

Gallucci's pseudo Copernican equatorium, Venice, 1593

Owen Gingerich

When a large package arrived from Germany a few years ago, I was filled with indignation. Why on earth had the auctioneers packaged Giovanni Paolo Gallucci's quarto volume in such an oversized carton?

But when I unwrapped the apparently oversized parcel, the joke was on me! I thought I had bid on Gallucci's relatively common celestial map book known for its volvelles, *Theatrum mundi et temporis* (1588), unaware that the Venetian author had in 1593 published a much larger-format volvelle book. What I had won at the auction was the very different folio, *Speculum uranicum*, a rarer production. It isn't a very thick volume, but it stands 42 cm high. It has [4] + 43 folios, and 16 of its pages have large circular woodcuts with moving parts.

Thus began a quest to understand how Gallucci's volvelles are supposed to work, and in fact how some of the moving parts were to be correctly assembled. So far I have examined a dozen copies, and with the exception of one hand-colored exemplar, apparently the dedication copy now in a private American collection, every copy has some fault: at least one loose, missing, or miss-positioned piece. The book was clearly intended to be used for computing the positions of the planets, undoubtedly for astrological purposes. My strong impression is that the book was more to be admired than actually used for computations.

But then again, the same could be said for Peter Apian's spectacular *Astronomicum Caesareum* of 1540. Apianus's masterpiece was surely the greatest work of astronomical printing in the 16th century. All the copies—probably around 150—were marvelously produced, hand-colored, and assembled in his Ingolstadt workshop. Twenty of the 146 pages contain volvelles, and the mechanism for the longitude of Mercury contains six rotating disks.

One reason that the *Astronomicum Caesareum* is a comparatively well-known book is that Edition Leipzig in 1970 issued a magnificent colored facsimile based on the copy in the Gotha (Germany) library, in an edition of 750 copies.¹ The printing was a tour de force, complete with a trompe l'oeil wrinkle on one of the final pages. But the assembly of the moving parts was a major disaster. For example, where Apianus had used multiple axes on some of the more complicated mechanisms, the facsimile has simply forced a single axis through the stack. And many of the disks are attached to the wrong pages. Having corrected more than a dozen copies of the facsimile, I feel familiar with the way it is supposed to work. When the paper discs are correctly positioned, it is possible to match the planetary longitudes calculated by the *Alfonsine Tables* to within a degree.

In order to appreciate what Gallucci has and has not accomplished with his *Speculum uranicum*, it is useful to examine first how planetary positions are established with a true equatorium. By an equatorium I mean a device that models the planetary motions, generally a paper instrument. To compare and contrast Gallucci's presentation with Apianus's *Astronomicum Caesareum*, I will show in detail with each volume how to compute the position of Mars for 1593 August 10 (Julian). This date is specially chosen because it was then that Tycho Brahe noticed that the

1 Owen Gingerich, Apianus's *Astronomicum Caesareum* and its Leipzig Facsimile, *Journal for the History of Astronomy*, 2 (1971), 168-177.

Ptolemaic (Alfonsine) tables and the Copernican tables predicted woefully inaccurate positions for Mars.² It is only a coincidence that this was noticed in 1593, the very year in which Gallucci's *Speculum uranicum* was printed. As Kepler was later to discover, this Martian catastrophe took place for a few weeks every 32 years (and had apparently gone unnoticed for centuries!). The catastrophe was of course for the tables, not for the ruddy planet. As we will see, Apianus's work is based on the *Alfonsine Tables*, whereas Gallucci generally uses the Copernican *Prutenicae Tabulae*, and both sets of tables were in trouble.

From Tycho's Observation Log for Mars, 1593 August 10 (Julian)

Copernicus	Psc 12° 0'	(calculated from <i>Prutenicae tabulae</i>)
Tycho	Psc 16° 7½'	(observed)
Alphonso	Psc 21° 26'	(calculated from the <i>Alfonsine Tables</i>)

In the technical details that follow, it will become apparent why I have chosen to refer to Gallucci's device as a "pseudo Copernican equatorium," for it is neither an equatorium nor is it heliocentric (but its numbers are Copernican!).

First, a brief orientation to the geocentric model for the superior planets (Mars, Jupiter, and Saturn). A large basic circle, called the deferent or carrying circle, is offset from the earth by an amount called the eccentricity and in a direction toward the farthest point called the apogee or aux. What the deferent or eccentric circle carries is the epicycle. Even if the epicycle is carried at a constant speed around the deferent, it will not appear to move at a constant angular speed as seen from the earth. Hence, in calculating the longitude of a planet, the first step must be to locate the direction of the apogee and to make an adjustment for the varying apparent speed of the epicycle. Then the apparent back-and-forth position of planet as it moves uniformly in its epicycle is the next step. Finally, a correction depending on the distance of the epicycle from the earth is required.

In Apianus's graphical device all of these corrections occur automatically in what is essentially an analog computer, where all the steps are visual. Gallucci's system, in contrast, is a less intuitive digital method, where it requires strict attention to know, for example, if a quantity is to be added or subtracted.

1593 August 10 (Julian): Mars in the Astronomicum Caesareum

In Ptolemy's epicyclic theory each planet's motion depends on three time-dependent motions: the moving position of the planet in its epicycle, the position of the epicycle on the deferent, and the position of the apogee. At first glance it might be assumed that the apogee is a fixed direction in space, but if it is locked with the starry frame, then there is a slow precessional motion as the entire sidereal frame moves with respect to the coordinate frame indexed to the sun's sidereal position at the time of the equinox.

Precession is such a sufficiently slow effect that Ptolemy was obliged to compare his current star positions with those made over two centuries earlier to derive the rate of precessional mo-

2 J. L. E. Dreyer, ed., *Tychonis Brahi Dani Opera Omnia*, vol. 12, (Copenhagen, 1925), 285.

tion, and in this he made one of the two major stumbles of his entire system (the other being the variation in the apparent size of the moon). In any event, he chose the convenient precessional rate of a degree per century, while actually it is closer to a degree every 71 years. Thus it didn't take too many centuries for repair work to be required on the rate of precession. However, in order to preserve the apparently slow rate deduced by Ptolemy between his time and that of Hipparchus, a variable supplementary rate was proposed, called trepidation. By the time of Copernicus and Apianus, the combined rate was failing, but Apianus stuck with the traditional precession-trepidation combination. He began his planetary calculations with precession and trepidation.

Because Apianus assumed that the precession-trepidation combination affected all the planets identically, he chose to deal with this by a single volvelle, a particularly beautiful rotating planisphere, the first moving disk in his book. (It will of course be desirable to have an *Astronomicum Caesareum* or its facsimile in hand in order to understand easily the following instructions.) To determine the precession for our example, open the book to its first volvelle, set the index tab M from the precession scale hidden under the right edge of Apianus's star map. Extend the thread from the middle of the planisphere across 1600 *post Christ* on the trepidation oval and rotate the planisphere until the index tab labeled *Y AUX Communis* lies under the thread. The index tab marked with the symbol for Mars should then be at Leo 16°, which will be used with the set of disks for Mars itself on f. D III.³ Note that this procedure has simultaneously set the starting point for each planet.

(Mean motion) We must next deal with the second and eventually the third time-dependent parameters; the starting points for these are given on the two tall tables on the page facing the Mars disks on f. D III of the *Astronomicum Caesareum*. Take the starting value for 1500 years after Christ for the mean motion of Mars, 8^s 5° 6', (where the superscript *s* refers to the zodiacal sign, which is labeled and numbered on the border of the disks on the facing pages). Take also the mean argument (which will refer to the epicycle), 1^s 14° 13'. Set the pointer tab (labeled M) on the lowermost disk to Sco 5° 6' (where Sco is labeled as the 8th sign).

The next step is a little tricky, because we will make the correction for the year by using the *completed* year, 1592, rather than the year in question, 1593. Find 92 on the border of the lower disk (hint: it should be near Libra 6°). Stretch the thread from *A mūdi* (= "mundi" or "world") over and past 92, and rotate the lower disk until the edge of the M index tab is directly under the thread. Finally, find August 10, which is concealed by the disk above it in the stack (the so-called AUX disk). Using the thread as before, rotate the M tab, which should now be at Aquarius 3°. This establishes the mean motion of Mars.

(Eccentricity) We must now take into account the eccentricity of Mars' orbit. Set the AUX disk at Leo 16°, the number found in the initial step from the planisphere. The next step is to transfer the mean motion indicated by the position of the M index pointer to the equant system which has its own axis, and which by definition carries uniform motion around the equant. In Apianus's ingenious arrangement, the set of diagonal lines on the AUX disk facilitates the transfer. The diagonal line from Aqr 3° leads to 5^s 17° on the AUX disk. Stretch the thread from *E Equant* to 5^s 17° and turn the Deferens Martis disk until the center of the epicycle coincides with the thread.

³ Readers using an uncorrected Leipzig facsimile will not find the index stub for Mars, which was carelessly trimmed off in the facsimile. Consult with the author of this essay concerning a repair kit.

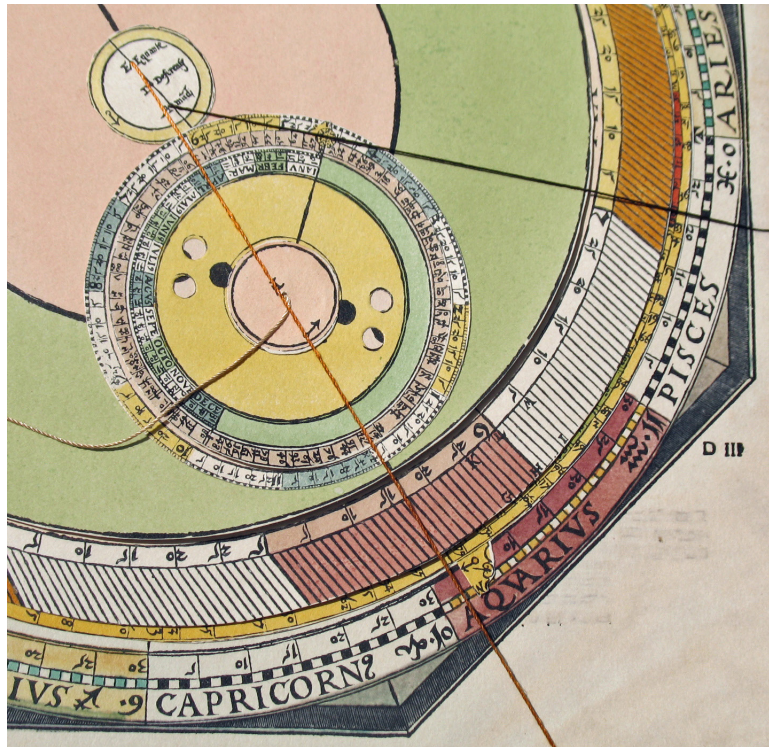


Figure 1. Detail of the Mars longitude calculation from the volvelles in Apianus's *Astronomicum Caesareum*. The setting is for 1593 August 10 (Julian), and the resultant longitude is Pisces 21°.

(Epicycle) Then stretch the thread from A through F (the center of the epicycle); rotate the lower epicycle disk so that index +AUX is also on the straight line.⁴ Next, rotate the upper epicycle disk so that tab Y with the small rosette sits at 1^s 14° (for 1500, from the table of mean arguments on the facing page). Then add the increment for 92 years (again using the *complete* year), and finally, add a further increment for August 10. Stretch the thread from A through the center of the rosette on the inner epicycle disk, and this should end up at Psc 21°, matching the calculation given by Tycho Brahe.

Examination of Leovitius' *Ephemeridum novum* (1557) (a huge compilation based on the *Alfonsine Tables*) shows that Mars was almost stationary at Psc 22° during July and the early part of August, and that by August 10 it had gone into retrograde motion and had dropped back to Psc 21°. Inspection of Figure 1, a detail of the Mars longitude page from the *Astronomicum Caesareum*, shows that as the Martian epicycle itself moves counterclockwise during this time, Mars in its epicycle is effectively cancelling this eastward motion as it approaches the earth. The derived position both from the *Alfonsine Tables* and Apianus's equatorium is 5° too large. It is difficult to see how some simple parameters could be changed to correct this without causing difficulties at another time. This was indeed the case, and it required a major geometrical rearrangement on Kepler's part to fix it (which was not done merely by introducing elliptical orbits).⁵

4 In this particular configuration the positioning of +AUX seems trivial, but it can be a significant correction elsewhere.

5 See Owen Gingerich, "The Great Martian Catastrophe and How Kepler Fixed It," *Physics Today*, vol. 64, no. 9 (September, 2011), pp. 50–54.

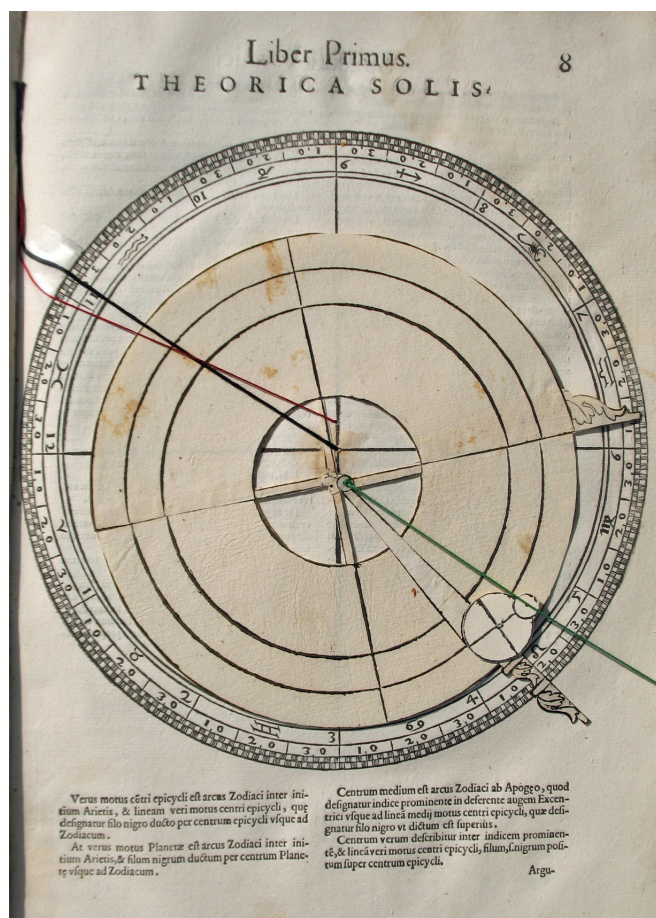


Figure 2. Volvelles used to teach the geocentric vocabulary for planetary theory in G. P. Gallucci's *Speculum uranicum* (Venice, 1593).

1593 August 20 (Gregorian):⁶ Mars in Gallucci's *Speculum Uranicum*

Because the *Speculum Uranicum* is not only full of volvelles covered with numbers, and because the geocentric cosmology is represented with a movable epicycle near the beginning of the book, one might readily suspect that this book is some sort of equatorium that has essentially fallen below the radar of the relevant experts. But such is not the case. The page in question (Figure 2) is simply a device for establishing the vocabulary for geocentric planetary astronomy, and the subsequent disks are an ingenious and not entirely successful way of presenting the requisite tables.

For Apianus, the great majority of additions (or subtractions) required in the calculation of a planetary position are carried out by sequential rotations of the disks. For Gallucci, these are pencil-and-paper calculations, and he provided a set of small printed tally sheets to guide the user (see Figure 3 for a worked example).

This section will explicate in detail how to use Gallucci's volume to calculate the position of Mars for the same day as before. Apianus's equatorium was based on the Ptolemaic/Alfonsine

⁶ In 1582 the Gregorian calendar reform took place for the Catholic countries, so Gallucci's computational scheme is entirely based on the newer calendar.

Computus Martis.

S	G	7	11	
11	0	16	41	Pro radice anni. 1584
3	1	9	31	Pro annis. 8
				Pro annis.
3	21	5	25	Pro toto mense. July
	10	28	45	Pro diebus. 20
				Pro horis.
				Pro minutis.
6	13	0	22	Summa centri medij Martis.
5	11	1	41	Pro radice anni. 1584
8	28	43	29	Pro annis. 8
				Pro annis.
3	7	57	15	Pro toto mense. July
	9	13	53	Pro diebus.
				Pro horis.
				Pro minutis.
5	126	50	18	Summa argum. medij Martis.
				Aequatio centri æquata.
5	24	13	24	Argumentum verum.
6	3	37	16	Centrum verum.
-5	16	12		Aequatio argum. primò correcta.
-4	34	12		Excessus correctus.
-1	9	50	24	Aequatio argum. absoluta.
4	10	19	42	Medius motus Solis.
5	16	23	0	Distantia σ à medio loco in præced.
10	13	56	42	Verus locus σ à cornu \surd .
	127	56	42	Præcessio æquinoctiorum.
11	11	53	24	Verus locus σ in decimo orbe.
				Latitudo σ .

Figure 3. Gallucci's tally sheet, filled in by the author for the longitude of Mars on 1593 August 20 (Gregorian). Here the resultant longitude is Pisces 12°. The actual observed longitude was just over Pisces 16°.

numbers, whereas Gallucci's numerical scheme turns out to be based on the Copernican numbers from Erasmus Reinhold's *Prutenicae tabulae*.

In general the Copernican system employs the same number of parameters as the Ptolemaic system, which means that the Copernican numbers will be very similar to those used for the *Alfonsine Tables*. Copernicus used the Ptolemaic observations to anchor the time-dependent parameters and followed up where possible with a set of modern observations to check if there had been any changes. In this pursuit he found that the apsidal lines of the superior planets did not simply follow the precessional changes. This meant that Copernicus, Reinhold, and Gallucci had to add another time-dependent table to handle the slow independent motions of the apsidal lines. Gallucci chose to tabulate the mean motion of the planet with respect to the moving solar position, and to include the moving precessional position of the apsidal line later in the calculation.

At the same time, Gallucci had to cope with the change in the calendar, which broke the rhythmic flow of years and months. Workers using the *Prutenic Tables* generally used the date of the Incarnation as the starting point (i.e., AD 0 as the initial complete year), and simply added the tabulated incremental motions to bring the setting to the desired date. For instance, the mean longitude of Mars for AD 0 was, according to the *Prutenic Tables*, $0^{\text{sex}} 34^{\circ} 7' 46''$. To find the "complete" position for 1593 August 20, the user of the tables had to add the relevant tabulated increments as follows:

Radix for AD 0	0 ^{Sex}	34°	7'	46''	
1500 yrs		4	4	12	41
80 yrs	3	12	13	29	
12 yrs	2	16	50	1	
July	1	51	5	41	
19 days		<u>0</u>	<u>9</u>	<u>57</u>	<u>24</u>
	0	8	27	2	

(As a quick reality check here, we know that the sidereal period of Mars is about two years, and therefore Mars must move about half a degree per day; hence 10 degrees in 19 days seems about right.) But there is a calendar problem! The *Prutenic Tables* were compiled long before the Gregorian calendar reform. A user working with the Gregorian calendar must remember to subtract 10 days to account for the short year, 1582. However, Gallucci has made it easy: he includes radices for 1584 in his tabulations, which nicely skips over the short year.

The procedure for Mars requires four of these calendar-dependent calculations, two for Mars itself, another for the mean sun, and a final one for “the eighth sphere” (which is tantamount to the precession). We turn now to the small tally sheet for Mars, filling it with numbers from the page facing the Mars volvelle on f. 15. (Note that Gallucci is using the zodiacal sign-degree system rather than the sexagesimal system of the *Prutenic Tables*, and the superscript S designates a zodiacal sign.) The calculations are for *complete* years, which means that for a date in August, the “complete” month of July applies.

Figure 3 shows the tally sheet, where the upper half of the sheet gives the sequential parts for the first two time-dependent quantities, the mean center for Mars and for the mean argument of Mars. The first moves with the sidereal rate of Mars, and the second with the synodic rate, just as in the *Prutenic Tables*, except Gallucci's radices for the first quantity are fixed to the mean sun (rather than the beginning of Aries), and without precession.

(Eccentricity) Then follow the corrections for the eccentricity and for the effect of the epicycle. (In the Apianus equatorium these effects are automatically taken care of by the varying distance of the actual epicycle.) These corrections are found on Gallucci's Mars disk on f. 15. Precisely why the disk is a rotatable volvelle is not at all clear except that the user can bring the appropriate part of the inscribed table to a convenient place where the requisite numbers are not upside down. There are two moving indices (see Figure 4); the first, labeled *CENT* and *S PRO* needs to be set on the mean center of Mars, $6^s 3^o 0' 22''$ which is just past the midpoint of the circumference, 183° . We will set this using the “aequationes” in the center of the volvelle, which is the principal clever thing about Gallucci's process. The central part of the disk has an outer ring with the signs 1 through 6 running counterclockwise and an inner ring with signs 7 through 12 running clockwise. As the cautionary caption below the disk indicates, numbers obtained from the inner signs (7–12) are positive, and the outer signs (1–6) negative. Inspection of the numbers in the double *CENT* band show they go from 0^s to $11^s 6^o$ and back down to 0^s (taking

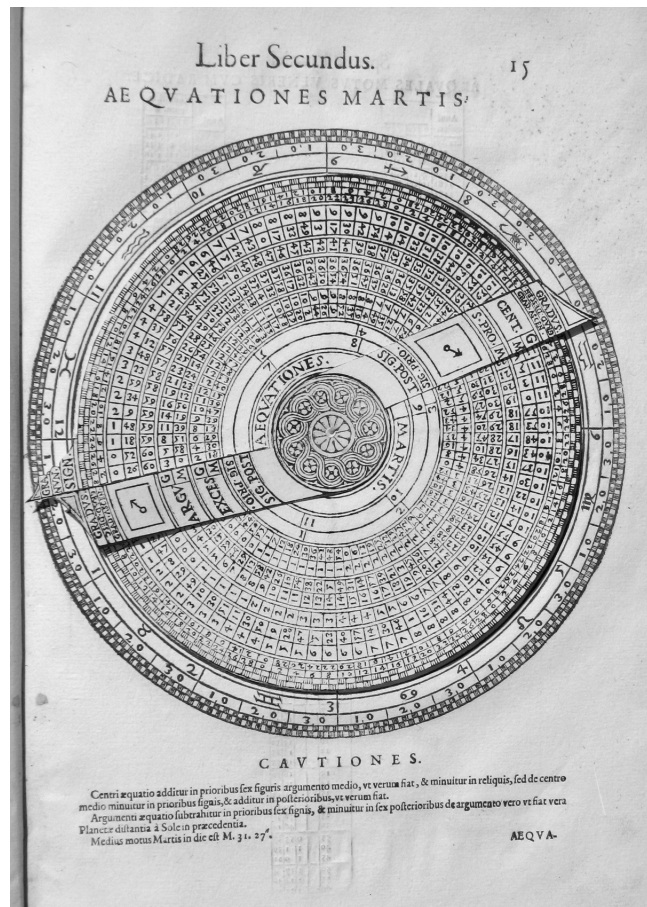


Figure 4. Detail from the disc for the longitude of Mars with the first index set for the example shown in the tally sheet (Figure 3). The correct setting for the second index would fall on top of the first, so it is simply randomly placed here.

care of the first six signs) and then in a symmetrical counterclockwise sense to $-11^{\circ} 6'$ and back to zero. This exactly matches the prosthaphaeresis columns in the *Prutenic Tables*.

Because in our particular case the epicycle is near the perigee of its orbit, the eccentricity effect is very small. We find this by setting the first pointer to $5^{\circ} 26'$, where for *CENT* we read $0^{\circ} 26'$. Of course we really want the result for $5^{\circ} 26' 50'' 18''$ and Gallucci is not much help with the interpolation over the two-degree step. We can say that the answer is roughly $-36' 54''$, which we enter on the tally sheet under *Aequatio centri æquata*. We subtract this from the argumentum and add it to the centrum, entering them into the next two lines of the tally sheet.

But wait! The first index points to a second quantity, labeled as *S Pro* or “scrupula proportio,” but with no place to enter it on the tally sheet. We can see on Figure 4 that the entry yields $60'$, which in the sexagesimal system equals unity, and as the process unfolds, a correction will have maximum effect with no intricate multiplication.

(Epicycle) What follows next both in the *Almagest*'s instructions and in the *Prutenicae tabulae* is one of Ptolemy's cleverest moves. Imagine that the planetary epicycle is at the apogee. Ptolemy could have tabulated the visual angle from the earth for every degree that the planet might have as it goes around in the epicycle, say 180 positions since symmetry reduces the total number of 360. As the epicycle moves away from the distant apogee point, the angles will be-

come gradually larger as the epicycle comes nearer. So another 180 tables each with 180 entries would be required to take into account the varying distance of the epicycle itself. This would make a truly huge size for a book of tables since such a 180×180 set would be required for each planet. What Ptolemy did was to reduce the two dimensional table to a product of pair of linear functions, one related to the synodic rate of the epicycle and the other to the sidereal rate of the deferent circle.

The functions related to the epicycle come with second index on Gallucci's Mars disk. These are abbreviated *Argu* and *Exces* for Argumentum and Excess, and the argument for the setting is the entry on the tally sheet labeled *Centrum verum*, $6^s 3^o 37' 16''$. At $6^s 2^o$, the respective numbers are $5^o 58'$ and $4^o 30'$. Once again a fairly tedious numerical interpolation is required, here between $6^s 2^o$ and $6^s 4^o$, leading to $5^o 16' 12''$ and $4^o 34' 12''$. As Gallucci explains in his text, the second number must be multiplied by the *S Pro* found with the first index, which in this case is unity and the number is unchanged. The label for the line on the tally sheet, *Excessus correctus*, gives the hint that a corrected number is required. (Gallucci included a foldout base-60 multiplication table in case that was needed.) The sum of these two corrections go into the tally sheet line *Aequatio argum. absoluta*. This quantity is then used to correct the *Argumentum verum*; for numbers determined from the second index, the sign convention is reversed with respect to the first index. Thus in our case the two corrections are negative, and hence they are subtractive. Note that their sum is marked with a minus sign when entered in the line *Aequatio argum. absoluta*, which is therefore subtracted from the *Argumentum verum* to obtain the *Distantia* etc.

(Mean motion and Precession) Two other time dependent quantities are needed to complete the calculation, the mean position of the sun, and the precession. Each of these have full openings with disks in Gallucci's scheme, although in the solar case, because only the mean sun's position is required, the disk is not used, and for the precession only a rudimentary use of the disk is required. The *Distantia* line is subtracted from the *Media motus Solis* to get the true location of Mars from the horn of Aries, and finally the precession is added to obtain $11^s 11^o 53' 24''$, that is $341^o 53' 24''$, the "true position of Mars in the tenth sphere." This position is in satisfactory agreement with Tycho's Copernican calculation, which misses his observed position by over 4^o and in the opposite sense from Apianus's even more erroneous Alfonsine calculation.

General Evaluation of the Speculum Uranicum

When Gallucci's *Speculum Uranicum* was published both Galileo and Kepler were alive. Both would become enthusiastic converts to the Copernican system, and the two most important figures in persuading a skeptical public that the heliocentric cosmology was a physically real description of the world. But in 1593 neither had yet published anything hinting of the earth as a spinning planet. In the closing decades of the 16th century the overwhelming majority of astronomers accepted *De revolutionibus* as a recipe book for computing the positions of the planets, but not as an actual physical cosmology. As the blurb at the center of Copernicus' title page put it, "You have in this recently created and edited work the motions of the stars, both fixed and planets, re-established from ancient observations and recent ones as well, and moreover embellished with new and admirable devices. You also have here the most expeditious tables from which you can very easily calculate for any time. Therefore buy, read, profit."

The expanded version of Copernicus' "most expeditious tables" was Erasmus Reinhold's *Pru-tenicæ tabulae* (Tübingen, 1551), and these were tastefully independent of cosmology. Paul Wittich's well-annotated copy of *De revolutionibus*, now in the Vatican Library, has supernumerary

pages with geocentric equivalences to Copernicus' heliocentric arrangement.⁷ In this category was J. A. Magini's *Novae coelestium orbium theoricæ congruentes cum observationibus N. Copernici* (Venice, 1589), a thoroughgoing geocentric text. These works provided the intellectual framework into which Gallucci's supposedly "handy version" of the *Prutenic Tables* fit.

My own encounter with these "handy tables" was exceedingly tedious because of the host of computational errors that beset my attempt to understand how it worked. It took some time to appreciate that this was not a volvelle book somehow distantly related to Apianus' masterpiece, and that it was in fact a distinct version of the *Prutenic Tables*. There was a great moment of triumph and relief when I discovered that the result from Gallucci's book agrees reasonably closely with the Psc 12° 8' in Magini's *Ephemeris* (Venice, 1582), a result which was more or less accidental because the time of day was not considered explicitly. It would have been much easier to use the *Prutenicæ tabulae* directly, and the whole process was in fact facilitated by having Reinhold's book immediately at hand.

While the circular disks show clearly how the corrections function, now additive, now subtractive, they hardly have enough significant figures for accurate interpolation, and even Gallucci remarked that it would have been better to have engraved plates rather than woodcut disks. In fairness to Gallucci, I must remark that besides the disks for planetary longitudes, there are similar disks for calculating the latitudes of the planets, and the volume concludes with a substantial amount of astrological instructions and star tables.

Nevertheless, the fact that only one tally sheet for each planet was provided suggests the set of small tally sheets was intended for one-time use as the owner personalized his copy by working out the planetary positions for his own horoscope. The volume could then take its place as a vanity press or trophy book. Today it remains as a trophy specimen for that transitional period when Copernicus was appreciated for his numbers but scarcely for his cosmology.

7 See Owen Gingerich and Robert S. Westman, *The Wittich Connection: Conflict and Priority in Late Sixteenth-Century Cosmology*, *Transactions of the American Philosophical Society*, vol. 78, part 7, 1988.