tells us more about the behaviour of the visual ray than about its physical nature. In the geometrical demonstrations of the circularity of halos and rainbows (373a4, 375b19), the ray is treated as equivalent to a straight line. Nevertheless it has properties indicative of a body: it has size (i.e. thickness) and strength (i.e. ability to push a way through a medium, cf. 373b9); but the ray grows weaker and smaller the further it is extended (374b11). Watery eyes have weak sight (374a21), which suggests that the ray is materially related to elemental fire.[27]

The colours that the ray perceives (or transmits to the viewer) are five discrete grades ranging from the pure positive quality of «bright» through crimson, leek-green, and violet, to «dark», the negation of bright (374a3, 374b30). Bodies have their own colours (for example, the sun and air are bright, water is dark), but the visual ray's perception of a body's colour may be shifted down the spectrum towards dark if the ray first passes through a dark medium or is reflected by a dark surface (372b7, 374a3), or if the ray is itself «weak» (372b7). Hence it is because of the weakening ray that objects are seen as darker and smoother and smaller with distance (374b18). This last point is of especial interest, since it implies that the 'perspective'

[26] Incidentally, this statement (recurring in *Meteorologica*, 374b22 and 377b11) does not mean that the direction of vision is irrelevant (as J. I. BEARE [1906], pp. 66-67 claims), but that the viewer cannot tell whether the apparent qualities (e.g. colour, movement) of a body are due to the body or to the visual ray.

[27] A textually corrupt passage (378a9) appears to assert that the ray becomes weak if cast upwards and away from the earth.

principles of Euclidean optics were not integrated into the optics of the *Meteorologica*.

Reflection takes place when a ray falls upon water, air, or anything having a smooth surface (372a29); but if the ray is too strong for the reflecting surface (as is usually the case with air), it may push it aside, while a very weak ray may fail to be reflected at all (378a2). The shapes of bodies are seen in larger mirrors, but such mirrors as have «no perceptible divisibility» exhibit only reflected colour (372a32). There is no hint that the author knows a specific geometric law of reflection, but he does assume that specific geometrical configurations of viewer, reflector, and object will consistently result in a reflection of the ray (373a3).

Theophrastus

However it is to be reconciled with Aristotle's theory of vision, *Meteorologica*, III, represents an early stage of Peripatetic optics. There are close ties between the much disputed Book IV and Theophrastus, and the same is perhaps true of Book III. Theophrastus's surviving writings give a few clues to his own opinions concerning the physics of vision (his *De sensu* reports his criticisms of some earlier doctrines, with much space devoted to Empedocles and Democritus). His brief work *De vertigine*, which attempts physiological explanations of dizziness (mostly as the effect of swirling fluid within the head), suggests an optical cause of vertigo:

> Moreover, people who look down on lofty, massive, and steep [places] become dizzy because the visual ray, being extended to great length, is shaken and knocked about; when thus shaken and moved, [the ray] disturbs and moves the [viewer's] inside. This does not happen to those who look upwards because [the visual ray] is not extended to great length but cut off in the light. (*De vertigine*, 8)

Both the general context of this work and parallels in the Aristotelean *Problemata* make it clear that this account is to be taken as a causal explanation, not a mere metaphor or phenomenological description.[28] The visual ray is apparently conceived as a rigid body

[28] *Problemata*, III, 9 and 20.

extending all the way to the seen object from within the eye, so that physical disturbance of the ray can set the viewer's internal fluid in dizzying motion. Similarly, Theophrastus explains the dizziness resulting from watching people running in a circle as a transfer of the circular motion from the runners through the rays to the viewer's inside (*De vertigine*, 6-7), while intent and fixed staring paradoxically disturbs the equilibrium of internal motions (*De vertigine*, 9). Secondly, the extinction of sight in light when one looks into the sky hints at a qualitative identification of the two with each other and with fire; this point is supported by passages from Theophrastus's other works:

> For the lesser fire is put out by the greater, which is also the reason why we cannot look back at the sun or anything that is intensely bright. (*De sensu*, 18)

> For the same and similar reasons fire burns less intensely in the sun than in the shade and lamps give less light when there is a blazing fire and are completely extinguished when placed above the flame, for then the greater fire consumes and interferes with the burning edge. (*De igne*, 11)[29]

The role of light in vision had been obscure in the *Meteorologica*, where there is only one passing reference to the sun's rays (ἀκτῖνεῃ) that diverge (ἐπαλλάττουσιν) to produce a shadow. But Theophrastus seems to have developed the putative similarity of nature of vision and light to the point where he attributed to light rays properties similar to those of visual rays. Thus in his *Meteorologica* (partially preserved in Syriac and Arabic versions) he explained the lunar halo as a wavelike thickening of the air induced by light from the moon.[30] The pseudo-Aristotelean *De coloribus*, which Gottschalk plausibly attributes to Theophrastus or a close follower of his teachings, gives copious further evidence of a belief in solar rays (ἀκτῖνες or αὐγαί) which are discrete material beams.[31] Light, which is itself the «colour of fire» (791b7), makes the colours of other bodies visible.[32] But Gottschalk is mistaken in claiming that «the Peripatetics rejected the notion of visual

[29] Translation by V. Coutant [1971].

[30] H. Daiber [1992], pp. 269-270.

[31] H. B. Gottschalk [1964], p. 63.

[32] There is no need here to go into the complicated theory of colour set out in the *De coloribus* (for which see H. B. Gottschalk [1964]). The doctrine of primary colours in this work is different from that of *Meteorologica*, III.

rays» and replaced it entirely with light rays. We have already seen that Theophrastus incorporated visual rays in his physiology, and abundant evidence will be given below to show that the rays were not abandoned by the later Peripatetics. If visual rays are not mentioned in the *De coloribus*, it is because they played no major part in the production of colour in Theophrastian optics. But the perception of a coloured surface (as distinct from a reflective surface) presumably required the incidence of both a light ray and a visual ray, the one to actualize the colour, the other to 'feel' it. The case of reflection is described in *De coloribus*, 793b23:

> For when light falls on certain things and is coloured, becoming reddish or greenish, and having been reflected, falls on some other colour, it is again mixed into by that [colour] and takes on a different combination of colour. And being thus affected continuously though not perceptibly, it sometimes reaches the visual rays [τὰς ὄψεις] mixed up with many colours and gives a perception of some one of the most dominant [colours].

The term ὄψεις in the foregoing passage is usually translated as 'eyes', but I suggest that, as usual in such contexts, the author has visual rays in mind. This would then mean that the solar ray reflects off smooth surfaces, coming to an end only on a non-reflective coloured body where it may or may not coincide with a visual ray. If the two kinds of ray are supposed to be similar in nature, however, a reflective surface ought to reflect either; and it seems that Theophrastus and his followers allowed some uncertainty about whether the light ray or the visual ray did most of the travelling and reflecting.

Transparency remained a problem. If sight is a material emanation from the eye, this material cannot occupy the same place as other matter. Transparency must signify the ability of a material to let visual rays through, either by giving way and letting its own substance be pushed aside, or by possessing channels through which the rays can pass. In the case of fluid media like air, the author of the *De coloribus* believed that rays forced the medium aside (791a27, 794a6).[33] This explanation would scarcely do, however, for transparent solids such as glass. Since Theophrastus revived the Empedoclean pore theory as an explanation of certain qualitative changes and intermixtures of solid

[33] The same principle is implied in the Aristotelean *Meteorologica*, 342b5 and 373b2.

bodies,[34] it is probable that he also attributed transparency to pores in solid media. But were the pores empty? Neither Empedocles nor Aristotle, to whom Theophrastus owed much of his physical thought, believed that void existed anywhere in the cosmos. If the pores were full of matter, how could other matter pass through them? The conclusion that the pores must be empty seems obvious; but prejudice against void was strong, and we have no evidence that Theophrastus ever spoke of the pores as empty of all matter.[35] He may have supposed that the pores contained air that was driven out of the other end as the rays pushed their way in.

STRATO

The step of introducing void was, however, taken by Theophrastus's successor, Strato, in his lost work *On Void*.[36] Our most substantial information about this book comes from the preface of Hero's *Pneumatica*, which in all probability takes much of its contents from a direct reading of Strato, and at one point tacitly quotes him. It was Strato's contention that all bodies, whether fluid or solid, consisted of small particles separated by still smaller voids, a theory that at one blow accounted for all the phenomena of expansion and contraction, intermixture and chemical change, qualitative change such as heating (by the intromission of fire particles), and transparency.[37] The transparency of water, indeed, gave Strato one of his arguments for the existence of the voids:

> For if there were no [voids], neither light nor heat nor any other corporeal power [δύναμις σωματική] would be able to fall through water or air or any other body. For how would the sun's rays [ἀκτῖνες] fall through water to the base of a vessel? For if the liquid did not have pores, but the beams [αὐγαί] pushed through the water by force, the result would be that full vessels would overflow; and this manifestly does not happen.
>
> It is obvious also in the following way. For if [the beams] pushed through

[34] E.g. *De Igne*, 28, 45, 73. For his reservations about Empedocles's use of pores to explain sense perception, see his *De Sensu*, 12ff.

[35] P. STEINMETZ [1964], p. 171.

[36] F. WEHRLI [1969], especially 20-25; H. B. GOTTSCHALK [1965].

[37] HERO, *Pneumatica* (edidit W. Schmidt, Lipsiae, [1899]), p. 6 = STRATO (F. WEHRLI, [1969]) frag. 56.

the water by force, it would not happen that some of the rays would be reflected to the place above and others fall through below. But in fact all the beams that strike against the particles of water are reflected, driven back, as it were, to the place above; while all that fall into the voids of the water, since they fall themselves in small particles, fall through to the base of the vessel. (Hero, *Pneumatica*, edidit W. Schmidt, 24-26; cf. Simplicius, *In Aristotelis Physica commentaria*, edidit H. Diels, 693)

Strato's colour theory seems to have been much like that of Theophrastus and the *De coloribus*.[38] The visual ray is not clearly attested in the most secure testimonia for his writings, but it is conspicuous both in Pseudo-Ptolemy's *De speculis* (and hence in Hero, whose dependence on Strato is established) and in several of the pseudo-Aristotelean *Problemata* that betray Stratonic influence. Gottschalk, who does not believe that Strato admitted visual rays at all into his optics, is compelled to regard the visual rays in these sources as intrusions.[39] I find it easier to believe that Strato maintained the Theophrastian bidirectional theory of vision.

Stratonic ideas are in evidence in the account of reflection (§5) in pseudo-Ptolemy *De speculis*, a Greek work extant only in William of Moerbeke's Latin version and heavily dependent on Hero's lost *Catoptrica*.[40] Bodies are said to have voids (*raritates*) on their surfaces which, being insubstantial, cannot reflect incident rays. When surfaces are polished, however, the surface voids are filled by small particles (*subtili substantia*), so that the resulting 'dense' surface can repell the rays. Liquids and glass have a special structure of voids and particles that allows rays either to be reflected or to pass through the whole body; this is obviously referring to aligned rectilinear pores.

In §§3-4 the *De speculis* describes the rectilinear path of the visual ray as a phenomenon of propulsion. The components of the argument are the following. (1) «Because of the propelling force (*violentiam emittentem*), what travels tries to travel in the shortest line in distance, because it has no time for slowness so as to travel in a longer line in distance, since the transmitting force does not allow it»; hence (2) «everything that travels with continuous speed [*continua uelocitate* = ? συνεχεῖ τάχει] travels in a straight line.» But (3) since the visual ray

[38] H. B. Gottschalk [1965], pp. 154-155.

[39] H. B. Gottschalk [1965], pp. 136, 161.

[40] Citations are from my forthcoming edition.

can reach the stars in an imperceptible time, it must travel with (practically) infinite speed. Therefore (4) the visual ray must travel in a straight line. As it stands the reasoning is not entirely coherent. It appears from Hero's testimony (*Pneumatica*, cit., pp. 12-14) that Strato retained the Aristotelean doctrine that a body is moved only so long as something else pushes against it; in the case of a thrown object, the air must follow and push it.[41] If the driving force fails to keep up, the object will slow down or change course; otherwise (according to the *De speculis*) it will take the shortest route of yielding to the force, which is rectilinear motion. Constant speed is manifest evidence that the emitting force is keeping up with the object. The *De speculis* seems to use the fact that the visual ray travels with «infinite speed» in order to establish that it is constant. One wonders whether Strato would not have considered the continuity of speed to be an obvious consequence of the continuity of the ray emitted from the eye; this is, after all, not a case of a projectile that becomes separated from the original source of movement. When an object that is being driven by a following force meets another body, according to §5, a reflection will occur if the following force has nowhere to «give way», so that it must continue to push the object somewhere else. How this principle applies to visual rays or light rays is not clear. We will return presently to the question of whether pseudo-Ptolemy's justification of the equal-angle law of reflection is due to Strato.

The *Problemata*

The Aristotelean *Problemata* consists of early Peripatetic speculations on a wide range of natural questions, and has several passages pertaining to optics. The specific date and authorship of individual 'problems' can only be guessed (contributions from the time of Theophrastus and Strato probably make up the bulk of the collection), so we cannot expect an entirely consistent theory. One problem at least, assumes the Theophrastian account of the transparency of fluids:[42]

[41] I am unable to see how H. B. Gottschalk [1965], pp. 147-148, can find in this passage evidence that Strato «abandoned Aristotle's doctrine [...] in favour of a simpler explanation closer to the modern concept of inertia».

[42] Remarked by H. B. Gottschalk [1965], p. 161 note 1.

Why is the water whiter in the Pontus than in the Aegean? Is it because of the reflection of the visual ray from the sea into the air? [...] Or is it because the visual ray does not quickly penetrate the fresh water, and is reflected towards the air, but it is not reflected up from the sea because the water is not smooth, and is exhausted with progressing downwards, and for this reason [the sea] appears black? But in [waters] of a lake-like character, the fresh water is on the surface and the salt water below, and hence the [the visual ray] does not penetrate, but is reflected towards the daylight; and for this reason its surface appears white. (*Problemata*, XXIII, 6)

Nevertheless other problems testify to several significant advances and refinements tending towards – or influenced by – both Strato's and Euclid's optics. For example, the following problems clarify the relation between rectilinear rays (solar and visual), transparency, and the pores:

Why does the visual ray not pass through solids, while sound does pass through? Is it because the visual ray has one motion, namely in a straight line (a sign of this being the sun's rays [ἀκτῖνες], and that we see only facing forwards), while sound has many [motions]? For we hear from every direction. So when [the ray] is prevented from falling in a straight line because the pores are not aligned, it cannot penetrate. But air and sound, inasmuch as they travel everywhere, penetrate everything and are heard. In liquids rays [ὄψεις] see through, while sounds are not heard, or just barely, the liquid having smaller particles than the earth, because the pores are small and dense and aligned, so that the visual ray is not prevented from travelling straight. Because of this glass too is seen through since it is dense [with pores], while fennel stalk is not seen through because it is sparse [with pores]. Also, the pores of the former are aligned, while those of the latter are out of line. Their being big is no help, if they are not in a straight line. (*ibid.*, XI, 58; cf. also XXV, 9)

Why is the sea more transparent than drinking water, though it is thicker? For drinking water is more delicate than salt water. Is it that delicateness is not the reason, but rather that there are many and very large straight courses of pores? Now drinking water is dense because it has delicate particles, while salt water has large internal voids [διάκενα]. (*ibid.*, XXIII, 8)

Why is night more conducive to hearing than day? [...] Or is it because hearing is better through an emptier [medium] than through a fuller one? The air of day is dense, inasmuch as it has been filled by the light and the rays, while that of night is thinner because the fire and the rays, which are bodies, have left it. (*ibid.*, XI, 33)

Pores in the surface membrane of the eye are necessary for vision to go forth at all, and it is plausible to assume that these pores gave rise to discrete visual rays. The idea is clearly described by an eclectic late antique writer on optics, Damianus:

One should know that this conical and light-like shape of the visual ray, travelling through the invisible pores of the pupil, is fragmented [σχίζεται] by necessity into rays [ἀκτῖνας] separated from each others, though they together irradiate [συναυγαζούσας] the whole space between them, that is the whole depth of the cone. (*Optica*, edidit R. Schöne, 8-10)

It also turns up in Philoponus's systematic refutation of all emission theories of vision:[43]

But if [the alleged visual rays] are bodies, then either the part of the cornea about the pupil must be perforated (which is false, since it is continuous, and if it is punctured, the liquids flow out immediately), or a body passes through a body (which too is impossible), or the visual rays must exit through certain pores. But it has been proved in [Aristotle's] *De Generatione* that bodies are not full of pores. And anyway there would certainly have to be certain particles [ὄγκους] and bodies in the membranes by which the pores are separated. Hence the visual rays cannot pass through these [particles], so that they ought to see neither the floor nor anything else as a continuum, but [they should] see some parts and not some others; for it is impossible for [the visual rays] to see the parts of the visible thing that the particles separating the pores are in a straight line with.

Philoponus, like Damianus, does not tell us whence he got this theory. But its Peripatetic provenance is assured by the *Problemata*:

Why does smoke cause the eyes to smart more? Is it because they alone are very weak? For the inner parts of the body are always the weakest. A sign of this is the fact that vinegar and anything pungent makes not the outer but the inner flesh smart, because [the inner flesh] is the rarest flesh in the body and contains most pores. For the rays are emitted through certain pores, so that what stings the inside most is drawn away [?] from the flesh.[44] (*Problemata*, XXXI, 21)

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[43] PHILOPONUS, *In Aristotelis de anima libros commentaria*, edidit M. Hayduck, in CAG, XV, p. 326.

[44] The last sentence is corrupt; the sense must be that the pores admit the acids to the inside flesh.

The following problem shows that a version of Euclid's cone of discrete visual rays was in fact part of Peripatetic theory, although Euclid needs to make no assumptions about the density and relative strength of the rays in the various parts of the cone. The passage also refers to the phenomenon, notorious from Epicurean discussions of vision, of the distant square's seeming round, which is also dealt with in Euclid, *Optica* 9; but where Euclid explains the rounding by having the corners of the square fall within the gaps between rays, our author assumes that the rays toward the periphery of the visual cone are too weak to perceive the corners.

Why is it that when the sunshine passes through quadrilaterals, it makes the shapes not rectilinear but circles, as in wickerworks? Is it because the emission of the rays [ἡ τῶν ὄψεων ἔκπτωσις] is a cone, and the cone's base is a circle, so that whatever the sun's beams [ἀκτῖνες] fall upon, they seem circular? [The author then argues that the rectilinear beams of the sun, cast upon a rectilinear orifice, must produce a rectilinear projection.] But because those [visual] rays that split off to the ends of the straight lines are weak [διὰ τὸ ἀσθενεῖς εἶναι τὰς ἀποσχιζομένας [ἀπὸ] τῶν ὄψεων πρὸς τὰ ἄκρα τῶν εὐθειῶν], the [parts] in the corners are not seen. But as much of the straight line as lies within the [visual] cone makes it [i.e. the appearance?], while the rest does not make it, but the incident rays are not noticed. For there are many things upon which the visual ray passes that we do not see, for example things in shadow.

Alike to this [phenomenon] is that of the square seeming to have more sides [than four], and if it stands farther away, [seeming to be] a circle. For since the emission of rays is a cone, when the shape stands further off, those rays that split off to the corners do not see because they are weak and few, when the distance is greater, while those that fall in the middle, being crowded and strong, persist. Now when the figure is near, [the rays][45] can see even the parts in the corners, while they cannot when it is farther away. (*ibid.*, XV, 6)

Others of the *Problemata* show concerns about the relation between geometrical appearances and the actual shapes of objects that resemble themes in Euclid's *Optica*:

Why is it that, although the moon is spherical, we see a straight line when it is halved? [cf. Euclid, *Optica*, prop. 22] Is it because the visual ray and the circumference of the circle that the sunshine makes by projecting

[45] Reading δύνανται.

on the moon are in the same plane? When this happens, the sunshine appears as a straight line. For since that which projects rays on a sphere must see a circle [cf. Euclid, *Optica*, prop. 23], and the moon is spherical, and the sun sees it, a circle must be what the sun produces. Hence when this [circle] is facing us, it is seen entire and a full moon appears to take place. But when it shifts because of the movement of the sun, its circumference falls along the visual ray, so that it appears straight. [cf. Euclid, *Optica*, props. 22, 23].

Why is it that, although the sun and moon are spherical, they appear flat? Is it because all things whose distance is not clear, when they are more or less distant, appear on an equal basis? So that in the case of one thing having parts, if they do not differ in hue, the parts necessarily appear on an equal basis, and what [is seen] on an equal basis necessarily seems to be uniform and flat. (*Problemata*, XV, 7-8)

Why are bubbles" bases pale in the water? And when they are put in the sunshine, they do not make a shadow: the rest of the bubble makes a shadow, but the base does not make one, but is in the sun right round [...] Is it that a shadow is not formed? But the shadow is defined [? διήρηται] by sunshine. Hence if shadow is what is not seen, then the body is seen also by the sun right round. But it is shown in the «optics» that this is impossible; for the smallest thing cannot be seen entire by the greatest. (*Problemata*, XVI, 1)

A final optical principle that is first unambiguously attested in the *Problemata* is the law of reflection at equal angles. Claims have, indeed, been made that this law was discovered as early as the mid fourth century, but the evidence is far from compelling.[46] Here, however, the law is clearly invoked:

Why is it, given that sound is air that has been shaped and is travelling, that its shape often disintegrates, but echo, which comes into being when such [a sound] strikes some solid, does not itself disintegrate, but we hear it clearly? Is it because it is a reflection, and not a scattering [κατάκλασις]? For in this way [i.e. by reflection] the whole persists, and two similarly shaped

[46] Boyer's arguments that Aristotle knew the law (C. Boyer [1959], pp. 39-41, and [1946]) are entirely based on passages from the *Problemata*, which Boyer regards as «derived largely from the master's teaching» ([1959], p. 326 note 24). For the conjecture that the law was discovered by Philip of Opus (a follower of Plato, c. 350 B.C.), see G. Simon [1988], pp. 22, 46, 56 note 43; this last gives references to theses by B. Saint-Pierre (1972) and G. Picolet (1982) which I have not seen. The only ancient testimony to Philip's book on mirrors reports his argument that the rainbow is a reflection, since, like an image in a mirror, it seems to move from side to side as the viewer does. But this is a far cry from the law of equal angles!

parts come into being out of it. For the reflection is at a similar [i.e. equal] angle. Hence the sound of the echo is similar to the original [sound]. (XI 23)

Why is it that when travelling [bodies] make a collision, they are turned away to travel in the opposite direction from the one that they were [travelling], and at similar angles? Is it because they are travelling not only with that motion that they travel to their proper place, but also with that to which the thrower gives rise? Now the proper [motion] comes to an end when they come to their proper place (for everything is stationary when it has come to the place that it travels to by nature); but according to the extraneous [motion] that they have, they still must be moved, not forwards because they are blocked, but either obliquely or perpendicularly. And all things bounce at similar angles because they travel where the movement [κίνησις] that the thrower caused travels; and there [i.e. at the blocking body] they chance to travel at an acute or a right angle [with respect to its surface]. Hence since the obstacle blocks the movement in a straight line, it likewise blocks the travelling [body] and its travel. So just as in mirrors the endpoint [?] of the straight line where the visual ray fell is seen, so in the case of travelling [bodies] the opposite [?] happens; for they bounce off at an angle as great as the vertical angle [made by the surface and the line of incidence]. For one must realize that the angle and the travel are transferred [μεταχινουμένην]. Since this occurs, it is clear that they have to rebound at similar angles. (*Ibid.*, XVI, 13)

It is not merely the knowlege of the law of reflection that is remarkable in these problems. Reflection of sight or light is here treated as only one case of a general class of reflections of corporeal entities (sounds, projectiles) that obey the law of equal angles.[47] The reflection, moreover, is fundamentally caused by the inability of the pushing force (here called κίνησις, 'motion') to proceed in a straight line. All this is strikingly reminiscent of the *De speculis*, and confirms the Peripatetic provenience of its theory of reflection. Strato again seems the most probable source.

The *De speculis* contains one prominent discovery that has no analogue in the extant Peripatetic texts: the reduction of the law of reflection to the law of shortest travel (§§6-8). It may be noted that this passage is unsuccessful in two ways. Firstly, there is the well-known limitation of the geometrical theorem on which it depends: an inflection at equal angles gives the shortest inflected path between two given points if the inflection is on a straight line or the exterior of a circle, but not always if it is on the interior of a circle. In other words, a ray reflected by a convex mirror may take the longest possible path from eye to object. This fact is suppressed in the *De speculis*. Secondly, there is no clear physical motivation for applying the law of shortest travel to a reflection. It seems likely that this is a case where the mathematical principle was discovered, and then forced upon the physical situation. In the original *Catoptrica* of Hero, according to Damianus, the geometrical proof was followed by the assertion that «if nature does not mean to lead our sight about pointlessly, it will reflect it at equal angles».[48] One suspects that to Strato, who had (so far as we know) no place for final causes in his physics, such an appeal to the agency of 'nature' would be no explanation at all. Is this, then, Hero's personal contribution to the theory of mirrors?

[47] ARISTOTELES, *Analytica Posteriora*, 98a27, classifies the questions «Why does it echo?», «Why is it mirrored?», and «Why is there a rainbow» as the same problem in genus, as all concerning reflection, but different in species. But there is no hint here of a quantitative law.

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[48] DAMIANUS, 14 (edidit R. Schöne, 20). A similar appeal to the cliché that «nature does nothing in vain» appears in Olympiodorus's version of Hero's proof for the case of the plane mirror (CAG, XII, pt. 2, p. 212).

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