The origins of the Ṭūsī-couple revisited

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Among the many contributions by James Evans to the history of astronomy is his clear and elegant paper on the origin of Ptolemy's equant.[1] As has been his hallmark, he there brought his considerable talent as a modern scientist together with his sophisticated historical sensitivity. The result was an important contribution to the vexed problem of the origins of this problematic device.[2]

The equant itself, despite its success in resolving observational issues related to the retrograde arcs of the planets, evoked considerable controversy among Islamic astronomers because of the violations resulting from it of the strictures of uniformity and circularity in the heavens. Among the devices proposed for dealing with these violations was the Ṭūsī-couple, put forth by the famous thirteenth-century astronomer and polymath Naṣīr al-Dīn al-Ṭūsī (1201-1274). Although it has been known for some time that Ṭūsī used the device in his lunar and planetary models found in his *al-Tadhkira fī ʿilm al-hayʾa* (Memoir on the science of astronomy), there has been a divergence of opinion about when Ṭūsī first proposed his new device and models. In this paper, I present new evidence that sheds light on the first appearance of the Ṭūsī-couple.

In an earlier paper,[3] I argued that Naṣīr al-Dīn al-Ṭūsī first *announced* his famous astronomical device, which we now refer to as the Ṭūsī-couple, in a Persian astronomical work entitled the *Risālah-i Muʿīniyya* (The Muʿīniyya treatise, named for one of Ṭūsī's patrons), which was completed in 632/1235.[4] He first *presented* it in the appendix to this work, which is called, among other things, the *Ḥall-i mushkilāt-i Muʿīniyya* and *Dhayl-i Muʿīniyya* (the resolution of difficulties in the *Muʿīniyya;* appendix to the *Muʿīniyya*). I maintained that there were compelling reasons for believing that the *Ḥall* predated a second version of the couple briefly presented in Ṭūsī's *Taḥrīr al-Majisṭī* (Recension of the *Almagest*), which was completed in 644/1247; however, there was still some question since no manuscript had yet been found that gave a date for the *Ḥall*. But thanks to an examination of a manuscript in Tashkent, which was brought to my attention by Sergei Tourkin, we now have a date for the *Ḥall* and therefore for the first publication of the Ṭūsī-couple. This new dating confirms my original chronology, but it also raises some new questions and puzzles, which I discuss in what follows.

Before presenting this new evidence, let me briefly summarize the information we have on the Ṭūsī-couple. The final and most complete presentation of Ṭūsī's models occurs in *al-Tadhkira fī ʿilm al-hayʾa*, written in Arabic, which first appeared in 659/1261 when Ṭūsī was the director of the Marāgha observatory that had been established under Mongol patronage in Azerbaijan. Ṭūsī presents them in the context of criticisms of the models that had been developed by Claudius Ptolemy in the 2nd century CE in Alexandria, Egypt, and brought forth in the latter's *Almagest*

1 Evans 1984.

2 For a review of several theories on the origin of the equant, see Duke 2005.

3 Ragep 2000.

4 When separated by a slash, the first date is lunar hijrī; the second is common era. Otherwise the date is common era.

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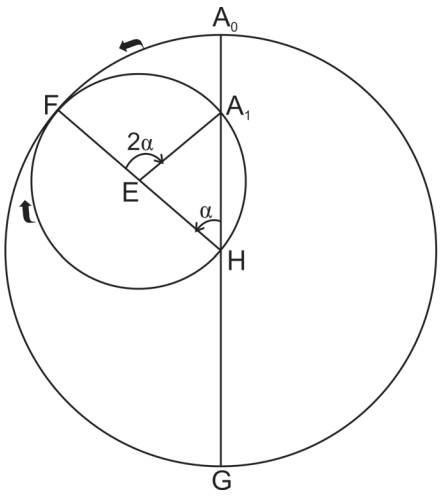

Figure 1. The Rectilinear Version of the Ṭūsī-couple.

and *Planetary Hypotheses*. Following a line of criticism that can be traced at least as far back as Ibn al-Haytham in the 11th century CE, Ṭūsī identifies 16 difficulties, or *ishkālāt,* that taint the Ptolemaic models. Rather than go through these individually, we can instead point to the general problem they highlight, namely that these models did not adhere to the recognized physics that required that all motion in the heavens be uniform and circular, and such that one uniformly rotating motion be brought about by a single spherical body called an orb [*falak*]. The two versions of the Ṭūsī-couple seek to resolve these problems by using a combination of uniformly rotating orbs that can, alternatively, produce either a straight-line oscillation in a plane [Rectilinear Version], or a curvilinear oscillation along a great circle arc [Curvilinear Version]. The Rectilinear Version was used by Ṭūsī to resolve irregular planetary motions in longitude by ingeniously decomposing Ptolemy's deferent (longitudinal) motions into two parts: one based on variable speed with respect to the observer and the other based on distance from the observer, this latter being brought about by the couple. The Curvilinear Version, which first appears in the *Tadhkira*, was used, among other things, to produce latitudinal (north-south) motion by having the couple create curvilinear oscillations by means of physical orbs. These latitudinal motions had been brought about in the *Almagest* by circles, but without an underlying physical explanation. Ṭūsī also notes that Ptolemy's latitude circles cause motions in all directions, whereas what is needed for the latitude models is an oscillation along a great circle arc.[5]

In the *Muʿīniyya*, when noting the irregular motion associated with the lunar epicycle center on its deferent, Ṭusī mentions "an elegant way" (*wajh-i laṭīf*) he has discovered to resolve the issue (Book II, Chap. 5). He refers to this solution at least twice more, when discussing the upper planets and Venus (Book II, Chap. 6) and when setting forth Mercury's configuration (Book II, Chap. 7). As for the models for latitude, Ṭūsī points out that Ibn al-Haytham had dealt with this in a treatise and gives a brief sketch of his theory (Book II, Chap. 8). But he finds this solution lacking and criticizes it without going into details, since "this [work, i.e. the *Muʿīniyya*] is not the place to discuss it." Despite this criticism, Ṭūsī does not claim to have a solution to the problem of latitude, unlike the case with the longitudinal motions of the moon and planets.[6]

5 Extended discussions of the Ṭūsī-couple occur in: Ragep 1987; Ragep 1993, 1.46-53 and 2.427-457; Ragep and Hashemipour 2006; and Ragep 2017.

6 The relevant passages from Book II, Chaps. 5, 6 and 8 of the *Muʿīniyya*, with English translation, can be found in Ragep 2000, 123-125.

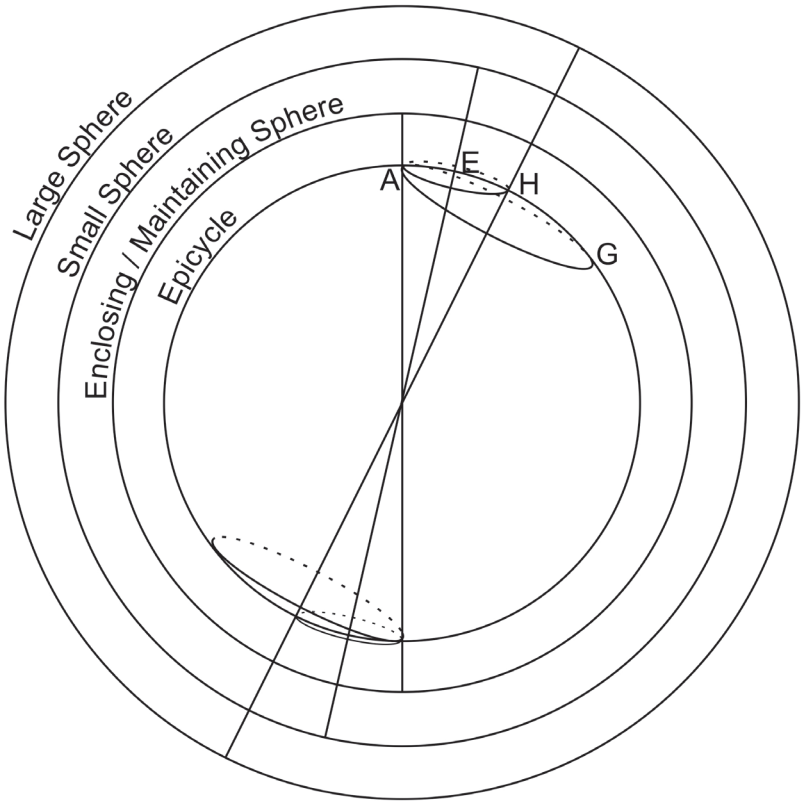

Figure 2. The Curvilinear Version of the Ṭūsī-couple.

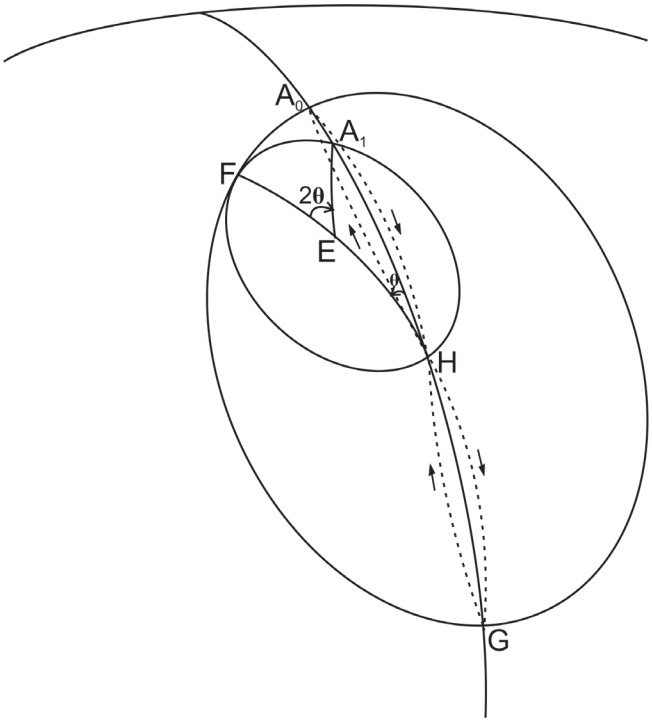

Figure 3. Polar View of the Curvilinear Ṭūsī-couple (dotted line represents actual path of pole A).

Ṭūsī promises to put his solution in a separate work if the "Prince of Iran...would be so pleased to pursue this problem," a reference to Muʿīn al-Dīn Abū al-Shams, the son of his patron Nāṣir al-Dīn Muḥtasham. And indeed, a solution is presented in the *Ḥall-i mushkilāt-i Muʿīniyya*. The *Ḥall* consists of 9 chapters:

Chapter 1: On the possibility of a fixed star whose colatitude is greater than the difference between the local latitude and the total obliquity, after having been either permanently visible or permanently invisible, becoming invisible or visible	فصل ۱: در آنکه چون تمام عرض کوکبی از ثوابت زیادت از فضل عرض بلد بر میل کلّی بود ممکن باشد که بعد از آنکه ابدی الظهور یا ابدی الخفا بوده باشد اورا خفائی یا ظهوری حادث شود
Chapter 2: On why the eccentric orb was chosen for the sun over the epicycle	فصل ۲: در آنکه فلك خارج مرکز جهت آفتاب چرا بر تدویر اختیار کرده اند
Chapter 3: On the solution of the difficulty occurring with regard to the motion of the center of the lunar epicycle on the circumference of the deferent, and the uniformity of that motion about the center of the World	فصل ۳: در حلّ شکی که بر حرکت مرکز تدویر ماه بر محیط حامل و تشابه آن حرکت بر حوالی مرکز عالم واردست
Chapter 4: On the explanation of the circuit of the moon's epicycle center and the manner in which the circuit of the center of the lunar epicycle orb comes about	فصل ٤: در شرح مدار مرکز تدویر قمر و چگونگی حدوث مدار مرکز فلك تدویر ماه
Chapter 5: On the configuration of the planets' epicycle orbs according to the doctrine of Abū ʿAlī ibn al-Haytham	فصل ٥: در هیأت افلاك تداویر سیارگان بر مذهب ابو علی بن الهیثم
Chapter 6: On the explanation for finding the stationary positions of the planets on the epicycle orb	فصل ٦: در شرح معرفت مواضع اقامت کواکب از فلك تدویر
Chapter 7: On clarifying the different circumstances of lunar and solar eclipses from the point of view of difference in latitude and other matters	فصل ۷: در بیان تفاوت احوال خسوف وکسوف از جهت تفاوت عرض وغیر آن

Chapter 8: On conceptualizing the equation of time [lit.: equation of days with their nights]	فصل ۸: در تصویر تعدیل الایام بلیالیها
Chapter 9: On depicting the Indian Circle, the direction of a locale and other matters	فصل ۹: در صورت دایرهٔ هندی و سمت بلاد وغیر آن

What is striking about the *Ḥall* is the variety of the contents (one might call it a hodgepodge) and the fact that the most innovative part of it, i.e. that devoted to the rectilinear version of the Ṭūsī-couple and its use to resolve the irregular motion of the moon's epicycle on its deferent, is relegated to Chapter 3. Furthermore, the curvilinear version, which is for resolving irregular motion resulting from Ptolemy's latitude theory, is not presented in any way in the *Ḥall*; rather, for the problem of latitude, for which Ṭūsī would later use his curvilinear version in the *Tadhkira*, he simply presents in Chapter 5 the solution that had been proposed by Ibn al-Haytham.[7]

Since it is sometimes referred to as an "Appendix" (*dhayl*), one might assume that the *Ḥall* must have been written soon after the *Muʿīniyya*, especially since there is nothing in it that is particularly new or that had not been promised in the *Muʿīniyya*. Thus it comes as something of a surprise that the *Ḥall* was completed over ten years after the *Muʿīniyya*. The evidence for this comes from a manuscript witness of the *Ḥall* currently housed at the al-Bīrūnī Institute of Oriental Studies in Tashkent, Uzbekistan [MS 8990, f. 46a (original foliation)]:[8]

تمت الرّساله والحمد لله

اتفق فراغ المصنف رفع الله مراتبه فی معارج القدس من تالیفه اوایل جمادی الآخرة سنه ۶۴۳ هجریه بمقام بلدة تون بالبستان المعروف بباغ برکه

> The treatise is completed, praise be to God. The author, may God elevate his stature on the ascents to the Divine, completed its composition during the first part of Jamādā II, 643 of the Hijra, within the town of Tūn in the garden known as Bāgh Barakah. [=late October 1245]

We should note here that Ṭūsī at this time was in the employ of the Ismāʿīlī rulers of Qūhistān in southern Khurāsān. As stated by Farhad Daftary: "The supreme Nezārī [Ismāʿīlī] leader, whether *dāʿī* or imam, selected the local chief *dāʿī*s to serve in the main Nezāri territories: Kūhestān (Qohestān) in southern Khorasan and Syria. The chief *dāʿī* (often called *moḥtašem* [as is the case here]) of the Kūhestān Nezārīs usually lived in Tūn, [in] Qāʾen, or [in] the fortress of Moʾmenābād, near Bīrjand."[9] Tūn, today called Firdaws, lay some 80 km/50 miles west-northwest of the main town of the region, Qāʾin.

7 For an edition, translation and discussion of this part of the *Ḥall*, see Ragep 2004.

8 I thank the Bīrūnī Institute for providing images of this valuable manuscript. On the side of the last page, the text is said to have been collated with a copy that had been collated with a copy in the hand of the author (i.e. Ṭūsī) on 4 Ramaḍān 825/late August 1422 (f. 46a). The page with the colophon and copy date is reproduced in the Appendix below.

9 Daftary 1993, 6.592 (col. 1). I have added a few clarifying remarks between square brackets.

As mentioned, the *Taḥrīr al-Majisṭī* (recension of Ptolemy's *Almagest*), written in Arabic, was completed on 5 Shawwāl 644/ 13 February 1247 and thus after the *Ḥall-i mushkilāt-i Muʿīniyya*. I have argued elsewhere that it is likely that Ṭūsī, for some reason, perhaps related to a falling out with his patrons in Qūhistān, relocated (or was relocated) to the Ismāʿīlī fortress of Alamūt in north-central Iran sometime before Ṣafar 644/June-July 1246. This was the date of the *Ḥall mushkilāt "al-Ishārāt"*, his commentary on Ibn Sīnā's philosophical treatise *al-Ishārāt wa-al-tanbīhāt*. Ṭūsī's work was dedicated to Shihāb al-Dīn Muḥtasham, who was most likely in Alamūt, thus providing us a probable location for Ṭūsī's residence at the time. Now that we know the date of the *Ḥall-i mushkilāt-i Muʿīniyya*, we can say with some degree of certainty that Ṭūsī's move to Alamūt occurred between Jamādā II 643 and Shawwāl 644, since the *Taḥrīr al-Majisṭī*, a major work of considerable consequence, is not dedicated to any of the Ismāʿīlī rulers.[10] The date of the move is further confirmed by the fact that Ṭūsī, after completing the *Ḥall-i mushkilāt-i Muʿīniyya,* no longer dedicated his works to anyone at the court in Qūhistān.[11]

There is another interesting aspect to Ṭūsī's writings after the move to Alamūt. The vast majority of Ṭūsī's works (but not all) appear now in Arabic. And we can perhaps better understand the context of his writing the *Taḥrīr al-Majisṭī.* It was the first of Ṭūsī's recensions; these would eventually include the Middle Books (*Mutawassiṭāt*, to be studied between the *Elements* and the *Almagest*), which were completed in 663/1265, as well as the recension of Euclid's *Elements*, completed in 646/1248. We can only speculate about Ṭūsī's motives for this monumental project, but it most likely involved both retrospective and prospective aspects: retrospective because of the desire to preserve the great mathematical and astronomical works of Hellenistic and early Islamic science, especially in the wake of the Mongol invasions; prospective because of the pedagogical importance of these works. Given the tumultuous times in which Ṭūsī lived, and the real danger that the great achievements of Islamic science might be lost, the recension projects can be understood as making available a body of textbooks, with commentary, that could provide both a record and a pedagogical tool even if the institutions of Islamic science were destroyed.

Now that the chronology between the *Muʿīniyya*, its *Ḥall*, the *Taḥrīr al-Majisṭī,* and *al-Tadhkira fī ʿīlm al-hayʾa* has been firmly established, we can make the following observations:

1) Ṭūsī's claim to having discovered an "elegant way" (*wajh-i laṭīf*) in the *Muʿīniyya* for resolving some of the problems of Ptolemaic planetary theory would seem to have been somewhat premature. That he waited over ten years to present this new model, and because none of the other material in the *Ḥall* is particularly new or creative, leads one to conclude that he had not finalized his model when he made his claim in the *Muʿīniyya*. Another bit of supporting evidence is that in the *Muʿīniyya* (II.7), Ṭūsī claimed that the solution for Mercury "is as for the other planets," something that he later contradicted in the *Tadhkira* (II.11[11]), where he admits to not having a solution for Mercury's complex model.

2) Another surprising point is that despite the many years between the *Muʿīniyya* and the *Ḥall,* the lunar model based on the Ṭūsī-couple has a mistake in it. In listing the orbs (*aflāk*) of the moon and their motions, Ṭūsī gave the wrong daily motion for the second (inclined) orb (13°11' instead of 13°14'). At some point he must have realized the error and corrected it in the *Tadhkira*, while at the same time dividing up the inclined orb of the *Ḥall* into an inclined and a deferent orb.[12]

10 The simple dedication is to a certain Ḥusām al-Dīn Ḥasan b. Muḥammad al-Sīwāsī.

11 For an elaboration of the points in this paragraph, see Ragep 1993, 1.9-13.

12 In the *Tadhkira,* the sum of the lunar inclined and deferent orbs comes to 13°14' (24°23'/day – 11°9'/day); cf. the *Ḥall*, where the equivalent motion of the inclined orb is given as the mean motion of the moon (*wasaṭ-i qamar*),

3) The criticism of Ibn al-Haytham's latitude model that Ṭūsī gave in the *Muʿīniyya* is not repeated in the *Ḥall.* Instead he presents Ibn al-Haytham's model without commentary. This seems another indication that in writing the *Ḥall* he still had not come up with the second, curvilinear version of his device.

4) The model for latitude that Ṭūsī describes in the *Taḥrīr al-Majisṭī* is schematic at best. In fact, it is a rather simplistic adaptation of the rectilinear Ṭūsī-couple and very different from the curvilinear version given in the *Tadhkira*, which Ṭūsī presented as an adaptation of Ibn al-Haytham's model.[13]

From this we can conclude that the Ṭūsī-couple, and its applications to various planetary models, emerged in stages and rather slowly. After coming up with the idea, apparently when writing the *Muʿīniyya*, it took many years before he felt comfortable enough to present it in the *Ḥall.* And at the time of writing the *Ḥall,* he still had not come up with the curvilinear version. A year later he tentatively put forth a kind of adaptation of the rectilinear version for a latitude model, but it was completely unsatisfactory since it produced straight-line motion, not the needed curvilinear oscillation along a great circle arc. Fifteen years later, he would bring forth both versions in their final form in his Arabic adaptation of the Persian *Muʿīniyya,* namely *al-Tadhkira fī ʿilm al-hayʾa*.

i.e. 13°11' (Naṣīr al-Dīn al-Ṭūsī 1335 H. Sh./1956-7 CE, f. 11). It is of great historical interest that it is the *Ḥall* version of Ṭūsī's lunar model that makes it into the Byzantine Greek work of Gregory Chioniades (d. ca. 1320) entitled the *Schemata of the Stars,* which would be available in Italy by the fifteenth century at the latest; see Ragep 2014, 242. For a listing of the parameters for the lunar model in the *Tadhkira*, see Ragep 1993, 2.457; a comparison of parameters between the *Tadhkira* and *Ḥall* can be found in Ragep 2017, 167.

13 Naṣīr al-Dīn al-Ṭūsī, *Taḥrīr al-Majisṭī*, Istanbul, Feyzullah MS 1360, ff. 199b-202a. This assessment of the model in the *Taḥrīr al-Majisṭī,* as well as the chronology of the development of the two versions of the Ṭūsī-couple, would tend to undermine the conclusions reached by G. Saliba 1987. A translation, edition, and analysis of the relevant parts of the *Taḥrīr* can be found in Ragep 2017, 168-171 and endnote 15. The *Taḥrīr* version appears in various European contexts, including Copernicus's *De revolutionibus*, for which see Ragep 2017, 182-184.

Appendix

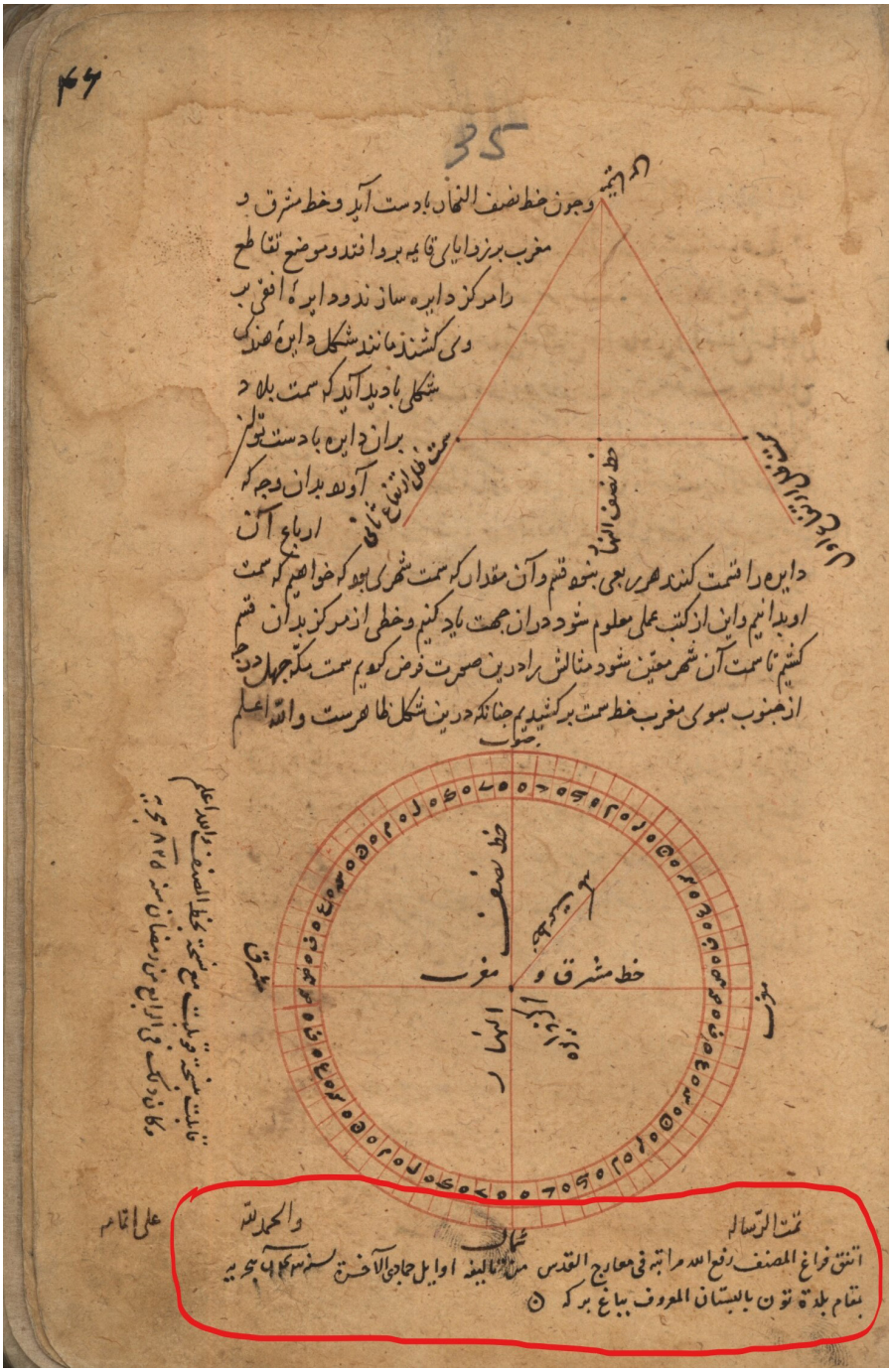

Figure 4. Colophon (boxed in red by current author) of *Ḥall-i mushkilāt-i Muʿīniyya,* Tashkent, al-Bīrūnī Institute of Oriental Studies, MS 8990, f. 46a (original foliation). Courtesy of the Institute.

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