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THE ECLIPSE OF HIPPARCHOS
AND
THE SIZE AND DISTANCE OF THE MOON AND THE SUN

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THE ECLIPSE OF HIPPARCHOS AND

THE SIZE AND DISTANCE OF THE MOON AND THE SUN

by

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1. Introduction

In the afternoon of November 20 of the year 128 BC a solar eclipse occurred which was total in the Attika peninsula, the island of Lesbos and in Nicaea (modern Iznik), the birthplace of Hipparchos. The astronomical elements of this eclipse can be taken from the Canon of Eclipses of Oppolzer[1] (his number 2566). In the following I will reproduce material indicating that Hipparchos may have used this particular eclipse for the measurement of the distances of the moon and the sun. This problem, as old as Hellenistic astronomy itself, goes back to Anaximander[2]. A treatise on the sizes and distances of the sun and the moon by Aristarchos has come down to us and has been translated by Heath[3]. Aristarchos worked out two geometrical constructions through which the lunar and solar distance can be determined by observation. No careful observations were made however and the data needed were estimated only. Yet this was a significant step forward with respect to earlier astronomers, who had not even estimated but just picked nice numbers. The constructions of Aristarchos were used by Hipparchos as the basis of his observations, and it looks as if he was the first to realize the importance of careful measurements. A survey of the results over the centuries is collected in the following table

		Mean distance of	
		moon	sun
1. Anaximander[2]	600 BC	19	27
2. Aristarchos[3]	270 BC	19	400
3. Hipparchos[4]	130 BC	67	2500
4. Poseidonius[5]	100 BC	52	13000
5. Ptolemy[6]	160 AD	59	1200
6. Copernicus[7]	1500 AD	60.3	1150
7. True values	1900 AD	60.4	23450

The distances are given in earth's radii $R_{\oplus}$.

Why the solar distance of the philosopher Poseidonius is so close to the correct value is not known. Hipparchos is closer to the value for the sun than Ptolemy, possibly because the measurements underlying his result were slightly more precise. The errors in these data are however so large that one should not attach too much value to the discrepancy between the two.
Some time ago I became interested in what Hipparchos may have done to reach his result, and to this effect I decided to repeat the observations. On May 20 1966, around noon, there was in the same region of the mediterranean an almost total eclipse, which followed

the same path as the one of 128 BC (Figure 1). I observed this eclipse near Athens and tried to use it to repeat the procedures of Hipparchos.

It is a pleasure to acknowledge here the help I received from Dr. E.T. Prothro, Dean of the School of Arts and Sciences of our University, who made my trip to Athens possible.

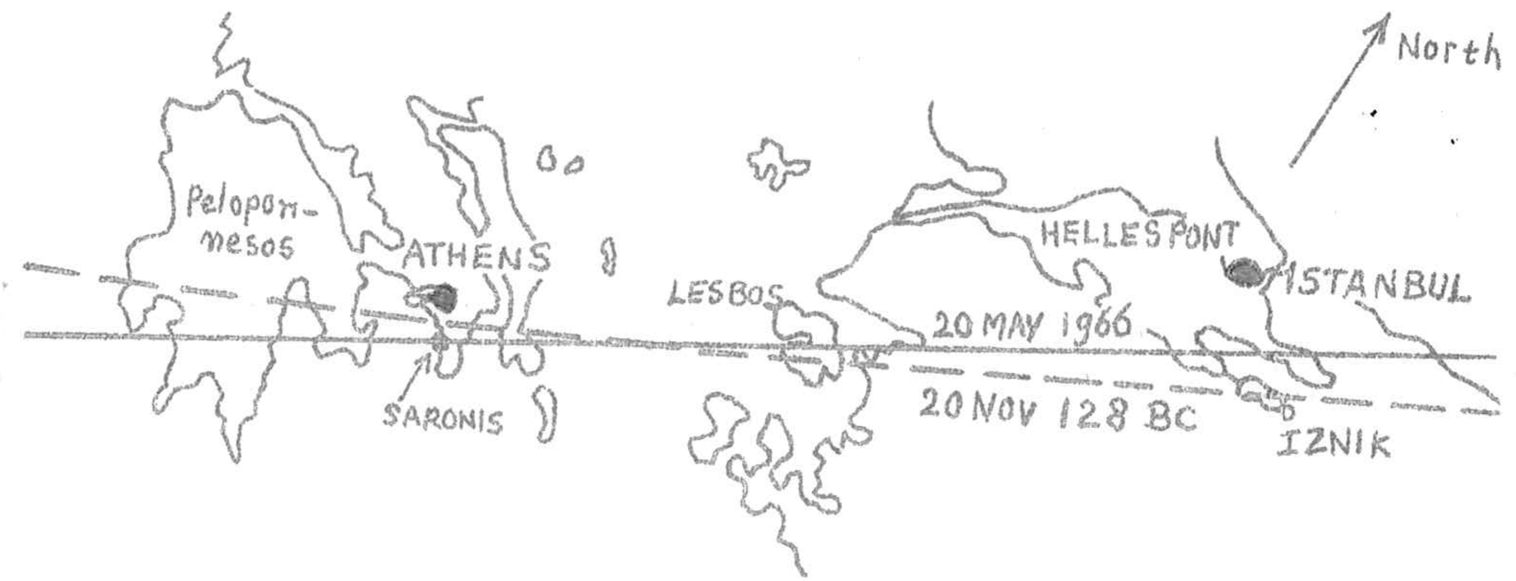

Figure 1. Map showing the path of centrality for the eclipses of 128 BC and 1966 AD.

2. Eye-witness reports on two solar eclipses

Before giving my own account on the eclipse of 1966 it is of some interest to reproduce a report made just over a century ago by Julius Schmidt, who like myself worked on naked-eye astronomy and also made his observation near Athens.

The solar eclipse on December 31, 1861

by J.F.Julius Schmidt
director of the observatory of Athens[8]

As a result of the accurate calculation of Dr. Weiss, I changed my original travel plan and decided, since I had given up already to go on a longer journey, to proceed only to the northern part of the Peloponnesos, specifically to the region of Nemea and Phlious, where the sun should set totally eclipsed. I left Athens on the 25th, traveled over the corinthian Isthmos on December 26th, where I experienced the severe earthquake of Aigion, and on December 29th went up the Longopotamos, via Kleonae and Nemea to Hagios Georgios at the southern foot of the Trikaranon. The weather had been predominantly bad all the time. On the 30th of December it cleared up completely and I climbed mount Polyphengos, which I had chosen for observation of the eclipse. The Koilossa (Megalo

Vouno), like the other naked rocky mountains, was completely inaccessible because of mighty snowlayers and the pathless uninhabited wild jungle. The 31st of December the sky was completely overcast and the sun did not show up for a second; often it rained at northern wind. Nevertheless I went up again on the 300 Toisen high Polyphengos and installed the telescope near the old ruin. Impervious clouds covered the sky, hiding all mountain peaks till 350 Toisen down. At 3h 53m (Athens mean time) in the far south-east rays from the sun slid through the plain of Argos. There sunlight remained in three flaming streaks, more and more paling, as the eclipse progressed, till the setting of the sun. In their light I saw Mykenai, Tiryns and Nauplia, from where one could have observed the event reasonably well, however also not in the most important phase. Between 1h and 5h the temperature on the Polyphengos decreased from 2.9 to 1.0 degree centigrade. At 4h 30m came a final blow of the eclipse wind from the south-west; at 4h 45m an explosion or a weak earthquake; at 4h 51m a dreary grey-brown darkness in which it was hard to read pencil writing. At 4h 55m it became quickly clearer and at the same time rain and hail poured down. Twilight remained clear enough to descend without lantern along the rocks into the Aropos valley. The night was starry bright and the two following days of an admiring clearness. On the 3rd and 4th of January I returned via Megara to Athens.

Athens, January 7, 1862.
(translated from the German)

The solar eclipse of May 20, 1966

The day before the eclipse, as I made the two hour flight from Beirut to Athens, the sky over the mediterranean was spotted with clouds and when I arrived in Athens it was quite cloudy. Meteorologists forecasted bad conditions. On the morning of the 20th however the sky was as clear and blue as could be. I joined a group of fifty amateur astronomers of the Swiss Astronomical Society, who the day before - by vote - had decided to choose a site 30 km south of Athens near the village of Saronis at the west coast of the Attika peninsula ($\lambda = 23°54'$, $\varphi = 37°45'$). A few of these had gone ahead as scouts so as to keep a claim on the selected area, since several other groups of Astronomers were expected with their telescopes. Indeed, when we arrived by bus, we found that Greeks and other nationals had already installed their instruments and were busy practising for the coming event. On the hills and houses around were perhaps altogether 200 of them. Most of the Swiss had in mind to take home a marvelous color slide of the corona at centrality and soon everyone was busy setting up his telescope and camera. A large and quite amusing variety of means was used to keep an eye on the sun, and one wondered what would be more interesting: the eclipse itself or the activities of those watching it. During the event also an airplane with observers repeatedly passed over low along the strip of centrality, which at our position was only 700 meter wide.

The eclipse proceeded exactly according to the data given in the Astronomical Ephemeris for 1966, further worked out for Greece by the Astronomical Institute of Athens and by Dr. R.A. Naef of Zuerich.[9]

The Swiss had brought with them a portable battery-powered quartz oscillator clock of high accuracy. They recorded the moments of first and last contact and a signal was given to everyone nearby at the moment of centrality at 11h 31m 15s zone time (9h 31m 15s GMT). Centrality lasted 1.5 second.
Before the eclipse had started it was hot to be out in the sun, but at 10h 30m the landscape got an unusual bluish-violet hue, as one may see in northern regions on a very cloudy and dark day before a thunderstorm. Near centrality it became quite chilly and for a moment it became so dark that it was hard to read a letter. At this moment of course no one was trying to do so. All were nervously strugling with their cameras and lenses. I myself was lying on my back and trying to fix the position of the sun in the ecliptic by means of the nearby star Aldebaran, which was 11 degrees East of the sun. I regret to report that I failed to see the star. Since I was well prepared and keen to see it, I would say that it is unlikely that observations with this purpose were made in antiquity, even when the eclipse lasted longer.
We all saw Venus very clearly for quite some time before and after centrality, but I believe that no stars were seen by anyone. For my own observations I measured time with a gnomon of 80 cm height having at the top a sphere of 2 cm diameter. It was interesting to follow the deformation of the shadow of this sphere as the eclipse progressed. According to this gnomon totality occorred 3/4 hour before local noon. Since the longitude of the place of observation was 23° 54' and the sun on May 20 was $3\frac{1}{2}$ minutes early at transit, it follows that according to this primitive sundial local transit took place at 12h - (1h $35\frac{1}{2}$m + $3\frac{1}{2}$m) = 10h 21m GMT and centrality at 10h 21m - 45m = 9h 36m, which is only 5 minutes more than the correct value. For the ancient Greeks it was certainly not very difficult to determine the moment of totality to within 15 minutes.

Image of total eclipse at Athens

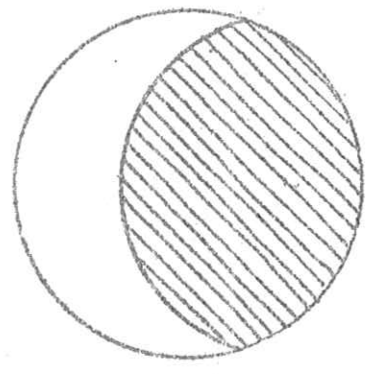

Image of maximum eclipse (70%) at Egypt

Figure 2

For the purpose of my investigations I had asked friends in Egypt and on Rhodos to report to me on the eclipse, and also someone in Iznik sent his observations to me. In Cairo Dr. M. El Nadi kindly advised some of his students to make careful drawings of the solar image projected from a circular orifice of 1.5 mm diameter, held about one meter above the plane of projection. From these drawings it can be concluded that the moon covered a maximum of 70% of the solar diameter around 9h 30m GMT, in accordance with the theoretical data. On Rhodos Mr and Mrs S.M. Hilmy climbed mount Stephanos near the town of Rhodos. On the spot where Hipparchos may have watched the eclipse of 128 BC they made a sketch of the solar image, like this was done in Cairo. Some time after returning home I received a letter from Mr N. İnceler, professor of French in Iznik (Hipparchos' birthplace) reporting on the eclipse, which was practically total there, with a set of accurate drawings of the successive phases. All these observations were made without the use of instruments involving optical lenses, in the combined effort to reconstruct the original situation as much as possible. The recordings were therefore simple and not very accurate. Yet from the egyptian observations it follows at once that in the ancient eclipse, like in the present one, the solar diameter as seen from Alexandria was not covered for 4/5th (see below) but for 70% or 3½/5th. Since the distance from Alexandria to Athens is about 1000 km, it follows from the simple theory given below that the lunar diameter is 10/3 x 1000 km ≈ 3300 km. This is exacly the correct value, but the outcome is fortuitous because the value of 70% is not very accurate. Taking ½ degree for the apparent diameter of the moon we find immediately for the lunar distance $(360/\pi) \times 3300$ km = 380 000 km or 58 R_e. The observation in Rhodos was somewhat difficult because of a strong north-western wind which prevails the year round on mount Stephanos. From the drawings and gnomon observations made there one deduces an eclipse of over 80% at around 9h 40m GMT, in accordançe with the Ephemeris. The unexpected illustrated letter from Mr İnceler was significant in that it confirmed that friends of Hipparchos would have informed him about the total eclipse they witnessed. I arrived in Turkey the day after the eclipse. I was told that the weather had been bad and that in several places the eclipse was seen directly through the clouds.

3. Hipparchos and Ptolemy on the lunar and solar distance

In first approximation the size of the moon follows directly from the observation of central lunar eclipses. As these can be observed more frequently than solar eclipses, it is likely that this is the way in which the Greeks first obtained an idea about the size and distance of the moon. A more accurate approach is through two observations in different places of the same solar eclipse. We have evidence for such a method used by Hipparchos for the eclipse of 128 BC. Although much of the work of Hipparchos is referred to in the Handbook of Astronomy (Almagest) of Ptolemy[6], one finds no explicit mention there of the eclipse of 128 BC. It was discovered by Hultsch[4] however that Pappos in his commentary on the Almagest mentions and quotes from two books (now lost) written by Hipparchos in which this eclipse is described. In his commentary on Book 5, chapter 11, of the Almagest Pappos writes:

"..... Hipparchos has carried out such an investigation inaccurately, in that he started primarily from the sun. He had observed that at the conjunctions, when the sun and moon are farthest away from the earth, their diameters are almost equal, and assumed further that the diameters of sun and moon are known in size - with which Ptolemy deals in a later section (of Book 5).

It follows from this that, when the distance of one of the two luminaries is given - like it is proved (by Ptolemy) in the 12th theorem (Book 5, chapter 15) - that when the distance of the moon and the diameter of the sun are given, also the distance of the sun is given. Therefore, in his final conclusions Hipparchos starts from the sun and tries to find its parallaxes and distances as well as the distance of the moon, whereas - like Ptolemy writes - as regards to the sun it is doubtful anyhow not only by what amount its apparent diameter varies, but also whether it varies at all. Whereas now Hipparchos was uncertain about the sun, not only (because he did not know) by what amount its apparent diameter varies, but also whether it varied at all, he assumed in his first book on the sizes and distances (of the sun and moon) that the earth with respect to (the orbit) of the sun is like a point or center (to a circle). With reference to the solar eclipse described by him, he took as a basis for his observations that the apparent solar diameter sometimes shows a very small, at other times a larger difference, so that also the calculations of the lunar distances turned out to be different. For in the first book on the sizes and distances he describes the following phenomenon: In the region of the Hellespont occurred a solar eclipse which was exactly total, whereas in Alexandria in Egypt only about 4/5th of the diameter was covered. On the basis of these observations he shows in the first book that, if one takes the earth's radius as a unit, the smallest distance of the moon amounts to 71, the largest to 83 and the mean to 77 earth radii. After having first solved the problem that was placed before him, he adds at the end of the same book: "In this treatise I have given proof of these results. In order that the reader will not think that the discussion on the distance of the moon has already reached a completely satisfactory conclusion, I note that to this end a further investigation should be carried out, according to which the distance of the moon will turn out to be smaller than the distance calculated". With this he admits himself that he can report absolutely nothing reliable about the parallaxes. He further shows extensively in the second book on the sizes and distances that the smallest distance of the moon amounts to 62, the mean to 67 1/3 earth radii and the distance to the sun to 2490 earth radiii From this[E] also follows that on the largest distance of the moon fit 72 2/3 earth radii."

The above extract from Pappos is a classical example of the tragic fact that the importance of most commentaries lies primarily in their quotations. Ignoring Pappos' critisisms and implications we are left with unique information about Hipparchos and his lost books on the sizes and distances of the moon and sun. Unfortunately the commentary does not disclose the methods of observation and calculation that were applied. These I think can be reconstructed from the book of Cleomedes. From the Canon of Eclipses[1] one would naturally select the eclipse of 128 BC as the one fitting the description. In the Almagest it is mentioned that Hipparchos did observations in Rhodos in May and July of 127 BC, so that it is quite possible that he saw the eclipse there in November 128. Concerning the method followed we note that Ptolemy, in his calculation of the distance of the sun, says (see below) that he follows the method of Hipparchos. Furthermore Cleomedes in his book 'The circular motion of the luminaries' describes in a paragraph how the eclipse of interest was used to find the size of the moon. Altogether we thus have a fair picture of what was done by Hipparchos. I reproduce the text of Cleomedes[10] for as far as it is concerned with our subject. It seems that Cleomedes did not always understand fully what he was writing about. Yet the treatise is clear enough to understand the meaning. In the translation I made three small corrections in conflicting statements, which do not affect the general bearing. The interested reader may look up the text in Greek for the exact wording[11]. When Cleomedes lived is not known, but it may have been around 50 AD, and it seems he lived before Ptolemy.

Cleomedes

The size of the moon and the stars

Book 2, chapter 3 of his treatise "The circular motion of the luminaries"

"That the moon is not as large as it appears to be, can be seen from what has been told already about the sun. For most of what has been said applies also to the moon. Best of all however it is shown by a solar eclipse. It is namely so that the sun is darkened only because the moon passes between the sun and us and covers the sun from our eyes. The eclipse is therefore not an event in the sun itself, but only an optical phenomenon to us. Therefore, as soon as the moon meets the sun and moves between it and the earth, it will send a conical shadow to the earth. The shadow must be smaller than 40 000 stadia (the earlier mentioned diameter of the moon). Any point on the earth, from which the sun cannot be seen because the moon is standing in between, is lying inside this moon shadow. When this conical shadow extends to the earth or further, it is clear that at its base, the radius of which equals the radius of the moon, it is many times as large (as it is on the earth).

Now the following observations have been made during a solar eclipse: Once when at the Hellespont the sun was totally eclipsed, 1/5th of its diameter, which seems as large as a little more than two fingerwidths, was not eclipsed in Alexandria. In comparison, the total apparent measure of the sun and the moon is 12 finger-widths. It is clear therefore that an apparent measure of two fingers in the size of the moon corresponds to the distance on the earth from Alexandria to the Hellespont. Now Alexandria and the Hellespont lie under the same meridian. If we assume that the eclipse lasted for a time long enough that meanwhile someone could travel from Alexandria to the Hellespont, then the part of the solar diameter that is seen in Alexandria would be reduced corresponding to the distance covered. The distance between Alexandria and Rhodos amounts to 5000 stadia, and the distance from there to the Hellespont to another 5000 stadia. Therefore the part of the solar diameter which can still be seen in Rhodos will have the apparent width of one finger. If one travels next from there to the Hellespont, then again the visible part of the solar diameter will decrease according to the measure of ones progress, till finally at the Hellespont the sun is totally eclipsed. It is clear therefore that when to such a distance on the earth corresponds two fingerwidths on the moon, the total diameter of the moon must correspond to the sixfold distance from Alexandria to the Hellespont."

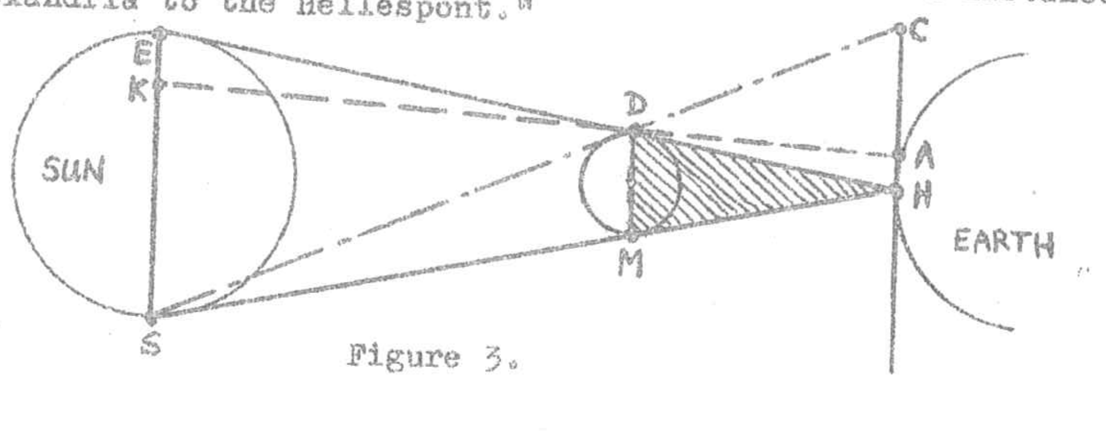

Figure 3.

The arguments of Cleomedes can be illustrated with figure 3. Let in this figure SE be the solar diameter, MD the lunar diameter, HAC the surface of the earth. In the Hellespont, H, sun and moon are seen as equal in apparent size and so the top of the shadow cone is just in H. In Alexandria, A, one sees only 1/5th of the sun's diameter. Now HA:HC = EK:ES = 1:5. Since the sun is much farther away than the moon MD ≙ HC, so that MD ≙ 5HA = 5000 km. Why Cleomedes at the end takes two fingerwidths and a sixfold distance is not clear. In the accurate calculation one should take into account tha MD is slightly less than HC.

The descriptions of Pappos and Cleomedes deal with the following section of the Almagest. At the end of chapter 11, Book 5, Ptolemy writes:

"In his efforts to find a parallax Hipparchos primarily started from the sun. From some other data appearing in relation to the sun and the moon - about which will be told in the following chapters - it can namely be concluded that, if the distance of one of the luminaries is given, also the distance of the other one can be determined. He therefore tries in this way by an approximate estimate of the solar distance also to determine the distance of the moon.

First he starts from the assumption that the sun shows only a just observable parallax, so as to determine from this its distance. Later however, presumably because the sun shows neither an observable nor a sufficiently large parallax, he tries to reach his aim by means of a solar eclipse that was reported by him. Since the distance of the sun remains doubtful throughout, not only with respect to the amount of its parallax, but also because it is questionable whether it has any at all, also his dataon the lunar distance comes out differently, depending on the assumption made."

We have now reached the following situation: Pappos reports that Hipparchos in his first book on sizes and distances describes the eclipse of the Hellespont. Cleomedes explains how this eclipse was used to find the lunar distance. Ptolemy confirms that Hipparchos used a solar eclipse for this purpose. Finally, it is shown by Hultsch that this was probably the eclipse of 128 BC. This being settled, let us now see how the distance of the sun was found.

In chapter 12 of the Almagest Ptolemy describes the so-called parallactic instrument, apparently designed by himself and used by him to measure the parallax of the moon. From it he computes as the maximum distance of the moon to the center of the earth 64 R_e and a mean distance of 59 R_e. In the absence of optical instruments his method is superior to Hipparchos' observation of the solar eclipse and it leads to data closer to the true values.

For the determination of the distance of the sun it is necessary to know the apparent diameter of the moon during an eclipse. Hipparchos measured the diameter with his dioptra. Ptolemy shows that during a total eclipse the moon is at maximum distance and not at the mean distance as was assumed before him. He also expresses the opinion that a more precise value of the apparent lunar diameter can be obtained by computation from the epicycle theory than from direct observation with Hipparchos' dioptra. He finds for the lunar diameter during a solar eclipse 31' 20", which is exactly the value measured for the eclipse of May 20, 1966. At the end of chapter 14, Book 5, Ptolemy writes:

"As we obtained the reported quantities in good agreement from a large number of observations we have used them not only for the theoretical investigations concerning the eclipse, but also to find the distance of the sun. This will be carried out following the same path as was taken already by Hipparchos."

Ptolemy then proceeds on the basis of a geometric construction (figure 4) which originates from the treatise of Aristarchos[5]. The results of Aristarchos were poor because he based his arguments on suitable numbers rather than on measurements. For the diamter of the moon he took 2 degrees. He may have well known however that the actual diameter was close to ½ degree. Ptolemy takes for the diameter 31' 20" and for the lunar distance TN = 64 R_e.

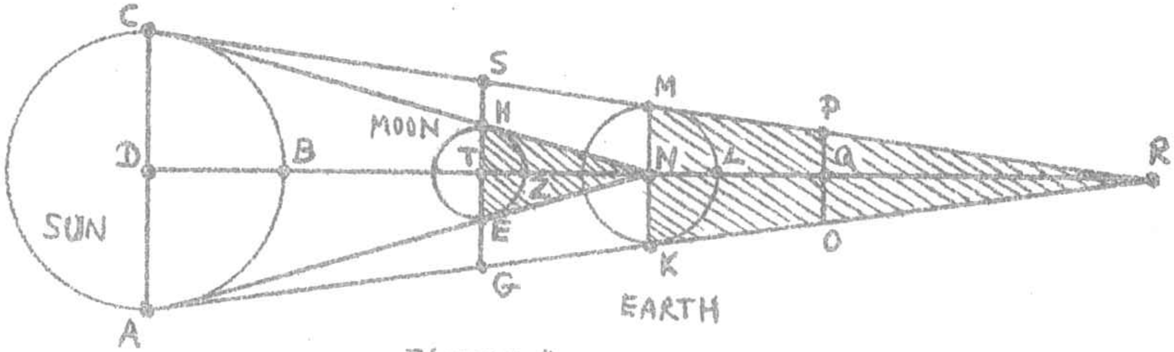

Figure 4.

Further, from the observations of lunar eclipses he finds that QP : TH = 2 3/5 : 1, whereas Aristarchos took a ratio of 2 : 1. With the points N, E, T, H, O, Q and P fixed, the figure is completely determined and the distance ND is found to be 1209 R_e. The correct value is 23000 R_e. The cause of the large error is that the lunar distance is about 400 times smaller than the solar distance. A small error in OP will therefore have a large effect on the final result. Ptolemy mentions (Book 4, chapter 9, line 8) that Hipparchos took for the measure of the shadow cone OP at the mean lunar distance 2½ times the lunar diameter. This is a little too large, but better than Ptolemy.

It should be noted that the ratio EH/OP, which is required to find the solar distance, is almost equal to EH/EP, and therefore in first approximation directly gives the size of the moon. This is probably how the lunar diameter was first determined. It was Anaximander who first realized that the phases of the moon and the eclipses were caused by sunlight and shadows.

In preparation for the eclipse of 1966 I observed the lunar eclipse of December 18, 1964 in New Haven, Conn., USA, from the start at 8 p.m. eastern standard time. The sky was clear, the moon high, and it was very cold. The moon in the umbral shadow was brownish red with what looked like a bluish border. The size of the earth's shadow could be estimated from making a drawing and extrapolating the curved edge. This indeed leads to roughly 2½ lunar diameters. A better value is obtained from the time it takes the moon to pass through the umbra, which was 2h 30m. The synodic motion of the moon amounts to 360 degrees per 29½ days, or 77" per hour. This gives an apparent diameter of the shadow cone of 2½ x 77' = 192'. Since the lunar diameter is 31 1/3' the shadow is 2.45 times as large.

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