Putting the astronomy back into Greek calendrics: the *parapegma* of Euctemon

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It is a pleasure to be able to offer a paper to our honorand. Many years ago James Evans established himself as a great teacher of the history of ancient Greek astronomy to many beyond the confines of his own lecture room through his book, *The History and Practice of Ancient Astronomy*. While in more recent years he has provided us with sophisticated papers on the more technical aspects of astronomy, especially as they pertain to the Antikythera Mechanism, it is to that earlier monograph, and its impact on myself and my own students, that I wish to pay homage in this small offering on ancient "observational" astronomy.

Parapegmata

Calendars across all cultures in the world have traditionally relied on the observed and/or measured motions of the celestial bodies: the sun, the moon, and the stars. From an early stage in historical Greek society the motions of the stars in particular played a significant part in time-measurement and the development of calendars. Alongside the phases of the moon (the fundamental basis of all Greek civil calendars) and the apparent movement of the sun (which provided seasonal markers), the Greeks also used the appearance or disappearance of stars at dawn or dusk to establish schedules for timekeeping.[1]

While we are aware of these uses of astronomy from the poems of Homer and Hesiod, from the fifth century BCE onwards there is evidence of an increase by Greek astronomers in the number of first and last star risings and star settings, and a formalisation of these data-sets into what were called *parapegmat*a. These survive from the Hellenistic and Roman periods as stone tablets inscribed with day-by-day entries for the appearance or disappearance of stars.[2] We may regard these as "star schedules," or more loosely as "star calendars." These were set up in public spaces in the cities, and therefore presumably had a civic significance beyond the narrowly astronomical, much as clocks did in Medieval and Early Modern Europe. Many of the leading astronomers of antiquity were credited with *parapegmata*—the list includes, from the fifth century, Democritus (more a philosopher than an astronomer, but who wrote a work called *Parapegma*)[3], Meton (associated with the 19-year "Metonic Cycle," which still governs the placement of Christian Easter in the western calendar, and the Jewish calendar as a whole), and Euctemon; and, from the fourth century, Eudoxus and Callippus.[4] *Parapegmata* continued in use until the Medieval period.[5] In literary form, they were combined into compilations and published either in their own

1 Hannah 2009, 2005, 2001.

2 Hannah 2002: figs. 6.1, 6.2.

3 Diogenes Laertius 9.48.

4 Dicks 1970: 81, 84–85.

5 McCluskey 1998.

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right by astronomers (e.g. in Geminus, *Eisagoge*; Ptolemy, *Phaseis*), or subsumed into agricultural "handbooks" by literary authors, especially by the Romans (e.g. Columella, *De re rustica*; Varro, *De re rustica*). Star lore of this kind pervades every aspect of Greek and Roman literature: it can be found in all the major authors, from Aeschylus to Euripides and Aristophanes, through Aratus to Plautus, Vergil and Ovid and beyond. Julius Caesar was credited with a "star calendar," which survives in later quotations (notably by Pliny, *Historia Naturalis* 18). In all, about 60 *parapegmata* survive in epigraphical and literary form.[6]

A generation before Democritus (b. ca. 470 BCE), the earliest of these authors, the philosopher Anaxagoras was already writing scientific treatises in prose, rather than in the poetic form that his predecessors had regularly used, so it seems to me likely that these early *parapegmata* appeared in prose form too. Whether these works were lists, as the term *parapegma* can signify and as they have come down to us through later compilations organised according to some temporal system (day-count, zodiacal months, calendars) or were embedded in treatises on broader topics from which the data have later been excerpted, it is impossible to tell now.[7] Equally whether there were more data than have survived in the compilations, or whether some data have been misattributed to these astronomers, is also impossible to know. Source criticism demands that such caveats be kept in mind, but they do not invalidate work based on what has survived; they simply make certainty impossible in the current state of affairs. In what follows, the "*parapegma* of Euctemon" signifies what authors in antiquity thought it meant, namely a list of star risings and settings, sometimes allied with other astronomical or meteorological data. Such a *parapegma* may have been a stand-alone entity (written in more or less perishable form), or it may have been derived from some larger work by Euctemon.

The *parapegma* was first dealt with in any significant manner in a series of publications by Rehm in the first half of the 20th century.[8] Several papers by van der Waerden treated the topic in the second half of the century,[9] but because he based his work on Rehm's, it is still the latter's views which lie behind his scholarship. Unfortunately, as others have noted, Rehm mixed fact and hypothesis indiscriminately in his conclusions,[10] and a rigorous critique of his work shows that there is a need to re-lay the astronomical foundations for work on the subject. Furthermore, Rehm and van der Waerden were primarily interested in simply reconstructing the *parapegmata* as lists of observations, rather than situating them into a wider context. The broader scientific and cultural context is what the most recent, and now fundamental, book specifically on *parapegmata* aims to provide, as it also presents all *parapegma*-style texts in full.[11] My own approach in this paper is to interrogate the accuracy of the *parapegma* as a time-marking instrument that once pervaded many aspects of Greek and Roman culture.

Accuracy

Despite the pervasiveness of data about star phenomena in the broader culture of Classical antiquity, the information that can be derived from *parapegmata* has been regarded as inaccurate

6 Lehoux 2007.

7 On the question of lists and literacy with regard to the *parapegmata*, see Hannah 2001.

8 Diels and Rehm 1904, Rehm 1913, Rehm 1949.

9 der Waerden 1960, Pritchett and van der Waerden 1961, van der Waerden 1984a, 1984b, 1984c, 1985.

10 Neugebauer 1975: 595 n.17, Lehoux 2007: 89, 2005: 125–26.

11 Lehoux 2007; cf. a similar approach for Babylonian astronomical texts in Rochberg 2004.

in itself or in the uses to which it was applied. This issue goes back to antiquity—Pliny the Elder (*Historia Naturalis* 18. 210–213) complained about the different dates given in his sources for the same star phenomena when observed in the same country. The complaint is picked up in modern scholarship.[12] This has been especially the case with regard to the dates for the phenomena given by the Roman poet Ovid in his calendar-poem, the *Fasti*.[13] But recent studies suggest that the astronomical basis of the star data relayed by Ovid has been misconstrued, and that modern parameters for accuracy have been imposed anachronistically on the ancient data.[14]

A *parapegma* afforded the facility to measure time through the year via the stars by pinpointing in the year when a star first became visible at dawn or dusk, or was last seen at dawn or dusk. For a *parapegma* to be able to state when a star would be first visible or last seen in the dawn or dusk sky, it has generally been assumed that ancient observations, like all modern ones, took the magnitude of each star into account, and therefore required the sun to be at different distances below the horizon, depending on the apparent brightness of the star. To be sure, Ptolemy himself says as much when discussing the methods by which one could calculate first and last visibility,[15] but this ignores the earlier practice in antiquity of calculating the times of star-rise and star-set by assuming a fixed angle for the sun below the horizon (on which, see below).

In the early 19th century Ideler calculated the angles for the solar depression on the basis of the 1st- and 2nd- magnitude stars, and the dates for first and last visibilty at the five different latitudes (*climata*) that were presented by Ptolemy in his *Phaseis*. Ideler's final angles were then the result of his averaging these calculations based on Ptolemy's dates.[16] In the early 20th century Boll, Ginzel, Vogt and Neugebauer presented the dates for star-rise and star-set in tables, again based on the assumption that solar angular depression was dependent on the magnitude of the star.[17] But even these tables depended on Ptolemy, as Ginzel noted. Thus, for morning risings and evening settings of 1st magnitude stars, the angle for the sun below the horizon was 11°–12°; for 2nd magnitude, 13°–14°, for 3rd magnitude, 14°–16°, and for fainter stars, 15°–17°. For evening risings and morning settings, the respective limits were 7°, 8.5°, 10°, and 14°. Ginzel added that the calculations agreed with naked-eye observations, with the angle of the sun below the horizon for (presumably) the morning rising and evening setting of Sirius being about 10°, for Aldebaran (1st magnitude) 10°–11.5°, for Regulus (magnitude 1.3) 12°, for α Arietis (2nd magnitude) 11°, and for the Pleiades (η Tauri, 3rd magnitude) 15°–16°. Neugebauer simplified these parameters for morning risings and evening settings to 11° for stars brighter than magnitude 1.5, 14° for magnitudes 1.5–2.5, 16° for magnitudes 2.5–3.5, 17° for magnitudes 3.5–4.5, and 17° for stars

12 The observations of the *parapegmata* are explicitly called "rough" by Bowen and Goldstein 1988: 56. More importantly, their accuracy is questioned in one of the standard modern editions of the work in which the *parapegma* of Euctemon is embedded (Aujac 1975), where one will find notes drawing the reader's attention to differences between the dates of observations given in Geminus's compilation and those calculated in modern times. Aujac (1975: 158) expressly includes among the possible causes for these discrepancies "manque de précision dans l'observation."

13 Ideler 1825: 137–69. J. G. Frazer's influential commentary (Frazer 1929) set Ideler's view in concrete for generations; his views are repeated and embellished in recent treatments of the poem, e.g. Bömer 1957–58, Herbert-Brown 1994, Nagle 1995, Newlands 1995, Fantham 1998, Gee 2000.

14 Hannah 1997a, 1997b, Fox 2004; Robinson 2009, Lewis 2014.

15 Ptolemy, *Almagest* H198–204 (Toomer 1998: 413–17).

16 Ideler 1819.

17 Boll 1909: 2429–30; Ginzel 1906–14: 2. 517–22 (on which Bickerman 1980: 112–14 was based); Vogt 1920; Neugebauer 1925.

fainter than magnitude 4.5; and for evening risings and morning settings to 7˚ for stars brighter than magnitude 1.5, 8.5˚ for magnitudes 1.5–2.5, 10˚ for magnitudes 2.5–3.5, 14˚ for magnitudes 3.5–4.5, and 17˚ for stars fainter than magnitude 4.5.[18] Later Neugebauer would update his parameters on the basis of naked-eye observations made by Schoch, and indeed Schoch's work is important in at least three respects in our present context: he made actual observations of first and last star phases to arrive at his angles of solar depression; he noted that previous calculations traditionally had ignored the effects of refraction and therefore assumed the stars were at the horizon rather than some degrees above it after clearing the atmosphere; and he was aware that reliance on Ptolemy's data was permissible only in Alexandria, where Ptolemy was based, because in places like Athens and Babylon the atmosphere was clearer and observations would reach markedly different dates for morning firsts and evening lasts[19]. Schoch's measurements "for places with a very clear sky (such as Babylon and Athens)" for first morning rising and last evening setting for stars near the ecliptic were: magnitude -1: (morning rise) 9.5˚, and (evening set) 8.5˚; magnitude 0: 10.5˚, and 9.5˚; magnitude +1.0: 11.5˚, and 10.5˚; magnitude +2: 13.5˚, and 12.5˚; magnitude +3: 16.0˚, and 15.0˚.

Later work addressed some of the difficulties inherent in naked-eye observations of these star phases, which Schoch had hinted at, in particular on calculating the "extinction angle" of a star at the horizon, i.e. "the smallest apparent altitude at which, in perfectly clear weather, it [the star] can be seen."[20] Matthew Robinson has provided a concise and lucid of history of some of these investigations, notably those of the astrophysicist Bradley Schaefer, so there is no need to enter into great detail here.[21] Let me instead note the practical approaches that stem from the work of Alexander Thom, who worked on the henges of Britain. Thom developed a rule of thumb according to which a star's extinction angle was "roughly equal to the magnitude of the star, so that a third-magnitude star cannot be seen below 3˚."[22] Mann, having noted that Thom's Rule was based on having "perfectly clear weather" and that such conditions were not common in Britain where both he and Thom worked, has suggested a more practical estimation would be "Thom's Rule +1," i.e. an angle computed from the value of the magnitude of the star plus an extra degree.[23] Interestingly, this revised rule may produce results not dissimilar from those derived from calculations using Schaefer's formulae for poor visibility conditions.[24]

18 Neugebauer 1925: 60 Tafel 28. I am taking Evans' point that the modern terminology of heliacal rising (for morning rising), heliacal setting (evening setting), achronychal rising (evening rising) and cosmical setting (morning setting) can be unhelpful and is largely anachronistic: Evans 1998: 197; see also Fox 2004: 104, Robinson 2009: 356–58.

19 Schoch 1924: 731–32. He notes that "in Alexandria there was always a layer of mist covering the lower part of the horizon," a comment that will resonate with anyone who has been near the Nile. For more recent "in the field" observations, but in a different context, in modern Australia, see Leaman, Hamacher and Carter 2016.

20 Thom 1967: 15.

21 Robinson 2009. See Schaefer 1985, 1986; cf. Ruggles 1999: 52. For an instance in Greece in which Schaefer's approach is taken into account, see Salt and Boutsikas 2005.

22 Thom 1967: 15. For a recent instance of "Thom's Rule" in action, see Hannah, Magli and Orlando 2017: 7 (in relation to the Temple of Juno at Agrigento).

23 Mann 2011: 252–53 n.11.

24 Using "Thom's Rule + 1" for the morning rising of Arcturus in Rome, 5 CE, I produce a date of 23 September if Arcturus was 1˚ above the horizon and the sun 11˚ below, which matches Robinson's calculation, based on Schaefer's criteria for bad visibilty with a limiting magnitude of 5 and an extinction factor of 0.3: Robinson 2009: 307 Table 15.

A different approach

However, in contrast to these approaches that depend on the magnitude of stars, Matthew Fox has used Pliny, *Historia Naturalis* 18. 218 to posit that ancient observations of the stars took the sun to be at a certain distance below the horizon, regardless of the brightness of the star being observed on the horizon.[25] Pliny was not referring here to an angular distance, such as the 15˚, or half a zodiacal sign, that we know Autolycus used at the end of the 4th century BCE.[26] Bowen and Goldstein once speculated that Eudoxus, as an older contemporary of Autolycus, "probably assumed in his *parapegma* that phenomena will be visible if the sun is ½ zodiacal sign below the horizon,"[27] but it seems unlikely that they would agree entirely with this now, given their later affirmation that artificial zodiacal signs of 30˚ are not attested in Greek texts before the third century BCE.[28]

As Neugebauer pointed out, a scheme based on a fixed visibility limit like that of 15˚ can produce symmetries between the phases, such as those produced by Autolycus:

(last) evening setting → (first) morning rising: 30 days

(first) morning rising → (last) evening rising: 5 months

(last) evening rising → (first) morning setting: 30 days

(first) morning setting → (last) evening setting: 5 months.[29]

Tannery had discerned elements of a similar symmetry in the *parapegma* associated with Eudoxus (for morning rising → evening rising, and for morning setting → evening setting) among those included in the *parapegma* of Geminus, but not in those attributed to Euctemon, Democritus or the little that is provided from Callippus.[30] This suggests that Eudoxus, like Autolycus, assumed a fixed visibility limit for the star phases,[31] whereas Euctemon did not, but whether Eudoxus used 15˚, however equivalently expressed, is not discussed. Tannery took it, anyway, that Euctemon did not subject the results of his star observations to a similar theory.[32]

The method adopted by Autolycus, and arguably by Eudoxus if Tannery was correct to discern a pattern in his *parapegma*, of using half a zodiacal sign as a guage of the sun's depression

25 Fox 2004.

26 E.g. Autolycus, *On Risings and Settings* 2.6. For examples derived from Autolycus's *Risings and Settings*, see Evans 1998: 190–7. Cf. Neugebauer 1975: 760–1: "From an astronomical viewpoint a universal 15° visibility limit is a rather crude simplification of facts which obviously are much more complex. It cannot have escaped notice that not all stars appear or vanish simultaneously or that the eastern and the western parts of the horizon are not the same in darkness near sunrise or sunset. Nevertheless the 15° limit—or the equivalent 15-day limit for the solar motion—was generally accepted."

27 Bowen and Goldstein 1988: 56.

28 Bowen and Goldstein 1991.

29 Autolycus, *On Risings and Settings* 2.6; Neugebauer 1975: 761.

30 Tannery 1912: 234.

31 As noted by Neugebauer 1975: 761, while dismissing Tannery's deduction that it also suggested that there was a (now lost) work by Eudoxus on spherics.

32 Tannery 1912: 234.

below the horizon, relies on an awareness of the sun's apparent path along the ecliptic. In Greece the discovery of the ecliptic as the sun's oblique pathway through the zodiac is accredited to Oenopides of Chios, who lived in the late fifth century BCE.[33] The use of its constituent constellations (rather than the artifically equal zodiacal signs, which were formulated later) as temporal stepping stones through the solar year is not impossible at this time, seeing as Oenopides' older contemporary, Cleostratus, is said by Pliny to have distinguished the constellations along the ecliptic, starting with Aries and Sagittarius.[34] Indeed Dicks went so far as to surmise that to judge from their activities Meton and Euctemon were also familiar with the ecliptic.[35] Whatever the case, we might keep this method in mind for future investigations of the star data not only of Euctemon but of Eudoxus too.

Two other options suggest themselves to me for how Euctemon produced his data: either he conducted simply naked-eye observations, and/or he used a different artificial method. The former has little to recommend it, as we shall see when I investigate a few of the data from Euctemon. However, one possibility for a different artificial method is provided by Pliny in the passage already referred to: a depression of the sun below the horizon that was measured by time. Pliny says that the sun must be "at least three-quarters of an hour" below the horizon. He clarifies at *Historia Naturalis* 18. 221 that he is talking of equinoctial hours, not the more usual seasonal hours used in sundials and daily life. Le Bonniec and Le Boeuffle translated this "three-quarters of an hour" into angular terms, suggesting that this corresponds to about 12° below the horizon.[36] This angle, which equates to modern "nautical twilight," is the minimum workable angle for the sun's depression below the horizon that would enable stars down to and including magnitude 2 to be observed, which would then allow the majority of the stars in the *parapegmata* to be seen. Pliny's modifier "at least" would permit the longer period needed to include the fainter Pleiades, Delphinus (the Dolphin) and Sagitta (the Arrow), assuming the last two have been correctly identified.

While Pliny's use of a period of "at least three-quarters of an hour" is primarily a time-related criterion, it is not impossible that this still relates, in a secondary fashion, to the brightness of the stars. Through the sequences of predawn twilight (we would say from "astronomical" through "nautical" to "civil"), the sun's depression below the horizon rises from about 18° to zero, with the result that stars of diminishing brightness are gradually lost to sight. But the fact that a time period is chosen as the criterion draws us away from magnitude and into the realm of timing mechanisms. This in turn suggests the prioritisation of an external timing device over naked-eye observation of the star.

33 Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utilium* [*Aspects of Mathematics Useful for the Reading of Plato*] 198.14–15 (2nd century CE) says that "Eudemus [late fourth century BCE] reported in his *Astronomies* that Oenopides was the first to discover the belt of the zodiac." Diodorus Siculus 1.98.3 (first century BCE) says that "Oenopides, while also spending time with the priests and astronomers [of Egypt], learned other things and especially about the circle of the sun, that it has a slanting route, and makes its movement opposite to the other stars." For an in-depth of analysis of what we might know, or not know, about Oenopides' discovery of the obliquity of the ecliptic, see Bodnár 2007: 4–8 (I am grateful to Professor Anthony Spalinger and Dr Dougal Blyth for this reference).

34 Pliny, *Historia Naturalis* 2.31: "Then [following Anaximander's discovery of the ecliptic] Cleostratus distinguished the signs in it, first those of Aries and Sagittarius." Bodnár 2007: 6 interprets Anaximander's "discovery" attested by Pliny here as nothing to do with the ecliptic *per se* but instead as concerning the slant of the paths of the stars and planets against the horizon.

35 Dicks 1970: 172.

36 Le Bonniec and Le Boeuffle 1972: 264 n. 3.

Timing

One possible explanation for the use of "three-quarters of an hour" as a basis for timing is that it reflects fairly well a whole "winter" hour for Athens or Rome. If we work with seasonal, rather than equinoctial, hours, then an hour in mid-winter in Rome—i.e. one twelfth of the time between sunset and sunrise—amounts to 45 minutes, or three-quarters of an equinoctial hour, while such an hour in Athens amounts to 47 minutes. Pliny's criterion might therefore signal that the night was measured by astronomers in "winter hours" regardless of the actual season, and that this could readily be equated with three-quarters of an equinoctial hour. Such a methodology is found elsewhere in Athenian culture, making it culturally plausible in the case of the star data associated with Euctemon: the whole of the legal day, regardless of season, was made to correspond to the length of the shortest days of the year, those of the mid-winter month Poseideon.[37]

Another possible explanation for the "three-quarters of an hour" is that it corresponds quite closely to twice the value of 24 minutes, which is a sixtieth of 24 hours. This is a fraction of the full day that was recognised in Babylonian (and later Indian) astronomy. In Babylonia one 24-minute measure was six UŠ, or one *mina*, and by the sixth century BCE waterclocks of the outflow type were being made that could measure this time-period or multiples of it.[38] The same measure of 24 minutes arises much later in Indian astronomy in historical times, and was measurable via simple holed bowls that would sink into buckets of water at this given rate of 24 minutes.[39] Such a bowl, made of copper and dating to perhaps the ninth century BCE, though of what measure is unknown, has been identified from the material from Nimrud.[40]

It may be that some similar type of bowl, made of metal or pottery, was used in fifth century Athens. Waterclocks were certainly used there. Ideally we would want one of the type that noted the Greek equivalent of the number of *minas* flowing out through a whole night, whose total measure in the equivalent of *minas* was known season by season.[41] The astronomer would just need to note the start of the last two *minas* to catch any observations in the crucial period of "three-quarters of an hour" before sunrise. The waterclocks used in Greece by soldiers on night watches might have offered such a means for measuring star-rise and star-set through the seasons. We read from a fourth century BCE source that, in order to ensure that the night watches were equal, the volume of water that the clocks held could be adjusted by coating the inner surface of the waterclock with different thicknesses of a layer of wax, and by changing the waterclock every ten days. The interior was waxed with a thinner coating to allow more water to be held when the nights were longer, and with a thicker layer to make the clock hold less water when the nights were shorter.[42] Alternatively, a fifth century BCE clay bucket excavated in the Athenian Agora was found to contain a volume of two *choes*, or 6.4 litres, which would empty

37 [Aristotle], *Athenian Constitution* 67.4; Harpokration, s.v. *hemera diamemetremene*; Hannah 2009: 102.

38 Fermor and Steele 2000. For short periods of time in Babylonia, see now Steele 2020.

39 Sarma 1994; Sarma 2015: 2008–2009; Sarma 2018: 3645–711. I am grateful to Professor Clemency Montelle for advice on the Indian material.

40 London, British Museum 91283; Brown 2000: 119–20.

41 Such measurements are mentioned in Babylonian tablets: Hunger and Pingree 1989: 163–64.

42 Aeneas Tacticus 22.24–25.

out in six minutes.[43] Seven *choes* could therefore empty out in 21 minutes, and two such periods would make 42 minutes, a span of time close to Pliny's criterion.

If appropriate artificial mechanisms were unavailable, we could still posit the use of natural time signals specifically for the predawn period, such as cockcrow. In his play, *Ekklesiazousai* (30–31, 82–85, 390–91), produced in 391 BCE, Aristophanes has characters noting the "second cockcrow" as a specific time signal that occurs before sunrise when the stars are still visible. This implies a first cockcrow earlier in the predawn period, and quite likely a third or more, to judge from cross-cultural examples.[44]

From the time of Eudoxus in the fourth century BCE, we might suppose use of some form of celestial globe.[45] Even some small time before then, however, Plato's description of the creation of the cosmos in his *Timaeus* suggests that something like the more skeletal armillary sphere was familiar to him.[46] We have, unfortunately, no physical evidence of such instruments from this period.

Euctemon's parapegma

So what happens if we apply the criterion of time rather than magnitude to the star data preserved from Euctemon? In what follows I deal only with a small section of the data set attributed to Euctemon by Geminus, namely the first part of the summer section from the morning rising of the Pleiades to the morning rise of Sirius. It is enough to give a flavour of the type of work which might be pursued further, and the relative strength of the argument regarding a time-limit rather than purely ocular observation. The Greek text followed here is Aujac's; the translation is my own.[47] However, I have not followed Aujac's Julian calendar equivalences for the zodiacal-month dates in Geminus. These dates were those supplied by Manitius, an earlier editor of Geminus,[48] and suited a date of ca. 45 BCE. Instead I have reset the Julian dates to suit the Julian equivalent of the date of the summer solstice used by Euctemon's colleague, Meton, namely 27 June in 432 BCE (in reality, the date was 28 June for that period). It is from the summer solstice and the entry of the sun into Cancer that the Geminus *parapegma* is calibrated.

I am not being mathematically precise about the figure of "three-quarters of an hour," but am allowing some leeway either side of it, given the likely imprecise method of measurement for a unit of this scale. If on a given date for a star phase it is found that the sun was around three-quarters of an hour below the horizon, then I regard this as an accurate datum. This does not necessarily mean, however, that the phase was "visible" in the ordinary sense of the word, any more than a star phase could be seen when rain was forecast.[49] For comparison I have also taken the astronomical data (RA, Dec, magnitude) from the computer planetarium programme,

43 Young 1939; Hannah 2013: 350–52, with Figure 23.2

44 For sequential cockcrows as a time signal for encroaching dawn in Roman, Medieval and modern cultures, see Birth 2011; he notes, for example, the first cockcrow in a Filipino context at 4 a.m., which would be about two hours before sunrise.

45 Evans 1998: 249.

46 Hannah 2009: 116–18.

47 Recent versions of the Geminus *parapegma* are those of Aujac 1975, Evans and Berggren 2006, Lehoux 2007.

48 Aujac 1975: 158.

49 On the disjunction in the astrometeorological *parapegmata* between weather events and star phases, see Lehoux 2007: 59.

Voyager 4.5.[50] Dates for the various stellar phases are then derived from my own calculations based on the trigonometrical formulae that underlie the examples in Neugebauer 1925; I have added here in Appendix 1 my own derivation of Neugebauer's method. I have entered the data derived from these calculations to the Voyager planetarium program to gain a visual impression. These readings have been set in the program against an horizon of Athens as seen from the excavated Pnyx, the site where Meton, Euctemon's colleague, was said to have set up a *heliotropion*—which might have been a means of identifying the solstices—and which has largely not changed since antiquity.[51] As it turns out, however, the readings suggest that this was not an observation point for the data that we have. I wonder whether Mount Lykabettos, at 300m high and on the outskirts of Athens, would have been a better observation site, epsecially as this is where Meton's teacher, Phaeinus, was said to have observed the solstices.[52]

> [Τὸν δὲ Ταῦρον διαπορεύεται ὁ ἥλιος ἐν ἡμέραις λβ.
>
> *The sun passes through Taurus in 32 days.]*

This could not be in Euctemon's *parapegma*, but the zodiac is used in Geminus as the organising principle; the usage stems probably from Callippus's *parapegma*.[53]

> Ἐν δὲ τῇ ιγ[η] Εὐκτήμονι Πλειὰς ἐπιτέλλει· θέρους ἀρχή· καὶ ἐπισημαίνει.
>
> *Day 13 (May 6), according to Euctemon Pleiades rise; beginning of summer; and there is sign of weather.*[54]

This is a difficult observation to start off with for several reasons, as will be seen shortly. It signals the first visible morning rising of the Pleiades. The date of 6 May puts the sun 52 minutes short of rising, which suits Pliny's criterion. But on 6 May when η Tau was rising, the sun was only 9°16' below the horizon. The rising of the Pleiades was therefore technically not visible by modern calculations—with a flat horizon first visibility would occur ca. 23 May, with the sun 16° below the horizon, more than an hour and a half before sunrise.[55] That being said, however, Ginzel, quoting calculations by Hartwig, has 15–19 May for 431 BCE, which presumably signals a smaller angle of solar depression than 16°.[56] Schoch was aware of the discrepancy between modern calculations and ancient records: while allowing for a solar arc of 15.5° for the morning rising of the Pleiades, he also noted, "The magnitude of η Tauri is only +3.0. But Greek and Babylonian observations seem to imply a greater brilliance for the Pleiades. Perhaps Alcyone was brighter

50 Voyager 4.5, Carina Software, 865 Ackerman Drive, Danville, CA 94526, USA.

51 On Meton's *heliotropion*, see Kourouniotes and Thompson 1932: 207–11, Bowen and Goldstein 1988: 72–77, and recently Jones 2017: 77–8.

52 Theophrastus, *De Signis* 4.

53 See Hannah 2002: 120–21.

54 On the meaning of ἐπισημαίνει, which I have translated as "there is sign of weather," see Evans and Berggren 2006: 230 n.1. They translate it as "it signifies"; Lehoux 2007: e.g. 233 proposes "there is a change in the weather." All understand the usage to refer to a meteorological event.

55 Ginzel 1906–14: 1.517; Neugebauer 1925: 60 Tafel 28 allows the solar arc to be 16° for stars of magnitude 2.5–3.5. Schoch 1924b: 4 has an arc of 15.5° for the Pleiades; I calculate this would result in a date of 21 May in Athens.

56 Ginzel 1906–14: 1. 27

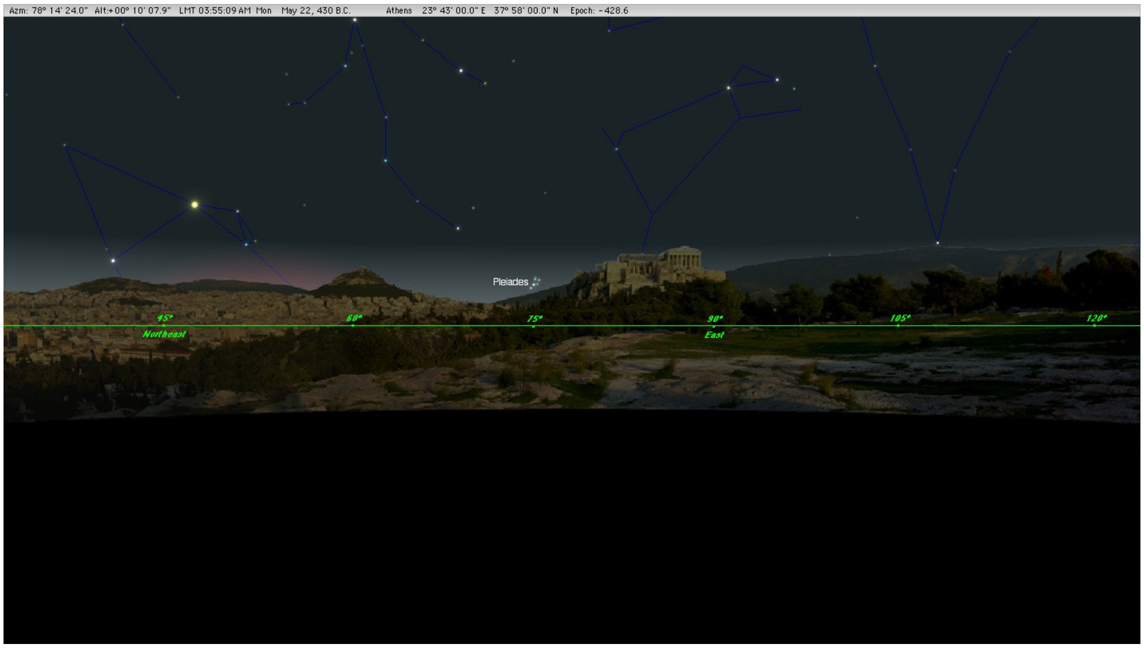

Figure 1. View from the Pnyx, Athens, of the first visibility of the rising Pleiades between the Acropolis and Mount Lykabettos, 22 May 430 BC.

2,500 years ago than now, or else the Pleiades, as a cluster, occupying an appreciable space on the sky, are more readily seen than an individual star of the same magnitude."[57] From my own observations I think there is truth in what Schoch says: the Pleiades seem to benefit from being a distinctive, twinkling cluster in a dark area of sky, that is most readily found by not looking directly at it at first (and this applies even to my home latitude at 46°S, where the Pleiades do not rise high in the sky as they do in the northern hemispehere). Ovid, *Fasti* 5.599–602 has the rising of the Pleiades in Rome on 13 May. Fox noted that for 5 CE, around the time Ovid composed his poem, "On May 13 in Rome the sun rises at 4:49 A .M . and the Pleiades at 3:43 A .M., probably enough time for them to be high enough in the eastern morning sky for an observer to see them before sunrise."[58] Fox is presumably allowing that the Pleiades, being so faint, would be some degrees above the horizon, to allow them to be visible in the predawn sky. But, although there is just over an hour between the rising of the Pleiades and that of the sun in Rome, and this suits the criterion of "at least three quarters of an hour" before sunrise, the sun lay only 10° below the horizon at the time of the rising of the Pleiades, and that seems to me too close to the horizon for the Pleiades to be visible. All the same, this arc is close to the one that I have calculated for 6 May 432 BCE, so perhaps the visibility of the Pleiades even with this small arc is within the bounds of possibility.

If the viewpoint was the Pnyx, then η Tau rose between the Acropolis and Mt Lykabettos, at an altitude of about 3°26' and an azimuth of 75°, and was not visible until about 22 May, with the sun 12.5° below a hypothetical flat horizon, but about 16° below the actual hilly horizon.[59] This late date would seem to count against viewing from the Pnyx. See Figure 1.

57 Schoch 1924b: 3.

58 Fox 2004: 120.

59 Cf. Salt and Boutsikas (2005) for a similar method of calculation for the much higher horizon at Delphi.

Ἐν δὲ τῇ λα^{η} Εὐκτήμονι Ἀετὸς ἑσπέριος ἐπιτέλλει.

Day 31 (May 24), according to Euctemon Eagle rises in the evening.[60]

This signals the evening rising of Aquila. The date of 24 May put the sun 44 minutes after setting, which suits Pliny's criterion. On 24 May when α Aql was rising, the sun was 7°26' below the horizon. The rising of Aquila would therefore have been visible and accurate—with a flat horizon last visibility would occur on 25 May, with the sun 7° below the horizon.

Ἐν δὲ τῇ λα^{η} Εὐκτήμονι Ἀρκτοῦρος ἑῷος δύνει· ἐπισημαίνει. ... Εὐκτήμονι Ὑάδες ἑῷαι ἐπιτέλλουσιν· ἐπισημαίνει.

Day 32 (May 25), according to Euctemon Arcturus sets at dawn; there is sign of weather. ... Hyades rise at dawn; there is sign of weather.

This signals the last visible morning setting of Arcturus. The date of 25 May puts the sun only 10 minutes short of rising, which does not suit Pliny's criterion. On 25 May when α Boo was setting, the sun was 1°52' below the horizon. The setting of Arcturus is therefore technically not visible—with a flat horizon first visibility would occur ca. 4 June, with the sun 7° below the horizon.[61] Indeed, this counts as a true, and therefore invisible, morning setting. As we shall see elsewhere, visible morning settings (by the criteria of time and optical visibility) are characteristically not this *parapegma*'s forte. This is curious, since last visible morning settings should have been relatively easy to note, because of the previous nights' observations of the star setting over the same horizon before dawn. This in turn stands in contrast to first morning risings, which could not be notified by previous morning observations and should therefore have been harder to determine, if one was actually physically observing the stars.[62]

This entry also signals the first visible morning rising of the Hyades. In this case, the date of 25 May put the sun 36 minutes short of rising, which practically suits Pliny's criterion. On 25 May when α Tau was rising, the sun was 6°23' below the horizon. The rising of the Hyades was therefore technically not visible—with a flat horizon first visibility would occur about 4 June, with the sun 11° below the horizon.[63]

[Τοὺς δὲ Διδύμους ὁ ἥλιος διαπορεύεται ἐν ἡμέραις λβ.]

[The sun passes through Gemini in 32 days.]

Ἐν δὲ τῇ κδ^{η} Εὐκτήμονι Ὠρίονος ὦμος ἐπιτέλλει.

Day 24 (June 18), according to Euctemon shoulder of Orion rises.

This signals the morning rising of Orion. Orion's shoulders are γ Ori (upper, so seen first) and α Ori (lower, so second).[64] According to calculation, the visible morning rising of both is on the

60 Aujac omits the manuscript reading, Ἐν δὲ τῇ κε^{η} Εὐκτήμονι Ἀετός ἑσπέριος ἐπιτέλλει [Day 25 (May 19), *according to Euktemon Eagle sets in the evening*], because Ἀετός (Eagle) makes no sense astronomically. Ἄιξ (Goat) would be better, but for the sake of consistency, as I have chosen to follow Aujac's text, I have omitted the line.

61 Vogt 1920: 55 gives angle of 8.5°, instead of Neugebauer's 7°, for the morning setting of Arcturus. This would push the date to 6 June or later.

62 As noted by Schoch 1924a: 733.

63 Ginzel 1906–14: 1.27, quoting Hartwig's calculations, has 3–7 June for 431 BC.

64 Kidd 1997: 303.

same date, 30 June. The date of 18 June puts the sun 40 minutes short of rising, which suits Pliny's criterion. On 18 June when γ Ori was rising, the sun was 6°43' below the horizon. The rising of the shoulders of Orion is therefore technically not visible—with a flat horizon first visibility would occur around 30 June, with the sun 14° below the horizon.

[Καρκίνον διαπορεύεται ὁ ἥλιος ἐν ἡμέραις λα.]

[The sun passes through Cancer in 31 days]

[<Ἐν μὲν οὖν τῇ> α^{η} ἡμέρᾳ Καλλίππῳ Καρκίνος ἄρχεται ἀνατέλλειν· τροπαὶ θεριναί· καὶ ἐπισημαίναι.]

[Day 1 (June 27). according to Callippus, Cancer begins to rise; summer solstice; and there is sign of weather.]

<Ἐν δὲ τῇ> ιγ^{η} ἡμέρᾳ Εὐκτήμονι Ὠρίων ὅλος ἐπιτέλλει.

Day 13 (July 9), according to Euctemon all of Orion rises.

This signals the morning rising of all of Orion—I assume that "all of Orion" means not just β Ori but also κ Ori, as these are the "two feet" of Orion in Aratus (*Phaenomena* 338). The date of 9 July puts the sun 46 minutes short of rising, which suits Pliny's criterion. On 9 July when κ Ori was rising, the sun was 7°34' below the horizon. The rising of Orion is therefore technically not visible—with a flat horizon first visibility would occur around 20 July, with the sun 14° below the horizon.

<Ἐν δὲ τῇ> κζ^{η} Εὐκτήμονι Κύων ἐπιτέλλει.

Day 27 (July 23), according to Euctemon Dog rises.

This signals the morning rising of Sirius. I assume "Dog" means α CanMaj, not β CanMaj. The date of 23 July puts the sun 36 minutes short of rising, which practically suits Pliny's criterion. On 23 July when α CanMaj was rising, the sun was 6°03' below the horizon. The rising of Sirius is therefore technically not visible—with a flat horizon first visibility would occur around 31 July, with the sun 11° below the horizon.[65]

<Ἐν δὲ τῇ> κη^{η} Εὐκτήμονι Ἀετὸς ἑῷος δύνει· χειμὼν κατὰ θάλασσαν ἐπιγίνεται.

Day 28 (July 24), according to Euctemon Eagle sets at dawn; storm at sea comes on.

This signals the morning setting of Aquila. I assume "Eagle" means α Aql (the last part of Aquila to set), not λ Aql (the first part), whose setting date is much earlier. The date of 24 July puts the sun 16 minutes short of rising, which does not suit Pliny's criterion. On 24 July when α Aql was setting, the sun was 3°14' below the horizon. The setting of Aquila is therefore technically not visible—with a flat horizon last visibility would occur around 30 July, with the sun 7° below the horizon, and the sun would be 42 minutes short of rising, which would suit Pliny's criterion. As we have seen already, however, morning settings are a weak point in this *parapegma*.

65 Ginzel 1906–14: 1. 27, quoting Hartwig's calculations, has 27–31 June for 431 BC, but this must be a misprint for 27–31 July.

[Τὸν δὲ Λέοντα διαπορεύεται ὁ ἥλιος ἐν ἡμέραις λα.]

[The sun passes through Leo in 31 days.]

Ἐν δὲ τῇ α$^{\text{η}}$ ἡμέρᾳ Εὐκτήμονι Κύων μὲν ἐκφανής, πνῖγος δὲ ἐπιγίνεται· ἐπισημαίνει.

Day 1 (July 29), according to Euctemon Dog is visible, and stifling heat comes on; there are signs of weather.

This signals the morning rising of Sirius. I assume "Dog" means α CanMaj: Evans and Berggren suggest that the two "observations" for the Dog signify a first fleeting visibility (27 Cancer / 23 July) and then an easy visibility (this present date).[66] This date of 28 July puts the sun 60 minutes short of rising, which suits Pliny's criterion. On 28 July when α CanMaj is rising, the sun is 9°57' below the horizon. The rising of Sirius is therefore technically just visible—with a flat horizon first visibility would occur around 31 July, with the sun 11° below the horizon.

Conclusions—and speculations

While acknowledging the caveats that inevitably surround the construct called "the *parapegma* of Euctemon," I have used a sample of the star phases attributed to Euctemon in the compilation attached to Geminus's *Eisagoge* so as to test the criterion provided by Pliny for the visibility of first and last risings and settings of stars. This criterion prioritises time relating to the sun over magnitude relating to the stars, requiring that the sun be "at least three-quarters of an hour" before rising or after setting, regardless of the magnitude of the star. This is not to say that magnitude does not matter, only that it is secondary, and in this I stand in contrast to previous scholarship from the 19th century on, which has focussed on magnitude as the only criterion.

By modern standards of calculation only two of the nine star phases attributed to Euctemon investigated in this paper appear to have been physically visible at the time assigned to them. On the other hand, of these nine star phases seven seem to suggest that a time-based measure lies behind the "observations." I have treated the time criterion loosely, since "three quarters of an hour" is anachronistic for fifth century BCE Greece—although hours, in the guise of "the twelve parts of the day," are among the things that Herodotus, in the fifth century BCE, says the Greeks learned from the Babylonians,[67] and therefore were presumably available to Euctemon and his colleagues, nonetheless part-hours do not appear until the late fourth century BCE.[68] The measure of "three quarters of an hour" may also be a later translation not of time but of quantity, i.e. of the amount of water that flowed out of a waterclock of some kind, or through a holed vessel, for a duration before or after the astronomical event being recorded. Or, probably less likely but suggested by developments under Eudoxus and Autolycus, the measure might reflect early attempts to measure time via the passage of the sun through the zodiacal constellations, which seem to have been distinguished by the Greeks in the fifth century BCE before Euctemon's time.

That the star phases themselves may not have been physically observed need not bother us for the period in question. The situation may be similar to that identified by Bowen and Goldstein with regard to the "observation" of the summer solstice by Meton and Euctemon in 432 BCE. In that case they deduced that the solstice was not fixed by a series of observations, since

66 Evans and Berggren 2006: 233 n.7.

67 Herodotus 2.109.3.

68 Hannah 2009: 82–3, 125–6.

there is no evidence of such a series, but rather by the placement in the Athenian civil calendar of a date for the solstice already identified by Babylonian astronomers.[69]

Nor does this tentative result of my investigation necessarily make Euctemon's data "wrong." We might imagine that the data were still regarded as correct, in the same sense that a medieval "scratch-dial" told the time "accurately" because everyone agreed that when it said "one o'clock" it was one o'clock for those who were to do something at that agreed time. Or perhaps it is like when people synchronize their watches but to the same wrong time: the "time" may be inaccurate according to an objective standard, yet it will be "correct" according to an internal agreement and therefore perfectly adequate for internally coordinating an activity.[70] Is it the same here with Euctemon's data, that the star phases, in some way, linked in with some external purpose that benefitted from the notices of the annual regularity not only of the weather signs but also of the star phases?

Was the Athenian calendar the point of the exercise? And—as I often come back to, in dealing with the Athenian *parapegma*—who was interested in these data? I usually propose the civic/religious authorities, intertwined as they were then, not the farmers and sailors, who did not really need to watch the stars to know when to sow and plough, or when to sail.[71] Nor, for that matter, did the officials, but they did want a calendar that allowed seasonal activities (governed by the sun and the weather) to correspond to the associated festivals (governed by timing with regard to the moon). The timing of religious festivals was a matter of public concern in Athens at the time when these star phases were organised. This meant that calibrating the festival calendar with the cycles of the sun and moon was necessary in order to get the timing of the festivals right. Meton is credited with the construction of a 19-year cycle (everyone seems to agree now that this was on the basis of exposure to Babylonian ideas, where such a cycle had long been in use). Bowen and Goldstein agreed with Neugebauer and Toomer in dismissing the notion that the Metonic cycle was devised in order to reform the Athenian civil calendar "for the good reason that there is no evidence that such cycles were developed to improve the civil calendar."[72] On this score they would have been better keeping their powder dry, as the use of the Metonic cycle to regulate the Athenian calendar is now demonstrable for the Hellenistic period, and may well have been experimented with from 432 BCE, although irregularities in its application would appear to have occurred to start with.[73] But on another score, I think Bowen and Goldstein were right to suggest that "one important purpose of the early Greek calendric cycles was to facilitate the use of almanacs or *parapegmata*. These cycles were used to correlate dates in the *parapegmata* with dates in a lunar calendar."[74]

69 Bowen and Goldstein 1988: 72.

70 Hannah 2009: 85–6.

71 For an example, see Hannah 2005: 62–70.

72 Bowen and Goldstein 1988: 52 n.62.

73 See Hannah 2009: 37 for further references, and Ossendrijver 2018 for the Babylonian evidence.

74 Bowen and Goldstein 52.

Appendix: P. V. Neugebauer's method for calculating the visibility of stellar phenomena (Neugebauer 1925)

	NEUGEBAUER'S TEXT page XXXVIII: Beispiel	MY COMMENTS
24a)	501 v.Chr. = -500	The given year (501 BCE) is transformed into the astronomical year.
24b)	Sirius -500: $a = 73^{\circ}.7$ $d = -16^{\circ}.4$	The *equatorial* coordinates for Sirius in 501 BC. a = Right Ascension, the distance in longitude from the vernal equinox (a) to the star d = Declination, the distance in latitude above or below the equator.
24c)	$l = 15^{\circ}$ östlich Greenwich, $f = 34^{\circ}$	l = geographical longitude f = geographical latitude of the observer (15°E, +34°).
24d)	Sirius ist ein Stern 1. Grösse. Tafel 28. $b = 11$ für heliakischen Aufgang und Untergang $b = 7$ für scheinbaren akronychischen Aufgang und scheinbaren kosmischen Untergang.	Sirius is a 1st magnitude star $b = -11^{\circ}$ for arc of visibility below horizon at heliacal rising and setting $b = -7^{\circ}$ for apparent acronychal rising and apparent cosmical setting. = sun's minimum negative altitude (in *horizon* coordinates) below the horizon for the star still to be visible just above the horizon. Formula:
24e)	Tafel 1. Vertikal Argument $d = -16^{\circ}.4$ Horizontales Argument $f = 34^{\circ}.0$ $t = 5^{h}.29$ oder in Grad = $79^{\circ}.4$	$t = \cos^{-1}\{-\tan f \tan d\}$ = the semi-diurnal arc, the time or distance between the rising or setting of a star on the horizon and its transit over the observer's meridian. = the hour angle (H) of a star at rising or setting. Hour angle (H) = how far the star has travelled since it crossed the meridian, so here t = time or distance, expressed in *equatorial* terms, between the meridian and the rising/setting point of the star on the horizon. The star's zenith distance = 90° at its rising/setting point, but this is in altazimuth terms, not equatorial.

Following the calculations for heliacal rising and apparent acronychal rising:

$a = 73^\circ.7$
$t = 79^\circ.4$
$y = a - t = 354^\circ.3$

y = the time elapsed/distance covered since the Vernal Equinox (VE) last crossed the meridian, and therefore also = where the meridian is.
Whenever Sirius rises, at any time of the day, VE lies about 24 minutes (0.4h) east of the meridian (i.e. a will cross the meridian about 24 minutes after Sirius rises). VE's precise position at Sirius's rising corresponds to the difference between the semi-diurnal arc of Sirius ($t = 79^\circ.4$) and the distance between Sirius and VE (= R.A. of Sirius, $a = 73^\circ.7$)
$= -5^\circ.7$ (= the approx. 24 minutes between VE and the meridian), to which 360° must be added to make it positive, i.e.
$y = 360^\circ - (t - a) = 360^\circ - t + a = 360^\circ + a - t$
$= 360^\circ + 73^\circ.7 - 79^\circ.4 = 360^\circ - 5^\circ.7 = 354^\circ.3$ (= 23h 37m 12s R.A.), which is effectively Neugebauer's formula.
[In the case of calculating heliacal / cosmical settings, $y = t + a$ = the sum of the semi-diurnal arc (t)—which takes us from the meridian westwards to the point of setting for Sirius—plus the R.A. of Sirius—which takes us beyond Sirius round to where a now lies; i.e.
$y = 79^\circ.4 + 73^\circ.7 = 153^\circ 1$ = 10h 12m 24s R.A.]
This means that the meridian lies at 354°.3 R.A. in the case of Sirius's rising, and at 153°.1 in the case of its setting.
In its turn, the R.A. of the meridian is also necessarily the R.A. of the zenith, since it lies on the meridian.

24f) Argument $y = 354^\circ.3$:
$p = -0^\circ.6$
$r = +2^\circ.6$
$s = +23^\circ.6$
$S = f + r = +36^\circ.6$

24g) Tafel 25.
Vertik. Arg. $s = +23^\circ.6$
Horiz. Arg. $S = +36^\circ.6$
$P = +16^\circ.6$
Tafel 26.
Dieselben Argumente:
$B = +33^\circ.2$
$y = 354^\circ.3$
$p = -0^\circ.6$
$P = +16^\circ.6$
$L = 10^\circ.3$

24f) - 24g):
y = R.A. of the meridian, and
f = observer's geographical latitude.

Formulae:
$L = \tan^{-1} \{(\sin y \cos e + \tan f \sin e) / \cos y\}$
$B = \sin^{-1} \{\sin f \cos e - \cos f \sin e \sin y\}$ where
e = the obliquity of the ecliptic.

y and f are converted from equatorial to *ecliptic* form (L, B).

f by extension = Declination of the zenith point. Converting f into ecliptic form (B) not only gives the latitude of the zenith point from the ecliptic, but more importantly the angle below the ecliptic, between it and the horizon (= 90° – B, since zenith-to-horizon = 90°). This will be used to calculate L_1: see 24h) below.

y is converted to ecliptic form to provide the sun's position on the observer's meridian (the sun's ecliptic longitude, L), when Sirius is rising or setting.

We need to maintain this angular relationship between Sirius and the sun. Putting the sun on the meridian as Sirius rises or sets gives a time (noon) and date when the sun and Sirius are in this angular relationship. But it is not the date of Sirius's HR, AR, HS or CS, because if we then move the sun of this date to either the eastern or the western horizon, Sirius will also have moved commensurately forwards or backwards from its position on the horizon, to maintain the angle between it and the sun.

We therefore still need to find when the sun is on or just below the horizon as Sirius rises or sets.

24h) Heliakischer Aufgang
$b = 11^\circ$
Tafel 27.
Spalte $b = 11^\circ$
Arg. $B = 33^\circ.2$
$L_1 = 13^\circ.2$

Heliacal Rising
The formula at Tafel 27 is:
$\sin L_1 = (\sin b \,/\, \cos B)$,
so $L_1 = \sin^{-1} (\sin b \,/\, \cos B)$
L_1 = the angular distance on the ecliptic from the horizon to the nearest position of the sun below the horizon at which the star is visible before sunrise / after sunset.

f = observer's geographical latitude, and by extension = Declination of the zenith point.

As noted above, converting f into ecliptic form (B) not only gives the latitude of the zenith point from the ecliptic, but more importantly the angle below the ecliptic, between it and the horizon (= $90^\circ - B$, since zenith-to-horizon = 90°).
This in turn is equal to the angle opposite (angle B), in the triangle bounded by the horizon above (side a) and side L_1 (side c) below, with side b (side b) opposite enclosing a right-angle (angle C) with the horizon. By use of the sine-formula:

$$\frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$\frac{\sin (90^\circ - B)}{\sin b} = \frac{\sin 90^\circ}{\sin L_1}$$

$$\frac{\sin L_1}{\sin b} = \frac{\sin 90^\circ}{\sin (90^\circ - B)}$$

$$\sin L_1 = \frac{\sin 90^\circ \cdot \sin b}{\sin (90^\circ - B)}$$

Since $\sin 90^\circ = 1$, and $\sin (90^\circ - B) = \cos B$:

$$\sin L_1 = \frac{\sin b}{\cos B}$$

hence, $L_1 = \sin^{-1} (\sin b \,/\, \cos B)$.

24i) $L = 10^\circ.3$
$L_1 = 13^\circ.2$
$S = L + L_1 + 90^\circ = 113^\circ.5$

We are now in *ecliptic* coordinates, with the sun on the meridian and of course on the ecliptic, while Sirius is rising or setting.

The angular distance *along the ecliptic* from the sun on the meridian to its position on the horizon is 90˚. (This is not its zenith distance, but a measure of how the horizon intersects with the great circle of the ecliptic.)

The ecliptic longitude is measured eastwards from *a*. So moving from the meridian to the point of rising increases the longitude of the sun, while moving from the meridian to the point of setting decreases it. Therefore, for HR and AR we must add 90˚ to move the sun to the rising point, but subtract 90˚ to move it to the setting point. The same applies to L_1.

$L + L_1 + 90^\circ$ = the ecliptic longitude of the sun at the time of the heliacal rising of the star. From this the date of the phenomenon can be ascertained.

NB: these are ecliptic coordinates, which have to be converted to equatorial.

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