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P. Stras. inv. 1097: An Astronomical Epoch Table for Jupiter

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P.Stras. inv. 1097 entered the Strasbourg University papyrus collection in 1906 as a gift of the egyptologist Balthasar Pörtner; its previous history is unknown.[1] Although hitherto unpublished, the papyrus is listed in Neugebauer's 1962 bibliographical inventory of astronomical papyri and ostraca, where it is described as "two small fragments from planetary tables, probably concerning the entry of Mars or Venus into the zodiacal signs", whereas in a still briefer mention some years later Neugebauer stated categorically that it gives "dates of entry of Venus into the signs".[2] Neugebauer's file on the papyrus, preserved in the University of Michigan Papyrology Collection, contains black-and-white photographic prints of the papyrus as two disconnected fragments, a letter from Jacques Schwartz to Neugebauer concerning the papyrus, dated August 23, 1967, and handwritten notes by Neugebauer. Among these notes is a sheet quoting a brief description, referring to two fragments, from an earlier letter from Schwartz, dated November 10, 1961. The extant 1967 letter, a response to two (presumably lost) letters from Neugebauer, contains an annotated transcription, from which it is evident that the two fragments had by then been correctly joined to form the present single fragment.

The papyrus has dimensions 57 mm (width) by 132 mm (height), and is broken on all sides. On the side with horizontal fibers are negligible remains of two lines in a documentary hand of the first or possibly the early second century AD (Fig. 2).[3] The other side (Fig. 1), which has a collesis about halfway across, bears parts of eleven rows and two columns of an astronomical table written, the other way up relative to the document, in a first-century cursive hand, with vertical black ruling separating the two columns and horizontal rulings between each row and the next (omitted in the transcription below).[4] The row height averages about 5 mm. Column i, which contains names of Egyptian calendar months, can be estimated to have been about 45 mm wide when intact. The original width of column ii, containing numerals, is indeterminate but certainly well over 20 mm.

Fig. 1. P.Stras. inv. 1097, back Collection and photo BNU de Strasbourg

Fig. 2. P.Stras. inv. 1097, front Collection and photo BNU de Strasbourg

[1] Daniel Bornemann, personal communication. I wish to thank the Bibliothèque nationale et universitaire, Strasbourg, for access to the papyrus and permission to publish it, and the University of Michigan Papyrology Collection for access to Otto Neugebauer's files on papyri.

[2] Neugebauer 1962, 388; Neugebauer 1975, 2.788.

[3] I owe the estimate of the hand's date to the New York papyrology seminar and particularly Roger Bagnall. According to Neugebauer's 1962 description, Schwartz saw a personal name in these traces; the seminar was unable to confirm this.

[4] In his 1961 letter to Neugebauer, Schwartz described the hand as "une écriture mal habile et difficile à dater à l'antérieur de l'époque romaine".

Transcription.

i	ii
— — — —	— —
[Ἐπε]ίπ	κ̣[
[Με]cωρ̣ή	κ ̣ [
[Θώ]θιϲ [	
[Φ]α̣ῶφι	ιζ [
[Ἁ]θ̣ύρ	κ ι[
Χοιάκ	κδ [
Μ̣ει̣κίρ	α [
[Φ]αμεν(ώθ)	̣ [
Ἁ̣θύρ	ιϲ [
[̣ ̣ ̣] ̣ν	κβ [
[̣ ̣ ̣ ̣]ι̣	κϲ [
— — — —	— —

Notes.

i 1. *l.* Ἐπείφ.

i 4. *l.* Φαῶφι.

i 7. *l.* Μεχείρ.

ii 2. To the right of κ a vertical stroke along the edge of the papyrus. If this letter formed a single numeral with the κ, it would have to be β, γ, or η; if a separate numeral, ι, κ, or ν would also be possible.

Initial observations and Neugebauer's interpretation.

Since the numerals preserved in col. ii are all less than thirty, an obvious assumption is that they represent dates within the months written in col. i, so that the table, so far as we have it, is an ordered list of dates of some recurring event or phenomenon. (Line 5 shows that something further was written following the day numbers in at least some lines.) In the absence of a column giving year numbers, the intervals of time from one row of the table to the next could be simply the number of days between the stated dates in the same calendar year, or in the case of lines 3–4, in consecutive years; or the intervals might be one or more calendar years plus that number of days. If we consider only the minimum possible intervals, we have for lines 1–7:[5]

Date	*Interval since previous date*
Epeiph 2x	
Mesore 2x	
Thoth 16	23, 28, 29, or 31 days
Phaophi 17	31 days
Hathyr 20	33 days
Choeac 24	34 days
Mechir 1	37 days

Phamenoth in line 8 is consistent with this pattern, but the big leap forward by eight months to Hathyr in line 9 is unexpected. Neugebauer's notes show that he was certain that Hathyr was an isolated mistake for Pharmouthi, with Pachon and Payni following in 10–11, as is consistent with the preserved traces of the ends of these month names. I am convinced that he was correct, since I am unable to think of any phenomenon likely to appear in an ancient astronomical table that could occur seven times at intervals in the 20

[5] These intervals assume that the calendar of the table was the unreformed Egyptian calendar lacking leap years, which was often employed in astronomical contexts. We will verify this assumption at a later stage.

to 40 day range (with or without additional calendar years) and thereafter at an eight-month interval. We may therefore extend the data thus:

Phamenoth xx	
Pharmouthi 16	75 days since Mechir 1
Pachon 22	36 days
Payni 26	34 days

Neugebauer supposed that the table was of the type now called a Sign-Entry Almanac, which comprises a list of calculated dates when each of the five planets known to ancient astronomy entered one of the twelve zodiacal signs in either direct or retrograde motion.[6] The usual format for a Sign-Entry Almanac presents the dates of all sign-entries during a single calendar year for each of the five planets in turn, in the order Saturn, Jupiter, Mars, Venus, Mercury. Our papyrus would have to have represented a variant format in which more than one year's data was provided all together for a single planet. As Neugebauer clearly realized, the only planet that comes into consideration given an extended sequence of dates at intervals around 30 days is Venus.[7]

For a large part of its synodic cycle, roughly from its greatest morning elongation to its greatest evening elongation, Venus travels in direct motion by more than 30° in 30 days. Intervals of more than 31 days between sign-entries are only possible within the roughly 60 days of direct motion immediately preceding evening station, the retrogradation, and the roughly 60 days of direct motion immediately following morning station. Given the trend of the intervals between the dates in the papyrus, the transition from direct motion faster than 30° in 31 days to slower than 30° in 31 days would have to fall between Thoth 16 and Phaophi 17, and thus the evening station ought to fall before Choeac 24, so that the sign-entry on that date has to be in retrograde motion. Since Venus's retrogradations are always well under 50 days, the morning station must be before Mechir 14, and the transition back to direct motion faster than 30° in 31 days must be before Pharmouthi 14. Yet if we adopt Neugebauer's correction and restorations of the month names in lines 9–11, the sequence has two intervals significantly longer than 31 days subsequent to the point when this transition should have happened. In other words, if the papyrus is indeed a Sign-Entry Almanac for Venus, the underlying model for Venus's motion would appear to have been rather poor. As we will see below, a much more satisfactory identification of the table's contents can be offered.

P.Stras. inv. 1097 as an epoch table.

Besides Sign-Entry Almanacs, another variety of table is known from the Greco-Egyptian astronomical papyri in which one encounters lists of dates at variable intervals, the Epoch Table.[8] In an Epoch Table, the tabulated dates are of occurrences of a particular stage of the synodic or anomalistic cycle of a heavenly body, and the columns giving the date are in almost all cases accompanied by columns giving the longitude of the relevant heavenly body on that date. Neugebauer knew of only a single example of an Epoch Table, P.Lund inv. 35a, which comprises computed dates and longitudes for the Moon at its minimum apparent speed at intervals of nine or eleven anomalistic months. During the last two decades many Epoch Tables have come to light for the Sun and the five planets as well as additional ones for the Moon; most but not all were excavated at Oxyrhynchus. The planetary Epoch Tables were apparently all computed using arithmetical algorithms that derive from the Babylonian mathematical astronomy known from cuneiform tablets of roughly the last three centuries BC excavated at Babylon and Uruk.

In layout and ruling the table of P.Stras. inv. 1097 (henceforth "*S*") resembles some Epoch Tables, and particularly P.Oxy. astr. 4160 + P.Berlin 16511 (henceforth "*OB*"), a table of computed synodic phenomena of Jupiter covering the years AD 30 through at least 79, in which consecutive columns contain respectively Egyptian month names and numerals representing day numbers followed by sexagesimal fractions of days.[9]

[6] Jones 1999a, 324–326.

[7] I have no idea why Neugebauer suggested Mars as an alternative candidate in his 1962 description. With isolated exceptions occurring around its retrogradations, the intervals between Mars's sign-entries are always longer than 40 days.

[8] Jones 1999a, 305–310.

[9] In addition to the edition of both fragments in Jones 1999b, 1.145–148 and 2.88–91, see Brashear–Jones 1999.

It was this resemblance that initially suggested the identification of *S* as an Epoch Table for Jupiter that I offer here, and the more extensively preserved contents of *OB* make a convenient basis for delineating the characteristics of such a table that made this identification possible.[10]

The time intervals separating consecutive occurrences of any synodic phenomenon of Jupiter are always one 365-day Egyptian year plus a number of days that can be as small as the high 20s and never as great as 40. In *S*, if we again assume that Hathyr in line 9 is a mistake for Pharmouthi, the dates as preserved are consistent with this rule, so that at first glance they look plausible as a sequence of dates of one of Jupiter's synodic phenomena. The synodic phenomena of Jupiter tabulated in epoch tables and in the Babylonian tablets that were their ancestors include, in order of occurrence in a synodic cycle, first morning visibility, morning station, first evening rising (slightly before opposition with the Sun), evening station, and last evening visibility.

Turning to *OB* now, the preserved tabular columns are, from left to right: (i) incompletely preserved longitudes of a sequence of a phenomenon (probably first morning visibility) in degrees and sexagesimal fractions; (ii) a vacant column; and (iii) dates of a sequence of evening stations expressed as emperor's regnal year, Egyptian month, and day number with sexagesimal fractions. The calendar of the dates is the old pre-Roman Egyptian calendar, which continued to be employed for astronomical and astrological applications long after the Augustan reform because of the convenience of its constant 365-day years for chronological calculations. The fractional parts of the dates have little astronomical significance but are necessary elements in the mathematical method by which the dates were computed.

The longitudes and dates of phenomena in *OB* were computed by a "System A" model, that is, a set of arithmetical algorithms of a type that was originally developed in Babylonia during the second half of the first millennium BC and transmitted to Greco-Egyptian astronomy by the Roman period. In a System A model, both the interval of longitude ("synodic arc") and the interval of time ("synodic time") separating two consecutive occurrences of a specific synodic phenomenon of a planet are functionally dependent solely on the planet's longitude at the first of the two occurrences. Specifically, the circuit of the zodiac is treated as comprising fixed intervals of longitude, within each of which the synodic arc and synodic time are either assumed to be constant or to increase or decrease linearly. Several System A models are attested for Jupiter in cuneiform tablets; the model employed in computing *OB*, however, was an especially refined one not known from any other source.[11]

Since *S* as preserved contains only dates, and is missing almost all the fractional data that might cast light on the details of computation, it is not susceptible to the kind of analysis by which the model underlying *OB* was reconstructed. The loss of the column for year numbers is a further inconvenience, since a complete date would allow us to identify the synodic phenomenon by comparison with Jupiter's historic motions as computed by modern theory. We can learn a certain amount, however, from the correlation of synodic times with the preserved months and days. This is because even in the unreformed Egyptian calendar the month and day are, over a time scale of several decades, an approximate indicator of the Sun's longitude, and each synodic phenomenon occurs when the planet is near a certain characteristic elongation from the Sun. The characteristic elongations for Jupiter are roughly as follows:

Phenomenon	*Elongation*
First morning visibility	345°
Morning station	245°
Acronychal rising	180°
Evening station	115°
Last evening visibility	15°

[10] Like the table of *S*, that of *OB* is written across the fibers in a first century hand, on the other side of a document; however, the hands of the tables, though somewhat similar, are not so much alike as to suggest that the same person wrote both, while the documentary side of *OB*, apparently court proceedings, is in a hand described by Brashear as "an elegant (but non-literary) documentary script of the mid to late first century A.D.", the same way up as the table, and quite unlike the traces on *S*.

[11] Britton–Jones 2000.

To eliminate the "blur" resulting from the gradual shift of the calendar years relative to the solar year, we will convert the dates in both papyri to the reformed Egyptian calendar, for this purpose provisionally assigning the dates in *S* to the years AD 48–59 which fall approximately in the middle of the range covered by *OB*.

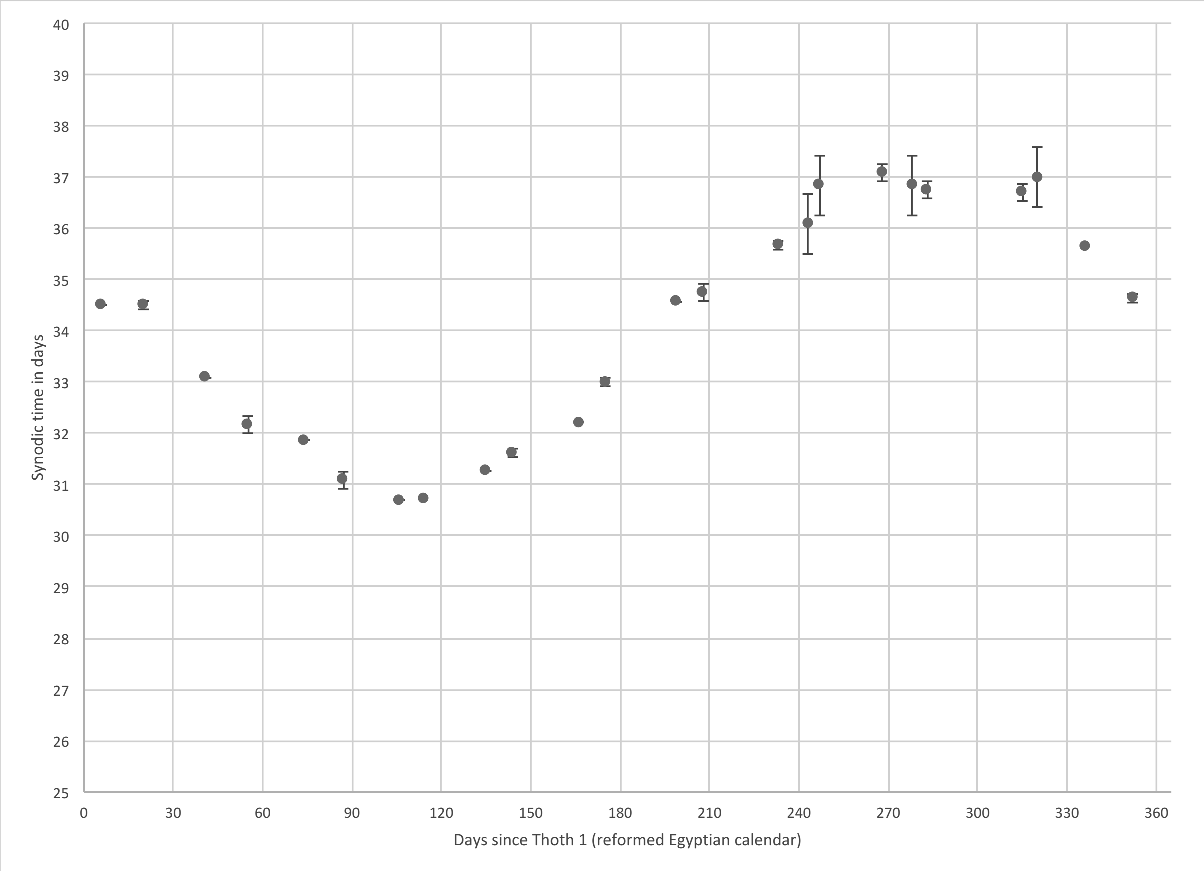

Fig. 3. Synodic times from P.Oxy. astr. 4160 + P.Berlin 16511 (solid gray circles) plotted against the reformed Egyptian calendar date of the preceding morning station. Error bars indicate the range of uncertainty resulting from loss of fractional places in the papyrus

In Fig. 3, the synodic times of *OB* are graphed as a function of the recorded calendar date of the phenomenon, counted as days from Thoth 1 according to the reformed Egyptian calendar. Error bars indicate the range of uncertainty in the synodic times due to loss of fractions in the papyrus. The minimum synodic times, which fall around 110 days after Thoth 1, should correspond to occurrences of the phenomenon having Jupiter at a longitude about half a synodic arc less than its apogee (near Virgo 10°), say Leo 25°, so that the planet would be at apogee around the middle of the synodic time.[12] In any year in the mid first century AD, the Sun's longitude 110 days after reformed calendar Thoth 1 was near Sagittarius 24°, so the planet's elongation from the Sun at the tabulated phenomenon must have been around 241°. This is close to the characteristic elongation for evening station. Hence we could have identified the tabulated phenomenon in *OB* cols. iii–v even if the year numbers had been lost, so long as we knew that the range of dates was around the first century AD.

[12] According to Ptolemy, *Almagest* 11.1, Jupiter's apogee was at Virgo 11° in the mid second century AD.

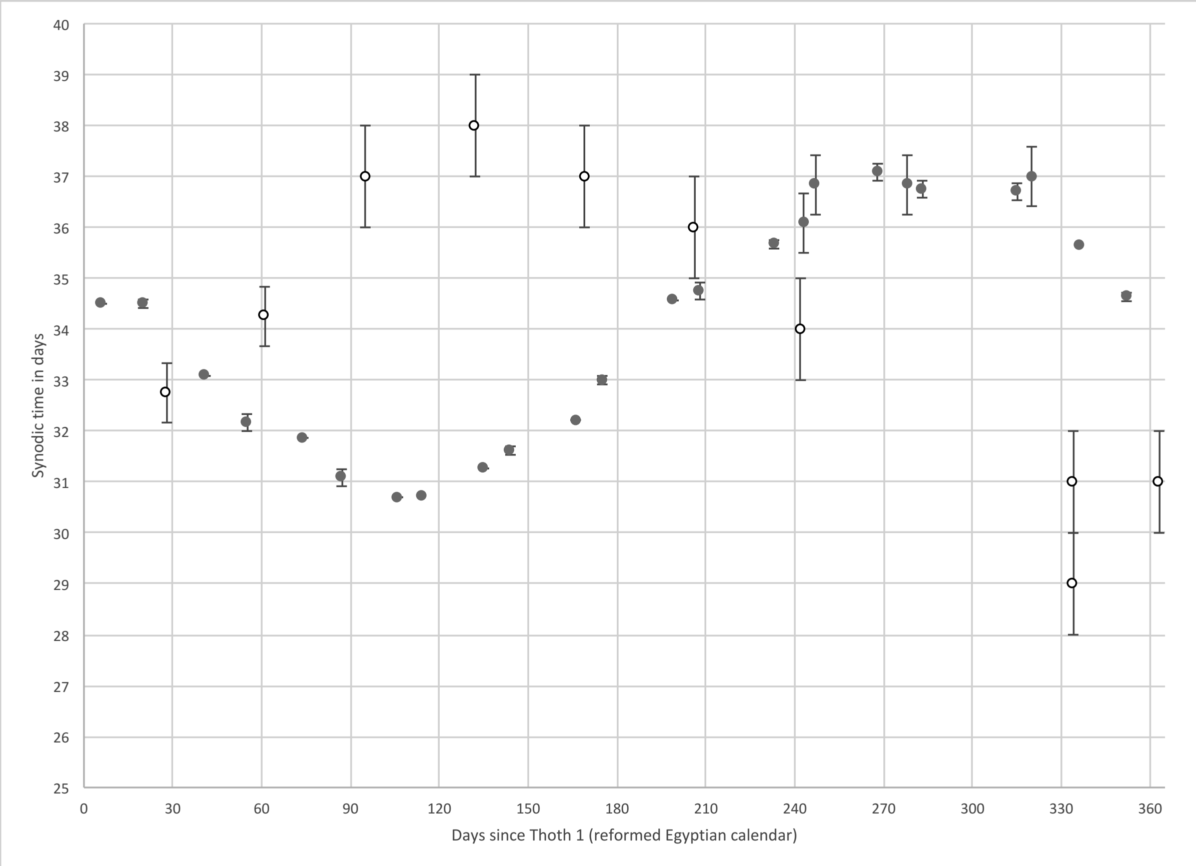

Fig. 4. Synodic times from P.Stras. inv. 1097 (hollow black circles) plotted against the estimated reformed calendar date of the preceding synodic event, superimposed on the graph of Fig. 3. The alternate data points labelled (1) and (2) in Figs. 4–7 correspond respectively to restoring Mesore 22 and Mesore 20 in line 2 of the papyrus

Fig. 4 superimposes on the graph of Fig. 3 the presumed synodic times of *S* graphed as a function of the recorded calendar dates expressed in the reformed calendar using the provisional assignment to the range of years AD 48–59.[13] The shape of the function resembles that of *OB* but with a large phase shift. As Fig. 5 shows, adding 140 days to the dates in *S* results in a pattern in phase with that of *OB*, so that we can estimate that the reformed calendar dates of minimum synodic time for *S* are about 140 days before those of *OB*, namely around 335 days after Thoth 1. The corresponding solar longitude is about Leo 3°, so that the elongation of Jupiter at the tabulated event would have been about 22°, close to the characteristic elongation of last evening visibility. We may be confident that *S*, so far as it survives, is part of an Epoch Table of Jupiter's last visibilities.

[13] For continuity, the illegible day number of line 8 is assumed to be 9, the only restoration that produces a plausible division of the 75 days between lines 7 and 9 consistent with the general pattern.

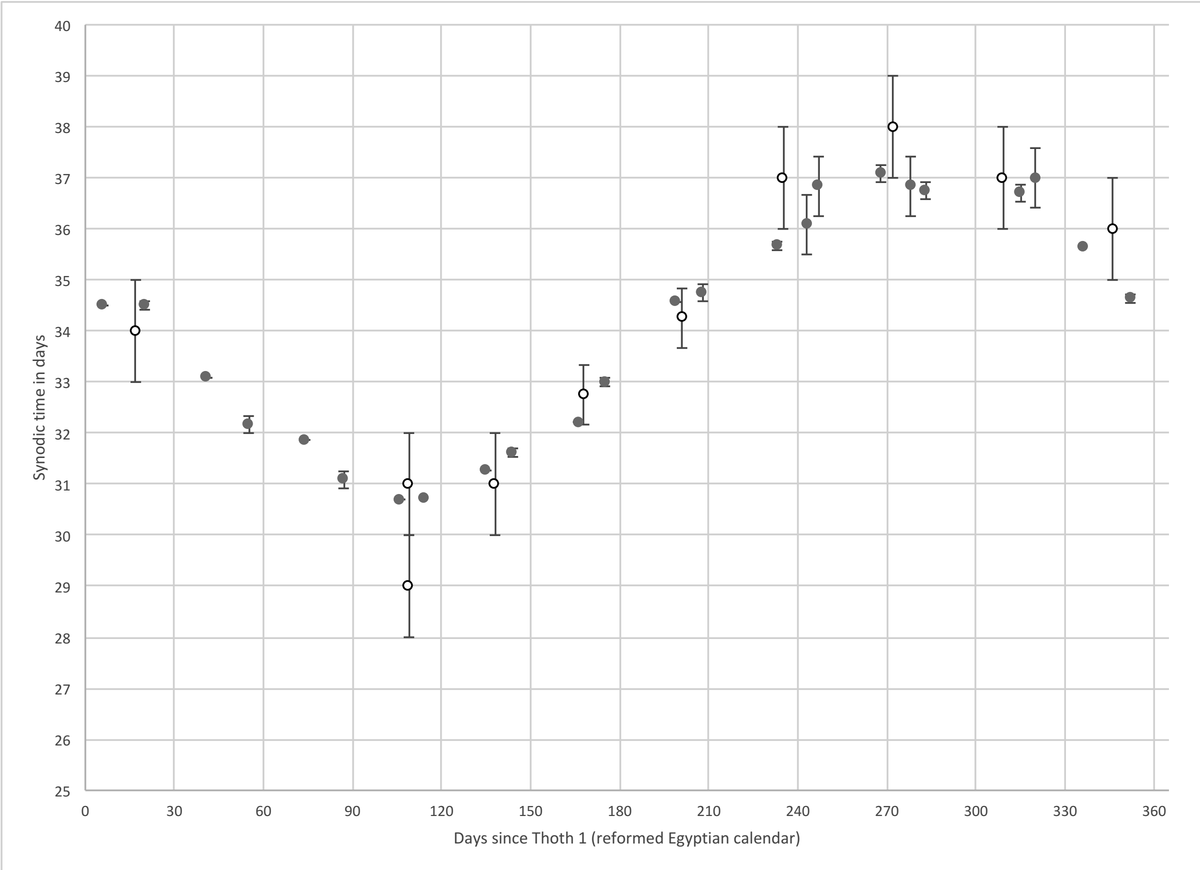

Fig. 5. Synodic times from P.Stras. inv. 1097 (hollow black circles),
with the estimated reformed calendar dates shifted 140 days later, superimposed on the graph of Fig. 3

We have been assuming up to this point that the dates recorded in *S* are according to the unreformed Egyptian calendar, as is expected for Epoch Tables and now we can confirm this assumption. If the dates in the papyrus were according to the reformed calendar, the reformed calendar dates of minimum synodic time would have been about 355 days after Thoth 1, corresponding to a solar longitude of about 142°, which is too close to the Sun's longitude to be either first or last visibility.

Jupiter's synodic phenomena return to roughly the same dates in the unreformed Egyptian calendar after 12 years (11 synodic cycles), 47 years (43 synodic cycles), and 59 years (54 synodic cycles), so that within the first century AD there are several sequences of years within which Jupiter's last visibilities would have fallen on dates reasonably close to those in *S*. Dates of first and last visibility of planets are subject to considerable uncertainty and cannot be modelled exactly by any modern theory, and besides, the algorithms used in the papyrus Epoch Tables for computing synodic phenomena were invented in Babylonia, not Egypt, and on the basis of observations made several centuries in the past. The years AD 22–33 and 69–80 offer perhaps the best fits, as shown below by comparison with dates of Jupiter's last visibilities computed using Ptolemy's *Almagest* and using Alcyone Software's *Planetary, Stellar and Lunar Visibility* (PLSV31.exe).[14]

[14] The *Almagest* calculations used Ptolemy's tables for planetary and solar longitudes together with the visibility table in Book 13.10, which is stated to be based on empirical data for the latitude of Phoenicia (i.e. roughly the latitude of Babylonia). The PLSV calculations were carried out for the latitude of Babylon. For the theoretical assumptions underlying PLSV, see http://www.alcyone.de/plsv/documentation/index.html.

Papyrus	*Year (AD)*	*Almagest*	*PLSV*	*Year (AD)*	*Almagest*	*PLSV*
Epeiph 2x	22	Epeiph 23	Epeiph 26	69	Epeiph 21	Epeiph 26
Mesore 2x	23	Mesore 23	Mesore 27	70	Mesore 22	Mesore 27
Thoth 16	24	Thoth 18	Thoth 23	71	Thoth 17	Thoth 23
Phaophi 17	25	Phaophi 19	Phaophi 23	72	Phaophi 17	Phaophi 23
Hathyr 20	26	Hathyr 22	Hathyr 25	73	Hathyr 19	Hathyr 25
Choeac 24	27	Choeac 27	Choeac 28	74	Choeac 23	Choeac 28
Mechir 1	29	Mechir 4	Mechir 4	76	Tybi 29	Mechir 4
Phamenoth 9?[15]	30	Phamenoth 11	Phamenoth 11	77	Phamenoth 6	Phamenoth 11
Hathyr! 16	31	Pharmouthi 19	Pharmouthi 18	78	Pharmouthi 14	Pharmouthi 18
Pachon? 22	32	Pachon 25	Pachon 25	79	Pachon 21	Pachon 25
Payni? 26	33	Payni 29	Payni 30	80	Payni 26	Payni 30

The computational model.
We have used the graph of synodic times plotted against reformed calendar dates as a means of identifying the planet and the synodic phenomenon tabulated in *S*, taking into consideration chiefly the amplitude and phase of the curve passing through the data points. The shape of the curve also provides evidence respecting the mathematical model according to which the dates in the papyrus were computed, though this evidence is less clear-cut than it would have been if more of the fractional parts of the day numbers had survived.

All the epoch tables on papyrus that are well enough preserved to allow identification of the underlying model were computed using arithmetical models attested in Babylonian cuneiform tablets or modifications of known Babylonian models. The models are either of the System A type, in which synodic arcs and synodic times are functionally dependent on the planet's longitude at each occurrence of the relevant synodic event (as described earlier in this article), or of the System B type, in which synodic arcs and times are successive values of linear zigzag functions, alternately increasing and decreasing by constant steps between a fixed maximum and minimum value.

Several System A models are known for Jupiter, in all of which the minimum and maximum synodic arcs are respectively 30° and 36°, while the minimum and maximum synodic times are respectively a little less than 365 + 31 days and 365 + 37 days. The majority of the data points from *S* are consistent with either the Babylonian model known as System A' which is well attested in cuneiform texts but not as yet found in the papyri (Fig. 6) or the model reconstructed from the data in *OB*, which is not attested in cuneiform texts (Fig. 7). But no known System A model for Jupiter allows for a synodic time as great as 38±1 days; and although the date Phamenoth 9 in line 8 that gives rise to this specific high value is hypothetical, the 75 days between the dates in lines 7 and 9 – presuming the month in line 9 should indeed be Pharmouthi – cannot be divided into two parts without having one of them at least 38.

[15] See note 13.

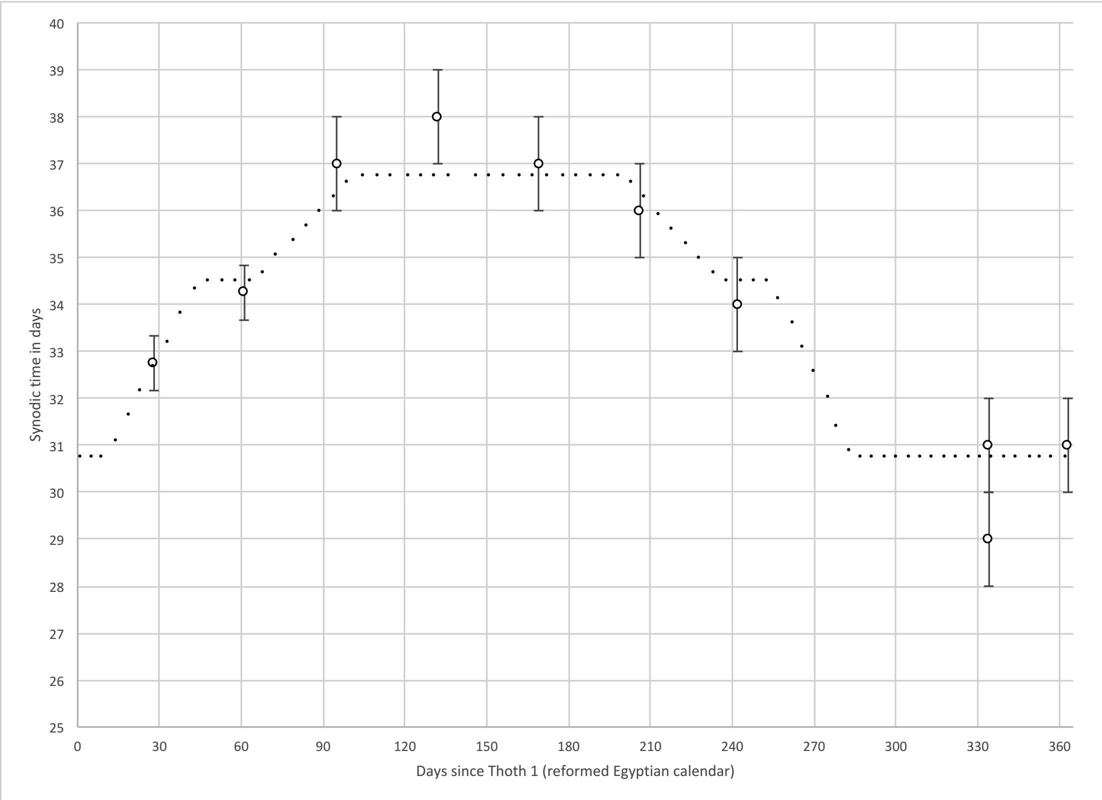

Fig. 6. Synodic times from P.Stras. inv. 1097 (hollow black circles) compared with values generated by System A'

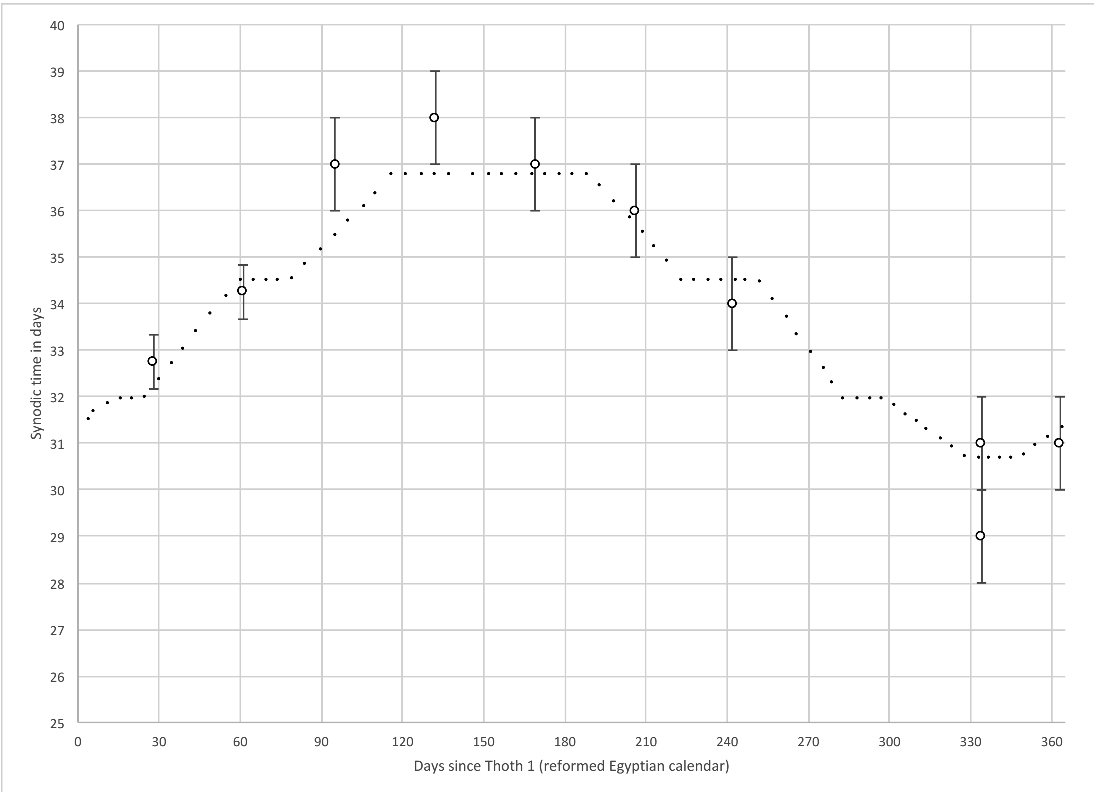

Fig. 7. Synodic times from P.Stras. inv. 1097 (hollow black circles)
compared with values generated by the model of P.Oxy. astr. 4160 + P.Berlin 16511

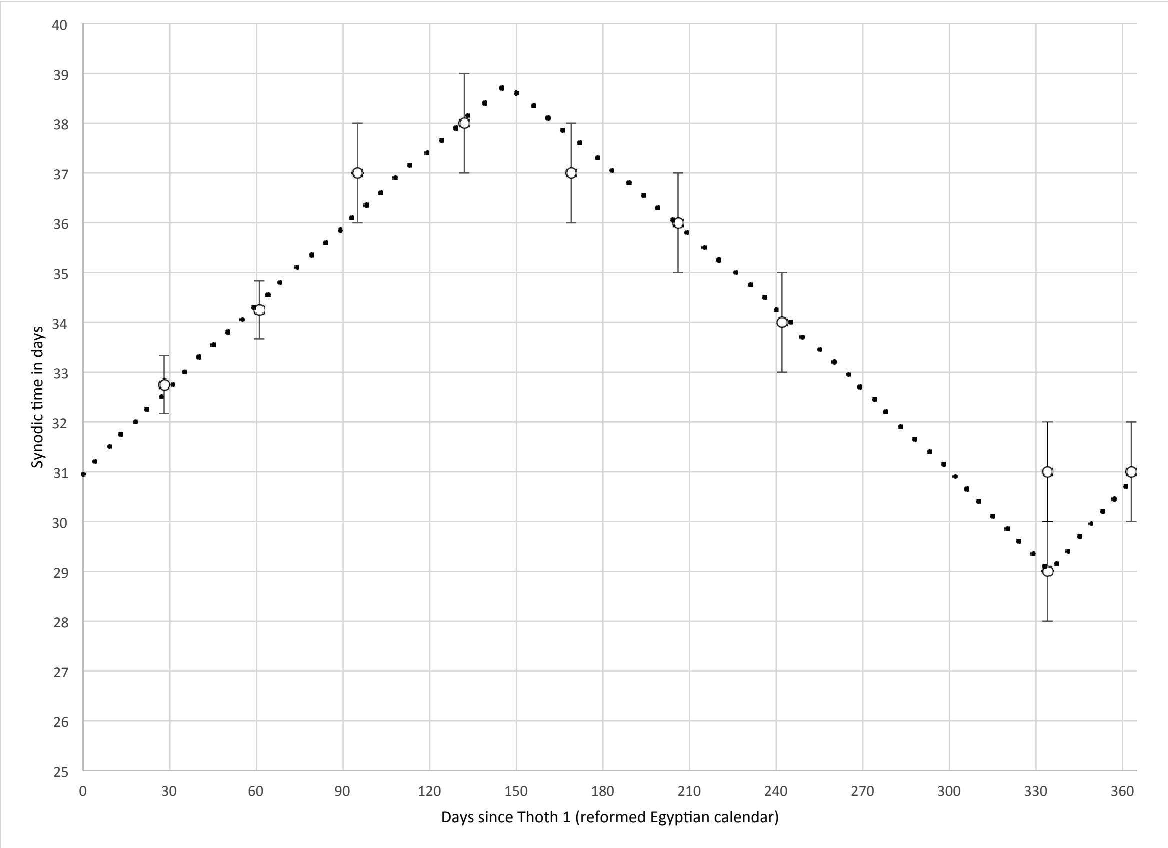

Fig. 8. Synodic times from P.Stras. inv. 1097 (hollow black circles) compared with values generated by System B

According to System B, on the other hand, synodic times vary between about 29 days and a little under 39 days; this model is well attested in cuneiform tablets and is found in P.Oxy. astr. 4160a, an Epoch Table listing first morning visibilities of Jupiter for AD 6–13.[16] System B fits the synodic times of *S* well (Fig. 8), accommodating the otherwise problematic high value, and suggesting that the day number in line 2 should be read as 20 followed by a trace of a sexagesimal fraction so that the smallest synodic time would be 29 days, a value smaller than any of the System A models could have generated.

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[16] A variant model, called System B', is attested in a small number of cuneiform tablets; the parameters are close to those of System B.