The Little Astronomy and Middle Books between the 2nd and 13th Centuries CE:

Transmissions of Astronomical Curricula

by

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ABSTRACT

This dissertation examines the transmission of two astronomical curricula: the Little Astronomy of Greek late antiquity and the Middle Books of the medieval Islamicate world. The Little Astronomy is usually understood to have comprised a group of approximately nine ancient Greek texts: Theodosius's *Sphaerica*, Autolycus's *On the Moving Sphere*, Euclid's *Optics*, Euclid's *Phaenomena*, Theodosius's *On Habitations*, Theodosius's *On Days and Nights*, Aristarchus's *On Sizes and Distances*, Autolycus's *On Risings and Settings*, and Hypsicles's *Anaphoricus*. All of these treatises were translated into Arabic by the end of the ninth century CE, and these translations came to serve as the core of the Middle Books – a grouping named as such because they were the books to be read between Euclid's *Elements* and Ptolemy's *Almagest*. The existence of a collection called the Middle Books is well-attested by contemporary sources; that of the Little Astronomy is less so. This dissertation therefore sets out to establish the evidence for these respective groupings, examining when they existed, what form they took, and how they developed over time. It determines that the Little Astronomy and Middle Books both comprised a persistent core series of treatises set out in a logically ordered arrangement, sometimes accompanied by other treatises at different points in time. The dissertation then turns to philological analyses to establish the influence of the curricular context on the transmission of the component texts. I argue that many of the changes introduced into these texts by late antique and medieval editors can be identified as motivated by the didactic use of these curricula, and that these contributions speak to how copyists, teachers, and editors in different contexts perceived of their own relationship to a long-lived astronomical tradition.

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INTRODUCTION

Modern scholarship on ancient Greek astronomy has, since the seventeenth century, often used the term 'Little Astronomy' to refer to a particular grouping of ancient Greek treatises on mathematics and spherical astronomy found in the manuscripts. The modern understanding often portrays the Little Astronomy as the curriculum a student worked through in preparation for the 'great' astronomy, Ptolemy's *Almagest*. The component texts of the Little Astronomy were translated into Arabic during the ninth century, in which language they also formed a grouping together, one known as the Middle Books. However, while in the Arabic transmission there exists contemporary evidence that speaks of the Middle Books as a curriculum, the evidence that has been put forth for the Little Astronomy as a curriculum that already existed in late antiquity is much less certain. Indeed, it is the Arabic sources which attest that the Middle Books are so named because they are the books read between Euclid's *Elements* and Ptolemy's *Almagest*.[1] None of the scarce Greek evidence scholars usually put forward today suggests that the supposed Little Astronomy was intended to prepare one for Ptolemy's great treatise. Such a characterization in the modern scholarship would seem to be retrojected from the later Arabic transmission.

This dissertation will show that both the Little Astronomy and the Middle Books did exist as deliberate groupings and did see didactic use. First, however, a reevaluation of the evidence available for the Little Astronomy is necessary, and a separation of it from the evidence for the Middle Books, to which it should be compared. It is also desirable to present a broader examination of these texts' combined manuscript transmissions. In the case of the Middle Books, it is necessary to determine when this curriculum coalesced. Naṣīr al-Dīn al-Ṭūsī produced an edition of the Middle Books in the 13th century,

[1] These Arabic sources will be presented in chapter 3.

but they certainly had their educational use before then, whether the works came to Arabic already as a curriculum or were shaped into one afterwards. Further, when these groupings of texts did serve as curricula, they did not do so in a vacuum. This dissertation examines the transmission and use of the Middle Books to determine how this curriculum intersected with astronomical scholarly activities in the Islamicate world from the seventh through the thirteenth century. The dissertation also leverages the evidence from the transmission of the Little Astronomy texts in Greek to illuminate late antique and Byzantine engagement with this curriculum.

Overview of the Little Astronomy and Middle Books in western scholarship

The earliest printed mention of a Little Astronomy in western scholarship appears in the 1621 publication of Henry Savile's thirteen preparatory lectures on Euclid's *Elements* as Geometry chair at Oxford. Savile spoke of a "μικρὸν ἀστρονόμον" or "μικρὸν ἀστρονομούμενον" – perhaps best translated as Little Astronomer – comprising nine texts in sequence.[2] These nine works will be introduced more fully at the end of this introduction, but they are the following:

1. Theodosius's *Sphaerica*, three books
2. Euclid's *Optics*
3. Euclid's *Phaenomena*
4. Theodosius's *On Habitations*
5. Theodosius's *On Nights and Days*, two books
6. Autolycus's *On the Moving Sphere*
7. Autolycus's *On Risings and Settings*, two books
8. Aristarchus's *On the Sizes and Distances of the Sun and the Moon*
9. Hypsicles's *Anaphoricus*

[2] Savile (1621) 40-41: "Hoc volumen integrum in Bibliothecis Galliae & Italiae saepe vidi, continetque nouem diversos tractatus sequentes: primo loco, Theodosii Sphaericorum libros tres: secundo, Euclidis Optica: Tertio eiusdem Phaenomena; Quarto, Theodosii libellum de habitationibus: quinto, Eiusdem de Noctibus & Diebus, libros duos: Sexto, Autolyci librum de Sphaera mota: Septimo, eiusdem de Ortu & Ocassu libros duos: Octauo, Aristarchum Samium de Magnitudinibus & distantiis solis & lunae: vltimo, Hypsiclis ἀναφορικὸν, siue de ascensionibus."

The same account of these works was picked up by Vossius 1650, who was subsequently cited by Fabricius in his own account of the grouping in the eighteenth century.[3] The latter evidently had other sources at hand, since he enumerated twelve works of the collection, adding Euclid's *Data* and *Catoptrics* and Menelaus's *Spherics*. The evidence cited by these early authors was twofold: the material in Pappus (4th century CE) *Collection* Book VI said by a scholion to discuss problems in the "μικρῶι αστρονομουμενωι", and the fact that these texts tended to appear together in manuscripts. The collection was attributed to the Alexandrian scholars.

The influence of the Arabic tradition can already be seen in the above accounts, which explain the Arabic name of Ptolemy's treatise — the *Almagest* — and discuss the Little Astronomy's relationship to this "Great Astronomy." Subsequent scholarship like Wenrich 1842 and the modern editions of texts claimed as members of the Little Astronomy present very similar narratives.

In the twentieth century, Pingree 1968 laid out the body of evidence which had become standard to support the existence of the collection. In addition to the above evidence, he presented the suspected references to the Little Astronomy by an anonymous late antique commentator (6th century CE?), by John Philoponus (6th century CE), and by Cassiodorus (6th century CE). He also presented a comparison of orders in the Greek and Arabic collections according to different manuscripts. His overview was brief, and he concluded it with a call for an examination of intra- and extra-corpus recensions of Little Astronomy works to understand how they were edited and circulated together.

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[3] Fabricius (1716) 88: "Theodosii Tripolitae Sphaericorum libri III.
Euclidis Data, Optica, Catoptrica ac Phaenomena.
Theodosii Tripolitae de habitationibus & noctibus ac Diebus libri II.
Autolyci Pitanaei de Sphaera mota, & libri II, de Ortu atque occasu stellarum inerrantium.
Aristarchi Samii de magnitidinibus & distantiis Solis ac Luna.
Hypsiclis Alexandrini Ἀναφορικός sive de ascensionibus.
Menelai Alexandrini Sphaericorum libri III."

Much of the evidence above is open to dispute, however, and the existence of a Little Astronomy in late antiquity has been questioned. Neugebauer 1975 in particular dismisses much of Pingree's evidence, arguing that the only factual evidence is the tendency for approximately the same mixture of elementary astronomy, mathematics, and optics to appear together in manuscripts. He sees this as a demonstration of the texts' usefulness to contemporary schoolmasters, but sees no evidence that indicates the earlier existence of a curriculum under the title μικρὸς ἀστρονομούμενος.

Certainly one of the claims about the Little Astronomy — that it was preparation for the *Almagest* — is not present in any of the currently known Greek evidence. This seems to have come from the Arabic tradition of the Middle Books, knowledge of which had certainly entered the Latin tradition by the fourteenth century at the latest.[4]

Despite these doubts, scholars afterwards have generally accepted the notion of a late antique Little Astronomy. Subsequent scholarship often alludes to the Little Astronomy as the context for a particular text.

Meanwhile, for the Middle Books the key piece of scholarship is Steinschneider 1865's article "Die 'mittleren' Bücher der Araber und ihre Bearbeiter." He notes some of the evidence for the Middle Books before al-Ṭūsī and explores the attested orders for the collection. He then discusses each work in turn, adducing not only the Arabic material but also medieval Latin and Hebrew translations from the Arabic (some of which were made before al-Ṭūsī's edition, e.g. Gerard of Cremona's twelfth century translations).

Subsequent nineteenth century scholarship usually covered ground quite similar to Steinschneider, citing the same evidence and offering similar presentations of the various attested orders

[4] Awareness of the Arabic tradition is shown by the manuscript Paris lat. 9335, which contains a listing of Middle Books titles under a heading which describes it as the order after Euclid's book according to the writings of Johanicus. This manuscript and note will receive further discussion in chapters 3 and 7.

(see Manitius 1888, Suter 1900). Some variety does appear in what treatises are listed among the Middle Books — Suter 1900 for instance includes also Euclid's *Elements*, Apollonius's *Conics*, Ptolemy's *Almagest* and *Tetrabiblos*, and the *Centiloquium*.

Overview of chapters

This dissertation will examine the continuing history of the Little Astronomy and the Middle Books at three points: (1) the Little Astronomy in Greek late antiquity, leading up to the ninth century; (2) the component texts in the ninth century, after their translation into Arabic; and (3) the Middle Books in the thirteenth century, when they received a new edition by Naṣīr al-Dīn al-Ṭūsī. It will be structured in four parts, the first addressing point (1) above, the second addressing point (2), the third offering some comments on the transmission of the two curricula between the ninth and thirteenth centuries, and the fourth addressing point (3). There will be nine chapters total, excluding the introduction, conclusion, and appendix.

A note on dates presented in this dissertation: the chapters which concern Greek or Latin material will use Gregorian dates. Chapters which concern Arabic material will use the combined Hijri / Gregorian dates.

In Part I, chapter 1 will argue that evidence does survive to say that a group of nine or so Greek texts were used in late antiquity as an ordered astronomical curriculum. This discussion disentangles references to or other evidence for the Little Astronomy from claims about the Middle Books. This grouping of texts should not be attributed to much later Byzantine redactors, as has sometimes been suggested,[5] but rather has roots as far back as the second century CE and so may have been the product of an increasing canonization of texts during that period.

[5] See e.g. Pingree (1968) 16 and Neugebauer (1975) 769.

In chapter 2, the Greek transmission of nine Little Astronomy texts will be examined in more detail, to identify what alterations occurring in these texts can be identified as motivated by or otherwise speaking to the curricular context. The chapter strives to focus on variations which were introduced before the ninth century CE – so, in the seven centuries after the curriculum's earliest possible attestation. It relies on manuscript evidence along with insights offered by contemporary scholars like Pappus.

Part II is headed by chapter 3 and its examination of these works' translations in the third / ninth century. It is firstly a general overview of which of the curriculum's texts were translated and of attestations regarding who patronized, produced, and corrected these translations. But one of its significant takeaways is that, despite these endeavors being attributed to many different translators with seemingly no unified effort to translate or correct the curriculum as a whole, the component works of the Little Astronomy quickly saw use as a didactic group in Arabic shortly after their translation. The name al-Mutawassiṭāt, "the Middle [Books]," is already attested by the title of a commentary by one of the translators in question; another one of the translators is credited with a list that declares the relevant works are the ones to be read after the *Elements*.

Chapter 4 follows the second chapter's model, laying out the alterations that are found in the Arabic manuscripts and seeking deliberate choices by the translators or early Arabic editors. Ultimately, at this stage many of the variants appear to be indicative more of the state of the Greek manuscripts that were available to the translators. There are some cases of material being added or expanded upon, or being rewritten in a clearer style. But in this early stage of work in Arabic with the texts, the greater focus seems to be grappling with the Greek tradition (and multiple variants thereof) which the translators and scholars had available to them.

Part III begins with chapter 5's overview of the Middle Books and their usage between the third and seventh centuries H / ninth and thirteenth centuries CE. It presents manuscript data from witnesses penned during this period, highlighting several codices in which the curriculum is transmitted in a whole or partial grouping. It then turns to contemporary bio/bibliographical and other outside sources to discuss the scholars whose intellectual activities intersected with the Middle Books.

Chapter 6 returns to the Byzantine world to inquire after usage of the Little Astronomy after the ninth century. In comparison to the preceding chapter, it ultimately describes a gap. While the component works of the Little Astronomy survived (and indeed there are multiple manuscripts extant from the thirteenth century to show that they received attention during the Palaiologan Renaissance), evidence of ongoing use of the curriculum is nonexistent. There is a general lack of information about education in mathematical astronomy during this period, so in light of this context the apparent absence of the Little Astronomy is not exceptional.

In chapter 7, the translations of the curriculum's works into Latin and Hebrew are sketched out. A detailed study of these translations and their subsequent transmissions, which occurred in the twelfth and thirteenth century, is beyond the scope of this present dissertation. The chapter instead delves into how fully the astronomical curriculum was translated into these languages and by whom. It is quite clear that interest in these texts was motivated by the active study they were receiving in the Islamicate world, and the story seen in this chapter of translations of the Arabic Middle Books rather than the Greek Little Astronomy reinforces the findings of the preceding two chapters.

The final two chapters comprise part IV. Chapter 8 gives a brief overview of Naṣīr al-Dīn al-Ṭūsī's life as it has been established in modern scholarship and as it pertains to mathematical astronomy. It then narrows its focus to his work as an editor and as a teacher, considering as a comparison the parallel efforts of a certain Muḥyī al-Dīn al-Maghribī, one of al-Ṭūsī's colleagues at Maragha Observatory, the new astronomical center of its age. Al-Ṭūsī produced new editions of the full sequence of the *Elements*, the Middle Books, and the *Almagest*; al-Maghribī produced or is reported to have produced new editions of the *Elements*, several Middle Books treatises, and the *Almagest*. These works came to be among the works studied by students at Maragha Observatory.

Chapter 9 concludes on the model of chapters 2 and 4, exploring what choices al-Ṭūsī made in producing his editions of the Middle Books. Where variants considered in the previous chapters may have had their source in the decisions of any number of possible known or unknown actors, here many of the alterations can be ascribed to the choices of a single editor.

Studying the transmission of a curriculum

The texts commonly named as components of the Little Astronomy and later of the Middle Books range in date from the fourth century BCE to the first century CE. The forms of these texts were not static throughout their subsequent transmissions. All of the texts possess at least one later recension in Greek. Modern scholarship usually acknowledges this multiplicity to be a result of late antique pedagogic programs, but no study has yet examined the varying versions of Little Astronomy / Middle Books works as a group to determine what they might reveal about intellectual or teaching practices in the relevant periods.

A key component of this dissertation is therefore philological, albeit not philological in the sense of standard textual criticism and its usual orientation towards an original text. Rather, several chapters examine the variances between the different versions of these works — whether these versions be Greek recensions or Arabic translations, corrections, and editions. The orientation of these inquiries is towards how the texts were received, used, and adapted by subsequent centuries of readers.

This work takes cues from Vitrac 2012, which approached this problem in the context of Euclid's *Elements*. Vitrac laid out a typology of deliberate alterations — a system distinct from the set of variant types frequently used for textual criticism and the construction of stemmata — and he brought it to bear on the various Greek/Arabic/Latin versions of the *Elements*. The subset of variants which Vitrac deemed deliberate alterations are ones introduced intentionally into the text by the individual responsible. He established the following set of alterations:

- Modification of Presentation
    - Alteration of Proofs
        - Global
            - Substitution of Proof
            - Double Proofs
            - Addition / Suppression of Cases
        - Local
            - Stylistic Interventions
            - Abridged Construction / Shortened Proof
            - Logical Interventions
    - Change in Order
    - Fusion / Division
    - Change of Status
    - Different Formulations
- Addition / Suppression of Material

This typology's origins in work with Euclid's *Elements* is very apparent, and nearly all of the potential alterations concern changes which could be found in the standard proposition-based genre of Greek mathematics. Several concern changes in the exterior ordering and structure of propositions and their adjacent units; several concern changes interior to the parts of the proposition. All concern changes which ancient and medieval editors of mathematical texts are known to have made, since they were often motivated to make choices based on considerations other than preserving the perceived original form of a text.

This approach is quite applicable to research with the Little Astronomy and Middle Books because they too comprise works of proposition-based Greek mathematics. This structure of ordered, clearly divisible textual units lends itself very well to identifying the kinds of alterations which are laid out in Vitrac's typology. The formulaic language of ancient Greek mathematics also facilitates such an investigation.

Since this dissertation examines a grouping of nine to ten treatises across at least three different versions and two different languages, there is not the time to survey this corpus for the whole set of deliberate alterations Vitrac lays out. Instead the study focuses on alterations that affect the global and structural form of the text. These are the following:

- Addition / Suppression of Material
- Substitution of Proof
- Double Proofs
- Addition / Suppression of Cases
- Change in Order
- Fusion / Division
- Change of Status

Chapters 2, 4, and 9 will survey the selected works of the Little Astronomy and the Middle Books and discuss what broader patterns and insights emerge from this set of alterations and how they speak to ways in which users interacted with these texts and contributed to their tradition.

<u>On "Curricula" in Late Antiquity and the Medieval Period</u>

This introduction has already used the term "curriculum" several times in reference both to the Little Astronomy and the Middle Books. But we must take care not to import the whole host of modern associations that the word might call to mind in the reader today. This study will use the term "curriculum" as a shorthand for the idea of treatises grouped together for didactic purposes.

In embarking upon this study of the Little Astronomy and the Middle Books, we must take care not to simply *assume* they possessed characteristics that we might find for modern curricula: late antique and medieval education looked quite different from today's modern educational systems. Nor should we assume late antique and medieval "curricula" looked the same across the temporal and geographic ranges covered in this study. Over the course of this dissertation we will uncover various characteristics of the Little Astronomy and the Middle Books and interrogate their continuity over the transmissions of these corpora, but we must not take any of these for granted merely because they come to mind when the modern reader thinks of curricula today.

Overview of works relevant to the Little Astronomy and Middle Books

The following will give a brief overview of each of the texts to be examined in this dissertation as part of or connected to the Little Astronomy and the Middle Books.[6] The works are presented chronologically by author to emphasize the range of time their production encompassed. From this, it is quite clear that whatever didactic groupings they came to form part of were ones which necessarily developed later.

Autolycus (4th century BCE)

On the Moving Sphere (Περὶ κινουμένης σφαίρας) is a work by the ancient Greek mathematician Autolycus on the subject of spherical geometry. Its propositions concern the movement of points and arcs on the surface of a sphere when the sphere is turned on its axis.

On Risings and Settings (Περὶ ἐπιτολῶν καὶ δύσεων) is another work of spherical geometry, but of an explicitly astronomical character. It concerns the risings and settings of stars as they occur or are seen to occur throughout the year.[7]

Euclid (fl. 300 BCE)

The *Elements* (Στοιχεῖα) of Euclid is the most well-known work of ancient Greek geometry. Its thirteen books take the reader through plane geometry, magnitudes, number theory, and solid geometry. This work has a long history of being used as a textbook for beginning students of geometry.[8]

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[6] See Berggren (1991) overall for discussion of the history and contents of many of these treatises and their relevance to astronomical spherical geometry.
[7] See Mogenet (1950) for the editions of the Greek texts for both of Autolycus's works. See Bruin and Vondjidis (1971) for translations into English.
[8] See Heiberg and Menge (1883-1916) for the edition of the Greek text. See Heath (1925) for a translation into English.

The *Data* (Δεδομένα) is a general geometrical work which examines what can be deduced when certain information is 'given' in geometrical contexts.[9]

The *Phaenomena* (Φαινόμενα) is a work of spherical geometry of a specifically astronomical character. Its propositions concern the risings and settings of stars and of particular arcs associated with the zodiac.[10]

The *Optics* (Ὀπτικά) is a work of geometrical optics, whose propositions consider vision projected as straight lines from the eye. It is often connected to the *Catoptrics* (Κατοπτρικά), also attributed to Euclid (though this is disputed), which is another work of geometrical optics but dealing specifically with reflections.[11]

Aristarchus (3rd century BCE)

On the Sizes and Distances of the Sun and the Moon (Περὶ μεγεθῶν καὶ ἀποστημάτων ἡλίου καὶ σελήνης) is a work of astronomical spherical geometry. As the title communicates, its propositions deal with calculating the sizes of the sun and the moon and their distances from the earth relative to the earth's radius.[12]

Theodosius (2nd century BCE)

The *Sphaerica* (Σφαιρικά) of Theodosius is a work on spherical geometry which served as an introduction for the topic. Theodosius evidently drew from a no longer extant corpus of spherics in producing this work, arranging the material in a more didactic manner.[13]

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[9] See Menge (1986) for the edition of the Greek text. See Taisbak (2003) for a translation into English.
[10] See Menge (1916) for the edition of the Greek text. See Berggren and Thomas (1996) for a translation into English.
[11] See Heiberg (1895) for the editions of both Greek texts.
[12] See Heath (1913) for both the edition of the Greek text and a translation into English. See also Noack (1992) for a study of the text.
[13] See Heiberg (1927) and more recently Czinczenheim (2000) for editions of the Greek text. A translation into French is also provided by Czinczenheim; see also the earlier translation into French in Ver Eecke (1927).

On Habitations (Περὶ οἰκήσεων) is another work of spherical geometry, but of a specifically astronomical character. Its propositions deal with astronomical phenomena as seen at different geographical latitudes.

On Days and Nights (Περὶ ἡμερῶν καὶ νυκτῶν) is again an astronomical work of spherical geometry. It concerns the lengths of days and nights according to the sun's position on the ecliptic.[14]

Hypsicles (2nd century BCE)

The Anaphoricus (Ἀναφορικός) is a work of astronomy concerning the rising times of the zodiac signs. It is a work of a more arithmetical character than the ones otherwise listed here.[15]

Menelaus (d. 140 CE)

The *Spherics* (Σφαιρικά)[16] of Menelaus is no longer extant in the Greek. It was a work on spherical geometry with applications for astronomy and was a more advanced approach to the topic, introducing techniques that did not exist when Theodosius wrote his *Sphaerica*.[17]

Ptolemy (d. c. 170 CE)

The *Almagest*, originally the *Mathematical Syntaxis* (Μαθηματικὴ Σύνταξις), was the major text of ancient Greek astronomy and one which had tremendous influence throughout late antiquity and the medieval period. It superseded the other astronomical writings that preceded it to such an extent that many of them are no longer extant today. It is therefore all the more notable that the above astronomical texts remained in circulation.

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[14] See Fecht (1927) for the editions of the Greek texts of *On Habitations* and *On Days and Nights*.

[15] See De Falco, Krause, and Neugebauer (1966) for the edition of the Greek text.

[16] Throughout the present study, Menelaus's *Spherics* and Theodosius's *Sphaerica* will be referred to by those separate names in order to distinguish what are otherwise similarly titled works in Greek.

[17] As this work is no longer extant in Greek, see the editions available for the various versions which survive in Arabic: Krause (1936) for Ibn ʿIrāq's version, and Rashed and Papadopoulos (2017) for Māhānī / al-Harawī's version and a fragment of an early translation. See also Acerbi (2015) for a study on the traces of this text which survive in Greek scholia to the *Almagest*.

PART I

Chapter 1

An Ordered Curriculum of Spherical Geometry?

1. Introduction

The introduction has already offered a sense of what ideas about the Little Astronomy circulate in modern scholarship, many of them descending from the early modern accounts in Savile, Fabricius, and Vossius which have been influenced by the later Arabic tradition of the Middle Books. This chapter will set out what evidence exists for the Little Astronomy in the Greek and Latin sources up until its contents' translation into Arabic in the ninth century. The Arabic material will not be used here, except where a work's inclusion in the Middle Books offers support for its ninth century inclusion in the Little Astronomy. The Arabic evidence for the Middle Books will be the subject of chapter 3: its omission here is intended to disentangle what can be known about the Little Astronomy from retrojected claims that better fit the Middle Books.

From this evidence, the present chapter will argue that an ordered curriculum of astronomy existed by the fourth century, seemingly descended from a curriculum of spherical geometry already in existence by the second century CE. This ordered curriculum persists through the sixth century to its translation in the ninth, and it is at some point in this later period that it starts to be referred to as the Little Astronomy. Over these many centuries certain works formed the core of the curriculum, while others found inclusion in Little Astronomy codices through their links to member texts. The chapter takes as its starting point *Collection* Book 6 of Pappus, a source which has been key since the early modern scholars' comments on the Little Astronomy.[1]

[1] Savile (1621) 40-41, Vossius (1650) 163, Fabricius (1716) 88.

2. Testimonies in Outside Sources

2.1 Pappus of Alexandria

The work which links together a selection of mathematical texts, a teaching context, and, indirectly, the name "Little Astronomy" is the *Collection*, written by Pappus of Alexandria in the fourth century CE. This is a work in eight books, each treating different mathematical topics. Book 6 is astronomy, and its introduction lays out the intent of the book as follows:

> "Many of those teaching the domain of astronomy, attending carelessly to the propositions, add some (things) as necessary, and pass over some (things) as not necessary. For they say concerning the sixth theorem of the third book of Theodosius's *Sphaerica*, that it is necessary that each of the two great circles cut the poles of the sphere at right angles. But this is by no means (necessary). Similarly they omit in the second theorem of the *Phaenomena* of Euclid how many (cases) the zodiac is twice perpendicular to the horizon. And they falsely prove Theodosius in the fourth theorem of *On Days and Nights*, and they omit various others of the following as not necessary, each of which I will demonstrate."[2]

Pappus is dissatisfied with how a collection of texts which he call the "Domain of Astronomy" (ἀστρονομούμενον τόπον) has been taught (διδασκόντων).[3] He cites problems in three named works as examples: Theodosius's *Sphaerica*, Euclid's *Phaenomena*, and Theodosius's *On Days and Nights*. This is not a comprehensive list of works in the Domain of Astronomy – with "the following" (τῶν ἑξῆς) Pappus alludes to an unspecified amount of further treatises. Indeed, beyond the introduction the main body of the book dwells on several more works, any number of which may also have been included in this collection.

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[2] Greek edition in Hultsch (1877) 474: "Πολλοὶ τῶν τὸν ἀστρονομούμενον τόπον διδασκόντων ἀμελέστερον τῶν προτάσεων ἀκούοντες τὰ μὲν προστιθέασιν ὡς ἀναγκαῖα, τὰ δὲ παραλείπουσιν ὡς οὐκ ἀναγκαῖα. λέγουσιν γὰρ ἐπὶ τοῦ ἕκτου θεωρήματος τοῦ τρίτου τῶν Θεοδοσίου σφαιρικῶν, ὅτι δεῖ τῶν δύο μεγίστων κύκλων ἑκάτερον ὑπὸ τοῦ διὰ τῶν πόλων τῆς σφαίρας τέμνεσθαι πρὸς ὀρθάς· τοῦτο δὲ οὐ πάντως. ὁμοίως δὲ παραλείπουσιν ἐν τῷ β' θεωρήματι τῶν φαινομένων Εὐκλείδου, ποσάκις ὁ ζῳδιακὸς [δὶς] ἔσται ὀρθὸς πρὸς τὸν ὁρίζοντα. κἀν τῷ δ' θεωρήματι τοῦ περὶ ἡμερῶν καὶ νυκτῶν ψευδογραφοῦσι τὸν Θεοδόσιον, καὶ ἄλλα δέ τινα τῶν ἑξῆς ὡς οὐκ ἀναγκαῖα παραλείπουσιν, ὧν ἕκαστον ἐπιδείξομεν ἡμεῖς."

[3] The translation "domain of astronomy" follows Jones (1986) 377-379, which concerns the similarly named "domain of analysis" (ἀναλυόμενος τόπος). He notes multiple cases where the word τόπος has been used to mean a division of knowledge.

Pappus's introduction offers little information about what this corpus is, especially when considered against his treatment of the "Domain of Analysis" (ἀναλυόμενος τόπος) in Book 7, to be compared below. The reader, removed from Pappus's context, knows only that it was a grouping of proposition-based works somehow used in astronomical education.

The sixty one propositions of Book 6 which follow expand on what works apparently numbered among the Domain of Astronomy. In some cases Pappus writes clearly in his text what work a set of propositions relies upon, in others it can be deduced from their contents. Additionally, a series of marginal scholia flag most of the relevant works. The texts which Pappus discusses in this book are as follows: the *Sphaerica* of Theodosius (propositions 1-26),[4] Autolycus (apparently only *On the Moving Sphere*) (prop. 27),[5] *On Nights and Days* of Theodosius (prop. 28-36),[6] *On Sizes and Distances* of Aristarchus (prop. 37),[7] the *Optics* of Euclid (prop. 38-52),[8] and the *Phaenomena* of Euclid (prop. 53-61).[9]

In the course of his commentary on certain concepts, Pappus also cites from the *Spherics* of Menelaus (in props. 1, 56),[10] the *Almagest* of Ptolemy (in props. 37, 59, 61),[11] and works by Hipparchus (in props. 37, 56).[12] These additional works are used to support Pappus's mathematical arguments rather

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4 Hultsch (1877) 474ff. Pappus's text does not explicitly note that these propositions concern Theodosius's *Sphaerica* but it follows from their content.
5 Hultsch (1877) 518ff. Pappus's text cites Autolycus, while the content seems to come only from his *On the Moving Sphere*. A scholion in Vat. gr. 218 f. 96r reads, "εις το πε(ρι) κεινουμεν(ης) σφαιρ(ας)."
6 Hultsch (1877) 530ff. Pappus's text cites Theodosius, while a scholion in Vat. gr. 218 f. 98v reads, "[ει]ς τὸ πε(ρι) ημερων (καὶ) νυκτῶν."
7 Hultsch (1877) 554ff. Pappus's text specifically cites Aristarchus's *On Sizes and Distances*. Additionally, a scholion in Vat. gr. 218 f. 103v reads, "εις τ(ο) πε(ρι) μεγεθ(ων) (και) (απο)στηματ(ων) αρισταρχου."
8 Hultsch (1877) 568ff. Pappus's text does not cite an author. A scholion in a different hand in Vat. gr. 218 f. 106v reads, "εις (τα) οπτικα ευκλειδου." Neugebauer (1975) 768 argues that these propositions do not specifically relate to Euclid's *Optics*.
9 Hultsch (1877) 594ff. Pappus's text specifically cites Euclid's *Phaenomena*.
10 Hultsch (1877) 476, 602.
11 Hultsch (1877) 558, 622, 632.
12 Hultsch (1877) 556, 600.

than as sources for the propositions under investigation. They therefore may not have numbered among works taught in the Domain of Astronomy – Pappus does not clarify either way.

Pappus's Domain of Astronomy is identified with the so-called Little Astronomy through scholia which accompany Book 6. These appear in several manuscripts of the *Collection*, including the oldest independent witness, the tenth century Vaticanus graecus 218.[13] On the folio which starts Book 6, a marginal comment reads "The sixth [book] of Pappus contains solutions to difficulties in the Little Astronomy."[14] On the last folio of the book, the same hand writes another scholion reiterating the matter: "*Collection* 6 of Pappus of Alexandria contains solutions to difficult theorems in the Little Astronomy."[15] Here, the Greek term μικρὸς ἀστρονομούμενος is taken to refer to the "Little Astronomy" – its odd phrasing will be considered later in this chapter alongside other attestations of the name. These scholia to Vat. gr. 218 are penned in the same hand as the other scholia in the manuscript. Hultsch simply identifies it as the hand of the scholiast (A3 in his edition) and comments on the material it contributes to the manuscript, but does not offer a date for it.[16] These scholia could have been copied by the scribe from his exemplar and have an origin from any time between the fourth and tenth centuries. Or, they might have been added by a later reader of this manuscript, in whose day the collection which Pappus discussed might have been known as the Little Astronomy. The other attestations of the name "Little Astronomy" make the first alternative the more likely one, as will be shown.

It is worth delving into how Book 6 and Book 7 of the *Collection* are similar and how they diverge. The naming structure of the respective subjects discussed in each book – Domain of Astronomy

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[13] Vat. gr. 218 may be viewed online in the Digital Vatican Library.
[14] Vat. gr. 218 fol. 87v: π(ερι)εχει τὸ ς̅ τ(ων) παππου απορι(ων) λύσεις τ(ων) εν τῶι μικρῶι αστρονομουμενωι.
[15] Vat. gr. 218 fol. 118r: "παππου αλεξανδρε(ως) συναγωγ(ης) ς̅ π(ερι)εχει δε των εν (τωι) μικρῶι αστρονομουμενωι θεωρηματ(ων) απόρων λυσεις."
[16] Hultsch (1876) xiii.

and Domain of Analysis (ἀστρονομούμενος τόπος and ἀναλυόμενος τόπος) – stands out for its similarity. While Pappus does not detail what the former actually is, he does do so for the latter: it was "material prepared after the production of the *Common Elements* for those wishing to acquire ability in lines (geometry) useful for the problems presented to them."[17] It comprises twelve works by four authors, set out in a particular order. In comparison, Book 6 does not speak of the purpose of the Domain of Astronomy and it does not explicitly set out all of its contents in order, though it does note three example texts by two different authors. This divergence could be read as a suggestion that the Domain of Astronomy and the Domain of Analysis were not actually similar kinds of collections.

However, the difference may arise instead from Pappus's different goals for Books 6 and 7, which are structurally very dissimilar. Book 7 has three parts, the first an overview of the Domain of Analysis, the second a series of introductions to the works in the collection, and the third a group of lemmas for those works. Book 6's structure is less clear. After its introduction it delves into correcting the errors others make in teaching the Domain of Astronomy, but Pappus additionally takes time for digressions such as summarizing a work by Autolycus and comparing Aristarchus's work with that of Ptolemy and Hipparchus. What results is not the thought-out, ordered arrangement of overviews as is seen in Book 7, but rather a collection of assorted comments pertaining to works in the Domain of Astronomy.

Perhaps the reader should not expect consistency between the books of the *Collection*: beyond 6 and 7 the remaining books also vary tremendously. Pappus may not have conceived of the *Collection* as one unified work.[18] If the variation is due more to Pappus than to legitimate differences between the

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[17] See Jones (1986) 83: "τίς ἐστιν ὕλη παρασκευασμένη μετὰ τὴν τῶν κοινῶν στοιχείων ποίησιν τοῖς βουλομένοις ἀναλαμβάνειν ἐν γραμμαῖς δύναμιν εὑρετικὴν τῶν προτεινομένων αὐτοῖς προβλημάτων, καὶ εἰς τοῦτο μόνον χρησίμη καθεστῶσα."

[18] Jones (1986) 15-18.

Domains of Astronomy and Analysis, then it is possible to think of the former as a similar kind of ordered educational collection like the latter.

Overall, the text of *Collection* Book 6 sets out a grouping of theorem-based works used in a didactic context to teach the Domain of Astronomy. Scholia from the tenth century identify this collection with a different name, the Little Astronomy, and it is unclear from the evidence in Pappus and his manuscripts when that name first originated. Regardless, Pappus is the linchpin that ties together the form, name, and purpose of an astronomical curriculum that existed in the fourth century. The sources in the following sections will both help to support what is argued from Pappus, and to expand what can be said about this curriculum.

2.2 Theon of Alexandria

Theon of Alexandria lived after Pappus in the second half of the fourth century. Like the other author, he was a mathematician and wrote on very similar topics – for instance, both Pappus and Theon authored commentaries to parts of Ptolemy's *Almagest*. Their similarities may extend to engagement with a contemporary astronomical curriculum: where Pappus focused Book 6 of his *Collection* on the Domain of Astronomy, one extant medieval source attributes to Theon a commentary on the Little Astronomy.

This reference appears in an anonymous work on isoperimetric figures included within the *Introduction to the Almagest*, which was produced sometime in the sixth century.[19] The author provides a lemma whose argument, he says, follows one in a particular work of Theon's: "it is proved by Theon in the commentary on the Little Astronomy" (δέδεικται μὲν Θέωνι ἐν τῷ ὑπομνήματι τοῦ μικροῦ

[19] Acerbi, Vinel, and Vitrac (2010) 55. Acerbi (2014) 136-141 also expands on the probably Alexandrian Neoplatonic context of the *Introduction to the Almagest*. Mogenet (1956) has argued for Eutocius (c. 480-540) as the author of the anonymous work on isoperimetric figures.

ἀστρονόμου).[20] In this case the Greek translated as "Little Astronomy" is "μικρὸς ἀστρονόμος," but this otherwise agrees with what is found in the scholia to Pappus.

What this suggests is that by the sixth century there existed the idea that Theon had written a commentary on something called the Little Astronomy. Whether Theon actually did so is a different matter, and this has been called into question by several scholars. No such commentary is extant. Neugebauer does not see the proof which cites Theon as providing evidence for any particular collection of treatises.[21] The lemma in question does have a long history, with versions appearing in multiple texts and scholia relevant to spherics.[22] Mogenet, considering what was meant by "the commentary on the Little Astronomy" referenced in the anonymous work, suggested that this was an error for Theon's commentary on the *Almagest* – a work which also contains a version of the lemma.[23] The anonymous author or a scribe, then, seemingly has substituted 'small' for 'big'.

While this reference has been called into question, other scholars have read the evidence more generously. Mansfeld points out that an error of 'small' for 'big' in this context would be easier to make if there did indeed exist something termed "ὁ μικρος ἀστρονόμος." He also notes the possibility that Theon wrote commentaries on both the *Almagest* and the Little Astronomy and used the lemma in both.[24] Pingree, meanwhile, turns to the tenth century *Suda* to pull out a potential parallel to the referenced "ὑπόμνημα τοῦ μικροῦ ἀστρονόμου": the encyclopedia's entry for Theon of Alexandria mentions a work titled "Εἰς τὸν μικρὸν ἀστρόλαβον." Pingree suggests that here "ἀστρόλαβον" is an error for

[20] Hultsch (1878) 1142. See also the more recent edition in Acerbi, Vinel, and Vitrac (2010) 121.
[21] Neugebauer (1975) 769.
[22] See Knorr (1985) for a study of this lemma, versions of which also appear in both recensions of Euclid's *Optics*, in Theon's commentary on the *Almagest*, in Theodosius's *Sphaerica* book III, and in Pappus's *Collection* book V.
[23] Mogenet (1956) 38-39
[24] Mansfeld (1998) 17-18.

“ἀστρονόμον,” a difference of only three letters.[25] This is not impossible – in Greek manuscripts one can find minuscule scripts where the letters nu and lambda look similar and where the only difference between a beta and a mu is the inclusion of an initial stroke from below the baseline for the latter. However, editions of the *Suda* make no mention of manuscript variants for this word.

Interpreting this attestation most generously: in the fourth century Theon, like Pappus, commented on a body of astronomical material known as the Little Astronomy.[26] However, it is still informative even when interpreting it narrowly: perhaps as early as the sixth century there existed something called the Little Astronomy, which influenced the author or scribe into recording the citation here erroneously.

2.3 Cassiodorus

Another problematic mention of a Little Astronomy appears in Latin in the sixth century. The source is the *Institutiones* of Cassiodorus, a Roman scholar who served as a statesman under Theodoric the Great. The text was written to offer an introduction to divine and secular learning. Astronomy is included among the topics of secular knowledge, and as part of his discussion on the topic Cassiodorus writes the following:

> In both languages volumes have been written on the discipline of astronomy; of which Ptolemy among the Greeks is considered preeminent. He published two books on this subject, of which he called one the Smaller (Astronomer), the other the Greater Astronomer. He also established the Tables, in which the courses of the stars are found...[27]

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[25] Pingree (1968) 15.

[26] Acerbi puts forth the suggestion that Theon’s commentary on the Little Astronomy may have comprised a range of material, portions of which may have been distributed through manuscripts of the Little Astronomy both within and outside the text: e.g. as scholia, introductions, added definitions, alternate proofs, etc. On this, see Acerbi (2014) 145-147.

[27] *Institutiones* II.7.3. See the Latin edition in Migne (1847) 1218: “De astronomia vero disciplina in utraque lingua diversorum quidem sunt scripta volumina; inter quos tamen Ptolomeus apud Graecos praecipuus habetur, qui de hac re duos codices edidit, quorum unum minorem, alterum maiorem vocavit Astronomum. is etiam canones, quibus cursus astrorum inveniantur, instituit...”

Ptolemy is the only authority on astronomy whom Cassiodorus names, and here he attributes to the astronomer three works: a codex called the "the Smaller Astronomer" (minorem… Astronomum), one called the "the Greater Astronomer" (maiorem… Astronomum), and the "Tables" (canones). The latter is almost certainly Ptolemy's *Handy Tables* and the second one must be his *Almagest*, leaving only the Smaller Astronomer uncertain.

This would appear to be another allusion to the Little Astronomy, but Cassiodorus attributes it to Ptolemy, which does not agree with what other evidence suggests about the collection. Ptolemy did not author any of the treatises thought to have been a part of it. Cassiodorus may instead be drawing upon a tradition that claims Ptolemy as the editor who compiled the Little Astronomy together, but this idea can be found nowhere else in the extant sources. Alternatively, Pingree suggests that Cassiodorus is grouping together Ptolemy's minor astronomical works (other than the *Handy Tables*) under the title "minorem Astronomum,"[28] and if so then this passage does not reference the Little Astronomy at all.

Unlike Pappus and Theon, Cassiodorus is not a mathematical scholar and he is no expert in astronomy. He may simply be aware of a codex that went by the name Little Astronomy and is mistakenly attributing it to Ptolemy. In any case, by the sixth century in the Latin tradition there was awareness of some kind of astronomical corpus called the Little Astronomy, which was transmitted as one codex.

2.4 John Philoponus

The sixth century lastly offers an important source in the writings of John Philoponus, a Christian philosopher who worked in Alexandria. Philoponus was well-read and trained in the natural sciences, including in astronomy, which shows in his work even when focused on other topics. This section will

[28] Pingree (1968) 15.

delve into two passages from his Aristotelian commentaries, both of which discuss philosophical classification. Philoponus is concerned with explaining the reasons why a treatise would be described as "more precise" (ἀκριβεστέρα) versus "more particular" (μερικώτερα). The examples he chooses to illustrate this spectrum are works which are relevant to this dissertation.

The first passage appears in his Commentary on Aristotle's *Posterior Analytics*, which has been argued to date from Philoponus's school days under Ammonius.[29] It reads as follows:

> "For example, the spherics of Theodosius is a more precise science than that of Autolycus, which treats of the moving sphere. For the one simply looks into the accidents of the sphere, without considering in addition whether it moves or not. But Autolycus studies the accidents of the moving sphere. In the sciences additions always make the subject more particular and for this reason less precise. And yet, Autolycus' science of the moving sphere is more precise than astronomy; for that, finally, studies the moving sphere with matter. For it studies this moving [sphere], I mean the heavenly [sphere]. And for this reason it lacks precision. So everything that is proved in astronomy does not offer the utmost precision, but the approximate. For example, they say that the sun stands from the moon more than eighteen times, but less than twenty times, the distance that the moon stands from the earth; for in these things we should be satisfied with approximating precision. And for all other things that are proved in astronomy the same argument holds. So that the spherics of Theodosius should not be taken as additional elements of astronomy, but as principles and causes of the things that are demonstrated in astronomy. For through them, as causes, those things are demonstrated. And the relative position that arithmetic has to harmonics and geometry to optics, that same relative position Theodosius' spherics has to the study of the sphere in motion, and the latter to astronomy. For the higher sciences are always the causes of the lower ones."[30]

[29] Golitsis (2008) 23-27. The commentary on the *Posterior Analytics* is one of four commentaries whose title in the manuscripts states that it comes from notes "from the lectures of Ammonius Hermiae" (ἐκ τῶν συνουσιῶν Ἀμμωνίου τοῦ Ἑρμείου).

[30] Translation from Goldin and Martijn (2012) 103. The Greek text is in Wallies (1909) 300-301: "οἷον τὰ Θεοδοσίου σφαιρικὰ ἀκριβεστέρα ἐστὶν ἐπιστήμη τῆς τῶν Αὐτολύκου περὶ κινουμένης σφαίρας· ὁ μὲν γὰρ ἁπλῶς τὰ συμβαίνοντα τῇ σφαίρᾳ σκοπεῖ, μὴ προσλογιζόμενος εἴτε κινεῖται εἴτε μή· ὁ δὲ Αὐτόλυκος τὰ τῇ κινουμένῃ σφαίρᾳ συμβαίνοντα θεωρεῖ· ἀεὶ δὲ αἱ προσθῆκαι ἐν ταῖς ἐπιστήμαις μερικώτερα τὰ πράγματα ἐργάζονται καὶ διὰ τοῦτο ἧττον ἀκριβέστερα. ὁμοίως τὰ Αὐτολύκου περὶ κινουμένης σφαίρας ἀκριβέστερά ἐστιν ἀστρονομίας· ἥδε γὰρ λοιπὸν μετὰ ὕλης τὴν θεωρίαν τῆς κινουμένης σφαίρας ποιεῖται· τήνδε γὰρ τὴν κινουμένην θεωρεῖ, λέγω δὴ τὴν οὐρανίαν. διὸ δὴ καὶ τοῦ ἀκριβοῦς λείπεται· πάντα γοῦν τὰ ἐν ἀστρονομίᾳ δεικνύμενα οὐ τὴν ἐσχάτην ἀκρίβειαν ἐπαγγέλλεται ἀλλὰ τὸ ἐγγύς. οἷον λέγουσιν ἀφεστηκέναι τὸν ἥλιον τῆς σελήνης, ὅσον ἡ σελήνη τῆς γῆς ἀφέστηκε, μεῖζον μὲν ἢ ὀκτωκαιδεκαπλάσιον ἔλαττον δὲ ἢ εἰκοσαπλάσιον· ἀγαπητὸν γὰρ ἐν τούτοις τὸ ἐγγὺς τῆς ἀκριβείας ἐλθεῖν. καὶ ἐπὶ πάντων τῶν δεικνυμένων ἐν ἀστρονομίᾳ ὁ αὐτὸς λόγος. ὥστε τὰ Θεοδοσίου σφαιρικὰ οὐχ ὡς στοιχεῖα προσλαμβάνονται ἀστρονομίας, ἀλλ' ὡς ἀρχαὶ καὶ αἴτια τῶν ἐν ἀστρονομίᾳ ἀποδεικνυμένων· δι' αὐτῶν γὰρ τῶν αἰτίων ἐκεῖνα ἀποδείκνυνται. καὶ ὃν λόγον ἔχει ἀριθμητικὴ πρὸς ἁρμονικὴν καὶ γεωμετρία πρὸς ὀπτικήν,

In these examples, Philoponus's spectrum spans from Theodosius's *Sphaerica* (more precise), through Autolycus's *On the Moving Sphere*, to indeterminate works on astronomy (more particular). While Philoponus does not explicitly name any of these more particular astronomical works, his example about the distances of the sun, moon, and earth can be clearly identified as coming from Aristarchus's *On the Sizes and Distances of the Sun and the Moon*.[31] So Philoponus directly names two Little Astronomy texts and paraphrases the text of a third.

Philoponus discusses this scheme of philosophical classification again a few years later[32] in his Commentary on Aristotles's *Physics*:

> "Theodosius at any rate in [his work] On Spheres when teaching [us] the attributes that hold true of a sphere does not add any calculations about matter, but separating the spherical shape from all substance considers in this way what holds true of it, [arguing] that if a sphere is cut by a plane it produces a circle, and so on. Autolycus, writing [his work] On the Moving Sphere and [writing about] what holds true of a sphere in motion, is more concerned with particular objects than Theodosius and approaches nearer to the natural philosopher (for [the idea of] movement is in a way close to [that of] substance); for even if he does not think of some substance [in the case of] the moving sphere he does at least take [into consideration] a combination of shape and movement and in this is close in a way to substance. Even more concerned with particulars than this is Euclid's Phaenomena and in general the whole of astronomy…"[33]

τοῦτον ἔχει τὸν λόγον τὰ Θεοδοσίου σφαιρικὰ πρὸς τὰ περὶ κινουμένης σφαίρας καὶ ταῦτα πρὸς ἀστρονομίαν· ἀεὶ γὰρ αἱ ἀνωτέρω ἐπιστῆμαι αἴτιαι τῶν ὑποκάτω."

[31] Compare the enunciation of proposition 7: "The distance which the sun is distant from the earth compared to the distance which the moon is distant from the earth is greater than 18 times and less than 20 times." See Heath (1913) 376: "Τὸ ἀπόστημα ὃ ἀπέχει ὁ ἥλιος ἀπὸ τῆς τῆς τοῦ ἀποστήματος οὗ ἀπέχει ἡ σελήνη ἀπὸ τῆς γῆς μεῖζον μέν ἐστιν ἢ ὀκτωκαιδεκαπλάσιον, ἔλασσον δὲ ἢ εἰκοσαπλάσιον."

[32] The date 517 CE appears in Comm. on *Physics* 4.10. Sorabji (2016) 379-380 argues that Books 1-3 were written afterwards, based on how Philoponus's writing shows further independence and criticism of Aristotle.

[33] Translation from Lacey (1993) 33. The Greek text is in Vitelli (1887) 220: "ὁ γοῦν Θεοδόσιος ἐν τοῖς Σφαιρικοῖς διδάσκων τὰ συμβαίνοντα πάθη τῇ σφαίρᾳ οὐδὲν προσλογίζεται ὕλην, ἀλλὰ χωρίσας πάσης οὐσίας τὸ σφαιρικὸν σχῆμα οὕτω τὰ συμβαίνοντα αὐτῷ ἐπισκέπτεται, ὅτι ἐὰν σφαῖρα ἐπιπέδῳ τμηθῇ κύκλον ποιεῖμ καὶ ὅσα ἄλλα. ὁ δὲ Αὐτόλυκος Περὶ κινουμένης σφαίρας γράψας καὶ ὅσα συμβαίνει τῇ κινουμένῃ σφαίρᾳ, μερικώτερός ἐστι τοῦ Θεοδοσίου καὶ μᾶλλον τῷ φυσικῷ προσεγγίζων [ἡ γὰρ κίνησις ἐγγός πως ἐστι τῆς οὐσίας]· εἰ γὰρ καὶ μὴ ἐπινοεῖ οὐσίαν τινὰ ἐν τῇ κινουμένῃ σφαίρᾳ, ἀλλ' οὖν σύνθεσίν τινα λαμβάνει τοῦ σχήματος καὶ τῆς κινήσεως, καὶ ταύτῃ ἐγγύς πως ἐστι τῆς οὐσίας. ἔτι τούτου μερικώτερα τὰ Εὐκλείδου Φαινόμενα καὶ ἁπλῶς πᾶσα ἀστρονομία."

Again the philosopher gives three examples, although in this case the most particular astronomical treatises are exemplified with Euclid's *Phaenomena*. What these two passages together suggest is that Philoponus is not selecting examples randomly. There are other works which could have served as examples for his discussion here: a few lines after the above passage, for instance, he notes "the thirteen books of Euclid" as the *Sphaerica*'s equivalent in the schema he has set out.[34] There are any number of astronomical works which could have exemplified the more particular category – the *Almagest* for instance is a text which looms large in all discussions of late antique astronomy. Instead, Philoponus draws from *Sizes and Distances* and names the *Phaenomena*.

Perhaps this was an unconscious choice on Philoponus's part: perhaps he was aware of an astronomical corpus comprising these texts, and that awareness influenced his selection subconsciously. Nowhere does he explicitly say that these works were members of a particular grouping. But the reader should not expect Philoponus to make any such mention; this would be outside his point for these passages. He is not writing about the Little Astronomy, he is writing about philosophical classification, and these works happen to make good examples.

The above is a narrower interpretation from the two passages, but when they are read in combination with the broader evidence for the Little Astronomy's contents and arrangement, it is possible to make a stronger claim that Philoponus consciously selected these works because he was aware of an contemporary astronomical curriculum whose works were arranged from more precise to more particular – a perfect fit for his purposes. This chapter will return to the question of the Little Astronomy's arrangement, but first it will consider the evidence from a final author, one who is important for what his allusions suggest about how far back the so-called Little Astronomy might be dated.

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[34] Vitelli (1887) 220: "τὰ Εὐκλείδου ιγ' βιβλία."

2.5 Galen

A much earlier source can be found in the second century Galen who, though a doctor by profession, held the study of mathematics in high esteem. In one of his treatises, *De animi cuiuslibet peccatorum dignotione et curatione*, he discusses the order of mathematical instruction, saying that the *Elements* of Euclid is to be followed by the study of spherics and then by conics and sundials.[35] In another treatise, *De placitis Hippocratis et Platonis*, Galen mentions the *Phaenomena* of Euclid as a text typically possessing students (οἱ μαθόντες), suggesting a didactic usage.[36] The statement that spherics followed the *Elements* is interesting in light of what will later be reported about the Middle Books: that they stand between Euclid's *Elements* and Ptolemy's *Almagest*.[37] At the same time, the *Elements* was the starting point of ancient Greek geometrical education, so Galen's statement here is not terribly surprising.

The most significant piece of evidence from Galen survives in an Arabic translation of his commentary to Hippocrates's *Airs, Waters, Places*. In a passage in Book III, Galen writes how he is unimpressed by the knowledge of the Roman horoscope casters and he goes on to disparage the mathematical and astronomical learning of the Romans in general.[38] What follows are some named texts and some topics which he apparently expects an educated person to be familiar with, some of which the Romans had read and some of which they had not:

> "Some of them know the thirteen *Elements* which Euclid set down <…> …and it is called the 'Dedomena' and this is the *Data*. And some of them know the science of the movement of the

[35] De Boer (1937) 42: "ἐπιστήμη γοῦν ἐστι τοῦ γεωμετρικοῦ τοιαύτη περὶ τὰ δεδιδαγμένα διὰ τῶν Εὐκλείδου στοιχείων, ὁποία τῶν πολλῶν ἐστι τοῦ τὰ δὶς δύο τέτταρα εἶναι. τὴν δ' αὐτὴν ἐπιστήμην ἔχει καὶ περὶ τῶν ἐφεξῆς τούτοις διδασκομένων σφαιρικῶν θεωρημάτων, ὥσπερ γε καὶ τῶν κατ' αὐτὰ ἀναλυομένων ἁπάντων, ἔτι τε τῶν κωνικῶν καὶ τῶν γνωμονικῶν."

[36] De Lacy (2005) 484: "διὰ τοῦτ' οὖν Εὐκλείδης μὲν ἑνὶ θεωρήματι τῷ πρώτῳ κατὰ τὸ τῶν Φαινομένων βιβλίον ἐπέδειξε δι' ὀλιγίστων ἐπῶν τὴν γῆν μέσην εἶναι τοῦ κόσμου καὶ σημείου καὶ κέντρου λόγον ἔχειν πρὸς αὐτόν, καὶ οἱ μαθόντες οὕτω πιστεύουσι τῷ συμπεράσματι τῆς ἀποδείξεως ὡς καὶ τῷ τὰ δὶς δυό τέτταρα εἶναι."

[37] Arabic reports describing the contents and function of the Middle Books will be discussed in chapter 3.

[38] §13 in Toomer (1985) 196.

sphere and the science of what is seen of the stars and the science of the inhabited earth and the science of night and day. And few know the science of geometry, all of it…"[39]

Euclid's *Elements* unsurprisingly tops the list and it is followed by his *Data*, a work whose general treatment of geometry makes it a fair fit after the previous work. Afterwards Galen calls attention to four topics, whose descriptors are close matches to the titles of four Little Astronomy works. He appears to allude to *On the Moving Sphere* (علم حركة الفلك), the *Phaenomena* (علم ما يرى من النجوم), *On Habitations* (علم الأرض المعمورة), and *On Days and Nights* (علم الليل والنهار).[40] Galen goes on to complain about the general ignorance of several other geometrical topics and works, but it is these four which are relevant to the present discussion, and it is notable they are presented together in the text as one unit.

3. The Timeline of an Astronomical Curriculum

Since the early modern scholars, the Little Astronomy has been thought of as a late antique curriculum and indeed, most of the references above offer evidence of it circulating in the fourth and sixth centuries. Galen's testimony is notable for pushing its origins centuries back. Though his curriculum of spherical geometry may not have been identical with the astronomical curriculum Pappus knew, the overlap of texts (and orders, as will be discussed) strongly points to the late antique curriculum evolving from the second century one.

If the four texts Galen names can be identified with the study of spherics that followed the *Elements*, then his expectations of the Romans suggest that such a curriculum of spherical geometry was already in use by his day. Further, because Galen's own mathematical study occurred during his youth,

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[39] Galen *Commentary on Hippocrates' Airs Waters Places*: see Toomer (1985) 196: "ومنهم مَن يعرف الثلثة عشر حرفاً التى وضعها اقليدس... <...> ويسمّى دادومنا وهو المعطى. وهو منهم مَن يعرف علم حركة الفلك وعلم ما يرى من النجوم وعلم الارض المعمورة وعلم الليل والنهار. وقلّ مَن يعلم منهم علم المساحة كلّه"

[40] §18 in Toomer (1985) 196. It is possible that he is still talking about astrologers, although he may have doctors in mind, whom he also thinks are bad at astronomy (*Opt. Med.* 1), or just the Roman people in general. The Arabic word used is simply "اهل" for "people."

this curriculum was already established and customary at the latest by fifteen years after his birth in 129 CE.[41]

This possibility is interesting for what would follow about the relationship between this curriculum and Ptolemy's *Almagest*: namely, that there was not one. The *Almagest* was written after 141 CE[42] – the curriculum which Galen knew would most likely predate it. This is in contrast to the Middle Books, which are explicitly described as preparation for the *Almagest*. No such report appears in the Greek. It is possible that at a later date the Little Astronomy did become linked to the *Almagest*; the evidence does include Cassiodorus linking a "minorem Astronomum" to Ptolemy and the close similarity in naming schemes for the Little and Greater Astronomies. But this would have been a later development if it did occur, not a factor in the curriculum's original composition.

The lifetime of this curriculum was evidently a long one, then. It remains actively in use in the fourth century from Pappus's report; it continues to be used in the sixth century when Cassiodorus and Philoponus wrote. Outside sources have less to say afterwards, but codices continue to be transmitted and in the ninth century awareness of the Little Astronomy persisted enough to encourage a parallel tradition in the Arabic.

4. The Name "Little Astronomy"

This name does not appear to have been a constant in the curriculum's lifetime. Rather, the earliest extant term attached to the corpus appears only in the fourth century, and it is Pappus's ἀστρονομούμενος τόπος. This forms a clear parallel with the ἀναλυόμενος τόπος in Book 7, but that is the term's only link. It does not appear outside of Pappus.

[41] Sidoli (2015) 395.
[42] Pedersen (2011) 12.

The English translation "Little Astronomy" comes variously from the attestations μικρὸς ἀστρονομούμενος, μικρὸς ἀστρονόμος, and *minorem Astronomum*. It is inexact in all cases: a more literal translation would be the "Little Astronomer." The term ἀστρονομούμενος appearing in the manuscripts of the *Collection* rather than ἀστρονόμος might be explained either by supposing a conflation between the collection's name and Pappus's ἀστρονομούμενος τόπος or by accepting ἀστρονομούμενος as a synonym of ἀστρονόμος.

This title would seem to be later than Pappus since it is not the one he uses, though he may simply have omitted it. While it is unclear when the scholia naming it so were added to the *Collection*, the title does appear to have been in use by the sixth century when Cassiodorus and Eutocius were writing.

5. An Ordered Collection?

5.1 Evidence from Outside Texts

Several of the sources which have been discussed give reason to believe that the works in the so-called Little Astronomy may have been arranged in a particular order. Although Pappus does not offer a thorough description of the Domain of Astronomy, if it is to be compared with the Domain of Analysis then it is notable that Pappus does set out a defined order for the latter. Two centuries later, Philoponus indirectly offers a possible rationale behind an ordered Little Astronomy: his discussion of more precise and more particular treatises is striking in light of the arrangement in which Little Astronomy works are most often found.

The ninth century manuscript Vat. gr. 204 is crucial for the manuscript evidence of the Little Astronomy, and this includes the order in which it presents its texts. Its contents (excluding commentaries and scholia) proceed as follows: *Sphaerica*, *On the Moving Sphere*, *Optics*, *Phaenomena*, *On Habitations*, *On Nights and Days*, *On Sizes and Distances*, *On Risings and Settings*, *Anaphoricus*, *Catoptrica*, and

Data. While Philoponus only uses a selection of these as his examples, it is clear that in both cases he discusses the works in an order which matches their relative position in Vat. gr. 204's arrangement. The *Sphaerica* is the most precise, *On the Moving Sphere* is less so, and either *On Sizes and Distances* or the *Phaenomena* is selected as the example of a more particular text. Setting aside the *Catoptrica* and the *Data*, which seemingly did not number among the Little Astronomy,[43] any of the works after the *Phaenomena* would fit as a more particular astronomical treatise.

The other allusions to works in an astronomical curriculum agree with this order in all but one case. In the main body of Book 6, Pappus's discussion moves from the *Sphaerica* to *On the Moving Sphere*, then from *On Nights and Days* and *On Sizes and Distances* to the *Optics* and lastly the *Phaenomena*. Those final two Euclidean works break from the order seen in Vat. gr. 204, though it can be said that Book 6 still broadly follows an arrangement from a more precise text (the *Sphaerica*), to an intermediate one (*On the Moving Sphere*), then the assorted particular astronomical treatises (excluding the *Optics*).

Pappus's introduction, however, contradicts the order in the main text of Book 6 and instead agrees with the order seen in Vat. gr. 204. The mathematician could have grouped together Theodosius's *Sphaerica* and *On Nights and Days* – one might have expected this on the basis of their shared author. Instead the *Phaenomena* stands between them, so that the order of works mentioned is the *Sphaerica*, the *Phaenomena*, and *On Days and Nights*.

Even the testimony from Galen stands in agreement, centuries earlier. The order of the astronomical texts is *On the Moving Sphere*, the *Phaenomena*, *On Habitations*, and *On Nights and Days*.

[43] This will be examined below.

Not only is the relative order the same as in Vat. gr. 204, but the actual order is a near match as well, except Galen does not list the *Optics*.

Centuries later, it would appear that the order in which the Arabic scholars found the Little Astronomy influenced the early arrangement of the Middle Books. The earliest manuscript containing a significant number of the Middle Books in Arabic (dated 25 years before al-Ṭūsī's edition) agrees almost exactly with the order in Vat. gr. 204. This is the manuscript Topkapi Seray Ahmet III 3464, and the only disagreements in order are its moving of the *Data* from the end to the very beginning, and placement of Menelaus's *Spherics* at the end.

5.2 Evidence from the Scholia

In an article on mathematical scholia, Acerbi calls attention to a particular category which appears liberally throughout manuscripts of Little Astronomy treatises.[44] These scholia are ones which supplement the main text with brief references to another text. Acerbi highlights how such referential scholia in Little Astronomy treatises are frequently citations of propositions from other Little Astronomy treatises. More importantly, these only cite propositions which have occurred previously according to the order in Vat. gr. 204. They are very concise and formulaic: examples from Vat. gr. 204 include "from the 9th [proposition] in the 2nd [book] of the *Sphaerica*" (ἀπὸ τοῦ θ' τοῦ ἐν τῷ β' τῶν σφαιρικῶν) on 61r and "by means of the 20th [proposition] of *On the Moving Sphere*" (διὰ τοῦ κ' τοῦ περὶ κινουμένης σφαίρας) on 62r.[45]

The following figure summarizes what texts are cited by such scholia for each supposed Little Astronomy treatise. Euclid's *Elements* is added as the first column since it is a work which is frequently

[44] Acerbi (2014) 141-151.
[45] The primary scholiast hand writes these scholia using both the ἀπὸ and διὰ formulae.

cited as well – this and the *Catoptrics* are the only outside works cited. Most works also cite their own propositions but the rule holds as expected: only propositions prior to the present one will be cited.

		Work Cited by the Scholia									
		El	**Sph**	**MS**	**Opt**	**Phaen**	**Hab**	**D&N**	**S&D**	**R&S**	**Ana**
Work Containing Scholia	**Sph**	✓	✓								
	MS	✓	✓								
	Opt	✓	✓		✓						
	Phaen		✓	✓	✓	✓					
	Hab	✓	✓	✓			✓				
	D&N		✓			✓		✓			
	S&D	✓	✓						✓		
	R&S			✓		✓				✓	
	Ana					✓					✓

El = *Elements*, Sph = *Sphaerica*, MS = *Moving Sphere*, Opt = *Optics*, Phaen = *Phaenomena*, Hab = *Habitations*, D&N = *Days and Nights*, S&D = *Sizes and Distances*, R&S = *Risings and Settings*, Ana = *Anaphoricus*

Table 1.1: Citations by referential scholia in the Little Astronomy

Acerbi consulted the edited scholia and Vat. gr. 204 for his study, and he included only those scholia which were written in the primary scholiasts' hands. The rule holds true, however, even when examining the referential scholia in later hands in Vat. gr. 204.

It is also informative to expand Acerbi's survey to include the works of dubious membership in the Little Astronomy, the *Catoptrics* and the *Data*. This expansion reveals that if the former were to be considered part of the collection, it would break the pattern seen here. A scholion to the *Optics* cites the

Catoptrics,[46] but the *Catoptrics* appears second to last in Vat. gr. 204. The *Data*, meanwhile, neither cites any other Little Astronomy text nor is it cited by them.

The edited scholia are heavily dependent on Vat. gr. 204, so it might be expected that they would follow its sequence. Looking to manuscripts outside this one, however, confirms that the implied order from the scholia persists even when texts are arranged differently or separately. The thirteenth century manuscript Vienna phil. gr. 31, for instance, contains only Euclid's *Elements*, *Optics*, and *Phaenomena*, but referential scholia in the *Phaenomena* cite the absent *Moving Sphere* and *Sphaerica*. Meanwhile, the thirteenth century Vat. gr. 192 places the *Phaenomena* after the *Anaphoricus*, but the scholion in the latter citing the former remains. The thirteenth century Vat. gr. 191 does the same for the *Optica* and *Phaenomena*, and the c.1300 Paris gr. 2448 does the same for *Moving Sphere* and *Sphaerica*.

It may not be surprising that scholia attached to these texts persist – they after all all become part of the work to be transmitted – but these manuscripts represent several different versions of the treatises and are not all descended from Vat. gr. 204. The manuscript Vienna phil. gr. 31, for instance, contains different recensions than those in Vat. gr. 204, so either the scholarly practice represented by these scholia was applied regardless of the recension that was at hand or it has its roots before the split.

Placing this type of scholia into a fuller context would require a greater foundation of edited mathematical marginalia. It is worth noting that Pappus's *Collection* does contain referential scholia and, in Vat. gr. 218, these scholia only appear in Book 6, pointing to the relevant Little Astronomy text.

[46] This appears in *Optics* proposition 19, on folio 48v of Vat. gr. 204.

6. Works in the Collection

With the evidence from the manuscripts and the references in outside texts laid out above, it is now possible to evaluate what works likely were members of the Little Astronomy and what works became attached more tenuously.

Theodosius *Sphaerica*[47]

There can be little doubt that the *Sphaerica* not only numbered among the treatises of the Little Astronomy but also stood at the head of the collection. Its inclusion is affirmed and its position strongly implied by Pappus and both passages in Philoponus, with support from Vat. gr. 204. Nearly all of the other works contain referential scholia citing it, while it lacks citations to any propositions except those earlier in its own books.

Autolycus *On the Moving Sphere*

Similarly, *On the Moving Sphere* need not be doubted as a member of the collection. Pappus, Philoponus, and Galen all mention it and the manuscript evidence supports this. It is cited by the scholia to three subsequent Little Astronomy treatises, while it itself relies only on the *Sphaerica* (and the *Elements*, preceding the curriculum as a whole).

Euclid *Optics*

Conversely, the position of the *Optics* is less clear. It falls immediately under suspicion for being a treatise neither on spherics nor on astronomy. There is evidence in its favor: it stands in Vat. gr. 204 in the midst of other certain Little Astronomy treatises, it is treated in *Collection* Book 6,[48] it is linked by

[47] As noted in the introduction, to distinguish between the work by Theodosius and the work by Menelaus, Theodosius's work will consistently be referred to as the *Sphaerica* in this dissertation. Menelaus's work will be referred to as the *Spherics*.
[48] Hultsch had indicated portions of Book 6 drew from the *Optics*, though Neugebauer disagreed with this: see Neugebauer (1975) 768. More recent scholarship has shown that Neugebauer likely erred here: Pappus does indeed

referential scholia to other Little Astronomy works. It is even named specifically in the introduction to the text of the *Phaenomena*, which follows after it.[49]

However, not all of this evidence is unshakeable. Its appearance in the text of the *Phaenomena* is in the introduction, which was the work of a later editor. Meanwhile, the links in the scholia are tenuous: while the *Optics* cites the *Sphaerica* before it, the *Phaenomena* is the only treatise which cites the *Optics*, and it does so only once.[50]

This chapter has not presented the evidence from the Arabic, as this will be the subject of chapter 3, but the *Optics* do appear as part of the Middle Books seemingly already in the ninth century.[51] This lends support to the idea that the *Optics* were a part of the Little Astronomy by the ninth century at the latest. The treatise may not have been included for the entire lifetime of the curriculum. When Galen alludes to Little Astronomy works the reader should not expect him to offer a full listing, but it is worth noting that he names the works from *Moving Sphere* to *Days and Nights* in order but skips the *Optics*.

Since the main link the *Optics* has is to the *Phaenomena*, it is likely that the work came to be added to the Little Astronomy as a text which was useful reading before the *Phaenomena*, the first work in the corpus which is dealing with more particular matters, as Philoponus deliminates it. Its addition could have been around the time when a later editor added the preface to the *Phaenomena*, since this is

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treat of material that overlaps with *Optics* propositions 34-35, though he introduces multiple lemmas and fully rewrites the proof. On this, see Jones (2001) 52-57.

49 This occurs in the *Phaenomena* preface; the line is “as is shown in the *Optics*.” See Menge (1916) 2: “...ὡς ἐν τοῖς ὀπτικοῖς δείκνυται.”

50 This is a scholion to proposition 1: see Menge (1916) 136.

51 It is included in a list attributed to Isḥāq ibn Ḥunayn and there is a possible reference to the work in al-Kindī already in the ninth century, both of which will be discussed further in chapter 3. Its inclusion among the Middle Books might reflect the usefulness various topics in the *Optics* had for Ptolemy’s *Almagest*. If, in the centuries after the second CE, the Little Astronomy came to have some connection with the *Almagest* (which would have replaced the other, more advanced astronomical works in circulation in Galen’s day), this may have also motivated the addition of the *Optics* to the corpus.

the part of the text which refers back to the *Optics* directly. There are potentially further overlaps between the *Optics* and *On Sizes and Distances*, since propositions 23-27 of the *Optics* concern spheres, their magnitudes, and their distances.[52]

Euclid *Phaenomena*

With Pappus, Philoponus, and Galen attesting to it, and the manuscript and scholia evidence supporting it, the *Phaenomena* is a return to far more certain Little Astronomy members. This treatise even includes references to other Little Astronomy works in its main text, though these are likely interpolations and will be discussed further in chapter 2.[53]

Theodosius *On Habitations*

The works after the *Phaenomena* have fewer sources attesting to them, but the astronomical ones among them also do not have evidence standing against their inclusion. Starting with *On Habitations*, the following astronomical works contain referential scholia that point back to preceding Little Astronomy treatises, but they are never the source of such citations in the subsequent texts.

Nevertheless, *On Habitations* is attested in important manuscripts such as Vat. gr. 204, and receives mention by Galen as well. It also will be seen to be a member of the Middle Books in chapter 3.

Theodosius *On Days and Nights*

The same can be said for *On Days and Nights* as was said for *On Habitations*. Additionally, Pappus names the text both in the introduction and the main body of *Collection* VI.

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[52] See Webster (2014) for the argument that these propositions involve material that significantly overlap with *On Sizes and Distances* propositions 1-3.

[53] In addition to the *Optics* in the preface, discussed above, recension A's proposition 12 points to a theorem from the *Sphaerica*: "As is written in the eighth theorem of the third book of the Sphaerica." See Menge (1916) 76: "γραφομένων ὥστε τῷ ἕκτῳ θεωρήματι τοῦ τρίτου βιβλίου τῶν Σφαιρικῶν."

Aristarchus *On Sizes and Distances*

Like Theodosius's astronomical works, *On Sizes and Distances* appears with the Little Astronomy texts in significant manuscripts, its scholia refer back to other works in the collection, and it will be seen to be one of the Middle Books. Pappus discusses it explicitly, while Philoponus does not name it but clearly has it in mind as one of the more particular astronomical works.

Autolycus *On Risings and Settings*

Compared to Autolycus's other work, *On Risings and Settings* receives no mention by outside sources. The manuscript evidence supports its inclusion among members of the Little Astronomy, its referential scholia are directed towards those texts, and it will number among the Middle Books. As a proposition-based astronomical work there is no reason to exclude it. This lack of further evidence should be noted, but *On Risings and Settings* likely formed part of the curriculum.

Hypsicles *Anaphoricus*

The *Anaphoricus* finds itself in the same position as *On Risings and Settings*. Though still an astronomical work, it does not have the same proposition-based structure as the previous treatises did. But while the work is more distinctive, its manuscript transmission is fully linked to the Little Astronomy, and so it likely did number among them.

Euclid *Catoptrics*

The *Catoptrics* is not an astronomical text, and as a work on mirrors it has even less than the *Optics* to recommend it as relevant to the subject. No outside source speaks of it in connection with other Little Astronomy treatises, and it seems to never have been translated into Arabic, unlike the rest of the works discussed here. Yet it appears near the end of Vat. gr. 204 and it does contain referential scholia pointing to the *Sphaerica* – though, as mentioned above, it is the target of a scholion in the *Optics* and so its inclusion would break the pattern of scholia citing only prior texts.

This work was very likely never conceived of as part of the Little Astronomy. Rather, its subject matter being similar to that of the *Optics* and its shared author likely contributed to it being transmitted very often together in the same manuscripts. When the *Optics* started to appear alongside other Little Astronomy texts, it may sometimes have dragged the *Catoptrics* along with it, which would explain why the latter appears in manuscripts such as Vat. gr. 204.

Euclid *Data*

The *Data* is the last non-commentary or -scholia work in Vat. gr. 204, but it otherwise has nothing to recommend it as part of the late antique Little Astronomy. It is not an astronomical work, it neither references nor is referenced by other Little Astronomy works via scholia, and it is never mentioned in connection with the collection. Instead, Pappus tells his reader that it was a member of a different grouping entirely, the Domain of Analysis, which shows no overlap with the works in the Little Astronomy.

It is possible that the *Data*'s contents were thought of as useful for geometry more generally, and so the work was added to supplement geometrical codices, including those of spherical geometry. In Vat. gr. 204, the *Data* is located at the end of the codex. Not only does this break from the otherwise clear pattern from more precise to more particular seen in the earlier Little Astronomy treatises (the *Data* is certainly a more precise work), this places it amongst various commentary works: it is preceded by Eutocius's commentary on Apollonius's *Conics*, and it is followed by Euclidean scholia and Marinus's introduction. The *Data* is even written in a smaller and more abbreviated hand compared to the previous treatises, further linking it with how scholia and commentary material is treated in the codex.

In the Arabic the *Data* does become one of the Middle Books, and interestingly it is placed at the head of the collection, as would be expected from its far more general geometrical contents. Even if the text was not a member of the late antique Little Astronomy, it is possible that it was attached often enough to such codices by the ninth century to influence this development in the Arabic.[54]

Menelaus *Spherics*

The last work to be considered does not appear in Vat. gr. 204: the *Spherics* is not extant anywhere in the Greek outside fragments. However it does later become one of the Middle Books, and the fact that it is cited by Pappus (though it is not an object of discussion) in *Collection* VI merits it being noted here.

Ultimately, the *Spherics* most likely was not part of the curriculum. Its failure to survive in Greek, despite the Little Astronomy being solely responsible for the preservation of other treatises attached to it like the *Anaphoricus*,[55] speaks strongly against its inclusion. Nor is it the target of referential scholia in any of the Little Astronomy treatises.

If Menelaus's *Spherics* had been a part of the curriculum, it would have been expected alongside Theodosius's *Sphaerica* as a more precise treatise. Since Menelaus explicitly cites the *Sphaerica* in the text several times according to its Arabic translation,[56] its position would most likely have been second, immediately after the other work but before the less precise *On the Moving Sphere*. If the *Spherics* had

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[54] However, as chapter 3 will show, one of the earliest reports in Arabic omits any mention of the *Data*. It may be that the *Data* was added to the Middle Books after the ninth century. Since the Middle Books were explicitly intended as preparation for the *Almagest*, the *Data* may have been chosen as an addition because of the relevance its theorems have for Ptolemy's text.
[55] Pingree (1968) 16-17.
[56] See e.g. references in Ibn ʿIrāq's version in Krause (1936) 241, 247; and references in al-Māhānī / al-Harawī's version in Rashed and Papadopoulos (2017) 684, 696, 768.

been included in the Little Astronomy, it would have had small odds of becoming lost: unlike the start or end of a codex, the second position is not likely to disappear.

Galen’s testimony is a possible aid in explaining why Menelaus’s *Spherics* was not a member of the Little Astronomy. If the curriculum were already established by Galen’s youth, perhaps it is not surprising that a work by Menelaus a generation or two before was too recent to be included.

7. Conclusion

Adding the evidence from Galen and reading the evidence in Philoponus in light of the canonical arrangement implied by the sources and scholia reveals that there was much more to the Little Astronomy than the “hodgepodge of treatises” Neugebauer considered it to be.[57] While this curriculum did develop over time, it had at its core an ordered set of works whose usage seems to go back to the second century.

At the same time, it is clear that some claims about the Little Astronomy lack support from the contemporary evidence. No source says that it was intended as preparation for the *Almagest*, showing that Fabricius’s claim is instead influenced by his knowledge of the Middle Books, as is his inclusion of Menelaus’s *Spherics* among the collection. Rather, the astronomical curriculum which would become the Little Astronomy appears to pre-date the *Almagest*, and the *Spherics* was perhaps too new of a text to be included.

This chapter has set out a series of treatises as members of the Little Astronomy, a pedagogical backdrop to the collection, and an extended period of time in which it was used. Chapter 2 now will proceed to delve into the text of these works and expand on them and their variations in light of these contexts.

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[57] Neugebauer (1975) 769.

Chapter 2

Greek Editors and Little Astronomy Texts

1. Introduction

Combined, the evidence discussed in chapter 1 supports a picture of an ordered grouping of mathematical and astronomical works being used for pedagogical purposes between the 2nd and 9th centuries. At some point these came to be called the Little Astronomy. The last chapter argued that the following works were or came to be members of the Little Astronomy during this period: Theodosius's *Sphaerica*, Autolycus's *On the Moving Sphere*, Euclid's *Optics* (perhaps a later addition), Euclid's *Phaenomena*, Theodosius's *On Habitations*, Theodosius's *On Days and Nights*, Aristarchus's *On Sizes and Distances*, Autolycus's *On Risings and Settings*, and Hypsicles's *Anaphoricus*. All of these works remain extant today, and so it is possible to study the surviving witnesses to determine how their usage in a didactic context influenced the ways these texts evolved and changed during late antiquity. This is the project of the present chapter.

Section 2 summarizes the evidence which is available for the investigation, noting both the coverage and the limitations of the manuscripts and indirect witnesses. The study in this chapter covers nine texts, so section 3 provides an overview of the results across the full grouping. These results are broken down in detail in the following section, and they are interpreted afterwards in section 5.

2. Overview of Evidence

The Greek texts of the Little Astronomy survive in numerous manuscripts. These offer evidence for what texts were transmitted together within individual codices, for different forms of the texts, and for scholia or marginal comments on those texts.

All of the Greek texts have been edited, so it is possible to draw upon critical editions and prior scholarship on these texts to support this chapter's study. While these critical editions are a useful foundation, the manuscripts themselves remain central, especially since there are multiple instances where modern editors have not fully preserved the textual variance present in the manuscripts or where they have limited themselves to only a small selection of manuscript evidence.

Of the Little Astronomy manuscripts, the earliest which survives today is from the 9th century. The other significant manuscripts are largely 13th century ones. So direct evidence for the period examined in this chapter – the 2nd through 9th centuries – is not available outside of one manuscript from the tail end of this range. Furthermore, where the Greek texts vary from each other, there are no comments attributing these variants to any named historical individuals. Pinpointing precisely when branches of the Greek tradition diverged before the 9th century is not usually feasible.

However, there is a key source that offers a lens into the forms of these texts in the 4th century. As noted in chapter 1, book 6 of Pappus's *Collection* concerns works from the Little Astronomy: in several cases Pappus offers quotations or describes the contents in such detail that these can be compared with the text according to the surviving manuscripts. This is a window into the forms of the Little Astronomy texts as Pappus had them in the 4th century. There are also sparse references to Little Astronomy texts in Theon's commentaries on the *Almagest*, which also date from the 4th century.

Outside of what can be gleaned from Pappus and Theon, then, this chapter does not seek to precisely date all of the changes in the Greek texts. It seeks rather to identify what changes can be attributed to the didactic context the texts would have been transmitted in versus what changes were accidents of transmission or motivated by other concerns.

The Arabic tradition can also offer insights into the forms of these texts prior to the ninth century, since the translations were made from versions that were in circulation in the ninth century and which might represent different traditions than what is preserved in the Greek. This material however is not addressed in this chapter, since it will be the project of chapter 4 to survey the alterations between the ninth century Arabic texts and the Greek texts and to try to disentangle which of these might be attributed to variants in the Greek and which might have originated in the Arabic.

Note that, unless stated otherwise, proposition numbers given correspond to those in the modern critical editions. Disagreements appear among the manuscripts and will be acknowledged when relevant.

3. Summary of Deliberate Alterations in the Little Astronomy

The introduction to the dissertation has already explained the reasoning behind the philological method used in this chapter, chapter 4, and chapter 9. Ancient and medieval editors of ancient mathematical texts are known to have made choices and changes based on considerations other than presenting an original text.[1]

The alterations which Vitrac has laid out in his typology of deliberate alterations have a particular relevance for thinking about how editors, teachers, students, scholars actively intervened in the text for their own purposes. The full listing is provided below, with emphasis added for the alterations examined in this dissertation.[2]

- Modification of Presentation
  - Alteration of Proofs
    - Global
      - **Substitution of Proof**
      - **Double Proofs**
      - **Addition / Suppression of Cases**
    - Local

[1] Cameron (1990) 126.
[2] Vitrac (2012) 89-92.

        - Stylistic Interventions
        - Abridged Construction / Shortened Proof
        - Logical Interventions
    - **Change in Order**
    - **Fusion / Division**
    - **Change of Status**
    - Different Formulations
- **Addition / Suppression of Material**

This dissertation focuses on alterations on the larger, structural scale, taking advantage of the proposition-based structure of Little Astronomy texts to efficiently examine variations across nine texts in three different periods (and two languages).

A note on terminology: since there are instances in the transmission of this astronomical curriculum where more than one alternate proof appears on a proposition, this study will use the more general phrasing “alternate proofs” rather than “double proofs.” It will also refer to addition or suppression of this material, especially in the Arabic tradition.

The below table summarizes where these alterations occur in the Greek tradition of the Little Astronomy before the ninth century. It lays out each proposition of the nine Little Astronomy works, using the proposition numbers as they are given in the relevant critical editions.

Sph. I	II	III	MS	Opt. B			Ph. B	Hab.	D&N I	II	S&D	R&S I	II	Ana.
d.1	d.1	1	d.1	intro	15	37	intro	1	d.1	1	d.1	d.1	1	1
d.2	1	2	d.2	d.1	16	38	1	2	d.2	2	d.2	d.2	2	2
d.3	2	3	d.3	d.2	17	39	2	3	d.3	3	d.3	d.3	3	3
d.4	3	4	d.4	d.3	18	40	3	4	d.4	4	d.4	d.4	4	4
d.5	4	5	1	d.4	19	41	4	5	d.5	assum	d.5	d.5	5	5
d.6	5	6	2	d.5	20	42	5	6	1	assum	d.6	d.6	6	
1	6	7	3	d.6	21	43	6	7	2	assum	1	d.7	7	
2	7	8	4	d.7	22	44	7	8	porism	assum	2	d.8	8	
3	8	9	5	1	23	45	8	9	3	5	3	d.9	9	
4	9	10	6	2	24	46	9	10	lemma	6	4	1	10	
5	10	11	7	3	25	47	10	11	4	7	5	2	11	
6	11	12	8	4	26	48	11	12	porism	8	6	3	12	
7	12	13	9	5	27	49	12		5	9	7	4	13	
8	13	14	10	6	28	50	13		6	lemma	8	5	14	
9	14		11	7	29	51	14		7	10	9	6	15	
10	15		12	8	30	52	15		8	11	10	7	16	
11	16			9	31	53	16		9	12	11	8	17	
12	17			10	32	54	17		10	13	12	9	18	
13	18			11	33	55	18		11	14	13	10		
14	19			12	34	56			12	15	14	11		
15	20			13	35	57				16	15	12		
16	21			14	36					17	16	13		
17	22									18	17			
18	23									19	18			
19														
20														
21														
22														
23														

Sph = *Sphaerica*, MS = *Moving Sphere*, Opt = *Optics*, Phaen = *Phaenomena*, Hab = *Habitations*, D&N = *Days and Nights*, S&D = *Sizes and Distances*, R&S = *Risings and Settings*, Ana = *Anaphoricus*

Rearranged propositions	Addition / suppression of cases
Fusion / division of propositions	Addition / suppression of material
Addition / suppression of alternate proofs	(Additions according to Neugebauer)

Table 2.1: Overview of alterations in core Little Astronomy works across the Greek manuscripts. In this table, propositions are indicated with numbers, definitions with the pattern "d.#," introductions with "intro," assumptions with "assum," and lemmas with "lemma."[3]

[3] Note this table considers the B recensions of the *Optics* and the *Phaenomena*, which will be discussed further in section 4.1.

This summary view brings some initial conclusions to light. Firstly, deliberate alterations are not equally dispersed across the Little Astronomy, nor are different types of alterations dispersed similarly to each other. A greater variety of alterations appear in the *Sphaerica*, *On the Moving Sphere*, and the *Phaenomena*: the first shows four types, the second three, and the last four. It may be that, as the three treatises on spherical geometry at the head of the curriculum, they received more editorial attention.

Inverted propositions are uncommon, and fusion / division of propositions are only slightly more frequent. The latter do not reorganize the material, just renumber it, so it is uncertain what they might reveal about didactic motivations. They may speak more to how individual editors conceived of the structure of a proposition.

Alternate proofs occur only in two of the nine texts. Seemingly few variant proofs were in circulation for these texts in the first place, but for those texts which had them, there was seemingly little interest from the editors of the Little Astronomy in compiling variants together. A closer look at the instances in the *Phaenomena* in section 4.2 below will point to how editors were more discerning in preserving proofs that had a certain usefulness.

The deliberate alterations which are most widespread across the Little Astronomy are addition / suppression of cases or larger material. Sections 4.3 and 4.7 below will show how – in nearly all instances – these are additions. The Little Astronomy certainly accumulated material over the course of its transmission. The purposes of these additions vary: they provide foundations for later arguments in the texts, they expand on cases that had been left unproved, and they provide general introductions to treatises which Philoponus would call “more particular.” The curricular context of these texts quite plausibly motivated the introduction of this material.

4. Deliberate Alterations and References in Detail

4.1 Substitution of Proof

In the Greek tradition of the Little Astronomy, substitutions of proof occur in two texts: the *Optics* and the *Phaenomena*. These alterations are in fact significant enough that modern editors consider both texts to have been transmitted in two recensions. Today, these are indicated as recension A and B in both texts.

Substitutions of proof are most extensive between the recensions of the *Optics*, where largely the same enunciations are provided but the proofs are rewritten. The understanding of the relationship between the two recensions of the *Optics* has evolved over time in the modern scholarship. The work's editor, Heiberg, argued that one recension (referred to here as recension B) was a revision by Theon of Alexandria and that the other (recension A) represented the more genuine Euclidean text. More recently, scholars have pushed back against this idea and see recension B as the earlier version. Recension A provides fuller proofs and is plausibly the work of an editor who was expanding and clarifying the text in recension B.[4]

The substituted proofs between the *Phaenomena* recensions are less extensive but do occur later in the treatise, starting with proposition 10. Propositions 11 and 12 in recension B expand on material that was left unexplained in A, showing similar motivations of clarifying the text.

These texts and their recensions are especially interesting because the recensions have distinct transmissions in the Greek manuscripts. For each recension, one is clearly transmitted within the Little Astronomy and one outside of it. Below are short summaries of manuscript contents of codices containing recensions A or B of the *Optics* and the *Phaenomena* from the thirteenth century or earlier.

[4] See the overview in Jones (1994) 49ff.

Optics recension A	*Optics* recension B

- Bibl. Med. Laurenziana Plut. 28.3 (10th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
- ÖNB phil. gr. 031 (12th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
- Bodleian Library Auct. F. 6. 23 (13th c)
  - Euclid *Elements*
  - Euclid *Optics*
- Vat. gr. 1038 (13th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
  - various Ptolemaic works (not the *Almagest*)
- Bibl. Med. Laurenziana Plut. 28.6 (13th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
	- Vat. gr. 204 (9th c)
	  - 9 Little Astronomy works
	- Vat. gr. 191 (13th c)
	  - 9 Little Astronomy works
	  - various astronomical, astrological, and mathematical works
	- Vat. gr. 202 (13th c)
	  - 7 Little Astronomy works
	- Paris gr. 2390 (13th c)
	  - various astronomical works, including Ptolemy's *Almagest*
	  - Theodosius *Sphaerica*
	  - Autolycus *On the Moving Sphere*
	  - Euclid *Optics*

Table 2.2: Comparison of contents between manuscripts of *Optics* A and B recensions

Phaenomena recension A	*Phaenomena* recension B

- Bibl. Med. Laurenziana Plut. 28.3 (10th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
- ÖNB phil. gr. 031 (12th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
- Vat. gr. 1038 (13th c)
  - Euclid *Elements*
  - Euclid *Optics*
  - Euclid *Phaenomena*
  - Ptolemaic works (not the *Almagest*)
- Bibl. Med. Laurenziana Plut. 28.6 (13th c)
  - Euclid *Elements*
  - Euclid *Phaenomena*
  - Euclid *Optics*
	- Vat. gr. 204 (9th c)
	  - 9 Little Astronomy works
	- Vat. gr. 192 (11-12th c)
	  - various mathematical works
	  - Euclid *Optics*
	  - Hypsicles *Anaphoricus*
	  - Aristarchus *On Sizes and Distances*
	  - Euclid *Phaenomena*
	  - various musical works
	- Vat. gr. 191 (13th c)
	  - 9 Little Astronomy works
	  - various astronomical, astrological, and mathematical works
	- Vat. gr. 202 (13th c)
	  - 7 Little Astronomy works

Table 2.3: Comparison of contents between manuscripts of *Phaenomena* A and B recensions

In fact, when it comes to recension A of the *Phaenomena*, it tends to appear in the same codices as recension A of the *Optics*, with both joined by the *Elements*. This Euclidean grouping preserved both recensions, while the Little Astronomy evidently preserved the B recensions of both texts.

The example manuscripts described above date to the thirteenth century and earlier. The pattern does start to break down when considering later manuscripts, presumably because by the fifteenth century there was no longer a strong conception of the Little Astronomy as a group or curriculum.

4.2 Alternate Proof

The alternate proofs present in the Little Astronomy appear in proposition 2 of *On the Moving Sphere* and in propositions 6, 12, 14, and 15 of the *Phaenomena* (recension B).[5]

Alternate proofs are a phenomenon which appear in a variety of mathematical texts: they are cases where the texts or scholia provide a different method of proving the enunciation. Usually only one other alternate proof will appear (hence Vitrac's phrasing "double proof"), but across the breadth of Greek mathematical works there are some cases where a larger number of alternate proofs will be provided.[6] In many cases, it can be assumed that the original ancient author did not write the text with these multiple variant proofs, although there are cases such as Apollonius's *Conics* where the ancient author had circulated several different "editions" (ἐκδόσεις) of his text. Outside of cases like these, alternate proofs tend to arise later in the mathematical text's transmission.

When considering alternate proofs, it is worthwhile to note a report from Eutocius, editing the text of Apollonius's *Conics* in the sixth century:

[5] There are also a significant number of alternate proofs in Euclid's *Data*, which are not discussed here but which will be relevant when discussing the Arabic *Data* in chapter 4. Propositions with alternate proofs are 24, 27, 30, 33, 45, 46, 54, 55, 67, 68, and 80.
[6] For example, proposition 30 of Euclid's *Data* has three alternate proofs.

> "Since there were several editions, as Apollonius himself says in the preface, I thought it better to put them together from whatever source came to hand and place the clearer version in the text to help the understanding of beginners; and to indicate the variations on the proofs outside [*sc*. in the margin]."[7]

In this situation, Eutocius is an editor faced with variants which he attributes to Apollonius himself. All of the proofs can be considered authentic, but still the editor is not interested in collecting and presenting all of these variants without judgment. For each case Eutocius chooses one proof which he deems to be "clearer" (σαφέστερα) and sets that as the proof used in his edited text. He states his motive: he wants the text to benefit "beginners" (εἰσαγομένων). Alternate proofs are still worth presenting, but are not placed in the text itself: they number among the scholia in the margins.[8]

The unknown editors of Little Astronomy texts have not left testimonies about their editorial choices, so Eutocius's account is a useful comparison. Eutocius kept students in mind when editing the *Conics*: did the presentation of alternate proofs have a similar purpose in the Little Astronomy?

Considering the curriculum in its entirety, there are actually very few alternate proofs in the Little Astronomy. As noted above, according to the extant evidence there are five propositions with this feature, but these five propositions are spread across only two of the nine Little Astronomy works. Evidently presenting alternate proofs was not a project that teachers and editors engaged in across the Little Astronomy as a whole.

Nor is it possible to say that texts within the Little Astronomy preserve more alternate proofs than texts outside of the grouping. As discussed above, recensions A of the *Optics* and the *Phaenomena*

[7] Translation from Cameron (1990) 117. The Greek text was edited by Heiberg (1893) 176: "πλειόνων δὲ οὐσῶν ἐκδόσεων, ὡς καὶ αὐτός φησιν ἐν τῇ ἐπιστολῇ, ἄμεινον ἡγησάμην συναγαγεῖν αὐτὰς ἐκ τῶν ἐμπιπτόντων τὰ σαφέστερα παρατιθέμενος ἐν τῷ ῥητῷ διὰ τὴν τῶν εἰσαγομένων εὐμάρειαν, ἔξωθεν δὲ ἐν τοῖς συντεταγμένοις σχολίοις ἐπισημαίνεσθαι τοὺς διαφόρους ὡς εἰκὸς τρόπους τῶν ἀποδείξεων."

[8] By Eutocius's time, the codex was the major format rather than the scroll. This had the advantage of allowing for exterior material to be written in the margins and therefore to be transmitted with the text.

circulated outside the Little Astronomy while recensions B of these texts circulated as parts of it. For the *Phaenomena*, recension B includes alternate proofs while the outside recension lacks them. But in the case of the *Optics*, the recension which circulates within the Little Astronomy lacks them while the one which circulates outside it includes them. This suggests that in editorial work with the Little Astronomy, there was not a particular interest in compiling multiple proofs and presenting them together.

Exploring the larger picture of alternate proofs does not lead to any hints about didactic motivations. A closer look at the texts is more informative. In the case of *On the Moving Sphere*, evidence for the alternate proof does not survive in Greek. It is known instead from the Arabic, Latin, and Hebrew traditions, and will be discussed further in chapter 4.[9]

The alternate proofs of *Phaenomena* B do survive in the Greek manuscripts, and so it is possible to observe how they are presented in the surviving sources. The following comments rely upon an examination of the manuscripts Vat. gr. 204, Vat. gr. 191, Vat. gr. 192, Paris grec 2342, Paris grec 2472, and BSB cod. graec. 361a: witnesses dating from the 9th through 14th centuries.

In all cases where they are present, the alternate proofs to the *Phaenomena* appear within the text itself, not among the marginal scholia. The following table summarizes how these alternate proofs are introduced in the manuscripts:[10]

[9] Mogenet (1950) 177ff.
[10] Note: the numbering of the manuscripts disagrees with Menge (1916)'s edition because the manuscripts number proposition 14's first and second parts as 14 and 15 respectively.

	Prop. 6	Prop. 12	Prop. 14 pt.1	Prop. 14 pt.2	Prop. 15
Vat. gr. 204	"Alternatively to this"[11]	"Alternatively to 12[12] : this is the clearer setting-out[13]"	"Alternatively to 14[14] : also this setting-out is clearer than the former[15]" + [in margins] "This is the clearer setting-out"	"Alternatively to 15"[16]	"Alternatively to this" + "16 additionally"[17]
Vat. gr. 191	[a new textual unit with a rubric initial]	"This is the clearer [one]"[18]	"Alternatively to 14"	"Alternatively to 15"	[not present]
Vat. gr. 192	[a new textual unit with a rubric initial]	"Alternatively to 12: this is the clearer [one]"	"Alternatively to 14: this is the clearer setting-out"	"Alternatively to 15"	"16 in addition" [19]
Paris grec 2342	"Alternatively"[20]	"Alternatively to 12: this is the clearer [one]"	"Alternatively to 14"	"Alternatively to 15"	"16 in addition"
Paris grec 2472	"Alternatively"	[no textual indicator, but "12"[21] is written a second time in the margin]	"Alternatively to 14: this is the clearer setting-out"	[no textual indicator, but a new textual unit with a rubric initial]	"16 in addition"
BSB cod. graec. 361a	"Alternatively"	"Alternatively to 12: this is the clearer [one]"	[not present]	"Alternatively to 15"	"16 in addition"

Table 2.4: Indications of alternate proofs across Greek manuscripts

[11] Translation of "ἄλλως τὸ αὐτό."
[12] Translation of "ἄλλως τὸ ιβ'."
[13] Translation of "Αὕτη δέ ἐστιν ἡ σαφεστέρα ἔκθεσις."
[14] Translation of "ἄλλως τὸ ιδ'."
[15] Translation of "Ἔστι δὲ καὶ αὕτη ἔκθεσις σαφεστέρα τῆς προτέρας."
[16] Translation of "ἄλλως τὸ ιε'."
[17] Translation of "ις' ἐκ περισσοῦ."
[18] Translation of "Αὕτη ἐστιν ἡ σαφεστέρα."
[19] Translation of "τὸ αὐτό ις' ἐκ περισσοῦ."
[20] Translation of "ἄλλως."
[21] Translation of "ιβ'."

The word "alternatively" (ἄλλως) is most commonly used to indicate an alternate proof, and this is the case across the breadth of mathematical texts, not just those among the Little Astronomy. It can usually be found at the head of the alternate proof, either immediately above it or in the margins alongside it.

Propositions 12 and 14 of the *Phaenomena* stand out for the very short notes that introduce them: commentary to the effect that the alternate proof is the "clearer" (σαφεστέρα) one. This is the same language used by Eutocius. Since Eutocius evaluated and arranged the proofs of his edition according to clarity for the purpose of helping beginners (students?), it is tempting to see these alternate proofs in the *Phaenomena* as serving a similar role.

The alternate proofs in *Optics* A, conversely, have no such claims. Examination of the manuscripts Österreichische Nationalbibliothek phil. gr. 31, Biblioteca Medicea Laurenziana Plut. 28.3, and Biblioteca Medicea Laurenziana Plut. 28.6 reveals alternate proofs indicated only with the word "alternatively" (ἄλλως) or through a new numeral, treating the alternate proof as a new one. Further, the alternate proofs themselves suggest more of a collation effort: many are slightly rephrased versions of the proofs in *Optics* recension B.[22] So where *Optics* A, outside the Little Astronomy, collects other versions of the proofs in circulation without comment, *Phaenomena* B, within the curriculum, presents proofs sometimes with the note that they are clearer. It might be reasonable to interpret this descriptor as indicating usefulness for teaching.

4.3 Addition / Suppression of Cases

Instances which fall under Vitrac's category of addition or suppression of cases appear in *Sphaerica* II.15, *Phaenomena* 2, *Phaenomena* 11, and *Phaenomena* 12. These are instances of addition rather than suppression.

[22] Acerbi (2014) 146 fn.102.

The cases discussed here were added before the ninth century, since they are well attested in the surviving manuscripts, including the ninth century Vat. gr. 204. They are sometimes identifiable through their language, sometimes through disagreements among the manuscripts, and in one notable instance through comparison with Pappus's comments about the proposition in question.

An example of the first and second instance can be found in *Sphaerica* book II proposition 15. Nearly all of the extant Greek manuscripts present two cases (Paris grec 2448 presents three[23]). The proposition in question presents the problem of how to draw a great circle on a sphere which is tangent to a given lesser circle and which passes through a given point that is located between the lesser circle and its parallel circle on the sphere. The original proposition considers the case where the arc between the given lesser circle and point is less than a quadrant. The second case declares that it concerns when that arc is equal to a quadrant.[24] This case is introduced with the phrase "if someone says" (εἰ δὲ τις λέγοι), phrasing which is found nowhere else in the text of the *Sphaerica* – indeed the text's editor, Czinczenheim, brackets it as a later addition. She also diverges from the text's prior editor, Heiberg, and places Paris grec 2448's interpolations (which lay out three cases at the start of the proposition and describe the third case at the end) into her edition of the scholia.[25] With the two cases described above, the third case is an obvious one: it concerns when the arc in question is greater than a quadrant.[26] Though

[23]This manuscript is a Byzantine recension from the fourteenth century that features numerous corrections and variants. Its alterations are later than the time period examined in this chapter, but it will be noted in this section to help highlight what alternate cases were being added by late antique scholars and what cases were left unspoken at the time (presumably because they were obvious ones).
[24] Czinczenheim (2000) 105: "Εἰ δέ τις λέγοι τὴν ἀπολαμβανομένην ἴσην τῇ τοῦ τετραγώνου πλευρᾷ τοῦ εἰς τὸν μέγιστον κύκλον ἐγγραφομένου εἶναι τὴν ΒΓ, ἀποδείξομεν οὕτως."
[25] Compare Heiberg (1927) 70-76 with Czinczenheim (2000) 102-105. The relevant scholia in Czinczenheim are 210 and 227. Note that these scholia do not actually appear in any extant manuscripts – they are only known via the interpolations in Paris grec 2448.
[26] See the summary of the three cases near the beginning of the proposition in Heiberg (1927) 70's text: "ἡ δὴ ΒΓ ἤτοι ἐλάττων ἐστὶ τῆς, ὑφ' ἣν ὑποτείνει ἡ τοῦ τετραγώνου πλευρὰ τοῦ εἰς τὸν μέγιστον κύκλον ἐγγραφομένου, ἢ ἴση ἢ μείζων."

obvious, it was a much later addition to the text: it does not appear even as scholia in Vat. gr. 204, nor does it appear in the early Arabic translations of the *Sphaerica*, and was instead presumably the work of the redactor behind Paris grec 2448's text.[27] The second case, conversely, does appear to have been added to the Greek text prior to the text's translation into Arabic, since the Arabic manuscripts also have both it and its introductory statement "if someone says."[28] So *Sphaerica* II.prop.15 is a circumstance where editors were interested in expanding on alternate cases, but not necessarily every case. The third case is perfectly obvious, but this was not enough to merit its inclusion in the text (outside of the much later Paris grec 2448).

One of the added cases in the *Phaenomena* is also worth further comment, because it is an instance where Pappus's testimony allows for a comparison between the text as it stands in the manuscripts extant today and the text as he encountered it in the fourth century. Pappus's account of *Phaenomena* proposition 2 is useful because he explicitly says that Euclid did not address two particular cases. The proposition concerns how frequently certain great circles on the sphere of the cosmos would be perpendicular to each other. Evidently the text of the *Phaenomena* Pappus possessed only discussed the case for locations on the earth where the zenith is situated between the summer tropic and the north pole. Pappus points out the other cases which could be addressed but were not: where the zenith is between the two tropic circles, or where the zenith lies upon one of the tropic circles:[29]

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[27] Sidoli and Saito (2009) 597 fn.57.
[28] See e.g. Kunitzsch and Lorch (2010b) 144.50.
[29] Technically there is also a fourth case, where the zenith appears between the winter tropic and the south pole, but this can be deduced from the first case.

> "Concerning the second theorem of Euclid's *Phaenomena* he leaves out also the proof: if the pole of the horizon is between the tropics or upon one of them, how many times the zodiac is orthogonal to the horizon in one revolution…"[30]

Manuscripts that survive of the *Phaenomena* today, however, explicitly add in these two cases in both the enunciation and the subsequent proof of the proposition. This is true for both recensions A and B of the text. The text of the enunciation available today is as follows:

> "In one revolution of the cosmos, the circle through the poles of the sphere will be perpendicular to the horizon twice; and the ecliptic will be perpendicular to the meridian twice but never to the horizon, when the zenith is between the summer tropic and the visible pole. **If the zenith is on either of the tropics, the ecliptic will be perpendicular to the horizon once; and, when the zenith lies between the tropics, the ecliptic will be perpendicular to the horizon twice.**"[31] [emphasis added]

It can be inferred that the enunciation in Pappus's text only spanned the unemphasized text above. Similarly, his text evidently lacked the proofs of those cases, which are located at the end of the proposition.[32]

It is possible that Pappus simply had a deficient copy of the *Phaenomena*, and that other versions in the fourth century (or the original version) covered all three cases. But, it is also plausible that an editor after Pappus found it useful to incorporate these cases into proposition 2, so that the proposition addressed the full range of cases. It is easier to envision an editor adding these cases after Pappus than it is to

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[30] See the Greek text in Hultsch (1877) 594: "Ἐπὶ τοῦ β′ θεωρήματος τῶν Εὐκλείδου φαινομένων παρεῖται καὶ διὰ τῆς ἀποδείξεως, ἐὰν ὁ πόλος τοῦ ὁρίζοντος μεταξὺ τῶν τροπικῶν ᾖ ἢ ἐπὶ τινος αὐτῶν, ποσάκις ὁ ζῳδιακὸς πρὸς ὀρθὰς ἔσται πρὸς τὸν ὁρίζοντα ἐν μιᾷ περιφορᾷ..."

[31] Trans. Berggren and Thomas (1996) 55. See the Greek text in Menge (1916) 12: "Ἐν μιᾷ κόσμου περιφορᾷ ὁ μὲν διὰ τῶν πόλων τῆς σφαίρας κύκλος δὶς ἔσται ὀρθὸς πρὸς τὸν ὁρίζοντα ὁ δὲ τῶν ζῳδίων κύκλος πρὸς μὲν τὸν μεσημβρινὸν δὶς ἔσται ὀρθός, πρὸς δὲ τὸν ὁρίζοντα οὐδέποτε, ὅταν ὁ πόλος τοῦ ὁρίζοντος μεταξὺ ᾖ τοῦ θερινοῦ τροπικοῦ καὶ τοῦ φανεροῦ πόλου. [ἐὰν δὲ ἐπί τινος τῶν τροπικῶν ὁ πόλος ᾖ τοῦ ὁρίζοντος, ὁ τῶν ζῳδίων κύκλος ἅπαξ ὀρθὸς ἔσται πρὸς τὸν ὁρίζοντα ὅταν δὲ ὁ πόλος τοῦ ὁρίζοντος μεταξὺ τῶν τροπικῶν κύκλων ὑπάρχῃ, δὶς ἔσται ὁ τῶν ζῳδίων κύκλος ὀρθὸς πρὸς τὸν ὁρίζοντα.]"

[32] See bracketed text in Menge (1916) 20-22 and in Berggren and Thomas (1996) 57-59.

envision both the end of the proposition and the summary of the cases in the enunciation being lost or intentionally removed.[33]

The other two additions of cases in the *Phaenomena* can be found in a comparison between recensions A and B of the text. Proposition 11 demonstrates that when considering two equal and opposite arcs of the ecliptic, one will set while the other rises and one will rise while the other sets. In recension A of the text, the second case is noted but the proof is not given: the text simply states without elaboration, "Similarly, we shall show that, while AD sets, GE rises" (ὁμοίως δὴ δείξομεν, ὅτι, ἐν ᾧ ἡ ΑΔ δύνει, ἐν τούτῳ ἡ ΓΕ ἀνατέλλει).[34] Recension B of the text provides the proof for this case. The same is true for proposition 12, which concerns the setting and rising times of arcs of the ecliptic. Recension A again ends with recognition of one of the cases but leaves it unproved: "Then, similarly, we shall show that they also rise in an equal time to one another" (ὁμοίως δὴ δείξομεν, ὅτι καὶ ἐν ἴσῳ χρόνῳ ἀλλήλαις ἀνατέλλουσιν). [35] Recension B provides the demonstration of the case.

In recension A the cases were known, but not proved in the text – their demonstrations were likely perceived to be clear enough from the preceding demonstrations. Recension B, which as discussed above circulated with the Little Astronomy, nevertheless expanded on these cases.

4.4 Change in Order of Propositions

The survey of the Little Astronomy reveals two inversions: one of *Sphaerica* II.prop.1-2 and one of *On the Moving Sphere* prop.8-9.

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[33] The editor of the Greek text, Menge (1916), favors the interpretation of addition rather than suppression, describing these passages as interpolations after Pappus.
[34] Trans. Berggren and Thomas (1996) 82. See Menge (1916) 62 for the Greek text.
[35] Trans. Berggren and Thomas (1996) 86. See Menge (1916) 78 for the Greek text.

The first instance is known from a scholion that states, "Some arrange the first theorem second and the second first" (Ἔνιοι τὸ α' θεώρημα β' τάττουσι καὶ τὸ β' πρῶτον).[36] The propositions in question are direct converses of each other, as can be seen from their enunciations below:

- *Sphaerica* 2.prop.1:
  - "In a sphere, the parallel circles are about the same poles" (Ἐν σφαίρᾳ οἱ παράλληλοι κύκλοι περὶ τοὺς αὐτοὺς πόλους εἰσίν)[37]

- *Sphaerica* 2.prop.2:
  - "The circles about the same poles in a sphere are parallel" (Οἱ περὶ τοὺς αὐτοὺς πόλους ὄντες ἐν σφαίρᾳ κύκλοι παράλληλοί εἰσιν)[38]

The instance in *On the Moving Sphere* is suggested by comparison of the manuscripts with the testimony of Pappus, who offers a summary of each of the propositions in that text. Pappus's account agrees for all twelve except propositions 8 and 9. A comparison of the texts makes it clear that his claimed proposition "8" is a match for proposition 9 in the extant manuscripts, as can be seen below.

The enunciation of *On the Moving Sphere* prop.9:

> "If in a sphere a great circle inclined to the axis cuts both the visible (half) of the sphere and the invisible, of the points rising at the same time, those towards the visible pole set later, and of those setting at the same time, those towards the visible pole set later."[39]

Pappus's summary of proposition "8":

> "…the points rising at the same time also set at the same time, and those setting at the same time also rise at the same time. For all the circles there cutting the horizon are cut by it in half, and

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[36] This appears in Paris grec 2448, fol. 100v. Czinczenheim (2000) 397 does not report any other manuscripts for this scholion. Unless it derives from earlier manuscripts that have since been lost, it may only be commenting on the situation in the fourteenth century and therefore not offer insights into the history of the text prior to the ninth century.

[37] Czinczenheim (2000) 82.

[38] Czinczenheim (2000) 83.

[39] See Mogenet (1950) 208: "Ἐὰν ἐν σφαίρᾳ μέγιστος κύκλος λοξὸς ὢν πρὸς τὸν ἄξονα ὁρίζῃ τό τε φανερὸν τῆς σφαίρας καὶ τὸ ἀφανές, τῶν ἅμα ἀνατελλόντων σημείων τὰ πρὸς τῷ φανερῷ πόλῳ ὕστερον δύνει, τῶν δὲ ἅμα δυνόντων τὰ πρὸς τῷ φανερῷ πόλῳ πρότερον ἀνατέλλει."

semicircles are both above the horizon and below the horizon, and for this reason those rising at the same time also set at the same time, and vice versa."[40]

There is however a complication in that what Pappus claims about the proposition he calls 9 does not agree with proposition 8 in the Greek manuscripts. The actual enunciation of the proposition is the following: "The great circles tangent to those which the horizon also touches, when the sphere turns they will fit on the horizon."[41] Pappus instead offers the description: "And similarly also the ninth... for he wishes those tangent to it to not touch any other (parallel circle) except the always visible (circle) alone." [42] The extant text in Pappus seems to be saying that on the sphere of the cosmos, great circles tangent to something will only be tangent to the always visible circle. If this is read in comparison with the actual enunciation of proposition 8, it may be that Pappus is explaining what is meant by "those (circles) which the horizon also touches" – namely, that regarding those great circles tangent to those which the horizon always touches, they are tangent only to the always visible (and always invisible) circle. But this would be oddly phrased for an explanatory statement, since it also stresses that these great circles cannot be tangent to any other parallel circles, a further clarification which would seem unnecessary. Compared with the otherwise accurate account of the propositions in Pappus's text, something has gone awry. Whether this was a mistake on Pappus's part or one arising later in the text's transmission is unclear.

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[40] The Greek text is edited by Hultsch (1877) 522: "...τὰ ἅμα ἀνατέλλοντα σημεῖα ἅμα καὶ δύνει, καὶ τὰ ἅμα δύνοντα ἅμα καὶ ἀνατέλλει· πάντες γὰρ ἐκεῖ οἱ κύκλοι οἱ τέμνοντες τὸν ὁρίζοντα δίχα τέμνονται ὑπ' αὐτοῦ, καὶ ἡμικύκλια ὑπέρ τε τὸν ὁρίζοντα ἔχουσιν καὶ ὑπὸ τὸν ὁρίζοντα, καὶ διὰ ταύτην τὴν αἰτίαν τὰ ἅμα ἀνατέλλοντα ἅμα καὶ δύνει, καὶ τὸ ἀνάπαλιν."

[41] See Mogenet (1950) 206: "Οἱ τῶν αὐτῶν ἐφαπτόμενοι μέγιστοι κύκλοι ὧν καὶ ὁ ὁρίζων ἅπτεται, στρεφομένης τῆς σφαίρας ἐφαρμόσουσιν ἐπὶ τὸν ὁρίζοντα."

[42] See Hultsch (1877) 522: "Ὁμοίως δὲ καὶ τὸ θ'... βούλεται γὰρ τοὺς τοῦ αὐτοῦ ἐφαπτομένους μὴ ἄλλου τινὸς ἐφάπτεσθαι ἢ μόνου τοῦ ἀεὶ φανεροῦ."

Mogenet reads this peculiar passage as evidence that Pappus may not have had the text of *On the Moving Sphere* in front of him and was instead summarizing from memory.[43] This is possible, and it would be a complication to any efforts to use Pappus as a reliable witness to the form of the Little Astronomy works in the fourth century. On the other hand, the text of the problematic summary may have rather been garbled in transmission. Pappus's summaries of Little Astronomy works elsewhere are detailed enough that in many cases he would seem to have a text in front of him.[44]

In any case, while inversions of propositions are interesting in the histories of these texts, they tend to lack hints of a didactic context that might have motivated them. This category of deliberate alterations therefore is not particularly informative for the question of this chapter.

4.5 Fusion / Division of Propositions

Fusion or division of propositions occurs in *Sphaerica* 2.props.11-12, *Optics* B prop.36, and *Phaenomena* B prop.14.

The instance in the *Sphaerica* is suggested by comparison of the Greek manuscripts, Arabic/Latin manuscripts, and Pappus's testimony. While the Greek manuscripts present propositions 11 and 12 of Book II separately, the Arabic and Latin present them in one proposition. In the broader context of the *Sphaerica*, these two propositions are clearly distinct from what is treated in the propositions immediately prior or immediately after. A comparison of the enunciations of each proposition clarifies why they may have been read together: both start from the same geometrical arrangement. The Greek texts, aligned below with emphasis added, agree nearly word-for-word in their first halves:

[43] Mogenet (1950) 168. He further thinks Pappus has erred in switching propositions 8 and 9, since the logical order is more sensible as they appear in the manuscripts.
[44] See for example his word-for-word quotations from *On the Moving Sphere* and *On Sizes and Distances*: Hultsch (1877) 518ff and 554ff.

Sphaerica 2.prop.11	*Sphaerica* 2.prop.12
Ἐὰν ἐν ἴσοις κύκλοις ἐπὶ διαμέτρων ἴσα καὶ ὀρθὰ τμήματα κύκλων ἐπισταθῇ, ἀπὸ δὲ αὐτῶν ἴσαι περιφέρειαι ἀποληφθῶσι πρὸς τοῖς πέρασι τῶν τμημάτων ἐλάττους ἢ ἡμίσειαι οὖσαι τῶν ὅλων, καὶ ἀπὸ τῶν γενομένων σημείων πρὸς τὰς τῶν ἐξ ἀρχῆς κύκλων περιφερείας ἴσαι εὐθεῖαι προσβληθῶσιν, ἴσας ἀπολήψονται περιφερείας τῶν ἐξ ἀρχῆς κύκλων πρὸς τοῖς πέρασι τῶν διαμέτρων.[45]	**Ἐὰν ἐν ἴσοις κύκλοις ἐπὶ διαμέτρων ἴσα καὶ ὀρθὰ τμήματα κύκλων ἐπισταθῇ, ἀπὸ δὲ αὐτῶν ἴσαι περιφέρειαι ἀποληφθῶσι πρὸς τοῖς πέρασιν ἐλάττους ἢ ἡμίσειαι οὖσαι τῶν ὅλων τμημάτων,** ἀπὸ δὲ τῶν κύκλων ἴσαι περιφέρειαι ἀποληφθῶσιν ἐπὶ τὰ αὐτὰ μέρη πρὸς τοῖς πέρασι τῶν διαμέτρων, αἱ ἐπὶ τὰ γενόμενα σημεῖα ἐπιζευγνύμεναι εὐθεῖαι ἴσαι ἔσονται ἀλλήλαις.[46]

Table 2.5: Comparison of enunciations in *Sphaerica* 2.prop.11 and 2.prop.12

This instance further serves as a useful example because it is another one where Pappus preserves relevant material. When comparing his discussion of *Spaherica* book II to what is extant in the Greek, the cited proposition numbers he gives do not agree with what is in the manuscripts. Proposition 23 is cited as proposition 22,[47] and 13 is twice cited as proposition 12.[48] So the version of the *Sphaerica* in front of Pappus seemingly had one fewer proposition in Book II, with this missing proposition occurring before proposition 13. Considering the Arabic and Latin tradition, one plausible explanation for this disagreement is that it was caused by Pappus's version of the *Sphaerica* presenting Book II's propositions 11 and 12 as one proposition: this proposition then divided in two later in the Greek transmission, but remained as one in the line of transmission that led to the Arabic and Latin traditions.[49]

[45] Czinczenheim (2000) 93.
[46] Czinczenheim (2000) 95.
[47] See Hultsch (1877) 610: Pappus cites 2.prop.23 as "the 22nd proposition in the 2nd (book) of Theodosius's Sphaerica" (τὸ ἐν τῷ β' τῶν σφαιρικῶν Θεοδοσίου κα' θεώρημα).
[48] See Hultsch (1877) 612 and 616: Pappus cites 2.prop.13 twice as "in the 12th (proposition) of the 2nd (book)" (τῷ ιβ' τοῦ β').
[49] Either this, or Theodosius originally wrote two propositions but Pappus had on hand a version in which they were already fused, even if other versions with the propositions separated circulated elsewhere.

The instances in the *Optics* and *Phaenomena* are also ones of division. *Optics* B proposition 36 is numbered separately in five parts in the manuscript tradition: in Vat. gr. 204, for instance, it is presented as propositions 36 through 40.[50] *Phaenomena* B proposition 14 meanwhile is presented as two propositions in many of the manuscripts: Vat. gr. 204 clearly labels it with the numerals 14 and 15.[51]

For the *Optics* and *Phaenomena*, these divisions attested in the numbering system are not so clear in the text alone. Rather, the various sections were clearly originally one proposition because they lack the internal structure that would be expected of a new proposition. None of them begin with a new enunciation, which would be required for a typical proposition.

Overall, it can be said that the fusion or division of propositions is not particularly enlightening when seeking didactic motivations. A change in the numbering of the propositions does not affect their order or the logic of the treatise. Nevertheless, these varied numerals are important to keep track of when looking at how these texts and their scholia cite each other.

4.6 Change in Status

The alteration which Vitrac calls "change in status" does not appear to occur in the Greek tradition of the Little Astronomy. An example of it can rather be seen outside the curriculum in *Phaenomena* recension A. There, the lemma in the work is sometimes promoted to being considered a proposition.[52] As with fusion and division, this affects the numbering of the propositions but does not affect their logical arrangement.

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[50] See Vat. gr. 204 fols. 52v-53v.
[51] See Vat. gr. 204 fols. 71r-72r.
[52] Menge (1916) 84.

4.7 Addition / Suppression of Material

Compared to the deliberate alterations discussed so far, addition or suppression of material is considerably more widespread across the Little Astronomy. Instances include *Sphaerica* I.def.6 and I.props.22-23, *On the Moving Sphere* defs.3-4, the introduction to the *Phaenomena*, and the definitions and assumptions in *On Days and Nights*. Neugebauer argues that the final three propositions of *On Habitations* and the final five of *On Days and Nights* may also be additions. His argument is based on how they diverge from the logic of their texts; it is unclear whether there is manuscript evidence to further support this interpretation.[53]

The examples across the Little Astronomy follow a general pattern: when larger units of text are added, they tend to appear at beginnings or endings. Preliminary material becomes attached to the start of the text, while added propositions or definitions appear after the other existing definitions or propositions.

The preliminary material proves to be especially relevant when examining these works in a curricular context. To summarize, there is one definition added to the definitions of the *Sphaerica*, two definitions that do not appear in all manuscripts of *On the Moving Sphere*, an anonymous preface added to *Optics* B, an anonymous preface added to both recensions of the *Phaenomena*, and definitions added to the beginning of *On Days and Nights*.

The final definition in *Sphaerica* Book I has a notably different character than the rest of the definitions: while the others all concern matters that have to do with spheres, it declares when a plane inclined to a plane is similar to another plane inclined to another plane.[54] This supposed definition also

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[53] Neugebauer (1975) 757-758.

[54] See Czinczenheim (2000) 52 for definition 6: "Ἐπίπεδον πρὸς ἐπίπεδον ὁμοίως κεκλίσθαι λέγεται καὶ ἕτερον πρὸς ἕτερον ὅταν αἱ τῇ κοινῇ τομῇ τῶν ἐπιπέδων πρὸς ὀρθὰς γωνίας ἀγόμεναι εὐθεῖαι ἐν ἑκατέρῳ τῶν ἐπιπέδων πρὸς τοῖς αὐτοῖς σημείοις ἴσας γωνίας περιέχωσιν."

appears later in the text, at the end of book II proposition 21: "for we learned that a plane inclined to a plane is said to be similar to another (plane) (inclined) to another (plane) when the straight lines drawn to the common sections of the planes at right angles in each of the planes contain equal angles."[55] The *Sphaerica*'s recent editor, Czinczenheim, argues that these are both interpolations and that the passage in 2.prop.21 occurred first, drawing on an archaic definition from a corpus that has been lost. She sees the definition as having been added afterwards to support the statement made in this proposition.[56]

So in the *Sphaerica*, the added definition provides information that will later be used in the treatise. The definitions in *On the Moving Sphere* are a different case. Definitions 3 and 4 of this treatise mostly do not appear in the Greek manuscripts, although they do appear in the Arabic translations. They are close matches for definitions 3 and 4 of the *Sphaerica*:

Sphaerica definitions 3-4	*On the Moving Sphere* definitions 3-4
"An axis of the sphere is some straight line lying through the center and terminating at both sides on the surface of the sphere, about which fixed straight line the sphere turns."[57]	"An axis of a sphere is the diameter of the sphere about which fixed (axis) the sphere rotates."[58]
"The poles of the sphere are the ends of the axis." [59]	"And poles of the sphere are the ends of the axis." [60]

Table 2.6: Comparison of *Sphaerica* defs. 3-4 and *Moving Sphere* defs. 3-4

[55] Czinczenheim (2000) 122: "ἐμάθομεν γὰρ ὅτι ἐπίπεδον πρὸς ἐπίπεδον ὁμοίως κεκλίσθαι λέγεται καὶ ἕτερον πρὸς ἕτερον ὅταν αἱ τῇ κοινῇ τομῇ τῶν ἐπιπέδων πρὸς ὀρθὰς γωνίας ἀγόμεναι εὐθεῖαι ἐν ἑκατέρῳ τῶν ἐπιπέδων ἴσας γωνίας περιέχωσιν."
[56] Czinczenheim (2000) 208ff.
[57] See Czinczenheim (2000) 52: "Ἄξων δὲ τῆς σφαίρας ἐστὶν εὐθεῖά τις διὰ τοῦ κέντρου ἠγμένη καὶ περατουμένη ἐφ' ἑκάτερα τὰ μέρη ὑπὸ τῆς ἐπιφανείας τῆς σφαίρας περὶ ἣν μένουσαν εὐθεῖαν ἡ σφαῖρα στρέφεται."
[58] See Mogenet (1950) 195: "Ἄξων σφαίρας ἐστὶν ἡ διάμετρος τῆς σφαίρας περὶ ἣν μένουσαν ἡ σφαῖρα στρέφεται."
[59] See Czinczenheim (2000) 52: "Πόλοι δὲ της σφαίρας εἰσὶ τὰ πέρατα τοῦ ἄξονος."
[60] See Mogenet (1950) 195: "πόλοι δὲ τῆς σφαίρας εἰσὶ τὰ πέρατα τοῦ ἄξονος."

Mogenet discusses several possibilities that may have led to this circumstance. These definitions see significant use in *On the Moving Sphere*, so it is not impossible that Autolycus included them in the composition. They do not actually receive use in the text of Theodosius's *Sphaerica* (which raises questions as to their inclusion there), but Mogenet still notes Theodosius may have adopted them from spherics material which preceded him. Alternatively, they may have been interpolated into the text of *On the Moving Sphere* at an early point in time because of their usefulness for the text.[61] Regardless of when these definitions entered in to *On the Moving Sphere*, it is possible that their exclusion in all of the Greek manuscripts (besides the manuscript Paris gr. 2448, which is curious[62]) is motivated by the fact that, when grouped in the Little Astronomy, these definitions are redundant: they appear immediately in the text prior. However, the testimony from the Arabic tradition conflicts with this narrative, since the third and fourth definition do appear in the translated text. Looking at *On the Moving Sphere* alone, then, is inconclusive, but this instance will be revisited when looking at the Little Astronomy as a whole.

A much clearer example of added definitions is provided by *On Days and Nights*. They are undoubtedly of a later origin because at their start they name the author in the third person: "Theodosius furnishes by hypothesis..." (Ὑποθέσει χρῆται ὁ Θεοδόσιος…). The definitions also refer to him in the third person twice more with "he calls" (καλεῖ).[63] The content of these definitions would support the argument that they were added to provide information which was required in the treatise proper.

[61] Mogenet (1950) 179-180.

[62] As noted above, the recensions present in Paris gr. 2448 are probably the work of later Byzantine redactors, and so the alterations present in it likely originated later than the period under consideration in this chapter. But here, the definitions present in Paris gr. 2448 are *also* present in the Arabic tradition, which raises the possibility of a common source in at least this particular case.

[63] Fecht (1927) 54.

The prefaces of the *Phaenomena* and the *Optics* are more significant additions. To start, most scholars agree that the preface of the *Phaenomena* was not original to the third century BCE treatise. It is unfortunately unclear when it was written. The scholars Berggren and Thomas suggest that it originates after the fourth century CE, since Pappus makes no mention of it.[64] Of course, it is possible that he simply did not find it useful to comment on. The preface would seem to predate the separation of the *Phaenomena* into recensions A and B, since both recensions carry it, but, alternatively, it could have had its origins with one branch of the text's transmission and have been added to the other later. Therefore, the evidence thus far is inconclusive.

The contents of the preface are summarized below:[65]

- On fixed stars: their risings and settings in the same place, their simultaneous risings, their constant distance, that they are carried on parallel circles with the same pole
- On stars which are always visible, stars which rise and set, stars which are always invisible: various points
- Defining the circles of the milky way and the ecliptic
- On a spherical cosmos: that it is not conical or cylindrical, that it turns uniformly on its axis, that there is one visible pole and one invisible
- On the horizon, meridian, tropic, and zodiac circles: their definitions, which of these are great circles
- Defining the time of revolution
- Defining an arc's passages across the visible and invisible hemisphere

Overall, this is a mix of astronomical points which comment on observable phenomena and make arguments about the geometric configurations of these features. It is not at all a list of definitions. This preface is interesting for being instead a general astronomical introduction, one not necessarily directed to the needs of the *Phaenomena* specifically. On the one hand, there are assumptions laid out in the preface which the text of the *Phaenomena* does make use of, such as the passage noting that the horizon and

[64] Berggren and Thomas (1996) 12.
[65] See the Greek of the preface in Menge (1916) 2-10, and an English translation in Berggren and Thomas (1996) 43-48.

ecliptic bisect each other or the one dealing with arcs' passages across the hemispheres. On the other hand, there are points made about the milky way, for example, which are never used in the text of the *Phaenomena*.

The preface of the *Phaenomena* can be compared to the definitions that begin the other astronomical texts in the Little Astronomy. The added definitions in *On Days and Nights* cover the day, night, and various motions of relevant parts of the cosmos. The introductory material in *On Sizes and Distances* provides the reader with the assumptions that will be used in arguing the relative sizes and distances of the sun and the moon. The definitions of *On Risings and Settings* cover risings and settings that are true or apparent and that are occurring in the evening or morning. It is clear that compared to the introductory material in the preface of the *Phaenomena*, these definitions are largely specific to the needs of these particular treatises and not so generalizable to the others.

Meanwhile the preface to recension B of the *Optics* is an interesting addition because there may be more information to help in dating it. Heiberg claims it was known by Nemesius, which would provide a *terminus ante quem* of the fourth century for the text. However, a comparison of the relevant passage of Nemesius and the preface reveals this is not definite: the agreement is that both the preface and Nemesius discuss how the eye might miss a small object lying on the ground, because of the gaps between the rays cast out by the eye. In the preface the small object is a needle (βελόνης) and in Nemesius it is a coin (νόμισμα).[66] While Nemesius may be drawing on the preface, there is also the possibility that this was simply a stock example used in optical discussions at this time.

[66] See Heiberg (1895) 144-154 for the Greek text of the *Optics*'s preface and Morani (1987) 58 for the Greek text of the passage of Nemesius.

More interestingly, there is an attestation (although only in one manuscript, and a late one at that) of a source for this preface. The manuscript Paris grec 2468 claims that "the preface is from the lectures of Theon" (Τὸ προοίμιον ἐκ τῆς τοῦ Θέωνός ἐστιν ἐξηγήσεως)[67] – that is, the fourth century Theon of Alexandria. Since this claim appears only in one late manuscript, scholars have acknowledged that this may simply have been a guess on the part of the copyist. The editor of the *Optics*, Heiberg, instead argued that not just this preface but also recension B of the *Optics* in its entirety was to be attributed to the work of Theon. He held recension A to be more faithful to Euclid's original text.[68] Heiberg's arguments about the relation of recensions A and B and Theon's involvement have since been called into question, as noted.[69]

But in the case of the preface alone, although the extant attribution to Theon is a late one, dismissing it outright might be too hasty. Theon was indeed known as a teacher. As Heiberg has already pointed out, where Theon cites the *Optics* in his commentary to Ptolemy's *Almagest*, his citations follow recension B rather than recension A.[70] The entire recension may not be the work of Theon as an editor; rather, it is possible that Theon taught the *Optics* and so some material from his lectures ended up being worked into a preface that was attached to the version of the text which he taught from.

It should be noted that, unlike the preface to the *Phaenomena*, the preface to the *Optics* does not seem to have been translated into Arabic.[71] The *Optics*'s preface further differs from the other in that it appears with its own title, "What Comes Before Euclid's *Optics*" (τὰ πρὸ τῶν Εὐκλείδου Ὀπτικῶν).[72]

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[67] Paris gr. 2468, fol. 1r. This manuscript may be viewed online in the Gallica digital library.
[68] Heiberg (1895).
[69] Jones (1994) 49-56.
[70] Heiberg (1882) xxx.
[71] Kheirandish (1999) xxi fn.6.
[72] See for example, Vat. gr. 191 fol. 11v; Vat. gr. 192 fol. 127r; and Vat. gr. 204 fol. 43v.

Manuscripts extant today tend to present it as part of the *Optics*: there is no separation between the end of the preface and the start of the *Optics*'s definitions in Vat. gr. 204, Vat. gr. 191, or Vat. gr. 192, for instance.[73] The codex Paris grec 2342, however, contains a rubricated "Euclid's *Optics*" between the two.[74] Perhaps its absence in Arabic speaks to it sometimes being considered a separate text earlier in its transmission.

<u>**4.8** References to the Curriculum within the Texts</u>

Signs of editorial work with these texts as a curriculum might be found in instances where the texts reference each other. It is worthwhile to distinguish between more ancient citations and those which arose later in its transmission.

Before the interventions of later editors, where texts reuse ideas from other texts, usually they make no mention of the title or the author referenced. Instead, they quote or paraphrase the text. The quote might not be word-for-word: studies have shown that ancient quotations did not always seek to be so precise.[75] Even so, they are usually close enough for recognition.

An example of this can be found between *On the Moving Sphere* and the *Sphaerica*. The text of the former's proposition 1 mentions in the course of its argument that "Circles about the same poles on a sphere are parallel" (Οἱ δὲ περὶ τοὺς αὐτοὺς πόλους ὄντες ἐν σφαίρᾳ κύκλοι παράλληλοί εἰσι).[76] This agrees with the enunciation of *Sphaerica* 2.prop.2: "Circles about the same poles on a sphere are parallel" (Οἱ περὶ τοὺς αὐτοὺς πόλους ὄντες ἐν σφαίρᾳ κύκλοι παράλληλοί εἰσιν).[77]

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[73] Vat. gr. 204, fol. 45v; Vat. gr. 191, fol. 12r; and Vat. gr. 192, fol. 128r.
[74] Paris grec 2342, fol. 109v.
[75] Netz (2012) 179.
[76] Mogenet (1950) 196.
[77] Czinczenheim (2000) 83.

Since the *Sphaerica* was written after *On the Moving Sphere*, the latter likely did not originally cite the former. Rather, Autolycus's text would seem to have been quoting from the corpus of ancient spherics which has since been lost, but which Theodosius relied upon when rearranging this material into the *Sphaerica*.[78] As noted above, there is a similar case in *Sphaerica* 2.prop.21 where an interpolation appears to have come from a lost work on spherics, which later motivated the introduction of definition 6.

In comparison to the more ancient citations, there are a few instances within the texts of these treatises with direct references to texts and, in one case, a specific proposition of a text. Modern scholars take these as interpolations and usually disregard them, but for the purposes of this chapter they serve as interpolations which are indicative of how this group of texts was thought of in late antiquity as a coherent unit. These citations are not original to the texts when they were first written (the *Sphaerica*, for instance, was written after the *Phaenomena*) but they do have relevance for the Little Astronomy. Most likely these were originally marginal comments that became incorporated into the texts in their transmission. The citations are the following:

- *Phaenomena* preface:
    - "as is demonstrated in the *Optics*" (ὡς ἐν τοῖς ὀπτικοῖς δείκνυται)[79]
- *Phaenomena* A prop.12:
    - "as in the 6th theorem of the 3rd book of the *Sphaerica*" (ὥστε τῷ ἕκτῳ θεωρήματι τοῦ τρίτου βιβλίου τῶν Σφαιρικῶν)[80]
- *On Days and Nights* 2.prop.10:
    - "as was demonstrated in the *Phaenomena*" (ὡς ἐν τοῖς Φαινομένοις δέδεικται)[81]

[78] See for example Berggren (1991) 241-247, which discusses the spherics materials and methods that presumably were available when Euclid and Autolycus were writing, and later also when Theodosius was writing. Berggren notes, however, that mathematical methods did not remain static over the centuries, and highlights several ways in which the use of particular theorems evolved over time.
[79] Menge (1916) 2.
[80] Menge (1916) 76.
[81] Fecht (1927) 126.

- *Optics* A prop 19:
    - “as it is said in the *Catoptrics*” (ὡς ἐν τοῖς κατοπτρικοῖς λέγεται)[82]

- *Optics* B prop 19:
    - “because this is demonstrated in the *Catoptrics*” (τοῦτο γὰρ δείκνυται ἐν τοῖς κατοπτρικοῖς)[83]

Interestingly, the citation which comes closest to looking like the referential scholia in the Little Astronomy – *Phaenomena* A prop.12 – still does not use the formulaic language that appears in the scholia. Had it appeared as a referential scholion, it would have taken a form like “according to the sixth [proposition] of the third [book] of the *Sphaerica*” (διὰ τοῦ ϛʹ τῶν γʹ τῶν Σφαιρικῶν). Further, this citation occurs in recension A of the *Phaenomena*, despite the fact that recension B is the one associated with the Little Astronomy.

Matters are also complicated by a case of an extra-corpus citation: the *Optics* cites the *Catoptrics*. Since chapter 1 of the dissertation has already posited that the *Optics* was a later addition to the Little Astronomy, this is perhaps not very surprising. The reference appears in both recensions, suggesting that it had its origins early, before the split.

4.9 Referential Scholia

As discussed in chapter 1, there exist a series of referential scholia on texts in the Little Astronomy which are notable for only citing other Little Astronomy texts. Further, they only cite propositions which have occurred previously according to the logical order of the Little Astronomy. Since the Greek referential scholia have already been summarized in the last chapter, that will not be repeated here. But this chapter’s counterparts, chapters 4 and 9, will return to this matter for the Arabic translations and al-Ṭūsī’s editions.

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[82] Heiberg (1895) 30.
[83] Heiberg (1895) 176.

5. Interpretation of Attested Alterations and References

Overall, the treatises of the Little Astronomy show a tendency towards expansion in their transmissions before the ninth century, and this can be read as influenced by the curricular context in which they were transmitted. Other deliberate alterations – changes in order, fusion or division, and changes in status – are fewer, and will be more informative in comparison with their frequency at other stages of these texts' transmissions. But the expansion of cases, presentation of clearer alternate proofs, and addition of various kinds of material can be interpreted as motivated by the didactic aims of the curriculum.

At the same time, the Little Astronomy does not show attempts to provide exhaustive and systematic collections of material that was in circulation. As section 4.2 pointed out, alternate proofs are limited. Of the ones that are present, several show signs of inclusion on the basis of their clarity, which suggests that they were evaluated as useful for teaching or study. In general, the anonymous editors do not explain their goals, but their actions make it clear that they were addressing perceived shortcomings in the treatises. The example of *Phaenomena* proposition 2 is illuminating, since Pappus specifically complains about the absence of cases from its proof. This omission has been addressed in all of the extant Greek manuscripts.

There is one text which trends towards preservation rather than expansion, and this exception adds further nuance. Of the two recensions of the *Optics*, manuscripts of the Little Astronomy preserve the older one. As noted in section 4.1, recent scholarship has shown that recension A shows editorial work and expansion to improve on the text in recension B. This editorial activity, however, occurred outside the context of the Little Astronomy, which instead seems to have contributed to the survival of the earlier recension B. The Little Astronomy was a curriculum of spherical geometry and astronomy: it is plausible that this curriculum encouraged editorial work on those subjects but not on optics, despite the fact that the *Optics* had been introduced into the grouping of texts. Individuals who dealt with the *Optics* outside the Little Astronomy may have been more likely to intervene in its text.[84]

To return to the variety of additions in the Little Astronomy: Acerbi has raised a possible source for these kinds of expansions and scholia. As chapter 1 noted, one of the allusions to a late antique Little Astronomy came from the mention of Theon having written a commentary on the curriculum. This mention is an uncertain one. However, Acerbi has put forth the suggestion that if this commentary existed, it may have served as a receptacle for the exegetic tradition up until his time; furthermore, it may have later been distributed across the Little Astronomy both within the texts themselves and in scholia.[85]

A few of the deliberate alterations take on more significance when viewed as alterations to the full grouping of the Little Astronomy, not just to the treatise in question. The absence of *On the Moving Sphere*'s definitions 3 and 4 from nearly all the Greek manuscripts discussed in section 4.7, for instance, may be because the *Sphaerica* had presented those exact definitions in the treatise prior. In the full curriculum, they were not only repetitive but immediately so. This change, however, may not have permeated all late antique codices of the curriculum: the definitions are ultimately translated into Arabic.

Meanwhile the preface of the *Phaenomena* is a significant addition not just to the *Phaenomena*, but to the Little Astronomy as a whole. Considering the arrangement of the full curriculum, it is notable that this general astronomical introduction stands at the turning point where the curriculum transitions

[84] There are admittedly other possibilities: recensions A and B may both have existed by the time the *Optics* was incorporated into the Little Astronomy, and recension B just happened to be the version available. In that case, if editors later became aware of A they still evidently were not interested in its expanded text, not even as alternate proofs.
[85] Acerbi (2014) 145-146. Acerbi notes the prefaces, alternate proofs, rewritten propositions, but also states that the scholia to recensions B of the *Optics* and *Phaenomena* have a more complex structure than those of recensions A.

from spherical geometry (and some preliminary optics) to astronomy proper. As chapter 1 argued, the Little Astronomy follows an arrangement starting with more general topics and ending with more particular ones. Philoponus made this clear when he twice referenced texts of the Little Astronomy to discuss this classification scheme. He twice notes the *Sphaerica* as an example of a more general, less particular science. He twice notes *On the Moving Sphere* as a more particular science than the *Sphaerica*. And he twice notes the topic of "astronomy" as more particular than *On the Moving Sphere*, in one case naming the *Phaenomena* as an example specifically.[86]

Even if the preface of the *Phaenomena* were not composed with the framework of the Little Astronomy in mind, this preface may have settled into its particular position at the head of the astronomical treatises in the Little Astronomy because of its usefulness for transitioning the student from general geometrical concepts to more specific astronomical ones. Its points have relevance outside the Little Astronomy: the definitions it sets out for the passages of arcs across the visible and invisible hemispheres, for instance, come up again as definitions 4 and 5 in *On Nights and Days*.[87]

The preface of the *Phaenomena* should also be compared with the preface of recension B of the *Optics*. Both prefaces use appeals to observable phenomena to justify hypotheses for their respective texts. In the overarching structure of the Little Astronomy, both the *Optics* and the *Phaenomena* mark shifts into treatises on the more particular sciences: the *Optics* in the context of optics and the *Phaenomena* in the context of astronomy. This particular addition, therefore, may have been motivated less by the *Optics* itself and more by the logical progression of the Little Astronomy.

[86] Wallies (1909) 300 and Vitelli (1887) 220.
[87] Compare Menge (1916) 10 and Fecht (1927) 55.

PART II

Chapter 3

Arabic Translations and Translators

1. Introduction

All of the works in the Little Astronomy were translated into Arabic by the end of the third / ninth century. Based on the dates known for the attested translators, the majority of these translations occurred during this century, as will be seen over the course of this chapter. Evidence for the Little Astronomy or translations of its treatises is scarce for the seventh and eighth centuries.[1] Greek manuscripts of the corpus evidently remained *available* through this period up to the ninth century, though it is not clear how many new copies might have been produced, nor is it clear how much study these treatises still saw during this time.[2]

Many of the works of the Little Astronomy have entries in subsequent biobibliographical sources like the 4th/10th century *Fihrist* of Ibn al-Nadīm and the 7th/13th century *History of Learned Men* (*Taʾrīkh al-ḥukamāʾ*) by Ibn al-Qifṭī.[3] These sources, however, offer almost no information on the translators of the texts in question. It is necessary instead to look to attestations in the manuscripts for information about the figures who translated and revised these works. Key resources are the prefaces

[1] This chapter also will touch on translations of related texts, such as Menelaus's *Spherics*, Euclid's *Elements*, and Ptolemy's *Almagest* – each of these were translated multiple times and scholarship has suggested that the earliest of these translations likely occurred in the second / eighth century. This will be further discussed below. Similarly, the chapter will survey the evidence for Syriac translations made of the relevant treatises: with the limited evidence available, it is difficult to say with certainty whether these Syriac translations occurred also in the third / ninth century alongside the Arabic translations, or whether they may have been produced in earlier centuries.

[2] The fact that the study of Little Astronomy treatises did persist in Arabic within the context of the Middle Books is notable, however. It is possible that the scholars who first put together the Middle Books were reviving an educational corpus that had fallen into disuse, but it may be instead that they were adapting into Arabic a Greek corpus that still saw study. Compare the continuing study of Aristotle's *Categories* in Greek, Arabic, and Latin, a situation which Hermans (2016) argued was the result of a late Roman educational curriculum that continued to see use across the three cultural zones.

[3] The original composition of Ibn al-Qifṭī is no longer extant, but the epitome by al-Zawanī written in 647/1249 is extant.

al-Ṭūsī later adds to the texts in his edition of the Middle Books. In them he usually identifies the translator (and, if relevant, the reviser) of the version of the text he is editing. Al-Ṭūsī sometimes notes when a text was translated multiple times, but more often evidence of this is found in the surviving manuscripts of Middle Books texts before him. Translators and revisers can be found noted in the titles, colophons, and sometimes marginal comments to the work.

Section 2 of this chapter will gather together the reports about individuals who translated and revised the works from the Little Astronomy in the 3rd/9th century. Since the Middle Books come to comprise both these and several other Greek works translated in this period, this section will note the relevant individuals for those further works as well.[4] Section 3 will offer overviews of the historical figures attested in the preceding section, placing their involvement with these texts in context with their broader scholarship. In section 4, this chapter will consider whether the Little Astronomy was recognized as a curriculum by the ninth century figures who translated the relevant works, and inquire whether there is any evidence that the translation of the Little Astronomy as a complete unit was sought after. The chapter will conclude by showing that already in the 3rd/9th century a grouping of texts called al-Mutawassiṭāt (translated throughout this study as "the Middle Books") had come into existence in the Arabic tradition with the purpose of serving as preparation for the *Almagest*.

1.1 Overview of Translations

The following table summarizes the translators who are attested in biobibliographic and manuscript reports for each of the nine Little Astronomy texts. Section 2 will delve into the sources for each of these claims in further detail, and will note cases where the attestation may be mistaken.

[4] Later in their transmission the Middle Books also sometimes include original Arabic works as well. These will receive note in chapter 5.

	Qusṭā ibn Lūqā	Isḥāq ibn Ḥunayn	Ḥunayn ibn Isḥāq[5]	Thābit ibn Qurra	Hiliyā ibn Sarjūn	Īsā b. Yaḥyā	(Anon)
Sphaerica	x		x				x
On the Moving Sphere		x	x	x			
Optics			?[6]		x		
Phaenomena						x	
On Habitations	x						
On Days and Nights	x	x					
On Sizes and Distances	?[7]						
On Risings and Settings	x						
Anaphoricus	x	x					

Table 3.1: Translators attested for Little Astronomy texts

This overview, combined with the overview below of translators attested for other texts which came to be added to the Middle Books, make it clear that Qusṭā ibn Lūqā and Isḥāq ibn Ḥunayn are the two individuals with significant involvement in the translation of Middle Books texts.

[5] The references to Ḥunayn ibn Isḥāq as a translator for these texts are the ones most often pointed to as likely to be erroneous, since he is far better known as a medical translator than a mathematical one. Often these attestations are considered to be errors for his similarly named son, Isḥāq ibn Ḥunayn, who did have more involvement translating mathematical and astronomical texts.

[6] Steinschneider (1896) 171 claims some manuscripts of the *Optics* note Ḥunayn as translator, but does not specify which ones and I have not been able to find this attestation.

[7] Steinschneider (1896b) 355 claims some manuscripts of al-Ṭūsī's edition of *On Sizes and Distances* name Qusṭā as the translator. As noted below, the manuscripts I have checked do not contain this attestation.

	Qusṭā ibn Lūqā	Isḥāq ibn Ḥunayn	Ḥunayn ibn Isḥāq[8]	Al-Dimashqī	Thābit ibn Qurra	(Anon)
Data	x	x				
Spherics		x	x	x		x
Sphere and Cylinder	x	x				
Measurement of the Circle						
Lemmata					x	

Table 3.2: Translators attested for additions to the Middle Books[9]

Qusṭā was responsible for five or six of the nine Little Astronomy texts and a further two of the five additions. Isḥāq was responsible for three of the Little Astronomy texts and a further three of the additions.

Since (as will be shown below) the Middle Books are defined by multiple contemporary Arabic sources to be the works read between the *Elements* and the *Almagest*, it is also useful to note the attested translators for these two endpoints. The two named translators of the *Elements* are al-Ḥajjāj b. Yūsuf b. Maṭar and Isḥāq. The names associated with the translation of the *Almagest* are al-Ḥasan ibn Quraysh, al-Ḥajjāj, Sarjūn ibn Hiliyā, Isḥāq, and Thābit.[10] Isḥāq ibn Ḥunayn's involvement with both combined with his involvement discussed above is notable. The translator Sarjūn ibn Hiliyā is also worthy of note considering the attested translator of the *Optics*, Hiliyā ibn Sarjūn. These will be further discussed in the sections below.

[8] As above, Ḥunayn ibn Isḥāq in these mathematical and astronomical contexts is usually thought to be an error for Isḥāq ibn Ḥunayn.

[9] Note that the columns are not the same between the two tables: Hiliyā ibn Sarjūn and Īsā b. Yaḥyā were not involved with translations of the five additions, and al-Dimashqī was not involved with the translations of the Little Astronomy.

[10] These are discussed further in section 2.3, below.

1.2 Overview of Ninth Century Revisions

Only two names are relevant for revisions of the translations of Little Astronomy texts during the ninth century. The table below lays out which are claimed as the work of Thābit ibn Qurra and which as the work of al-Kindī. The attestations for these revisions will similarly receive further elaboration in section 2 below.

	Thābit ibn Qurra	al-Kindī
Sphaerica	x	
On the Moving Sphere	x	x
Optics		
Phaenomena		
On Habitations	x	
On Days and Nights		
On Sizes and Distances	x	
On Risings and Settings	x	
Anaphoricus	x	x

Table 3.3: Revisers attested for Little Astronomy texts

For the Greek works added to the Middle Books, Thābit ibn Qurra is also attested as the reviser of the *Data* and *On the Sphere and Cylinder*. Thābit furthermore revised both the *Elements* and the *Almagest*. The *Spherics* of Menelaus was partially revised by al-Māhānī in the ninth century as well (and it received several further corrections in the subsequent centuries, but these are beyond the scope of this chapter).[11]

[11] Al-Māhānī's incomplete revision, for instance, was later revised and completed by al-Harawī in the tenth century, though this edition introduced a number of problems into the text. On this see Sidoli and Kusuba (2014).

These individuals and their work with Middle Books and related texts will be further elaborated upon in the following sections.

2. Details of Translations and Revisions

2.1 Ninth Century Versions of Little Astronomy Texts

Sphaerica[12]

The *Sphaerica* has the greatest number of attestations for individuals involved in its translation. Ḥunayn ibn Isḥāq is credited with a translation, Qusṭā ibn Lūqā with one version and part of another, and an unnamed translator with the completion of Qusṭā's incomplete translation. An anonymous individual may have been responsible for another version which was revised by Thābit ibn Qurra.

Kunitzsch and Lorch have edited the 3rd/9th century translation which was said to have been revised by Thābit but whose translator goes unnamed.[13] They later also edited and discussed a selection of the translation attributed to Qusṭā.[14]

This work receives one of the more informative reports in the introductions of al-Ṭūsī's editions. The editor relates that this translation from Greek into Arabic was ordered by "Abū l-ʿAbbās Aḥmad ibn [Muḥammad ibn] Muʿtaṣim bi-llāh." Qusṭā ibn Lūqā completed it up until the fifth proposition in the third book and an unnamed translator completed the remainder.[15] The identification of the patron is problematic – this was either the son or the grandson of Caliph al-Muʿtaṣim. This will be discussed further in section 3.

[12] Cf. Steinschneider (1896b) 344 and Sezgin (2974) 154. Listed in *Fihrist* 7.2 (Dodge (1970) 642) and Ibn al-Qifṭī (Lippert (1903) 108).
[13] Kunitzsch and Lorch (2010b).
[14] Kunitzsch and Lorch (2019).
[15] Hyderabad (1939-40) *Kitāb al-Ukar* 2: "وقد امر بنقله من اليونانية الى العربية ابو العباس احمد ابن المعتصم بالله فتولى نقله قسطا بن لوقا البعلبكي الى الشكل الخامس من المقالة الثالثة ثم تولى نقل باقيه غيره واصلحه ثابت بن قرة"

The witness in the Kraus manuscript, which is a version separate from al-Ṭūsī's edition, has an introduction with nearly the same wording, including the omission of his patron's father's name: Qusṭā translated the *Sphaerica* for Aḥmad, son (sic) of Caliph al-Muʿtaṣim.[16] Kunitzsch and Lorch see this as indicating that this version either descends from al-Ṭūsī's edition or uses the same source that al-Ṭūsī did.[17] Interestingly, unlike al-Ṭūsī's introduction, the introduction in the Kraus manuscript does not include details about Qusṭā only partially completing the translation.

This partial translation would appear to be separate from the translation also attributed to Qusṭā which can be found in two Judeo-Arabic manuscripts. The codices Florence Laur. Med. 124 and Cambridge University Library Add. 1220 present an Arabic translation of the *Sphaerica* in Hebrew script which is distinct from the versions that survive in the Arabic manuscripts. Qusṭā is named as the translator in the titles of these copies.[18]

The last of the named translators can be found attested only in the manuscript Leiden Or. 1031, whose title names Ḥunayn ibn Isḥāq.[19] Kunitzsch and Lorch doubt this identification. They see this version as a reworking and credit it to Ibn al-Ṣalāḥ,[20] since the colophon declares this exemplar is twice removed from a copy written in Ibn al-Ṣalāḥ's own hand.[21] We will return to this in chapter 5.

The *Sphaerica* is a work which received significant attention based on the number of versions in circulation. Besides the attested translations above, there is also at least one more anonymous translation, the one edited by Kunitzsch and Lorch (2010b). This appears in the manuscripts Seray Ahmet III 3464,

[16] Kunitzsch and Lorch (2010b) 2.
[17] Kunitzsch and Lorch (2010b) 5.
[18] Kunitzsch and Lorch (2019) 123-5.
[19] MS Leiden Or. 1031, fol. 22b.
[20] Kunitzsch and Lorch (2010b) 5.
[21] MS Leiden Or. 1031, fol. 72a: "This book was copied from a copy copied from a copy in the hand of the great teacher Najm al-Din al-Sari" (نسخ هذا الكتاب من نسخة انتسخ من نسخة بخط الأجل الإمام نجم الدين ابن السري).

Lahore private library M. Nabī Khān, and Paris hebr. 1101. The colophon of the witness in the Lahore manuscript identifies it as a reworking by Thābit ibn Qurra.[22]

On the Moving Sphere[23]

The extant reports for *On the Moving Sphere* mention Isḥāq ibn Ḥunayn, Ḥunayn ibn Isḥāq, and Thābit ibn Qurra as translators and mention Thābit also as corrector. Ibn al-Qifṭī reports that al-Kindī corrected it.[24] The 3rd/9th century version of this work has not been edited.

The colophon of Seray Ahmet 3464 is the source for the former translator[25] and Bodleian Huntington 237 is the source for the second.[26] Manuscripts Istanbul Ayasofya 2671 and London Institute of Ismaili Studies Hamdani Collection, 1647 claim Thābit as translator – these manuscripts have also been identified as representing a different version of the text.[27] Al-Ṭūsī reports the text was corrected by Thābit.[28] It is possible that the Bodleian manuscript has erred in naming Ḥunayn instead of his son Isḥāq – it would be valuable to compare the two witnesses to see whether these manuscripts contain the same version or not.[29]

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[22] Kunitzsch and Lorch (2010b) 3.
[23] Cf. Steinschneider (1896b) 337-8 and Sezgin (1974) 82. Listed in *Fihrist* 7.2 (Dodge (1970) 640) and Ibn al-Qifṭī (Lippert (1903) 73).
[24] Lippert (1903) 73: "Book of the Moving Sphere, correction of al-Kindī" (كتاب الكرة المتحرّكة إصلاح الكندىّ) – this is the only one of the Little Astronomy works to receive any further detail from the extant biobibliographers. The report about al-Kindī's correction is also included in the much later encyclopedia by Kâtip Çelebi.
[25] Lorch (2008) 22. Mogenet (1950) 173 notes that the reading Isḥāq ibn Ḥunayn is a correction – the colophon actually records an otherwise unknown Isḥāq ibn al-Ḥasan.
[26] Steinschneider (1896b) 338.
[27] See Nikfahm-Khubravan and Eshera (2019) 13 and 46-47.
[28] Hyderabad (1939-40) *Kitāb al-kurah al-mutaḥarrikah* 2.
[29] Sezgin (1974) 82 identifies the version in Seray Ahmet 3464, the Kraus manuscript, and Ayasofya 2671 as a translation by Isḥāq and correction by Thābit, but he has nothing to say about the witness in Bodleian Huntington 237.

Optics[30]

The name associated with the translation of the *Optics* is Hiliyā ibn Sarjūn.

Kheirandish 1999 offers an edition of this translation. The early Arabic translation exists in two versions, one by the name of *Ikhtilāf al-manāẓir* and one by the name of *al-Manāẓir*. She concludes that while these two early versions are clearly different, their differences suggest a restoration or revision rather than two separate translations.[31] Only one of the manuscripts offers the translator's name: the colophon of the Kraus manuscript specifies that the *Ikhtilāf al-manāẓir* was translated by Hiliyā ibn Sarjūn.[32]

Al-Ṭūsī and other later editors have no comments to offer about the translator.

It should be noted that Steinschneider claims there are some manuscripts which name the translator as Ḥunayn, but he does not specify which manuscripts these might be.[33] None of the manuscripts examined in this study contain this claim.

Phaenomena[34]

The only translator named in connection with this text is Īsā ibn Yaḥyā, who produced it for Abū al-Ḥasan ʿAlī ibn Yaḥyā. The 3rd/9th century version of this work has not been edited.

Two manuscripts contain this early Arabic version: Leiden Or. 1031 and Seray Ahmet III 3464. The colophon of the former gives the information about the translator and patron.[35] According to Lorch,

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[30] Cf. Steinschneider (1896a) 171 and Sezgin (1974) 117. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and Ibn al-Qifṭī (Lippert (1903) 65).
[31] Kheirandish (1999) xxii and xxviii.
[32] Kraus manuscript, fol. 32b. Plate of the folio published in Kheirandish (1999) 240.
[33] Steinschneider (1896) 171: "Uebersetzer ist nach einigen mss. Honein."
[34] Cf. Steinschneider (1896a) 170-1 and Sezgin (1974) 118-119. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and al-Qifṭī p.65.
[35] Leiden Or. 1031, fol. 99b: "translation for Abū l-Ḥasan ʿAlī ibn Yaḥyā by ʿĪsā ibn Yaḥyā, student of Ḥunayn ibn Isḥāq" (ترجمة لابي الحسن علي بن يحيى عيسى بن يحيى تلميذ حنين بن اسحق). The Leiden manuscript catalogue erroneously merges these two names and attributes the translation to an ʿAlī b. Yaḥyā b. ʿĪsā b. Yaḥyā. However, Īsā ibn Yaḥyā

the latter manuscript offers no information about who translated it.[36] Al-Ṭūsī names no translators or revisers.

<u>On Habitations</u>[37]

This translation is attributed to Qusṭā ibn Lūqā by multiple sources. Thābit ibn Qurra is named as a reviser. The 3rd/9th century Arabic translation of this work has been edited by Kunitzsch and Lorch.[38]

Seray Ahmet III 3464 and the Kraus manuscript agree in naming Qusṭā as the translator.[39] Al-Ṭūsī's edition corroborates this report.[40] This work is also extant in Lahore, private library M. Nabī Khān, and this witness does not name a translator but does state that the work is the revision of Thābit.[41]

<u>On Days and Nights</u>[42]

Several manuscript sources name Qusṭā ibn Lūqā as the translator of this text, though one source instead credits Isḥāq ibn Ḥunayn. The early Arabic version of this work has not been edited.

The translator attributions come from the manuscript titles and colophons; al-Ṭūsī has nothing to say on the matter. Seray Ahmet III 3464 and the Kraus manuscript agree in claiming Qusṭā as the translator.[43] The manuscript Bodleian Or. 365 presents an incomplete version of the same text and in its

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and Abū al-Ḥasan ʿAlī ibn Yaḥyā are both known historical figures, and it is not surprising to see that the former, who was a student of Ḥunayn, produced a translation for the latter, a man who served as a patron for Ḥunayn.

36 Lorch (2008) 22.

37 Cf. Steinschneider (1896b) 344-5, Sezgin (1974) 156, and Sezgin (1978) 81. Listed in *Fihrist* 7.2 (Dodge (1970) 642) and Ibn al-Qifṭī (Lippert (1903) 108).

38 Kunitzsch and Lorch (2010a).

39 Kunitzsch and Lorch (2010a) 9: this report is given in the beginning of Seray Ahmet III 3464 and in both the beginning and colophon of the Kraus manuscript.

40 Hyderabad (1939-40) *Kitāb al-Masākin* 2: "Qusṭā ibn Lūqā of Baʿlabakk translated it" (نقل قسطا بن لوقا البعلبكي)

41 Kunitzsch and Lorch (2010a) 9.

42 Cf. Steinschneider (1896b) 345, Sezgin (1974) 156, and Sezgin (1978) 81. Listed in *Fihrist* 7.2 (Dodge (1970) 642) and al-Qifṭī p.108.

43 Kunitzsch and Lorch (2011) 13: this report is given in the beginning of Seray Ahmet III 3464 and three times in the Kraus manuscript (the beginning and end of Book I and the beginning of Book II).

title attributes the translation to Isḥāq ibn Ḥunayn.[44] The close similarity between the Bodleian version and the Qusṭā manuscripts (where the Bodleian manuscript is not incomplete) indicates that these are not two separate translations, so one of the attested translators is in error.

<u>On the Sizes and Distances of the Sun and the Moon</u>[45]

The translator of this text is unknown, though a revision by Thābit is attested. Its early Arabic version has not been edited.

The version prior to al-Ṭūsī's edition is extant in the Kraus manuscript and Columbia Or. 45. It is the colophon of the Kraus manuscript which notes the reviser as Thābit.[46] Steinschneider makes a claim that al-Ṭūsī names a translator, but this study has not found any such evidence.[47]

<u>On Risings and Settings</u>[48]

The translation of this work is attributed to Qusṭā ibn Lūqā in the manuscripts, and its revision to Thābit ibn Qurra. The 3rd/9th century Arabic version of this work has not been edited.

Leiden or. 1031 and the Kraus manuscript agree regarding the translator: the former names Qusṭā in titles of both books and colophon of the first,[49] and the latter in the title.[50]

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[44] Bodleian Or. 365, fol. 33b.
[45] Cf. Steinschneider (1896b) 354-5 and Sezgin (1978) 75. Listed in Fihrist 7.2 (Dodge (1970) 644) and Ibn al-Qifṭī (Lippert (1903) 70).
[46] Lorch (2008) 28.
[47] Steinschneider (1896b) 355. According to Steinschneider, al-Ṭūsī mentions the translator as Qusṭā ibn Lūqā. The manuscript Arch. Selden. A. 45 is named for example, but I have not found this attestation there. Noack (1992) offers some discussion on this question: see p.37-38 fn.6 for an overview of scholars who assert Qusṭā to have been the translator, and p.40-41 for obscure references to potential Arabic translators that can be found in the Latin tradition.
[48] Cf. Steinschneider (1896b) 338, Sezgin (1974) 82, and Sezgin (1978) 73. Listed in *Fihrist* 7.2 (Dodge (1970) 640) and al-Qifṭī p.73.
[49] Leiden Or. 1031 fol. 1b, 10b, 11b.
[50] Lorch (2008) 28.

Al-Ṭūsī and the manuscript Seray Ahmet III 3464 only mention Thābit as reviser and do not identify a translator.[51] There has been a claim in the secondary scholarship about a patron for this text, but this study has not found any evidence for this.[52]

<u>Anaphoricus</u>[53]

The *Anaphoricus* is reported to have two translations, one by Qusṭā ibn Lūqā and one by Isḥāq ibn Ḥunayn, as well as revisions of each by al-Kindī and Thābit ibn Qurra respectively.

An edition has been produced by Krause.[54] Qusṭā's version was revised by al-Kindī according to al-Ṭūsī,[55] Isḥāq's version by Thābit according to Paris arabe 2457.[56] Sezgin reports that the manuscript Mešhed Riḍā 5412 has the Qusṭā and al-Kindī version as well.[57]

2.2 Ninth Century Versions of Greek Texts added to the Middle Books

The *Data* of Euclid, *Spherics* of Menelaus, and several (pseudo-) Archimedean works will be noted in this section, since they can be seen sometimes forming part of the Middle Books after the 3rd/9th century.[58]

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[51] Hyderabad (1939-40) *Kitāb al-Ṭulūʿ w-l-Ghurūb* 2: "from the correction of Thābit" (من اصلاح ثابت). Lorch (2008) 22 reports the attestation from Seray Ahmet III 3464.
[52] Gabrieli (1912) 353. Gabrieli claims that like the translation of the *Sphaerica*, this translation by Qusṭā was on the order of al-Mustaʿin, though I have not encountered this report and he does not mention manuscripts. He notes Kâtip Çelebi and Steinschneider but this claim is present in neither.
[53] Cf. Sezgin (1974) 145. Listed in *Fihrist* 7.2 (Dodge (1970) 637) and Ibn al-Qifṭī (Lippert (1903) 73).
[54] De Falco, M. Krause, and O. Neugebauer (1966).
[55] Hyderabad (1939-40) *Kitāb fī al-Maṭāliʿ* 2: "مما اصلحه الكندي وهو من نقل قسطا بن لوقا البعلبكي"
[56] Paris arabe 2457, fol. 162a
[57] Sezgin (1974) 145.
[58] This evolution in the contents of the Middle Books can be seen both in later manuscripts of the Middle Books (which will be discussed in chapter 5) and in the treatises included in al-Ṭūsī's edition of the Middle Books (which will be seen in chapters 8 and 9).

Data[59]

Two names are associated with the translation of Euclid's *Data*: Qusṭā ibn Lūqā and Isḥāq ibn Ḥunayn. Thābit ibn Qurra is associated with a revision, and this revision of the *Data* has been edited by Sidoli and Isahaya.[60]

Qusṭā as this work's translator is only attested by the opening of its witness in the Kraus manuscript.[61] Isḥāq is named in the opening of al-Ṭūsī's revision of the *Data*.[62] Both report that Thābit revised the text.

One of these attested translators may be in error, unless Thābit produced his revision of the *Data* from two separate translations. Sidoli and Isahaya point out that the Kraus manuscript claims Qusṭā for many of the translations it contains, so it is not impossible that its scribe (or an earlier one) mistakenly assumed Qusṭā to have been responsible for the text.[63]

Spherics[64]

The manuscripts variously speak of Isḥāq ibn Ḥunayn, Ḥunayn ibn Isḥāq, or Abū Uthman al-Dimashqī as having translated the *Spherics* in the 3rd/9th century. Another manuscript version survives by an anonymous translator. Al-Māhānī revised the text in the 3rd/9th century.

[59] Cf. Steinschneider (1896a) 171-172 and Sezgin (1974) 116. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and al-Qifṭī p.65.
[60] Sidoli and Isahaya (2018).
[61] Sidoli and Isahaya (2018) 23 and Lorch (2008) 28.
[62] Hyderabad (1939-40) *Kitāb al-Muʿtīyāt* 2: "تحرير كتاب المعطيات لاقليدس ترجمه اسحاق واصلحه ثابت"
[63] Sidoli and Isahaya (2018) 29.
[64] Cf. Steinschneider (1896a) 196-197 and Sezgin (1974) 161. Listed in *Fihrist* 7.2 (Dodge (1970) 638).

Rashed and Papadopoulos have edited an extant fragment of an anonymous early translation along with al-Māhānī / al-Harawī's revision.[65] They set out evidence for there having been three translations made in the 2nd/8th and 3rd/9th centuries.[66]

Evidence for two of those translations appear in marginal comments to al-Ṭūsī's edition and al-Harawi's revision. These glosses speak of both an "ancient translation" (النقل القديم) and a translation by al-Dimashqī (نسخة / نقل الدمشقي).[67]

Rashed and Papadopoulos posit the above as the first and the third translations to occur; the second was a translation made by Isḥāq or his father Ḥunayn. Menelaus's *Spherics* was translated from Arabic into Hebrew by Jacob ben Makhir: two of the extant manuscripts state that the translator of the Arabic predecessor had been Ḥunayn, while the colophons of six other extant manuscripts declare that it was his son Isḥāq.[68]

Al-Ṭūsī names no translators, just comments on the variety of versions and revisions – this preface will be considered in chapter 9.

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[65] Rashed and Papadopoulos (2017). Another important version of the text (though a later one) was that of Abū Naṣr Manṣūr ibn ʿIrāq (d. c. 428/1036), which has been edited in Krause (1936).
[66] Rashed and Papadopoulos (2017) 19-20.
[67] See Rashed and Papadopoulos (2017) 19-20: their translations of the glosses are the following. In Haci Selim Aga 743, fol. 196b: "Gloss by the hand of Naẓīf ibn Yumn: in the translation of al-Dimashqī rectified by Yūḥannā ibn Yūsuf on the gloss in the Greek text; this is the end of the first book" (حاشية بخط نظيف بن يمن في نقل الدمشقي وإصلاح يوحنا بن يوسف على الحاشية في اليوناني وهذا آخر المقالة الأولى). In MS. Teheran Sepahsalar 4727, p.138: "In the ancient translation and in the translation of al-Dimashqī with the rectification of Yūḥannā, the converse was presented" (في النقل القديم وفي نقل الدمشقي وإصلاح يوحنا قدّم العكس). In British Library no. 13127, fol. 6a: "In the ancient translation and in the translation of al-Dimashqī (an illegible word) with the rectification of Yūḥannā also, the converse was stated first,and this is mentioned at the end" (في النقل القديم وفي نسخة الدمشقي وفي إصلاح (اسم غير مقروء) وإصلاح يوحنا أيضاً قدّم العكس أولاً وجعل المذكور هاهنا أولاً أخيراً عند منالاوس).
[68] See Krause (1936) 20-22 and Rashed and Papadopoulos (2017) 19.

On the Sphere and Cylinder[69]

Both Qusṭā ibn Lūqā and Isḥāq ibn Ḥunayn are attested as translators, and Thābit ibn Qurra as a reviser. The early Arabic of this work has not been edited.

The preface offered by al-Ṭūsī states that there was a translation corrected by Thābit and another translation by Isḥāq.[70]

The manuscript Fatih 3414 names the translator as Qusṭā.[71] Lorch suggests a possible Qusṭā-Thābit transmission on the basis of agreement between Fatih 3414's witness and al-Ṭūsī's reports about the Thābit version.[72] Qusṭā's version is also attested in the Hebrew translation of the Arabic: Cambridge Add. 1220 and Bodleian Laud 93 name Qusṭā in the title of the text.[73] Ibn Abī Uṣaybiʿa also attributes to him a work titled *On the figure of the sphere and the cylinder* (*Kitāb fī shakl al-kurah wa-l-usṭuwānah*).[74]

On the Measurement of the Circle[75]

The translator of this text is unclear. The 3rd/9th century Arabic of this version has not been edited, but English translations have been made of Fatih 3141's text by Knorr and also by van Lit.[76]

Sezgin suggests that Thābit translated the text, but neither the manuscripts nor al-Ṭūsī offer any indication of this.[77]

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[69] Cf. Steinschneider (1896a) 173-4 and Sezgin (1974) 128-9. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and Ibn al-Qifṭī (Lippert (1903) 67).
[70] Hyderabad (1939-40) *Kitāb al-kurah wa-al-usṭuwānah* 2. This preface will be examined further in chapter 9.
[71] Fatih 3414, fol. 9a.
[72] Lorch (1989) 96-7.
[73] Lorch (1989) 99.
[74] Ibn Abī Uṣaybiʿa chapter 10, biography 44.
[75] Cf. Steinschneider (1896a) 174-5 and Sezgin (1974) 130-1. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and al-Qifṭī p.67.
[76] Knorr (1989) and Van Lit (2012).
[77] Sezgin (1974) 131.

Al-Kindī wrote a commentary on this text.[78] He is reported to have corresponded with Ibn Māsawayh on the work, suggesting that *Measurement of the Circle* was available in Arabic before Ibn Māsawayh's death in 242/857. Rashed speculates that Qusṭā may have translated Archimedes's text, since he was attested for *On the Sphere and the Cylinder* and since al-Kindī revised several of his translations.[79] Outside scholarly speculation, however, the translator of this text remains unclear.

<u>Archimedean Lemmata</u>[80]

Thābit ibn Qurra is claimed as the translator. This work has been edited by Coşkun.[81]

Both al-Ṭūsī and MS Fatih 3414 identify the translator as Thābit.[82]

2.3 Eighth/Ninth Century Versions of the *Elements* and the *Almagest*

<u>Elements</u>[83]

Between the biobibliographical sources and the manuscript evidence, the individuals claimed as translators of the *Elements* in the 2nd-3rd/8th-9th centuries are al-Ḥajjāj b. Yūsuf b. Maṭar and Isḥāq ibn Ḥunayn.[84] Thābit ibn Qurra is named as the reviser of Isḥāq's translation.

[78] See discussion in Rashed (1993).

[79] Rashed (1993) 15-16.

[80] Cf. Steinschneider (1896a) 176-7 and Sezgin (1974) 131-3. Listed in *Fihrist* 7.2 (Dodge (1970) 636) and Ibn al-Qifṭī (Lippert (1903) 67).

[81] Coşkun (2018).

[82] For al-Ṭūsī's report, see Hyderabad (1939-40) *Kitāb Maʾkhūdhāt* 2: "ترجمة ثابت بن قرة وتفسير الأستاذ المختص ابى الحسن على بن أحمد النسوي". Thābit is named by Fatih 3414 on folio 68a.

[83] Cf. Steinschneider (1896a) 165 and Sezgin (1974) 103-4. Listed in *Fihrist* 7.2 (Dodge (1970) 634-5) and al-Qifṭī p.62.

[84] Disentangling the translations attributed to these individuals, however, is a complicated task. Brentjes (2018b) has shown that textual analysis of the manuscripts results in a picture that contradicts what is claimed about the translators in the manuscript titles and colophons. Analysis of books III-IX shows that – for these books, at least – the version that has been understood to be the Isḥāq/Thābit edition rather appears to be a misattribution of al-Ḥajjāj's work. She notes that it is possible, though uncertain, that books I, II, and perhaps X in the relevant manuscripts do belong to the Isḥāq/Thābit tradition: see Brentjes (2018b) 52. Thus the question of transmission is further complicated by manuscripts combining fragments of different versions into one text. These kinds of challenges should be kept in mind when considering Graeco-Arabic transmissions of texts more generally – they are likely not unique to the *Elements*.

Al-Ḥajjāj is reported to have translated the *Elements* twice: first for Caliph Hārūn al-Rashīd (r. 170-193 / 786-809) or his vizier Yaḥyā ibn Khālid al-Barmakī (d. 189 / 805), and a second time for Caliph al-Maʾmūn (r. 197-218 / 813-833), though there is disagreement in the sources regarding whether the second is a second translation or a revised edition of the first translation. Isḥāq ibn Ḥunayn's translation was produced later in the third / ninth century.[85]

Almagest[86]

Between the biobibliographical sources and the manuscripts, the translators claimed for the *Almagest* are al-Ḥasan ibn Quraysh, al-Ḥajjāj b. Yūsuf b. Maṭar, Sarjūn ibn Hiliyā, and Isḥāq ibn Ḥunayn. Isḥāq's translation was corrected by Thābit ibn Qurra.

Ibn al-Ṣalāḥ, in his work on Ptolemy's star catalogue, reports that there were five versions of the *Almagest* available in his time, four of which were translations – all of these would have been produced between the 2nd-3rd/8th-9th centuries.[87] The earliest was an anonymous Syriac translation. In the early 3rd/9th century al-Ḥasan ibn Quraysh composed a translation for Caliph al-Maʾmūn; only fragments of this translation survive.[88] A second rendition for al-Maʾmūn was completed by al-Ḥajjāj and Sarjūn ibn Hiliyā in 212/827-8; this is extant in the manuscript Leiden Or. 680 with both translators named. Isḥāq translated the text a fourth time in the second half of the 3rd/9th century and this was later corrected by Thābit. The Isḥāq-Thābit version is extant in its entirety in the manuscript Tunis Bibl. Nat. 07116.

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[85] An overview of the *Elements*'s transmission into Arabic is provided by Brentjes and De Young "Euclid" in EI3.
[86] Cf. Steinschneider (1896a) 200-2 and Sezgin (1978) 88-89. Listed in *Fihrist* 7.2 (Dodge (1970) 639) and Ibn al-Qifṭī (Lippert (1903) 97).
[87] Kunitzsch (1975) 155 (Arabic), 40 (German translation).
[88] Fragments of this translation survive in al-Battānī: see Kunitzsch (1974) 60-64. On the possible fragments of this translation in Ibn al-Ṣalāḥ's critique of al-Fārābī's commentary on the *Almagest*, see Thomann (2020).

Additionally, several historical sources between the tenth and fifteenth centuries mention a version of the *Almagest* solely attributed to Thābit (rather than it being Thābit's correction of Isḥāq's translation). The earliest of these is a report by Abū ʿAlī al-Muḥassin al-Ṣābiʾ (4th/10th century), preserved in al-Qifṭī. Al-Ṭūsī is another source: he mentions a version by Thābit in his own edition of the *Almagest*.[89] The possibility has been put forth that this was a separate translation produced by Thābit himself, but it alternatively may have been another adaption by the scholar of an existing translation.[90]

2.4 A Note on Syriac Translations[91]

This chapter has focused on translations into Arabic because these are what survived and persisted through the 7th/13th century. Syriac translations of these texts are not extant, but there is historical evidence to suggest that at least some of them existed by the 3rd/9th century at the latest. Nothing is known about the Syriac translators or the context of these translations, whether they occurred during the ninth century or were in circulation already before it.

The texts which are reported to have had a Syriac translation by the 3rd/9th century are Euclid's *Phaenomena*, Menelaus's *Spherics*, Archimedes's *On the Sphere and Cylinder*, and Ptolemy's *Almagest*.

[89] See Grupe (2012) 149-151 for an overview of these sources and further.

[90] Grupe (2012) and Grupe (2020) argues that there is reason to believe this version was Thābit's own translation. Compare Kunitzsch (1974) 25-34, which sees this evidence as more likely pointing to a second adaption (but not translation) by Thābit.

[91] For a general overview of the mathematical sciences in Syriac, see Takahashi (2011) and Hugonnard-Roche (2014). For an overview of the astronomical sciences in Syriac in the 6th and 7th centuries, see Takahashi (2014) and Villey (2014). Villey highlights several ancient and late antique Greek treatises which can be seen to have received study in the West Syriac monastery, Qenneshre, among them commentaries by Theon and the *Handy Tables* of Ptolemy. But evidence of the works which comprised the Little Astronomy have not been found in this context. The *Almagest*, meanwhile, appears to have been known by reputation but may not have been transmitted in Qenneshre before the eighth century: see Villey (2014) 173. Hugonnard-Roche comments on the evidence for Syriac translations of the *Sphere and Cylinder*, the *Spherics*, and the *Almagest* in the context of the early ʿAbbāsid translation movement: see Hugonnard-Roche (2014) 75-77 – as has been noted, however, the *Sphere and Cylinder* and the *Spherics* were transmitted separately from the Little Astronomy and/or the Middle Books in this period.

There is also an extant fragment of Euclid's *Elements* in Syriac, but scholars disagree on whether this was an early translation from the Greek or a late one from the Arabic.[92]

The reference to a Syriac version of the *Phaenomena* appears in the text of the Arabic translation. After the tenth proposition, the text offers an alternate proof of that proposition, introduced with "Proof of the tenth figure according to what we found in another copy."[93] What follows is the proof of proposition 10 corresponding to the Greek recension A of the *Phaenomena* (the Arabic translation otherwise follows recension B). At the end of the exposition of that proposition there is an aside saying, "it is found in the Syriac."[94] It should be noted that there were further propositions than this one which diverged in proof between recensions A and B of the *Phaenomena*, but this is the only portion of the Arabic following recension A. So if recension A existed in its entirety in Syriac, the Arabic translation was evidently not interested in presenting every alternate proof. Alternatively, this might suggest that the Syriac translation already presented a text that was a melding of Greek recensions A and B. In either case, what is clear is that the *Phaenomena* existed in some form in Syriac.

Scholars disagree on whether there was a Syriac translation of Menelaus's *Spherics* – the report is a later one and ambiguous. Ibn al-Qifṭī records in the biographical entry for Menelaus that "his books were once translated into Syriac, then into Arabic."[95] Krause (1936), apparently following Ibn al-Qifṭī, conjectured that the first translation of the *Spherics* into Arabic was made from a Syriac translation. Rashed and Papadopoulos (2017) have argued that outside Ibn al-Qifṭī there is nothing to support this idea, and they cast doubt on the report in the *History of Learned Men* because no other biographical

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[92] This fragment has been published by Furlani (1924). It is preserved in Cambridge University Library Gg 2.14, fols. 355-363. See Hugonnard-Roche (2014) 80-83 for discussion of its dating problems.
[93] Leiden Or. 1031, fol. 86a: "برهان الشكل العاشر على ما وجدنا في نسخة أخرى"
[94] Leiden Or. 1031, fol. 86a: "وجد في السرياني"
[95] Trans. Rashed and Papadopoulos (2017) 12. See Lippert (1903) 321: "وخرجت كتبه مرة إلى السرياني ثم إلى العربي"

source supports it.[96] Instead they consider the anonymous early translation into Arabic to be the work of a translator inexperienced with the language, and suggest his first language was Syriac.[97]

Conversely Hogendijk saw evidence for a Syriac translation of the *Spherics* in Ibn Hūd's *Book of Perfection*, portions of which draw from an Arabic version of the *Spherics*. Hogendijk's argument sees the early usage of the letters ط ي ق for mathematical labels as suggesting that the source *Spherics* text had seen a transmission through the Syriac.[98] Sidoli and Kusuba later found that the witness of Menelaus's *Spherics* in the manuscript London, British Library Or. 13127 has a second set of diagrams for the text whose labels shows the same feature. These diagrams are labelled in the manuscript as "Figures of the Treatise of Spheres by Menelaus, transcribed from a copy that was not corrected, but was translated based on the first composition."[99] They note, however, that this evidence does not necessarily indicate that a Syriac translation was already in circulation prior to the Arabic one, since there were many Syriac translations produced alongside Arabic ones as part of the early translation efforts.[100] In any case, even if the *Spherics* had been translated into Syriac, it is difficult to judge what this might suggest about awareness of an astronomical curriculum like the Little Astronomy / Middle Books in Syriac, since Menelaus's work appears to not have been included in the Little Astronomy and it is unclear when it was added to the Middle Books.

The same uncertainty is present for Archimedes' *On the Sphere and Cylinder*, which was not a member of the Little Astronomy but was added to the Middle Books at an unknown date. The existence of

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[96] Rashed and Papadopoulos (2017) 12-13.
[97] Rashed and Papadopoulos (2017) 486.
[98] Hogendijk (1996) 26.
[99] Trans. Sidoli and Kusuba (2014) 159. See British Library Or. 13127, fol. 52a: "اشكال كتاب الاكر لمنالاوس نقل من نسخة لم يصلح بل ترجمت على الوضع الاول"
[100] Sidoli and Kusuba (2014) 192, following Gutas (1998) 20-22 on the Arabic and Syriac translations in the early ʿAbbāsid translation movement..

a Syriac translation for this work however is less ambiguous. The manuscript Fatih 3414 contains an Arabic translation of *On the Sphere and Cylinder*, and it starts this text with two different versions of its introduction. A colophon after the first version includes a quotation reading, "I found in the copy that the translator of this book from Greek into Syriac mentioned that he omitted in this place a small passage which he did not translate from the Greek book because of its difficulty for him."[101] Nothing more is known about this Syriac translation of Archimedes.

Since Arabic sources claim that the Middle Books were supposed to be read between the *Elements* and the *Almagest*, it is worthwhile to note the Syriac translations of both these texts. As mentioned, the date and source of the *Elements* translation is uncertain. It is usually considered to be a translation from the Arabic made in the thirteenth century, but others suggest it was an early translation from the Greek that influenced the subsequent Arabic translations.[102]

No trace of the Syriac translation of the *Almagest* is extant, but its existence is reported by several sources. This translation was evidently still extant in the twelfth century, because Ibn al-Ṣalāḥ uses it as part of his work on Ptolemy's star catalogue – he reports that it was translated from the Greek.[103] The translation may even have survived to be used by al-Ṭūsī: in a manuscript containing his edition of the *Almagest*, a marginal comment present is reported to have been copied from a Syriac version.[104]

These reports are sparse, limiting what conclusions can be drawn beyond an awareness that there was more to the transmission of texts relevant to the Little Astronomy / Middle Books than has survived.

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[101] Trans. Lorch (1989) 109. See Fatih 3414, fol.7a: "وجدنا في النسخة المترجم لهذا الكتاب من اليوناني الى السرياني ذكر أنه قد خلف في هذا المكان معنى يسيرا لم ينقله من الكتاب اليوناني بصعوبته عليه"

[102] The former suggestion appears in Furlani (1924) 233; scholars who follow it include Saliba (2004) 29 and Takahashi (2013) 85. Baudoux (1935) 73-75 examines the latter possibility, arguing that it was used in the production of the Isḥāq-Thābit Arabic version. Sezgin (1974) 88-90 offers a summary of the various arguments.

[103] Kunitzsch (1975) 155 (Arabic), 40 (German translation).

[104] Saliba (1987) 10.

It is significant that many of the individuals reported as Arabic translators above were trilingual in Syriac, Arabic, and Greek. The 3rd/9th century reception of these ancient Greek texts was not a wholly Arabic reception. Further, as attestations from Ibn al-Ṣalāḥ and al-Ṭūsī suggest, some Syriac translations continued to be used in parallel with Arabic copies even centuries after the translation movement brought these texts into the latter language.

3. Translators, Correctors, and Patrons

This section will elaborate upon the individuals named above. It will offer biographical and bibliographical details relevant to understanding their scholarship in the 3rd/9th century, especially in connection with the works that came to comprise the Middle Books. Each subsection will also note what sources are available for further biographical and bibliographical details.

The historical biobibliographical sources used are the following. The earliest is the *Risāla* of Ḥunayn ibn Isḥāq, which first appeared in 241/855-6 and in a revised form in 250/864.[105] The second is the *Fihrist* of Ibn al-Nadīm, a catalogue of Arabic literature and translated works completed in 377/987-8.[106] Both of the following biobibliographers depended on the *Fihrist*, although they incorporated material from other sources as well.[107] Ibn al-Qifṭī (d. 646/1248) was the author of the *History of Learned Men* (*Taʾrīkh al-ḥukamāʾ*). This is a considerably later resource, and its original does not survive: rather, what

[105] Edited by Bergsträsser (1925), later edited and translated into English by Lamoreaux (2016). While its transmission has been complicated by two different recensions and a variety of scribal additions after Ḥunayn's death, it still remains an important contemporary resource for Ḥunayn and Isḥāq, and for detailed information about their medical translations and patrons.

[106] Edited by Flügel (1872) and translated into English by Dodge (1970), though there are issues with the currently available editions, as noted in Stewart (2006) 10-11. While not contemporary with the individuals discussed here, the catalogue was within a century of their lifetimes and benefited greatly from Ibn al-Nadīm's experience with the manuscripts that passed through his father's bookshop. The author does not provide sources for the biographical reports he presents, but frequently mentions an authority or details he read in a manuscript when discussing translated treatises.

[107] Dodge (1970) xxiii.

is extant is an epitome of the text by al-Zawzanī that was written in 647/1249.[108] Ibn Abī Uṣaybiʿa (d. 668/1269-70) is a similarly late source. His *History of Physicians* (*Uyūn al-anbāʾ fī ṭabaqāt al-aṭibbāʾ*) nonetheless draws on earlier sources to provide a massive amount of information on medical scholars.[109]

Qusṭā ibn Lūqā (d. c. 308/920)[110]

The Melkite Christian Qusṭā ibn Lūqā appears most frequently: as noted, his name is recorded with five or six of the nine Little Astronomy texts as well as two of the five texts added to the Middle Books. He receives entries in the biobibliographies of Ibn al-Nadīm, Ibn al-Qifṭī, and Ibn Abī Uṣaybiʿa.[111] A native of Baʿlabakk, he is reported by Ibn al-Qifṭī to have travelled the Byzantine Empire and returned with manuscripts he acquired there. Much of his scholarly career occurred in Baghdad, though he spent the end of his life until his death in Armenia. He was experienced in Greek, Arabic, and Syriac. Together, the three biobibliographers present a list of translations and original works that span a range of subjects: medicine, mathematics, astronomy, and philosophy. Qusṭā is described as a physician as well as an author, translator, and reviser.

It is known from the biobibliographical sources that Qusṭā composed and translated many works on the patronage of officials working for the ʿAbbāsid caliphs. Further, as noted above, Qusṭā's (partial) translation of the *Sphaerica* is reported to have been for Abū l-ʿAbbās Aḥmad ibn [Muḥammad ibn]

[108] Edited by Lippert (1903). See also Dietrich "Ibn al-Ḳifṭī" EI2.
[109] Edited and translated into English by Savage-Smith, Swain, and van Gelder (2020). The individuals discussed in this chapter were frequently involved in both medical and astronomical scholarship, as we as other topics.
[110] See Gabrieli (1912) for an overview of Qusṭā ibn Lūqā's life and works. A short biography by Hill is also available in EI2: "Ḳusṭā b. Lūḳā."
[111] See Ibn al-Nadīm (1988) 295, Ibn al-Qifṭī (Lippert (1903) 262), and Ibn Abī Uṣaybiʿa (2020) chapter 10, biography 44.

Muʿtaṣim bi-llāh. Qusṭā also dedicated Heron's *Mechanics* to a similarly named Aḥmad ibn [Muḥammad ibn] Muʿtaṣim.[112] This individual will be discussed in the following subsection.

The historical evidence preserves indirect connections between Qusṭā and other individuals discussed in this chapter. The only direct connection is between Qusṭā and ʿAlī ibn Yaḥyā: according to Ibn Abī Uṣaybiʿa, Qusṭā composed a work titled *Introduction to geometry in question and answer format* (*Kitāb fī l-Mudkhal ilā ʿilm al-handasah ʿalā ṭarīq al-masʾalah wa-l-jawāb*) for him.[113] Qusṭā therefore shared ʿAlī ibn Yaḥyā as a patron with Ḥunayn, Isḥāq, ʿĪsā, and Thābit.

Qusṭā may additionally have shared a second patron with Thābit: he is reported to have composed *On the use of the celestial globe* (*Kitāb fī l-ʿAmal bi-l-kurah al-nujūmiyyah*) for Ismāʿīl ibn Bulbul.[114]

Thābit and al-Kindī are both said to have been responsible for correcting several of Qusṭā's translations – it is unclear whether they would have directly interacted with Qusṭā as part of this process.

Aḥmad ibn [Muḥammad ibn] Muʿtaṣim (?)

As noted, the name of this patron of Qusṭā's which is found in the medieval sources is problematic. There was an Aḥmad ibn Muʿtaṣim, the son of the caliph al-Muʿtaṣim, who is also known to have been tutored by al-Kindī. There was also the grandson of this caliph, Abū l-ʿAbbās Aḥmad ibn Muḥammad ibn Muʿtaṣim, who himself ruled as Caliph al-Mustaʿin (r. 248-251 / 862-866). The name given for Qusṭā's patron blends the names of the son Aḥmad and the grandson Abū l-ʿAbbās Aḥmad. Kunitzsch and Lorch take the patron of the *Sphaerica* to have been the grandson, but do not indicate why they see this as the more likely interpretation.[115] Without further evidence, it is difficult to pin down which

112 Kheirandish (2013) 95.
113 Ibn Abī Uṣaybiʿa (2020) 10.44.5.
114 Gabrieli (1912) 348-9.
115 Kunitzsch and Lorch (2010b) 2-3.

individual is meant by these attestations of Qusṭā's patron – and, if the patron *was* Caliph al-Mustaʿin, whether this patronage occurred before or during his reign as caliph.[116]

Isḥāq ibn Ḥunayn (d. 298/910-11)[117]

The other key figure in translating this astronomical corpus is Isḥāq ibn Ḥunayn, the son of East Syrian Christian and renowned translator Ḥunayn ibn Isḥāq. His biography is reported in Ibn al-Nadīm,[118] al-Bayhaqī,[119] Ibn al-Qifṭī,[120] Ibn Abī Uṣaybiʿa,[121] and Ibn Khallikān.[122] The information presented is ultimately brief. All sources recognize him as the son of the famous Ḥunayn ibn Isḥāq and note his accomplished skill in languages: Greek Syriac, and Arabic. Both Ibn Abī Uṣaybiʿa and Ibn Khallikān make a point of mentioning that Isḥāq was more productive in his translations and commentaries of Aristotle than he was in his work on medical material. According to al-Nadīm, Ibn al-Qifṭī and Ibn Abī Uṣaybiʿa he served the same patrons as his father.

As noted above, Isḥāq shared ʿAlī ibn Yaḥyā as a patron with Qusṭā, ʿĪsā, his father, and Thābit. Additionally, several of the treatises translated by Isḥāq later saw revision by Thābit.

[116] Gutas argues that these references should be interpreted as indicating the son, not the grandson: see Gutas (1998) 125-126. Knorr (1986) 233, n.7 thought that even Caliph al-Mustaʿin was too early to reasonably be the patron of Qusṭā. There is further discussion of the problem in Kheirandish (2006) 216-221. Both son and grandson were attested to be involved in the sciences. Aḥmad ibn Muʿtaṣim was a patron of al-Kindī, meanwhile, Caliph al-Mustaʿin was associated with figures like Muḥammad ibn Mūsā and Abū Maʿshar. If Caliph al-Mustaʿin is intended in these references, it is curious that his regnal name is not used, though perhaps if his patronage occurred before his reign as caliph it would be omitted. Ultimately, the presently available sources do little to clear up the uncertainty.
[117] A short modern biography is available by Strohmaier in EI3: "Isḥāq b. Ḥunayn."
[118] al-Nadīm (1988) 343 and 356. Dodge (1970) 672 notes that the first account is omitted from Istanbul, Köprülü Library MS 1135.
[119] al-Bayhaqī (1932/3) 4-5.
[120] Ibn al-Qifṭī (2005) 65.
[121] Ibn Abī Uṣaybiʿa (1995) 201.
[122] Ibn Khallikān (1978) 205-206.

Isḥāq shares a further direct connection with Thābit: there survives a fragment of a letter sent to him by the latter on astronomical topics, asking for Isḥāq to send particular observations if he is able to find them.[123] Evidently the two at the very least corresponded about their astronomical work, though this might even point towards collaboration.

Ḥunayn ibn Isḥāq (d. 260/873)[124]

Ḥunayn ibn Isḥāq is the most well-known of the translators during the translation movement, and is renowned for his skills in Greek, Syriac, and Arabic. He is more commonly known for his work in the field of medicine, so it is curious that he is named with two or three of the Little Astronomy texts as well as Menelaus's *Spherics*.

It is quite possible that the attributions to Ḥunayn noted in this chapter are mistaken. Even Ibn al-Qifṭī commented on the tendency for others' translations to be attributed to the famous Ḥunayn.[125] In the case of *On the Moving Sphere* and Menelaus's *Spherics*, the other attested translator is his son Isḥāq. Reversing the name Isḥāq ibn Ḥunayn is an error that is even easier to make when the name Ḥunayn ibn Isḥāq is so well known.[126] The claim of Ḥunayn for Theodosius's *Sphaerica* is more difficult to explain, since there are no extant sources that claim Isḥāq to have been responsible for that text. It is given only by one source (Leiden Or. 1031), so it may simply be an error.

123 Carmody (1960) 45-46.
124 See the biography in Lamoreaux (2016) xii-xviii. A short modern biography is also available by Strohmaier in EI3: "Ḥunayn b. Isḥāq."
125 Lippert (1903) 177.
126 Further, there is the example of a manuscript of Aristotle's *Physics* which claimed its translation to be the work of Ḥunayn for "the vizier al-Qāsim ibn 'Ubaydallāh." As Gutas (1998) 131 fn.31 points out, Ḥunayn must be an error for Isḥāq because al-Qāsim became vizier only after Ḥunayn's death.

Outside his son, the only direct connection Ḥunayn has with the individuals discussed in this chapter is with his patron ʿAlī ibn Yaḥyā. He does mention Thābit in the *Risāla* as a translator of Galenic texts, but it is unclear whether they would have had any direct interaction.[127]

Hiliyā ibn Sarjūn (Elias, son of Sergius)

This individual is largely unknown. It is unclear whether the two instances of Hiliyā ibn Sarjūn and one instance of Sarjūn ibn Hiliyā should be taken as one individual or as father and son.

Outside the *Optics*, Hiliyā is attested as a translator of Cassianus Bassus's *Geoponika*.[128] Sarjūn is named in the title of Leiden Or. 680 as a co-translator with al-Ḥajjāj.[129] Ibn al-Ṣalāḥ agrees in naming both as co-translators of the *Almagest*.[130] His only known connection to other individuals in this chapter, therefore, is with al-Ḥajjāj.

ʿĪsā b. Yaḥyā (3rd / 9th century)

This was one of the students of Ḥunayn; he receives multiple mentions in the latter's *Risāla*. Ḥunayn translated several Galenic works into Syriac for him and reports that ʿĪsā made several translations into Arabic and at least one into Syriac.[131] He certainly operated within Ḥunayn's circle and sometimes shared a patron in Abū l-Ḥasan ʿAlī ibn Yaḥyā. He would seem to have been another one of the translators trilingual in Greek, Syriac, and Arabic.

[127] Lamoreaux (2016) 84-85 and 114-115.
[128] Kheirandish (1999) v.1 xix and v.2 xxii fn.20-21.
[129] Leiden Or. 680, fol.2b.
[130] Kunitzsch (1975) 15.
[131] Lamoreaux (2016) 143.

Abū l-Ḥasan ʿAlī ibn Yaḥyā (d. 275/888-9)[132]

This individual is a member of the Banū l-Munajjim, a family with noted interest in the ancient sciences (especially the astral sciences) and involvement with the ʿAbbāsid courts.[133] He is commonly known as a patron of translators – Ibn Abī Uṣaybiʿa characterizes him as such.[134]

ʿAlī ibn Yaḥyā has the most direct connections with translators in this chapter because he often patronized the translation and composition of various scientific works. We have already seen that the *Phaenomena* is reported to have been translated for him by ʿĪsā b. Yaḥyā. We also find that for him Qusṭā wrote a work on geometry and Thābit wrote a work on music. Ḥunayn wrote the Arabic *Risala* for him as well as translated numerous Galenic works. Isḥāq translated two Galenic works for him as well.[135]

Abū ʿUthmān al-Dimashqī (d. after 302/914)[136]

Al-Dimashqī was a physician and a translator of Greek scientific texts into Arabic well regarded for his translation style,[137] though not as highly renowned as Ḥunayn.[138]

The *Fihrist* attributes to him a partial translation of Euclid's *Elements* and manuscript glosses note his translation of the *Spherics*. He otherwise has no known contributions to texts related to the Middle Books, and no known connections to the other individuals discussed in this chapter.

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[132] See the overview of the Banū l-Munajjim family in EI3 by Berggren: "al-Munajjim, Banū."
[133] Dodge (1970) 313.
[134] Ibn Abī Uṣaybiʿa (2020) chapter 9, biography 41.
[135] Lamoreaux (2016) 92 fn.2 and 108 fn.2.
[136] A short modern biography is available by Endress in EI3: "Abū ʿUthmān al-Dimashqī."
[137] See for example the report from Miskawayh on al-Dimashqī (Trans. Rashed and Papadopoulos (2017) 20-21): "Thus, it is in these terms that the Philosopher (Aristotle) expressed himself. I have transcribed them according to the translation of Abū 'Uthmān al-Dimashqī. The latter uses fluently both languages, Greek and Arabic, and all those who know these two languages appreciate his way of translating. Furthermore, he brings out a strict requirement in rendering the Greek words and notions by their rigorously exact pendant in Arabic."
[138] Ibn Abī Uṣaybiʿa (2020) chapter 8, biography 29: "Works that were translated by some other translator, such as Usṭāth, Ibn Baks, al-Biṭrīq, Abū Saʿīd ʿUthmān al-Dimashqī, or others, are less highly prized and are deemed less desirable than those that were translated or revised by Ḥunayn."

Thābit ibn Qurra (d. 288/901)[139]

Thābit ibn Qurra is as involved as Qusṭā ibn Lūqā with the works discussed in this chapter. As noted above, the combination of manuscript titles and colophons along with other reports suggest that he revised six out of the nine Little Astronomy texts, a further two of the additions, and both the *Elements* and the *Almagest*. He furthermore is attested as a translator of *On the Moving Sphere* and of the pseudo-Archimedean *Lemmata*. He receives entries in the biobibliographies of Ibn al-Nadīm, Ibn al-Qifṭī, and Ibn Abī Uṣaybiʿa and is mentioned in the *Risāla*.[140] The biographies agree that he was a member of the Sabian religious sect and originally from Ḥarrān; after meeting the eldest of the Banū Mūsā, Abū Jaʿfar Muḥammad ibn Mūsā, he travelled with him back to Baghdad. He composed, translated, and revised scientific texts, and was trilingual in Syriac, Greek, and Arabic.

He shared three patrons with the other figures discussed here: for ʿAlī ibn Yaḥyā he composed a work on music and for Ismāʿīl ibn Bulbul he wrote a treatise on geometry. His patron Muḥammad ibn Mūsā additionally patronized translations from Ḥunayn (as he did from multiple other translators of the period).[141] However, while Ḥunayn comments on the Galenic translations produced by Thābit, it is unclear whether Ḥunayn and Thābit had much direct interaction.[142]

However, as mentioned, he did interact directly with Ḥunayn's son Isḥāq, as evidenced by the fragmentary letter on astronomical matters Isḥāq sent to him.[143]

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[139] A short modern biography is available by Rashed and Morelon in EI2: "Thābit b. Ḳurra." See also Rashed (2009) 15-24 for further biography.
[140] See *Fihrist* 7.2 (Dodge (1970) 647-648), Ibn al-Qifṭī (Lippert (1903) 115-122). and Ibn Abī Uṣaybiʿa (2020) 10.3.
[141] See Mimura (2020) for one investigation into what the relationship between Thābit and the Banū Mūsā may have looked like considering the reports about the Banū Mūsā's involvement in fostering and educating him.
[142] Lamoreaux (2016) 139.
[143] Carmody (1960) 45-46.

Al-Kindī (d. mid 3rd/9th c)[144]

Compared to several of the names above, al-Kindī appears less frequently in connection to the works under discussion. He is reported to have revised translations by Qusṭā, including that of the *Anaphoricus*. The treatise *On the Moving Sphere* also features among his corrections. He additionally wrote a short treatise titled *Correction of the Optics* (*Iṣlāḥ al-Manāẓir*) and a commentary on the *Measurement of the Circle*.[145]

He taught Aḥmad ibn Muʿtaṣim, the son of the caliph – there is extant for example a letter by him to Aḥmad on mathematics.[146] As discussed above, this may be the same patron who commissioned the *Sphaerica* from Qusṭā, but the identification is unclear.

al-Ḥajjāj b. Yūsuf b. Maṭar (fl. 169-218/786-833)[147]

Nothing known of his life besides his work as an early Arabic translator of the *Elements* and the *Almagest*. The only connection he has with other scholars involved in work on Middle Books texts is his collaboration on the *Almagest* with Sarjūn ibn Hiliyā.

4. From the Little Astronomy to the Middle Books

4.1 Collaboration in Translating?

The Little Astronomy existed as an ordered unit before the translation movement. What awareness did Arabic or Syriac scholars have of this? Was there any initiative or coordination among them to translate this corpus into Arabic in its entirety?

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[144] A short modern biography is available by Jolivet and Rashed in EI2: "al-Kindī."
[145] Al-Kindī's *Correction of the Optics* is edited and translated in Kheirandish (1999) 226-229.
[146] Rosenfeld and Ihsanoğlu (2003) 38, entry M4.
[147] A short modern biography is available by De Young and Brentjes in EI3: "al-Ḥajjāj b. Yūsuf b. Maṭar."

After all, the fact that the Little Astronomy was not translated wholly by one person should not preclude the possibility that its translators understood it as a unified curriculum. According to Ḥunayn's testimony in the *Risāla*, for instance, the Galenic medical curriculum was translated into Arabic through the joint efforts of Ḥunayn and Hubaysh ibn al-Ḥasan.[148] Another example can be found in the curriculum of the Aristotelian Organon, whose translators were even more varied according to the *Fihrist*. Ḥunayn and his son Isḥāq contributed to its translation into Arabic, but they were joined by other figures like Abū Bishr Matta and Yaḥyā ibn ʻAdi.[149]

There were certainly connections between the individuals summarized in the section above, especially those with a heavier involvement with the works examined in this chapter. ʿAlī ibn Yaḥyā stands as a key link: he served as patron for many of the translators and himself patronized the translation of the *Phaenomena*. With the exception of the *Optics*, the translation of all of the Little Astronomy works is covered by Qusṭā ibn Lūqā, Isḥāq ibn Ḥunayn, and ʿĪsā b. Yaḥyā – and ʿAlī b. Yaḥya was known to be a patron to all these men. It is tempting to suggest a coordinated effort, but while there would be overlap between ʿĪsā b. Yaḥyā, Ḥunayn's student, and Isḥāq, Ḥunayn's son, there is no extant evidence of Qusṭā working with them or in the same circles. As translators of medical, mathematical, and philosophical texts in this period and city, a shared patron is not surprising – so whether there was a unified effort between the three remains unfortunately unclear.

4.2 A Third / Ninth Century Middle Books

Regardless of whether there was a dedicated project to translate the Little Astronomy as a corpus, evidence in the Arabic strongly suggests that these works found use as a curriculum already in the ninth

148 Lamoreaux (2016) 8-39.
149 Dodge (1970) 598-602.

century and in the lifetimes of their translators. Significantly, Qusṭā ibn Lūqā, who received credit for so many of the translations above, is also credited with a work titled *Treatise on what Middle (Books) it is necessary to read before the Almagest* (*Risāla fī mā yajibu an yuqra'a min al-Mutawassiṭāt qabl al-Majisṭī*).[150] Unfortunately this work has not been found in any surviving manuscripts, but the title is already suggestive.

This is the only reference to "Middle Books" (al-Mutawassiṭāt) in the 3rd/9th century, but sources from the subsequent centuries confirm and explain the name al-Mutawassiṭāt: they are so named because they are the works that come between Euclid's *Elements* and Ptolemy's *Almagest*. The fifth / eleventh century Alī ibn Ahmad al-Nasawī describes it in those terms,[151] and later in the seventh / thirteenth century Ibn al-Qifṭī and al-Ṭūsī reiterate that the Middle Books stand between the *Elements* and the *Almagest*.[152] The latter specifically notes this as an "educational arrangement" (الترتيب التعليمي), a description which undoubtedly suggests the idea of a curriculum. The Middle Books' position is also implicit in an anecdote about Ibn al-Haytham which speaks of him copying out three codices over the course of a year for 150 Egyptian dinars: the *Elements*, al-Mutawassiṭāt, and the *Almagest*.[153]

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[150] Sezgin (1978) 182. This work is noted by al-Samawʾal b. Yaḥyā (6th/12th c) in his *Report on the Defects of the Astrologer*. See for example MS Leiden Or. 98, folio 43a.

[151] As reported by al-Ṭūsī. See Hyderabad (1939-40) *Kitāb Makhūdhāt* 2: "المتوسطات التى يلزم قراءتها فيما بين كتاب اقليدس والمجسطى"

[152] Lippert (1903): 108: "الكتب المتوسطات بين كتاب إقليدس والمجسطي"

[153] Lippert (1903) 167: "قال سمعت أن ابن الهيثم كان ينسخ في مدّة سنة ثلثة كتب في ضمن اشتغاله وهي أوقليدس والمتوسّطات والمجسطى ويستكملها في مدّة السنة فاذا شرع في نسخها جاءه من يعطيه فيهم مائه وخمسين دينارا مصريّة وصار ذلك كالرسم الذي لا يحتاج فيه إلى مواكسة ولا معاودة قول فيجعلها مؤنته لسنته"

Qusṭā ibn Lūqā's commentary is not the only evidence for a ninth century Middle Books. There is a note on the arrangement of books to be read after Euclid that appeared in a sixteenth century manuscript – significantly, this note was attributed to attributed to Isḥāq ibn Ḥunayn. It reads as follows:[154]

> "Arrangement of what is to be read after Euclid, found in a copy in the handwriting of Isḥāq ibn Ḥunayn. The *Optics* of Euclid, one book; the *Sphaerica* of Theodosius, three books; *On the Moving Sphere* of Autolycus, one book; the *Phaenomena* of Euclid, one book; *On Habitations* of Theodosius, one book; *On Risings and Settings* of Autolycus, two books; *On Nights and Days* of Theodosius, two books; the *Anaphoricus* of Hypsicles, one book; *On Sizes and Distances* of Aristarchus, one book. So these are thirteen books."

> "ترتيب ما يقرأ بعد إقليدس وحد في نسخة بخط إسحاق بن حنين
> كتاب المناظر لاقلودس مقالة واحدة الاكر لثاودوسيوس ثلاث مقالات الكرة المتحركة لاوطولوقس مقالة واحدة الظاهرات لاقليدس مقالة واحدة المعمورة[155] لثاودوسيوس مقالة واحدة الشروق والغروب لاوطولوقس مقالتان الليل والنهار لثاودسيوس مقالتان المطالع لابقلاوس[156] مقالة واحدة ابعاد الكواكب وعظامها لارسطوخس[157] مقالة واحدة
> فتلك ثلث عشرة مقالة هـ"

This Arabic witness is a late one, but it has an earlier counterpart in a fourteenth century Latin manuscript, Paris lat. 9335.[158] This Latin witness will be discussed further in chapter 7, but it presents the same material – an introductory statement about the order after Euclid, the same listing of works and book counts in the same order – with only small differences. The Latin version lacks the final statement summing up the list as thirteen books, and it attributes the list to "Johanicus," which scholars have previously interpreted to mean Ḥunayn ibn Isḥāq. It would seem that at some point in the transmission of

[154] The manuscript was Beirut MS St. Joseph University, BO 223A, which was lost during the Lebanese civil war. I thank Nathan Sidoli for bringing this manuscript and its surviving microfilm to my attention. The relevant Arabic occurs on p.64 of the microfilm. See also the discussion in Brentjes (2018a) 39-40.

[155] Note the title *al-Maʿmūra* here differs from the more commonly seen *Kitāb al-Masākin* (or variants thereof) for Theodosius's *On Habitations*. It does, however, agree with the phrasing "علم الأرض المعمورة" used in the Arabic translation of Galen's commentary to Hippocrates' *Airs, Waters, Places*, discussed in chapter 1.

[156] Hypsicles's name is more commonly transliterated "إبسقلاوس", but is missing the sīn here.

[157] This rendering of Aristarchus's name has mistaken the second rāʾ for a wāw (usually "أرسطرخس").

[158] This is a manuscript containing several of Gerard of Cremona's translations of Middle Books works (or translations by his school). See fol. 28v: the list of works begins, "Ordo qui est post librum Euclidis secundum quod invenitur in scriptis Iohanicii."

this list, then, Isḥāq was mistaken for his famous father, an error which was not uncommon. In any case, the similarities between these two lists are striking enough that they surely derived from a shared lost ancestor. And this ancestor is attributing to Isḥāq ibn Ḥunayn, the other major translator of the relevant works, an ordered listing of treatises to be read after (the *Elements* of) Euclid. Though the name al-Mutawassiṭāt does not appear in this note, it is clear enough that we are dealing with the same corpus. So this grouping of works is specifically understood to be an ordered one and to have a position after the *Elements* already in the lifetimes of its Arabic translators.[159]

This list also lends support to the idea that the *Optics* was already being grouped with the Little Astronomy when it was translated in the ninth century and so was included among the Middle Books from their start as well. Other works, like the *Data* of Euclid and the *Spherics* of Menelaus, are not present in this list and so presumably saw addition to the curriculum either at a later stage or under different authorities.[160]

Beyond the above, there is one other potential allusion to the Middle Books coming from a ninth century source. Although the term al-Mutawassiṭāt does not appear, al-Kindī's text *On the Great Art* lists

[159] Comparing the order declared in the two witnesses to this list with the Greek order seen in Vat. gr. 204 does reveal some differences. The *Optics* appears at the start of the Arabic list, and the final four works appear in a different order. The instability of the *Optics* may lend support to the idea that it was a later addition to the Little Astronomy, and, being outside the subject of spherics, was introduced at various early points in the curriculum before its turn to the particulars of astronomy. The reshuffling of *On Nights and Days*, *On Sizes and Distances*, *On Risings and Settings*, and the *Anaphoricus*, meanwhile, might indicate that the order of this subgroup – all more particular astronomical treatises, without dependencies among themselves – was not as solidified as the progression from more general to more particular treatises seen at the start of the curriculum.

[160] The fact that this list also explicitly notes the collection as comprising thirteen books is additionally worthy of note, considering how the *Elements* and the *Almagest* themselves comprise thirteen books each. It may have been an intentional decision (at some point in either the later transmission of the Little Astronomy or the establishment of the Middle Books) to have this curriculum proceed through the same number of books as the *Elements* before it and the *Almagest* after it.

nine works that would serve as useful preliminaries to the *Almagest*.[161] The first five of these works are noteworthy considering the texts which were known to comprise the Middle Books. Clearest are the third, fourth, and fifth on the list: *Book on the Motion of the Sphere* (*Kitāb fī Ḥarakat al-kura*), *Book on Habitations* (*Kitāb fī l-Masākin*), and *Book on Optics* (*Kitāb fī l-Manāẓir*). These appear to be direct references to *On the Moving Sphere*, *On Habitations*, and the *Optics* respectively.[162]

The second work on the list is given a lengthy title: *On the Sphere and the Solids and Immediate Primary Plain [Figures] the Knowledge of which is connected with that of the Sphere*.[163] Rosenthal suggests that this might be identical with a later mentioned *Book on the Sphere* (*Kitāb fī l-ukar*).[164] This bears some similarity to the *Sphaerica* of Theodosius: if not the translation itself, perhaps it is a related work.

The title of the first work on the list is corrupted: Rosenthal reads the unknown *Kitāb fī l-ʿlmʾt*. He raises the possibility that this once referred to a work of al-Kindī's recorded in the *Fihrist*: *On the Data* (*Risāla fī l-muʿṭayāt*).[165] This would seemingly bear some relation to Euclid's *Data*, which did come to stand at the head of the Middle Books.[166]

After listing these first five works, al-Kindī notes that "these all are arranged after the Book of *Ustuquṣṣāt* in geometry" (ومرتبة هذه جميعا بعد كتاب الأستقصات في المساحة).[167] This is likely Euclid's *Elements*, and the word al-Ustuquṣṣāt a transliteration its Greek title, the Στοιχεῖα. So here al-Kindī presents an

[161] Here al-Al-Kindī seems to be responding to a comment in the beginning of the *Almagest*, where Ptolemy notes that his work expects a reader who has already made progress in astronomy. See Heiberg (1898) 8.
[162] Rosenthal (1956) 441. Titles can vary in the Arabic, but these are more commonly named as *Kitāb al-Kura al-Mutaḥarrika*, *Kitāb al-Masākin*, and *Kitāb al-Manāẓir*.
[163] *Kitāb fī l-kurah wa-mā ttaṣal ʿilmuhū bi-ʿilmihā min al-mujassamāt wa-awāʾil qarībah min al-basīṭat*
[164] Rosenthal (1956) 440-441.
[165] Rosenthal (1956) 440 and 443.
[166] The title of Euclid's *Data* in Arabic was usually *Kitāb fī l-muʿṭayāt*.
[167] Rosenthal (1956) 441.

arrangement that begins with the *Elements*, possibly proceeds through the *Data* and the *Sphaerica* (or texts by al-Kindī related to them), more certainly proceeds through *On the Moving Sphere*, *On Habitations*, and the *Optics*, all for the purpose of preparing a reader for Ptolemy's *Almagest*. The works which follow afterwards on al-Kindī's list diverge to arithmetic, algebra, and Indian numerals, but the first five works stand out as separate grouping and a close match for the Middle Books in both title and arrangement.

Together therefore, Qusṭā ibn Lūqā, Isḥāq ibn Ḥunayn, and al-Kindī show that the Middle Books served astronomical learning very swiftly after its component works' translations into Arabic.

A hint might also be found in the work of al-Yaʿqūbī (d. 284/897-8): part of his *History* (*Taʾrīkh*) takes the time to describe various ancient Greek philosophers and scholars, and in his description of Euclid and his *Elements* he does not describe the *Elements* simply as work of geometry, but as the "key to the science of the book *Almagest* on the stars."[168] While al-Yaʿqūbī makes no mention of the Middle Books, it is notable that already in the ninth century there was a conception of certain ancient Greek texts – here, the *Elements* – as preparing one for the *Almagest*.

[168] Translation from al-Yaʿqūbī (2018) 398. Al-Yaʿqūbī goes on to summarize the thirteen books of the *Elements*; he also describes Euclid's *Optics* (p.402) and Ptolemy's *Almagest* (p.416ff).

Chapter 4

A Comparison of Greek and Arabic Versions

1. Introduction

The prior chapter showed that not only did the grouping of Little Astronomy texts end up translated into Arabic in the ninth century, but that already in that century they were being grouped together under the name Middle Books and as preparation for the study of the *Almagest* and/or as works to be read after the *Elements*. The collection's subsequent transmission saw the occasional addition of other works and the more consistent addition of Euclid's *Data*, as will be seen in chapter 5. The present chapter will examine what deliberate alterations might be identified among the ninth century Arabic translations and corrections of the core grouping of nine texts, and it will add the *Data* to its examination as well.

Of course, variations at this stage could be introduced through a number of avenues. The Arabic texts might depend on a Greek version different from the one(s) which survived. The Arabic translations might have drawn from multiple exemplars and reflect material from each. The translators may have misinterpreted the Greek or otherwise unintentionally rendered it with a different sense. The Arabic text surviving today may present not the original translation, but a deliberately corrected version or other recension, often the work of a different scholar. And, of course, the versions surviving today may be shaped by any number of accidental variations or intentional attempts at correction introduced in the copying process.

This chapter endeavors to highlight variations that were introduced into the Arabic versions early in their circulation. It avoids editions known to have been produced by later scholars and focuses on sources that purport to transmit the translations and corrections from the third / ninth century. However, since none of the extant manuscripts date from that period (most are instead four centuries later), there will inevitably be some alterations introduced into these witnesses as part of the subsequent transmission. So, even when seeking after more deliberate alterations in these ninth century texts, what we find might be shaped by a variety of different processes, and disentangling these will not always be possible. Nevertheless, this chapter will delve into the plausibly deliberate alterations that can be found between the Greek and the ninth century Arabic versions.

2. Overview of Evidence

Like chapter 2, the focus of this chapter remains the following nine works: the *Sphaerica*, *On the Moving Sphere*, the *Optics*, the *Phaenomena*, *On Habitations*, *On Days and Nights*, *On Sizes and Distances*, *On Risings and Settings*, and the *Anaphoricus*. The *Data* is added as a tenth core member of the Middle Books.[1] Of these ten works, five have had editions produced for their Arabic translations: the *Data*, the *Sphaerica*, the *Optics*, *On Habitations*, and the *Anaphoricus*.[2] *On Days and Nights* has a partial edition (proposition I.1 in full, then enunciations only until propositions II.20 and II.21) and the translations of *On the Moving Sphere* and *On Sizes and Distances* have been studied.[3] This leaves only the translations of Euclid's *Phaenomena* and Autolycus's *On Risings and Settings* both fully unedited and unstudied.

[1] Several works by Archimedes also came to be added to the Middle Books, but their inclusion occurred after the ninth century examined in this chapter – it is the tenth century al-Nasawi who speaks of an already extant Middle Books curriculum to which his contemporaries sometimes add Archimedes.

[2] The Arabic *Data* is edited in Sidoli and Isahaya (2018); the *Sphaerica* in Kunitzsch and Lorch (2010b) and its witness in Seray Ahmet III 3464 in Martin (1975); the *Optics* in Kheirandish (1999); *On Habitations* in Kunitzsch and Lorch (2010a); and the *Anaphoricus* in De Falco, Krause, and Neugebauer (1966).

[3] For *On Days and Nights*, see Kunitzsch and Lorch (2011). For *On the Moving Sphere*, see Nikfahm-Khubravan and Eshera (2019), who broadly discuss the several versions of the text with a focused study on proposition 7. For *On Sizes and Distances*, see Berggren and Sidoli (2007), who discuss agreements and disagreements between the Greek text, the Arabic witness in the Kraus manuscript, and al-Tusi's edition.

The translation of the *Phaenomena* is preserved in the manuscripts Seray Ahmet III 3464 and Leiden or. 1031 – the present study considers Leiden or. 1031's witness to the text. The translation of *On Risings and Settings* is preserved in Seray Ahmet III 3464, the Kraus MS, and Leiden Or. 1031 – again, the latter witness is the one examined here.

Manuscript witnesses have additionally been consulted for each of the works which have priorly been studied but which lack editions. *On Days and Nights* is preserved in Seray Ahmet III 3464, the Kraus MS, and Bodleian Or. 365 (Book I only). The partial edition was produced using the first two witnesses. The latter manuscript, being the one available, is consulted here. Since this is an inferior and incomplete witness to the text, and the full text was not otherwise available for this study, comments on *On Days and Nights* will be limited in this chapter. *On the Moving Sphere* is preserved in several manuscripts including Seray Ahmet III 3464, the Kraus manuscript, and Bodleian Huntington 237; Bodleian Huntington 237 is considered in this chapter.[4] *On Sizes and Distances* is preserved in the Kraus MS and Columbia Or. 45; Columbia Or. 45 receives focus in the present chapter.

3. Summary of Deliberate Alterations in the Arabic Translations

3.1 Concordances of Propositions[5]

One general impression of the relationship between the Greek and Arabic versions of these texts can be gleaned through concordances of their propositions. It must be noted that there is some variety

[4] These codices contain what Nikfahm-Khubravan and Eshera call version I of *On the Moving Sphere*; this version also appears in MS Tehran, Dānishgāh-i Tihrān, Kitābkhāna-yi Markazī 1063. They report that version II is represented in MS Istanbul, Köprülü Kütüphanesi, Fazıl Ahmed Paşa 932 and version III in MS Istanbul, Süleymaniye Kütüphanesi, Ayasofya 2671 and MS London, Institute of Ismaili Studies, Hamdani Collection 1647. See Nikhfahm-Khubravan and Eshera (2019) 44-48 for a description of manuscripts for each of the versions.

[5] Note: definitions are included and numbered in the following tables to show their alignment, but the reader should be aware that the Arabic manuscripts do not actually number the definitions in any way.

between proposition numbering in Arabic manuscripts.[6] For this reason, this section will include the testimony from several sources. It will also, in the footnotes, establish which proposition numbering scheme will be the default for this chapter's study (unless indicated otherwise, for e.g. examinations of particular manuscripts).

Euclid's *Data*[7]

The *Data* as edited by Menge comprised 94 propositions. Its translation in the manuscript Seray Ahmet III 3464 contains 91 propositions.[8] In the Kraus manuscript[9] and in the list of Middle Books provided by Bodleian Thurston 11,[10] it contains 95 propositions.

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[6] Indeed, chapter 2 already noted that there is some variety in numbering among the Greek manuscripts as well. But that chapter had the option of using established numerals from critical editions, which were available for all the texts under study. That same luxury is not available here.

[7] References to the Arabic translation of the *Data* will use the numbering system of Sidoli and Isahaya's edition, which follows the manuscript Seray Ahmet III 3464.

[8] This manuscript's colophon, however, writes that it contained 95 figures, written with the numeral ٩٥. Sidoli and Isahaya (2018) 200 fn.a note that the symbol for ٥ is oddly written and may have been changed. They also note that while the propositions were clearly numbered up to 91 in the manuscript's margins, there were 95 diagrams in the text, and this may be what is being counted.

[9] The Kraus manuscript's colophon reemphasizes the count of 95 propositions: "وهو خمسة وتسعون شكلا" – see Sidoli and Isahaya (2018) 201.

[10] The list of Middle Books provided by Bodleian Thurston 11 will receive discussion in chapter 5.

A	K	𝔾	A	K	𝔾	A	K	𝔾	A	K	𝔾	A	K	𝔾	A	K	𝔾
1	1	1	20	21	20	33	34	34.1	47	50	49	63	67	66	77	81	81.1
2	2	2			21	33	34	34.2	48	51	50	64	68	67a			81.2
3	3	3	21	22	22	34	35	35	49	52	51			67b	78	82	82
4	4	4	22	23	23			36	50	53	52			67c	79	83	83
5	5	5			24a	35	36	37	51	54	53			67d	80	84	84
6	6	6	23	24	24b	36	37	38	52	55	54a	65	69	68a	81	85	85
7	7	7	24	25	25	37a	38	39			54b			68b	82	86	v87a
8	8	8	25	26	26	37b	39		53	56	55a	66	70	69			v87b
9	9	9	26	27	27a	38	40	40			55b	67	71	70	83	87	86
10	10	12			27b	39	41	41	54	57	56	68	72	71	84	88	87
11	11	10	27	28	28	40	42	42	55	58	57	69	73	72	85	89	88
12	12	11.1	28	29	29	41	43	43	56	59	58	70	74	73	86	90	89
		11.2	29	30	30a	42	44	44.1	57	60	59	71	75	74	87	91	90
13	13	13			30b			44.2	58	61	60	72	76	75	88	92	91a
14	14	14			30c	43a	45	45b	59	62	61	73	77	76			91b
15	15	15			30d	43b	46	45a	60a	63	62	74	78	77	89	93	92
16	16	16	30	31	31			46a				75	79	78	90	94	93a
17	17	17	31	32	32	44	47	46b	60b	64				79			93b
18	18	18	32	33	33a	45	48	47			63			80a			93c
19a	19	19a			33b	46	49	48	61	65	65	76	80	80b	91	95	94
19b	20	19b							62	66	64						

Table 4.1: Concordance of propositions from the *Data*, adapted from Sidoli and Isahaya (2018) 320. A = Seray Ahmet III 3464, K = Kraus MS, and 𝔾 = the Greek according to Menge (1896).

<u>Theodosius's *Sphaerica*</u>[11]

In its Greek version the three books of the *Sphaerica*, as edited by Czinczenheim, totalled 60 propositions.[12] Seray Ahmet III 3464 has 58 propositions in total. Lahore, private library M. Nabī Khān and the list in Bodleian Thurston 11 both attest to 59 propositions in total. Gerard's Latin translation is 58 propositions in total.[13]

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[11] References to the Arabic translation of the *Sphaerica* will use the numbering system in Kunitzsch and Lorch's edition, which follows the manuscript Seray Ahmet III 3464.

[12] This includes the 22nd and 23rd propositions at the end of Book I, which have been identified to be later additions.

[13] The proposition counts are reemphasized in some of the colophons to the different books. At the end of Book I, Seray Ahmet III 3464 notes "وهى اثنان وعشرون شكلا" and Gerard's translation notes "viginti duas continens figuras." At the end of Book II, Seray Ahmet III 3464 notes "وهى اثنان وعشرون شكلا" and Lahore, private library M. Nabī Khān notes "وهى ثلثه وعشرون شكلا". At the end of Book III, these two manuscripts respectively notes "وهى أربعة عشر شكلا" and "وهى أربعة عشر شكلا وعدد أشكال المقالات الثلثة تسعة وخمسون شكلا".

A	H	𝔾		A	H	𝔾
Book I						
d.1	d.1	d.1		8	8	8
d.2	d.2	d.2		9	8	
d.3	d.3	d.3		10	9	9
d.4	d.4	d.4		11	(*)[14]	10
d.5	d.5	d.5		12	10	11
d.6	d.6			13	11	12
d.7	d.7			14	12	13
d.8	d.8			15	13	14
d.9	d.9			16	14	15
d.10	d.10	d.6		17	15	16
d.11	d.11			18	16	17
1	1	1		19	17	18
2	2	2		20	18	19
3	3	3		21	19	20
4	4	4		22	20	21
5	5	5				22
6	6	6				23
7	7	7				

A	H	𝔾		A	H	𝔾
Book II						
d.1	d.1	d.1		11	11	12
1	1	1		12	12	14
2	2	2		13	13	13
3	3	3		14	14	15
4	4	4		15	15	16
5	5	5		16	16	17
6	6	6		17	17	18
7	7	7		18	18	19
8	8	8		19	19	20
9	9	9		20	20	21
10	10	10		21	21	22
11	11	11		22	22	23

Table 4.2: Concordance of *Sphaerica* propositions, adapted from Kunitzsch and Lorch (2010b) 9-10. A = Seray Ahmet III 3464, H = Paris hebr. 1101, and 𝔾 = the Greek according to Czinczenheim (2000).

Book I of the *Sphaerica*, as edited by Czinczenheim, comprised 21 original propositions and the two later additions. The witnesses in Seray Ahmet III 3464 and Lahore, private library M. Nabī Khān contain 22 propositions – neither include the propositions which have been identified to be later additions in the Greek. Gerard's translation in Paris lat. 9335 is also 22 propositions, as is the Judeo-Arabic version. The Greek Book II of the *Sphaerica* comprised 23 propositions – this sees agreement with the Lahore private library witness. Seray Ahmet III 3464 contained 22 propositions (as did Gerard's translation in Paris lat. 9335). Proposition counts agree between the Greek, Arabic, and Latin for Book III, which contained 14 propositions. The above is a concordance of the definitions and propositions in books I and II (omitting book III since its contents agree). Leiden or. 1031 is omitted since it agrees with Seray Ahmet

[14] This proposition is skipped in the manuscript and instead added back in at the end of the book.

III 3464, and the same is true for Lahore, private library M. Nabī Khān, which fully agrees except for accidentally skipping a proposition numeral in book II and writing 13 instead of 12.

Autolycus's *On the Moving Sphere*[15]

The Greek *On the Moving Sphere*, as edited by Mogenet, had 12 propositions. This count is seen also in the list in Bodleian Thurston 11. Bodleian Huntington 237 has 12 propositions.[16] Since the structure is consistent, no concordance is necessary for this work.

Euclid's *Optics*[17]

The Greek *Optics*, as edited by Heiberg in two recensions, comprised 58 and 57 propositions respectively, though in the latter half of the work these are not ordered the same between the two recensions.[18] In the Arabic, Kheirandish notes two slightly different versions of the text which can be found between five manuscripts – Seray Ahmet III 3464, Leiden Or. 133, Cairo Dār al-Kutub al-Miṣriyya Dr 260, Cairo Dār al-Kutub al-Miṣriyya Dr 720, and the Kraus MS.[19] All of these present the *Optics* in the same structure of 64 propositions and are edited together in her edition.[20] The list in Bodleian Thurston 11 agrees: the *Optics* is attested to have 64 propositions. There are several Latin translations from the Arabic

[15] There are no disagreements between numbering systems for *On the Moving Sphere*.
[16] This is Nikfahm-Khubravan and Eshera's version I of the Arabic text, and they make no indication of this proposition count differing in the other codices they consulted.
[17] References to the Arabic translation of the *Optics* use the numbering system in Kheirandish's edition, since the five manuscripts her edition relies upon agree in this structure.
[18] These texts in the manuscripts showed greater variety, up to 61, 64, and 65 propositions, as some were split and separately numbered as multiple propositions. See Heiberg (1985) 244.
[19] Kheirandish (1999) xxviii and xxxiv.
[20] In two of their colophons this proposition count of 64 is reemphasized. Seray Ahmet III 3464 has "وهو اربعة وستون شكلا" and Cairo Dār al-Kutub al-Miṣriyya Dr 260 has "وهو اربعة ستون شكلا".

which have been identified; one of them, in MS Vatican Urb. Lat. 1329, agrees with this count of 64 propositions.[21]

Kh	𝔸	𝔹		Kh	𝔸	𝔹		Kh	𝔸	𝔹		Kh	𝔸	𝔹
d.1	d.1	d.1		11	10	10		29	28-1	28E		47		40-3
d.2	d.2	d.2		12	11	11		30	28-2	28D		48	38	42E
d.3	d.3	d.3		13	12	12		31	29	29		49	41	39
d.4-1	d.4-1	d.4-1		14	13	13		32	30	30		50	42-1	43
d.4-2	d.4-2	d.4-2		15	14	14		33	31	31		51	43D	44D
d.4-3	d.7	d.7		16	15	15		34	32	32		52	44-1	
d.4-4	d.4-3	d.4-3		17	16	16		35	33	33		53	46	
	d.5	d.5		18	17	17		36	34-1	34		54	45	46
	d.6	d.6		19	18	18		37	34-2	35-1		55	48E	48E
1	1	1		20	19	19		38	35-1	36-1		56	49D	
2	2	2		21	20	20		39	35-2	36-2		57	50	49
3	3	3		22	21	21		40	35-3	36-4		58	51	50
4	4	4		23	22-1	22E		41	35-4	36-5		59	53	52
5	5	5		24	23	23		42	36	37		60	54-2	53E
6	6.1	6.1		25	24	24		43	37	41		61	55	54
7	6.2	6.2		26	25	25		44	39	38		62	56	55
8	7	7		27	26	26		45	40-1	40-1		63	57	56
9	8	8		28	27	27		46		40-2		64	58	57
10	9	9												

Table 4.3: Concordance of propositions from the *Optics*, adapted from Kheirandish (1999).
Kh = Kheirandish (1999), 𝔸 and 𝔹 = the Greek recensions A and B according to Heiberg (1895).

<u>Euclid's *Phaenomena*</u>[22]

The Greek *Phaenomena* (recension B), as edited by Menge, contained 18 propositions. If recension A differed from this count, there is no way to tell from the incomplete version which survives. The manuscript Leiden Or. 1031 has this text with 20 propositions, whereas the list in Bodleian Thurston

[21] Theisen (1972) 324. This translation is titled *Euclidis de aspectuum diversitate*. Other Latin translations go by the titles *Liber de aspectibus* and *Liber de radiis visualibus*; they, however, circulated with 38 and 54 propositions respectively.
[22] References to the Arabic translation of the *Phaenomena* in this study will use the numbering of MS Leiden Or. 1031.

11 records the proposition count as 22. No Latin available to compare. This text is also extant in Seray Ahmet III 3464 but that witness has not yet been consulted.

L	𝔾		L	𝔾		L	𝔾
intro	intro		8	8		14	14
1	1		9	9		15	14[23]
2	2		10	10[24]		16	14[25]
3	3		10	10[26]		17	15
4	4		11	11		18	16
5	5		12	12		19	17
6	6		13	13		20	18
7	7						

Table 4.4: Concordance of propositions for the *Phaenomena*.
L = Leiden or. 1031 and 𝔾 = the Greek according to Menge (1916).

Theodosius's *On Habitations*[27]

The Greek *On Habitations*, as edited by Fecht, had 12 propositions. The list in Bodleian Thurston 11 attests to an Arabic version with 12 propositions, and the Arabic and Latin edited in Kunitzsch and Lorch have 12 propositions. Since the structure is consistent, no concordance is necessary for this work.

Theodosius's *On Days and Nights*[28]

The Greek *On Days and Nights*, as edited by Fecht, had 32 total propositions: 12 in its first book and 19 in its second book. According to Kunitzsch and Lorch, the Arabic version has 12 propositions in the first book and 21 in the second for a total of 33. They consulted Seray Ahmet III 3464 and the Kraus Manuscript and note the possibility that there is another witness in the Lahore private library witness. The

[23] This is part two of the proposition.
[24] The first part of Leiden or. 1031's proposition 10 corresponds to the proof in the B recension.
[25] This is the alternate proof to the proposition.
[26] The second part of Leiden or. 1031's proposition 10, and al-Ṭūsī's edition, corresponds to the proof in the A recension.
[27] There are no disagreements between numbering systems for *On Habitations*.
[28] References to *On Days and Nights* will number the propositions according to Kunitzsch and Lorch's edition.

list in Bodleian Thurston 11, meanwhile, attests to 37 propositions total – this is an unusually high number, and it is unclear what could have added an additional four propositions. Kunitzsch and Lorch's study of this text did show that the second book especially has several parts lacking any clear correspondence with the Greek; it may be that this part of the text was more unstable and its material sometimes totalled even more propositions.

K&L	𝔾		**K&L**	𝔾
Book I				
d.1	d.1		5	5
d.2	d.2		6	7
	d.3		7	6
	d.4		8	8
d.3	d.5		9	10
1	1		10	9
2	2		11	11
3	3		12	12
4	4			

K&L	𝔾		**K&L**	𝔾
Book II				
1	1			12
2	2		12[29]	
3	3		13[30]	
4	4		14	13
5	5		15	14
6	6		16	14[31]
7	7		17	15
8	8		18	16
9	9		19	17
	10		20	18
10[32]			21	19
11	11			

Table 4.5: Concordance of propositions for *On Days and Nights*.
K&L = Kunitzsch and Lorch (2011) and 𝔾 = the Greek according to Fecht (1927).

There is additionally a witness to this text in Bodleian Or. 365, but it is incomplete. It contains only book I in 9 propositions, and it both labels the final proposition as 10 instead of 9 and skips three propositions that are found in the other Arabic witnesses and in the Greek.

[29] Per Kunitzsch and Lorch (2011) 12, there is no clear correspondence between the Greek and Arabic of proposition 12.
[30] Same as above for proposition 13.
[31] The Arabic partly corresponds to the end of Greek proposition 14.
[32] Same as above for proposition 10.

K&L	d.1	d.2	d.3	1	2	3	4	5	6	7	8	9	10	11	12
B	d.1	d.2	d.3	1	2			3	4	5	6	7	8		10

Table 4.6: Comparison of propositions between Kunitzsch and Lorch's edition (K&L) and the incomplete manuscript Bodleian Or. 365 (B).

Aristarchus's *On Sizes and Distances*[33]

K	1	2	3	4	4	5	6	6	7	8	9	10	11	12	13	14	15	16	17
C	1	2	3	4	4	5	6	7	7	8	9	10	11	12	13	14	15	16	17
𝔾	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	

Table 4.7: Concordance of propositions from *On Sizes and Distances*.
K = Kraus MS, C = Columbia Or. 45, and 𝔾 = the Greek according to Heath (1913).

The Greek of *On Sizes and Distances*, as edited by Heath, contains 18 propositions. The witness in the Kraus Manuscript includes 17, and this number agrees with what is attested in Bodleian Thurston 11. The witness in Columbia Or. 45 also comprises 17 propositions. Despite the lower proposition count, *On Sizes and Distances* includes more material in the Arabic than in the Greek. Propositions are fused twice, but one proposition is added to the end of the text.

[33] References to *On Sizes and Distances* will use the proposition numerals according to Columbia Or. 45.

Autolycus’s *On Risings and Settings*[34]

L	𝔾		L	𝔾
Book I				
1	1		7	7
2	2		8	8
3	3		9	9
4	4		10	10
5	4[35]		11	11
(*)[36]	5		12	12
6	6		13	13

L	𝔾		L	𝔾
Book II				
1	1		11	
2	2		12	11
3	3		13	12
4	4		14	
5	5		15	13
6	6		16	14
7	7		17	15
8	8		18	16
9	9		19	17
10	10		20	18

Table 4.8: Concordance of propositions for *On Risings and Settings*.
L = Leiden or. 1031 and 𝔾 = the Greek according to Mogenet (1950).

The Greek of *On Risings and Settings*, as edited by Mogenet, comprises 13 propositions in its first book and 18 propositions in its second book. This is to be compared with the Arabic witness in Leiden Or. 1031, which also contains 13 propositions in its first book but 20 propositions in its second book. This total of 33 propositions in Leiden Or. 1031 agrees with the count of 33 propositions attested in Bodleian Thurston 11’s list. Both the Greek and Arabic agree in presenting nine definitions in book I.

Hypsicles’s *Anaphoricus*

The *Anaphoricus* is an unusual case since it is not a standard proposition-based text. In Vat. gr. 204, though different sections of the text are demarcated through new paragraphs and emphasized initials, they are not numbered as separate propositions. It can, however, be divided into five parts – the first three

[34] References to *On Risings and Settings* will use the proposition numerals according to MS Leiden Or. 1031.
[35] This is cases two and three of the proposition.
[36] This is not numbered as a separate proposition in Leiden or. 1031, but it is preceded by the proposition before it ending with the usual QED.

proceeding in a proposition-like way through different mathematical results, and the second two also doing so through astronomical results. In the Arabic these are treated as five propositions, being numbered as such and receiving formulaic language that would be expected in them, such as the QED at the end.[37] The list in Bodleian Thurston 11 similarly attests to five propositions.

3.2 Summary of Potential Deliberate Alterations

The following table presents the definitions and propositions according to their count in the Arabic translations. Compare the concordances of Greek and Arabic material, above, for how this relates to the Greek. The overview presented below is a very general one, intended to give an impression of how different kinds of potential deliberate alterations are distributed through the different texts of the astronomical curriculum. Fuller details on particular instances of these deliberate alterations follow in the sections below.

[37] See De Falco, Krause, and Neugebauer (1966) 70-75 and MS Paris arabe 2457, fol. 162a-164b.

Data				Sph. I		II	III	MS	Opt.			Ph.	Hab.	D&N I	II	S&D	R&S I	II	Ana.
d.1	16	[-] 44	73	d.1	20	d.1	1	d.1	[-intro]	21	51	intro	1	d.1	1	d.1	d.1	1	1
d.2	17	45	74	d.2	21	1	2	d.2	d.1	22	[-] 52	1	2	d.2	2	d.2	d.2	2	2
d.3	18	46	75	d.3	22	2	3	d.3	d.2	[-] 23	53	2	3	[-d]	3	d.3	d.3	3	3
d.4	19	47	[-p]	d.4	[-p]	3	4	d.4	d.3	24	54	3	4	[-d]	4	d.4	d.4	4	4
d.5	20	48	[-] 76	d.5	[-p]	4	5	1	d.4-1	25	55	4	5	d.3	[-a]	d.5	d.5	5	5
d.6	[-p]	49	[-] 77	d.6		5	6	2	d.4-2	26	56	5	6	1	[-a]	d.6	d.6	6	
d.7	21	50	78	d.7		6	7	3	d.4-3	27	57	6	7	2	[-a]	1	d.7	7	
d.8	22	51	79	d.8		7	8	4	d.4-4	28	58	7	8	3	[-a]	2	d.8	8	
d.9	[-] 23	[-] 52	80	d.9		8	9	5	[-d]	29	59	8	9	4	5	3	d.9	9	
d.10	24	[-] 53	81	d.10		9	10	6	[-d]	30	[-] 60	9	10	5	6	4	1	10	
d.11	25	54	82	d.11		10	11	7	1	31	61	10	11	6	7	5	2	11	
d.12	[-] 26	55	83	1		11	12	8	2	32	62	11	12	7	8	6	3	12	
d.13	27	56	84	2		12	13	9	3	33	63	12		8	9	7	4	13	
d.14	28	57	85	3		13	14	10	4	34	[-] 64	13		9	[-p]	8	5	14	
d.15	[-] 29	58	86	4		14	X	11	5	35		14		10	10	9	6	15	
1	30	59	87	5		15		12	6	36		15		11	11	10	7	16	
2	31	60	[-] 88	6		16			7	[-] 37		16		12	[-p]	11	8	17	
3	[-] 32	[-p]	89	7		17			8	38		17			12	12	9	18	
4	[-] 33	61	[-] 90	8		18			9	39		18			[-p]	13	10	19	
5	34	62	91	9		19			10	40		19			13	14	11	20	
6	[-p]	63		10		20			11	41		20			14	15	12		
7	35	[-] 64		11		21			12	42					15	16	13		
8	36	[-] 65		12		22			13	43					16	17			
9	37	66		13					14	44					17				
10	38	67		14					15	45					18				
11	39	68		15					16	46					19				
12	40	69		16					17	47					20				
13	41	70		17					18	48					21				
14	[-] 42	71		18					19	49									
15	43	72		19					20	[-] 50									

Rearranged propositions	Presence / absence of cases
Fusion / division of propositions	Presence / absence of material
Presence / absence of alternate proofs	

Table 4.9: Overview of alterations in core Middle Books works between their extant Arabic translations and their extant Greek forms. In this table, entries indicated with "[-]" represent material present in Greek which does not appear in the Arabic translation.
[-] = cases or proofs, [-d] = definitions, [-p] = propositions, [-a] = assumptions

A note on phrasing: here we discuss "potential" deliberate alterations because, while these variants in the Graeco-Arabic tradition would have arisen at some point through deliberate action, the discussion in the present chapter aims at those which arose in the Arabic part of this tradition. But the Arabic translations in the ninth century occurred not at the endpoint of the Greek tradition, but midway through it. Some amount of the above table's variants between the Arabic translations and the Greek appear not because of deliberate action on the part of the Arabic translators, correctors, and other scholars, but because the Greek manuscripts which they relied upon presented older or otherwise different versions of the texts than the ones which survive in Greek today. Hence we consider "potential" deliberate alterations here as a catch-all for those which are present; the sections below will seek to further disentangle which of these may be assigned specifically to the Arabic tradition. The terminology "presence / absence" rather than "addition / suppression" is used when comparing the Arabic and Greek material for the same reason.

The immediate impression from the table above is the variety and density of potential deliberate alterations that are present between the Greek texts and their Arabic translations. For features which may be present or absent (and so potentially deliberately added or deliberately suppressed), it stands out how many of these are absences, and across multiple texts. This is notable in comparison to the greater tendency towards addition seen in the Greek transmission, and this is one of the pieces of evidence to strongly suggest that many of these differences are because of material which saw addition to the Greek either later, after the Arabic translation, or in separate versions than the ones which appeared in the manuscripts the translators used.

Another feature which appears often is fusion / division of propositions, across five of the ten texts. Some of these similarly result because the Arabic manuscripts follow what is found in the Greek manuscripts – though modern editors treat the *Optics* B proposition 36 as a single proposition, for example, manuscripts tend to divide it into as many as five propositions, and it is seen divided and separately numbered in the Arabic as well.

The slight increase in rearranged propositions is worth noting. Where in the Little Astronomy this was seen only in two of the nine texts under study, here it occurs in a third, along with the added *Data* as well.

3.3 The Arabic Translations and the Greek Recensions of the *Optics* and *Phaenomena*

Both the translations of the *Optics* and the *Phaenomena* show signs of interaction with both recensions of the Greek texts, though this occurs to a far greater extent in the *Optics* than in the *Phaenomena*. This mixed transmission has a definite influence on the alterations that will be discussed in more detail in the sections below, and so it is useful to sketch out here an overview of how the different Greek recensions intersect with the Arabic translation.

Overall, the translation of the *Phaenomena* presented in Leiden Or. 1031 follows the B recension of the Greek text. This includes supplementary material that was attached to the B recension such as that recension's alternate proofs. See also the chapter's appendix for a comparison of the expositions between the Arabic, recension A, and recension B. Material from the A recension does appear in this manuscript, but it is clearly denoted as coming from another copy and potentially via the Syriac. The greater adherence to the B recension over the A recension is unsurprising, since the B recension was indeed the one more closely associated with the Little Astronomy, as discussed in chapter 2. We can see this was evidently the case by the time this translation was produced in the ninth century.

The *Optics* presents a much more complicated case. Thorough details of correspondence between the Arabic translation and the A and B recensions of the Greek text are presented by Kheirandish, who offers a careful study of each proposition. The following table offers a very general impression of how each of the propositions of the Arabic *Optics* relate to the two Greek recensions.

Propositions of the Arabic Optics						
1	11	21	31	41	51	61
2	12	22	32	42	52	62
3	13	23	33	43	53	63
4	14	24	34	44	54	64
5	15	25	35	45	55	
6	16	26	36	46	56	
7	17	27	37	47	57	
8	18	28	38	48	58	
9	19	29	39	49	59	
10	20	30	40	50	60	

Table 4.10: Propositions of the Arabic *Optics* and their agreement with the Greek recensions, according to Kheirandish (1999).[38]

■ Recensions A and B differ and the Arabic is closer to A ■ Recensions A and B differ and the Arabic is mixed ■ Recensions A and B differ and the Arabic is closer to B

Overall, there are fifty-six propositions in the Arabic that have their source in propositions which noticeably differ between the Greek recensions. In thirty-one of these, the Arabic is closer to the A recension; in seventeen the Arabic shows definite influence from both recensions, and in eight the Arabic is closer to the B recension.

While these numbers suggest a larger proportion of this translation of the *Optics* was based on the A recension, the greater takeaway should be how much of a mixed tradition the text presents. The case of the *Phaenomena*, above, was much more straightforward: that translation solidly derives from the B recension of the Greek text. Where material from the A recension appears, it is supplementary and clearly marked as coming from another copy – further, a separate translation. That material is presented as an alternate proof to the main proposition, whereas the main proposition is the B recension's rendition.

[38] Further details, including specifications on what features of the propositions differ and agree between the recensions the Arabic translation can be found in the commentary on the propositions in Kheirandish (1999) v.2 xxxiv.

Meanwhile the Arabic translation of the *Optics*, as we have it, cannot be said to derive from one recension, with perhaps supplementary additions from another. This mixed tradition is notable in light of what was seen in chapter 2, where the A and B recensions of the *Optics* were seen to have distinct manuscript traditions from at least the ninth century on, according to the extant manuscripts. Perhaps they had not diverged so thoroughly in the ninth century when they were translated, or perhaps this speaks to a wider variety of manuscripts used in the translation, or of multiple translations.

The more complicated case of the *Optics* is plausibly the result of its separate, optical subject matter – this was a text receiving study and editorial work outside the astronomical tradition of the Little Astronomy, and so ninth century translators could have encountered the text in different forms reflecting this. Subsequent work with the text in its Arabic tradition likely complicated its transmission history further.

4. Deliberate Alterations and References in Detail

4.1 Substitution of Proof

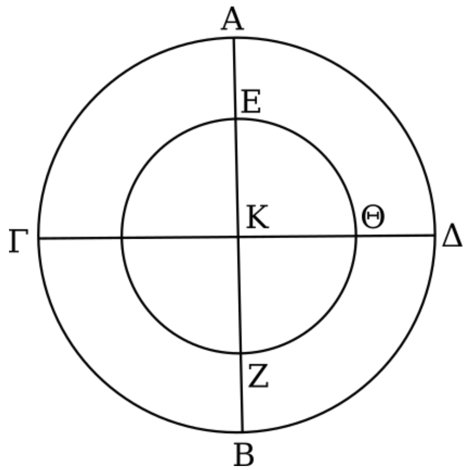
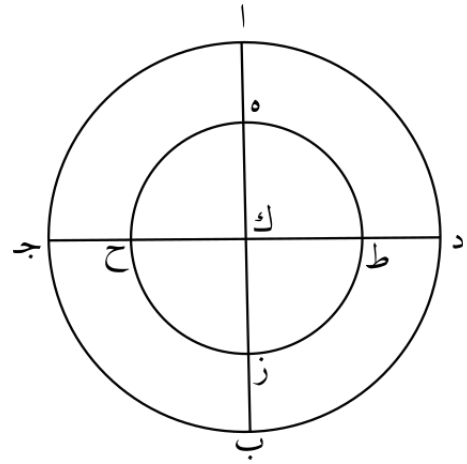
Already in the ninth century, shortly after their translation, several of the curriculum's texts are reported to have been corrected by Thābit ibn Qurra. Scholarship has shown that his editorial process sometimes involved rewriting proofs and redrawing diagrams. Corrections attributed to Thābit ibn Qurra are the following: the *Data*,[39] the *Sphaerica*,[40] *On the Moving Sphere*,[41] *On Habitations*,[42] *On Sizes and*

[39] According to the Kraus manuscript, for which see Sidoli and Isahaya (2018), and to al-Ṭūsī, for which see Hyderabad (1939-40) *Kitāb al-Muʿṭīyāt* 2.
[40] According to the Lahore manuscript, for which see Kunitzsch and Lorch (2010b) 3, and to al-Tusi, for which see Hyderabad (1939-40) *Kitāb al-ukar* 2.
[41] According to al-Ṭūsī, for which see Hyderabad (1939-40) *Kitāb al-kurah al-mutaḥarrikah* 2. Nikfahm-Khubravan and Eshera (2019) 13 report that he is attested as corrector in the manuscripts Istanbul Süleymaniye Kütüphanesi, Ayasofya 2671 and London Institute of Ismaili Studies, Hamdani Collection 1647 as well.
[42] According to the Lahore manuscript, for which see Kunitzsch and Lorch (2010a) 9.

Distances,[43] *On Risings and Settings*,[44] and the *Anaphoricus*.[45] Note that, while *On Habitations* is claimed to have been corrected by Thābit, the wording actually follows the Greek quite closely.[46] Nikfahm-Khubravan and Eshara have also shown that while al-Ṭūsī reports on Thābit's correction of *On the Moving Sphere*, the version which his edition is based on appears to be separate from the version whose witnesses attest to Thābit's involvement.[47] This suggests that there was some contamination between versions regarding which translators and correctors were claimed as responsible for the text.[48] This section will look at some samples of proofs in texts attributed to the correction of Thābit to highlight some of the ways these rewritten proofs vary.

However, we consider *On Habitations* first since its language is actually quite close to Greek, despite it being claimed as one of Thābit's corrections. This offers a point of comparison that will make clearer the more significant substitutions of proofs in other texts – substitution of proof does not actually occur in the Arabic translation of this text, and what local alterations are made are fewer in number. An example of the proximity between the Greek and Arabic *On Habitations* can be seen in proposition 2, below. While there are some slight differences, overall the wording, diagrams, labels, and logic of this proposition is quite similar.

[43] According to the Kraus manuscript, for which see Lorch (2008) 28.
[44] According to Seray Ahmet III 3464, for which see Lorch (2008) 22, and to al-Ṭūsī, for which see Hyderabad (1939-40) *Kitāb al-Ṭulūʿ w-l-Ghurūb* 2.
[45] See Paris arabe 2457, fol. 162a
[46] Kunitzsch and Lorch (2010a) 11. It may be that the attribution to Thābit is erroneous, or it may be that in this case the process of making a "revision" of the text happened to involve reviewing it and making only minor changes.
[47] Nikfahm-Khubravan and Eshera (2019) 13.
[48] Compare the similar problems of contamination in the Arabic transmission of Euclid's *Elements*, for example, as shown in Brentjes (2018b).

<table>
<tr><th>Greek On Habitations proposition 2:</th><th>Arabic On Habitations proposition 2:</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>For those who live under the equator all the stars will both set and rise and they will be borne in an equal time both above the horizon and below the horizon.

Let there be in the cosmos a meridian circle ABGD, and in the Earth circle EZHT, and the diameter of the equator line AB, and let there be habitations at point E. Therefore the point at the zenith of the habitation E is point A.

Indeed I say, that for those who live at point E all the stars will both set and rise and they will be borne in an equal time both above the horizon and below the horizon.

Let the center of the Earth be point K and through point K let axis GD be drawn on line AB at right angles; therefore it is line GD. And since the circle drawn on diameter GD, which stands at right angles on line AB, is the horizon for those who live at E; but the circle drawn on diameter GD standing at right angles on line AB passes through the poles; therefore the horizon for those who live at E passes through the poles of the sphere. And since the stars are borne on circles parallel to the</td><td>For those whose places of habitation are under the circuit of the equator, all the fixed stars rise above them and set from them, and the time of their movement above their horizon is equal to the time of their movement below it.

Example of this: we assume for those whose places of habitation are under the equator a line of the meridian: in the sphere of the cosmos circle ABGD and in the sphere of the earth circle EZHT, the diameter of the circuit of the equator line AB, and we assume some place of habitation at point E. So the zenith of the place of habitation E is point A.

I say: for those whose places of habitation are at point E, all the fixed stars rise above them and set from them and the time of their movement above their horizon is equal to the time of their movement below it.

Proof of this: we assume as center of the earth point K, so point K is indeed the center of the cosmos. And we draw through point K a line standing on line AB, and it is line GKD. So it is clear that line GKD is the axis of the sphere, and</td></tr>
</table>

equator, and the circle passing through the poles of the sphere bisects the parallel circles on which the stars are borne, all the stars for those who live at E will set and rise. And it is clear, that they will be borne an equal time both above the horizon and below the horizon, for each of them will be borne on a semicircle.	that the circle drawn on diameter GD standing on line AB is the horizon of place of habitation E, and the circle drawn on diameter GD which stands on circle ABGD passes through the two poles of the sphere. So the horizon of place of habitation E passes through the two poles of the sphere. Since the fixed stars travel on parallel circuits parallel to the circuit of the equator, and the circle passing through the two poles of the sphere cuts the parallel circles in half, and the horizon of place of habitation E passes through the two poles of the sphere, then the horizon of place of habitation E cuts in half the parallel circuits on which the fixed stars move. So the time of the movement of the fixed stars above the horizon of place of habitation E is equal to the time of their movement below it, because each of them in place of habitation E travels a semicircle above the earth and a semicircle below the earth. And this is what we wanted to demonstrate.
Τοῖς ὑπὸ τὸν ἰσημερινὸν οἰκοῦσιν πάντα τὰ ἄστρα καὶ δύσεται καὶ ἀνατελεῖ καὶ τὸν ἴσον χρόνον ὑπέρ τε τὸν ὁρίζοντα ἐνεχθήσεται καὶ ὑπὸ τὸν ὁρίζοντα. Ἔστω ἐν κόσμῳ μεσημβρινὸς ὁ ΑΒΓΔ, ἐν δὲ γῇ ὁ ΕΖΗΘ, ἰσημερινοῦ δὲ διάμετρος ἡ ΑΒ, οἴκησις δὲ ἔστω πρὸς τῷ Ε τῆς ἄρα Ε οἰκήσεως τὸ κατὰ κορυφὴν σημεῖόν ἐστι τὸ Α. Λέγω δή, ὅτι τοῖς πρὸς τῷ Ε οἰκοῦσι πάντα τὰ ἄστρα καὶ δύσεται καὶ ἀνατελεῖ καὶ τὸν ἴσον χρόνον ἐνεχθήσεται ὑπέρ τε τὸν ὁρίζοντα καὶ ὑπὸ τὸν ὁρίζοντα. Ἔστω κέντρον τῆς γῆς τὸ Κ σημεῖον καὶ ἀπὸ τοῦ Κ τῇ ΑΒ πρὸς ὀρθὰς ἤχθω ἡ ΓΔ ἄξων ἄρα ἐστὶν ὁ ΓΔ. Καὶ ἐπεὶ ὁ περὶ διάμετρον τὴν ΓΔ κύκλος γραφόμενος ὀρθὸς ὢν πρὸς τὴν ΑΒ ὁρίζων ἐστὶ τοῖς πρὸς τῷ Ε οἰκοῦσιν, ἀλλὰ ὁ περὶ διάμετρον τὴν ΓΔ κύκλος γραφόμενος ὀρθὸς ὢν πρὸς τὴν	الذين مساكنهم تحت فلك معدل النهار الكواكب الثابتة كلها تطلع عليهم وتغرب عنهم ويكون زمان مسيرها فوق أفقهم مساوياً لزمان مسيرها تحته، مثل ذلك أن نفرض الذين مساكنهم تحت معدل النهار خط نصف النهار أما من كرة الكل فدائرة (ا ب جـ د) وأما من كرة الأرض فدائرة (ه ز ح ط) ونفرض قطر فلك معدل النهار خط (ا ب) ونفرض مسكناً ما على نقطة (ه) فيكون سمت الرأس لمسكن (ه) نقطة (ا)، فأقول إن الذين مساكنهم على نقطة (ه) الكواكب الثابتة كلها تطلع عليهم وتغرب عنهم ويكون زمان مسيرها فوق أفقهم مساوياً لزمان مسيرها تحته، برهان ذلك أن نفرض مركز الأرض نقطة (ك) فنقطة (ك) مركز لكرة الكل ونخرج على نقطة (ك) خطاً قائماً على خط (ا ب) وهو خط (جـ ك د) فظاهر أن خط (جـ ك د) هو محور الكرة وأن الدائرة المرسومة على قطر (جـ د) القائم على خط (ا ب) هى أفق لمسكن (ه) والدائرة المرسومة على قطر (جـ د) هى قائمة على دائرة (ا ب جـ د) تجوز على قطبى الكرة فإذاً أفق مسكن (ه) يجوز على قطبى الكرة ومن أجل أن الكواكب الثابتة تسير على أفلاك

AB ὁ διὰ τῶν πόλων ἐστίν, ὁ ἄρα διὰ τῶν πόλων τῆς σφαίρας ὁρίζων ἐστὶ τοῖς πρὸς τῷ E οἰκοῦσιν. Καὶ ἐπεὶ πάντα τὰ ἄστρα κατὰ παραλλήλων κύκλων φέρεται τῷ ἰσημερινῷ, ὁ δὲ διὰ τῶν πόλων τῆς σφαίρας δίχα τέμνει τοὺς παραλλήλους κύκλους, καθ ὧν φέρεται τὰ ἄστρα, πάντα τὰ ἄστρα τοῖς πρὸς τῷ E οἰκοῦσιν δύσεται καὶ ἀνατελεῖ. Καὶ φανερόν, ὅτι τὸν ἴσον χρόνον ἐνεχθήσεται ὑπέρ τε τὸν ὁρίζοντα καὶ ὑπὸ τὸν ὁρίζοντα ἕκαστον γὰρ αὐτῶν ἐνεχθήσεται κατὰ ἡμικυκλίου.[50]	متوازية موازية لفلك معدل النهار والدائرة التى تجوز على قطبى الكرة تقطع الدوائر المتوازية على أنصافها وأفق مسكن (ه) يجوز على قطبى الكرة فإن أفق مسكن (ه) يقطع الأفلاك المتوازية التى تسير عليها الكواكب الثابتة على أنصافها فيكون زمان مسير الكواكب الثابتة فوق أفق مسكن (ه) مساوياً لزمان مسيرها تحته إذ كان كل واحد منها فى مسكن (ه) يسير نصف دائرة فوق الأرض ونصف دائرة تحت الأرض، وذلك ما أردنا أن نبين.[49]

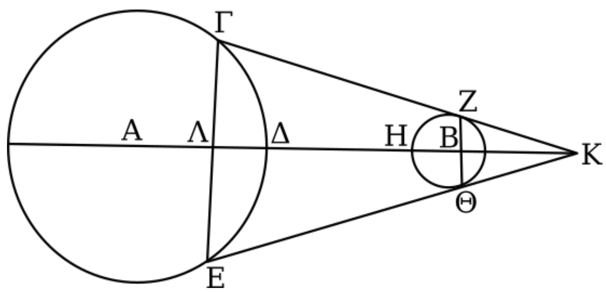
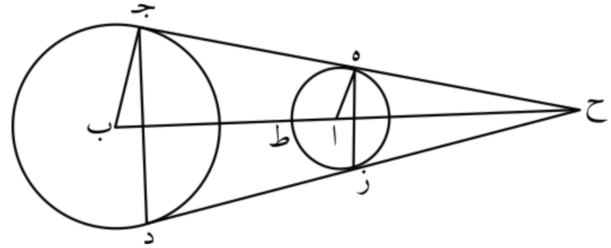
Table 4.11: Comparison of Greek and Arabic texts of *On Habitations* proposition 2

So while Thābit's process could involve rewriting and restructuring proofs, this was not a constant. The above is not an example of substitution of proof. There are some local alterations – the Arabic emphasizes that the center of the earth is the center of the cosmos, it lacks the conclusion in the proof that all stars will rise and set, and at the end it reemphasizes that the conclusions are only such for habitations under point E, i.e., on the equator. But the overall thrust of the proof is the same.

This situation with *On Habitations* stands in stark contrast to what has been found for the translation of Theodosius's other short text, *On Days and Nights*. For that text, Kunitzsch and Lorch report that in general, only the enunciations follow the Greek closely. The proof, diagrams, and labelling schemes tend to diverge, with few exceptions.[51] Since the available edition and manuscript present only a partial text (and Bodleian Or. 365 is a deficient witness in several regards), we cannot delve into this further at this time, but it is deserving of further study.

[49] Kunitzsch and Lorch (2010a) 20-22.
[50] Fecht (1927) 16.
[51] Kunitzsch and Lorch (2011) 11.

An example of a rewritten proof can be seen in *On Sizes and Distances*. Proposition 2 offers a short illustration for our purposes, and it is aligned with the Greek below:

Greek *Sizes and Distances* Proposition 2:	**Arabic *Sizes and Distances* Proposition 2:**
	
If a sphere be illuminated by a sphere greater than itself, then the illuminated portion of the former sphere will be greater than a hemisphere. For let a sphere the center of which is B be illuminated by a sphere greater than itself the center of which is A. I say that the illuminated portion of the sphere the center of which is B is greater than a hemisphere. For, since two unequal spheres are comprehended by one and the same cone which has its vertex in the direction of the lesser sphere, let the cone comprehending the spheres be (drawn), and let a plane be carried through the axis; this plane will cut the spheres in circles and the cone in a triangle. Let it cut the spheres in the circles CDE, FGH and the cone in the triangle CEK. It is then manifest that the segment of the sphere towards the circumference FGH, the base of which is the circle about FH as diameter, is the portion illuminated by the segment towards the circumference CDE, the base of which is the	When a smaller sphere receives light from a larger sphere, then the seen (part) is greater than a hemisphere. Let there be a sphere whose center is point A which receives light from a sphere greater than it whose center is point B. I say that the part of the sphere which receives light is greater than a hemisphere. And this is when the surrounding for each of the two unequal spheres is one cone. It is necessary that the surrounding for these two spheres be one cone. So let us connect line AB and let us draw on both sides and let us also draw one of the planes which pass through line AB. So the segments which result from it are two great circles in the two spheres and in the plane of the cone two straight lines. So in the two spheres let these be the two circles EZ and GD and in the plane of the cone the two straight lines HG and HD. And let us connect the lines GD and EZ. And the segment of the sphere on which is arc ETZ whose base is the circle on diameter EZ on

circle about CE as diameter and at right angles to the straight line AB; for the circumference FGH is illuminated by the circumference CDE, since CF, EH are the extreme rays. And center B of the sphere is within the segment FGH; so that the illuminated portion of the sphere is greater than a hemisphere.[52]	line AH at right angles is the part which receives light from the segment of the sphere whose center is B which is passing between the two points D and G whose base is the circle whose diameter is GD at right angles. And this is (because) the two rays GE and DZ are the extreme of the rays which we draw from one of the two spheres to the other. And the center of the sphere is in the section in which arc ETZ is. So the part of the sphere which receives light is greater than a hemisphere. And this is what we wanted to demonstrate.
Ἐὰν σφαῖρα ὑπὸ μείζονος ἑαυτῆς σφαίρας φωτίζηται, μεῖζον ἡμισφαιρίου φωτισθήσεται. Σφαῖρα γάρ, ἧς κέντρον τὸ Β, ὑπὸ μείζονος ἑαυτῆς σφαίρας φωτιζέσθω, ἧς κέντρον τὸ Α· λέγω ὅτι τὸ φωτιζόμενον μέρος τῆς σφαίρας, ἧς κέντρον τὸ Β, μεῖζόν ἐστιν ἡμισφαιρίου. Ἐπεὶ γὰρ δύο ἀνίσους σφαίρας ὁ αὐτὸς κῶνος περιλαμβάνει τὴν κορυφὴν ἔχων πρὸς τῇ ἐλάσσονι σφαίρᾳ, ἔστω ὁ περιλαμβάνων τὰς σφαίρας κῶνος, καὶ ἐκβεβλήσθω διὰ τοῦ ἄξονος ἐπίπεδον· ποιήσει δὴ τομὰς ἐν μὲν ταῖς σφαίραις κύκλους, ἐν δὲ τῷ κώνῳ τρίγωνον. ποιείτω οὖν ἐν μὲν ταῖς σφαίραις κύκλους τοὺς ΓΕ, ΖΗΘ, ἐν δὲ τῷ κώνῳ τρίγωνον τὸ ΓΕΚ. φανερὸν δὴ ὅτι τὸ κατὰ τὴν ΖΗΘ περιφέρειαν τμῆμα τῆς σφαίρας, οὗ βάσις ἐστὶν ὁ περὶ διάμετρον τὴν ΖΘ κύκλος, φωτιζόμενον μέρος ἐστὶν ὑπὸ τοῦ τμήματος τοῦ κατὰ τὴν ΓΕ. περιφέρειαν, οὗ βάσις ἐστὶν ὁ περὶ διάμετρον τὴν ΓΕ κύκλος, ὀρθὸς ὢν πρὸς τὴν ΑΒ εὐθεῖαν· καὶ γὰρ ἡ ΖΗΘ περιφέρεια φωτίζεται ὑπὸ τῆς ΓΕ	إذا كانت كرة صغرى تقبل الضوء عن كرة أعظم +فكا المنتظير أعظم+ من نصف كرة فليكن كرة مركزها نقطة (ا) يقبل الضوء عن كرة أعظم منها مركزها نقطة (ب) فاقول ان الجزء من الكرة الذى يقبل الضوء أعظم من نصف الكرة وذلك انه لما كان المحيط بكل كرتين غير متساويتين مخروط واحد وجب ان يكون المحيط بهاتين الكرتين مخروط واحد فلنوصل خط (ا ب) ولنخرج في كلتي الجهتين ولنخرج أيضا أحد السطوح التي تمر بخط (ا ب) فالقطوع التي تحدث عنه يكون في الكرتين دائرتين عظيمتين وفي سطح المخروط خطين مستقيمين فليحدث في الكرتين دائرتي (ه ز) (جـ د) وفي سطح المخروط خطي (ح جـ) (ح د) ولنوصل خطا (جـ د) (ه ز) والقطعة من الكرة التي عليها قوس (ه ط ز) التي قاعدتها الدائرة إلى على قطر (ه ز) القائمة على خط (ا ح) على زوايا قائمة هو الجزء الذي يقبل الضوء من قطعة الكرة التي مركزها (ب) التي جيزت فيها بين نقطتي (د) (جـ) التي قاعدتها الدائرة التي قطرها (ح د) القائمة على خط (جـ د) على زوايا قائمة وذلك ان شعاعي (جـ ه) (د ز) بها اخر اشعاعات التي نخرج من

[52] Translation from Heath (1913) 359-361.

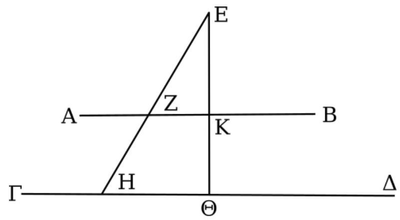
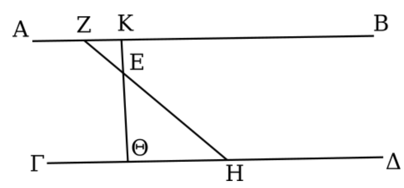
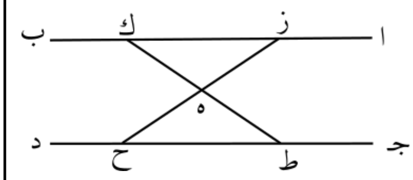
περιφερείας· ἔσχαται γὰρ ἀκτῖνές εἰσιν αἱ ΓΖ, ΕΘ·	احدى الكرتين الى الاخر
καὶ ἔστιν ἐν τῷ ΖΗΘ τμήματι τὸ κέντρον τῆς σφαίρας τὸ Β· ὥστε τὸ φωτιζόμενον μέρος τῆς σφαίρας μεῖζόν ἐστιν ἡμισφαιρίου.[54]	ومركز كرة في القطعة التي فيها قوس (ه ط ز) فالجزء من الكرة الذي يقبل الضوء أعظم من نصف الكرة وذلك ما اردنا ان نبين [53]

Table 4.12: Comparison of Greek and Arabic texts of *On Sizes and Distances* proposition 2

From the start and looking just at the diagrams, it is clear that the labelling has been redone, though this is by itself not a significant enough change to make this a case of a full substitution of proof. It is in the text itself, however, that the proof can seen to have been rewritten and expanded. Overall, however, the logic of the proof remains the same.

The *Data*, meanwhile, has an example where the substituted proof follows a different logic that what is found in the Greek. This is Arabic proposition 33 (= Greek 34). To start off, the Arabic proof considers a different case than the main one in the extant Greek sources. The proposition concerns how "if a straight line be drawn from a given point to parallel straight lines given in position, it will be cut in a given ratio" – in the main proof in Greek, this given point is outside the two parallel lines. In the Greek's second case and in the Arabic, it is located between them. Further, the Arabic is rewritten to offer a more general proof. Below is a comparison of the main Greek proof, the alternate case in the Greek, and the Arabic:

[53] MS Columbia Or. 45, fol. 32b-33a.
[54] Heath (1913) 358-360.

Greek *Data* 34, main:	**Greek *Data* 34, alternate:**	**Arabic *Data* 33:**
		
For, on the parallel straight lines given in position AB, CD let the straight line EZH have been drawn from the given point E. I say that the ratio EZ:ZH is given. For, from the point E let EKQ have been drawn perpendicular to CD. Since from the given point E to the straight line given in position CD the straight line EQ have been drawn, making the given angle EQH, therefore EQ is [given] in position. And each of AB, CD is [given] in position. Therefore each of K, Q is given. And E is given; therefore each of the lines EK, KQ is given; therefore the ratio EK:KQ is given. And EK:KQ :: EZ:ZH; therefore the ratio EZ:ZH is given.[55]	For, on the parallels given in position AB, CD let the straight line drawn from the given point E be EZH. I say that the ratio HE to EZ is given. For let a perpendicular EQ be drawn from point E upon CD and let it be extended to K. Since from the given point E upon the given in position straight line CD a straight line was extended, EQ, making a given angle which is EQH, then QEK is (given) in position; and both AB, CD are (given) in position; so both of the points Q, K are given. And also the (point) E is given; then both of the (lines) QE, EK are given; then the ratio of QE to EK is given; and as the (ratio) QE to EK, so thus the ratio HE to EZ; then also the ratio of HE to EZ is known.	For, let the known point be point E. And lines AB, GD, known in position, are mutually parallel. And let line ZEH be produced from point E. Then, I say that the ratio ZE to EH is known. Its proof: We designate a known point, which is T, on line GD, and we produce TE, and we prolong it to K. Then, TK is known in position, and AB is known in position, so point K is known. And each of points T, E is known, so each of lines KE, ET is known, so the ratio of one to the other is known. And the ratio ZE to EH is as the ratio KE to ET, so the ratio ZE to EH is known. And that is what we wanted to show.[56]
εἰς γὰρ παραλλήλους τῇ θέσει δεδομένας εὐθείας τὰς ΑΒ, ΓΔ ἀπὸ δεδομένου σημείου τοῦ Ε εὐθεῖα γραμμὴ ἤχθω ἡ ΕΖΗ.	Εἰς γὰρ παραλλήλους τῇ θέσει δεδομένας τὰς ΑΒ, ΓΔ ἀπὸ δεδομένου σημείου τοῦ Ε εὐθεῖα γραμμὴ ἤχθω ἡ ΕΖΗ· λέγω, ὅτι	فلتكن النقطة المعلومة نقطة (ه)، وخطا (ا ب) (جـ د) معلوما الوضع متوازيان، وليخرج من نقطة (ه) خط (ز ه ح). فأقول إنّ نسبة (ز ه) إلى (ه ح) معلومة.

[55] Translation from Taisbak (2003) 106.
[56] Translation from Sidoli and Isahaya (2018) 84.

λέγω, ὅτι λόγος ἐστὶ τῆς ΕΖ πρὸς ΖΗ δοθείς. ἤχθω γὰρ ἀπὸ τοῦ Ε σημείου ἐπὶ τὴν ΓΔ κάθετος ἡ ΕΚΘ. ἐπεὶ ἀπὸ δεδομένου σημείου τοῦ Ε ἐπὶ θέσει δεδομένην εὐθεῖαν τὴν ΓΔ εὐθεῖα γραμμὴ ἦκται ἡ ΕΘ δεδομένην ποιοῦσα γωνίαν τὴν ὑπὸ τῶν ΕΘΗ, θέσει ἄρα ἐστὶν ἡ ΕΘ· θέσει δὲ καὶ ἑκατερα τῶν ΑΒ, ΓΔ· δοθὲν ἄρα ἐστὶν ἑκάτερον τῶν Κ, Θ. ἔστι δὲ καὶ τὸ Ε δοθέν· δοθεῖσα ἄρα ἐστὶν ἑκατέρα τῶν ΕΚ, ΚΘ. λόγος ἄρα τῆς ΕΚ πρὸς τὴν ΚΘ δοθείς. καί ἐστιν ὡς ἡ ΕΚ πρὸς τὴν ΚΘ, οὕτως ἡ ΕΖ πρὸς τὴν ΖΗ. λόγος ἄρα καὶ τῆς ΕΖ πρὸς τὴν ΖΗ δοθείς.[57]	λόγος ἐστὶ τῆς ΗΕ πρὸς τὴν ΕΖ δοθείς. ἤχθω γὰρ ἀπὸ τοῦ Ε σημείου ἐπὶ τὴν ΓΔ κάθετος ἡ ΕΘ καὶ ἐκβεβλήσθω ἐπὶ τὸ Κ. ἐπεὶ ἀπὸ δεδομένου σημείου τοῦ Ε ἐπὶ θέσει δεδομένην εὐθεῖαν τὴν ΓΔ εὐθεῖα γραμμὴ ἦκται ἡ ΕΘ δεδομένην ποιοῦσα γωνίαν τὴν ὑπὸ τῶν ΕΘΗ, θέσει ἄρα ἐστὶν ἡ ΘΕΚ· θέσει δὲ καὶ ἑκατέρα τῶν ΑΒ, ΓΔ· δοθὲν ἄρα ἐστὶν ἑκάτερον τῶν Θ, Κ σημείων. ἔστι δὲ καὶ τὸ Ε δοθέν· δοθεῖσα ἄρα ἐστὶν ἑκατέρα τῶν ΘΕ, ΕΚ· λόγος ἄρα τῆς ΘΕ πρὸς ΕΚ δοθείς· ὡς δὲ ἡ ΘΕ πρὸς τὴν ΕΚ, οὕτως ἡ ΗΕ πρὸς ΕΖ· λόγος ἄρα καὶ τῆς ΗΕ πρὸς ΕΖ δοθείς.[58]	برهانه: إنّا نتعلّم على خط (جـ د) نقطة معلومة، وهي (ط)، ونخرج (ط ه)، وننفذه إلى (كـ). فـ(ط كـ) معلوم الوضع، و(ا ب) معلوم الوضع، فنقطة (كـ) معلومة. وكل واحدة من نقطتي (ط) (ه) معلومة، فكل واحد من خطي (كـ ه) (ه ط) معلوم، فنسبة احدهما إلى الآخر معلومة. ونسبة (ز ه) إلى (ه ح) كنسبة (كـ ه) إلى (ه ط) المعلومة، فنسبة (ز ه) إلى (ه ح) معلومة. وذلك ما أردنا أن نبيّن.[59]

Table 4.13: Comparison of Greek and Arabic texts of *Data* proposition 34

Unlike the example from *On Sizes and Distances* above, the Arabic here uses the same configuration of labels.[60] It is clear that the Arabic agrees more with the second case in the Greek. But further, the Arabic proof presented here shows that it is not necessary for the line KQ (ΚΘ / ك ط) constructed in the course of the proof to be a perpendicular on the parallel lines, unlike how it was presented in the Greek. It thus takes a somewhat more general approach to proving this case.

[57] Menge (1896) 56-58.
[58] Menge (1896) 198-200.
[59] Sidoli and Isahaya (2019) 85.
[60] With the exception that the Arabic diagram is horizontally flipped relative to the Greek diagram, but it is not uncommon to see this reversal in the shift from left-to-right to right-to-left scripts.

4.2 Addition / Suppression of Alternate Proofs

The works for which we can consider the presence or absence of alternate proofs in their Arabic translation are those which either have alternate proofs or alternate recensions in Greek. The works considered here, then, are the *Data*, *On the Moving Sphere*, *Optics*, and *Phaenomena*. Alternate proofs here may be present or absent due to accidental factors – what manuscripts and what versions of these texts happened to be available to the ninth century translators – or may have been added or suppressed more intentionally.

In the case of the *Data*, whose Greek transmission saw a number of alternate proofs in circulation, we find multiple cases in the Arabic translation where one or another of the multiple proofs is absent. To summarize:

- Arabic 23: Greek 24b, not Greek 24a
- Arabic 52: Greek 54a, not Greek 54b
- Arabic 53: Greek 55a, not Greek 55b
- Arabic 64: Greek 67a, not Greek 67b, c, or d
- Arabic 65: Greek 68a, not Greek 68b
- Arabic 76: Greek 80b, not Greek 80a
- Arabic 88: Greek 91a, not Greek 91b
- Arabic 90: Greek 93a, not Greek 93b or c

The translation does not omit alternate proofs entirely – we still see one from the Greek in proposition 19. This suggests that there probably was not a conscious decision to excise them. It is more likely that the manuscript(s) used for the translation from the Greek lacked the alternate proofs missing above – perhaps they were instead added to the Greek tradition at a later date.

The *Data* furthermore has two alternate proofs which may have been added in the Arabic, as they do not appear in the Greek. These appear in Arabic propositions 37 and 60 (= Greek 39 and 62). In the case of proposition 37, this is a simplification of the proof, skipping the explicit construction of a triangle seen in the main proof by invoking the process through reference to *Elements* I.22, which accomplishes this.[61] Proposition 60, meanwhile, is either corrupted or was intended to demonstrate the simpler proof that was possible for a special case of the proposition.[62]

Meanwhile, for *On the Moving Sphere*, the Arabic reveals an alternate proof (as was noted in chapter 2). The manuscript Seray Ahmet III 3464 presents this as the primary proof, and the proof extant in the Greek follows after under a header saying it was found in another copy. This doubling of proofs was in circulation by the sixth / twelfth century by the latest, since they are both translated into Latin.[63] Not all Arabic manuscripts, however, included both proofs – the MS Bodl. Hunt 237, for instance, contains only one proof, and that is the proof not extant in Greek.[64] This would seem to suggest that the doubling of proofs in this case is to be attributed to the different versions of *On the Moving Sphere* in circulation, either in the Arabic or already in the Greek, though this alternate proof does not survive in what is extant in Greek today.

The *Optics* presents a similar situation as was seen in the *Data*: the mixed tradition results in a translation which includes some, but not all, of the alternate proofs extant in the two Greek traditions. To summarize:

- Arabic 23 = Greek A22a, not A22b or A22c (= B22)
- Arabic 37 = Greek B35a (= A34b), not B35b
- Arabic 50 = Greek A42a (= B43), not A42b
- Arabic 52 = Greek A44a, not A44b
- Arabic 60 = Greek A54b, not A54a (= B53) or A54c

[61] See the discussion in Sidoli and Isahaya (2018) 263. The proof is introduced with “And this proposition is demonstrated in another way” (وبيّن هذا الشكل على جهة أخرى) – see their edition on p.93.
[62] See Sidoli and Isahaya (2018) 282-3 for further discussion. The proof is introduced with “And this proposition is worked in another way as well” (ويعمل هذا الشكل على وجه أخرى أيضاً) – see their edition on p.137.
[63] Mogenet (1948) 149.
[64] MS Bodl. Hunt 237, fol. 76b-77a.

Furthermore, in one manuscript (Seray Ahmet III 3464), there is an alternate proof included not in the main text but in the margins to proposition 3. This proof has no connection with the Greek and seemingly was introduced in the Arabic transmission.[65] At this stage, however, we do not see codices incorporating it into the main text – we will return to it in chapter 9, as part of the discussion on al-Ṭūsī's edition.

Leiden or. 1031's witness of the *Phaenomena* preserves the alternate proofs that appear in the B recension of the Greek text:

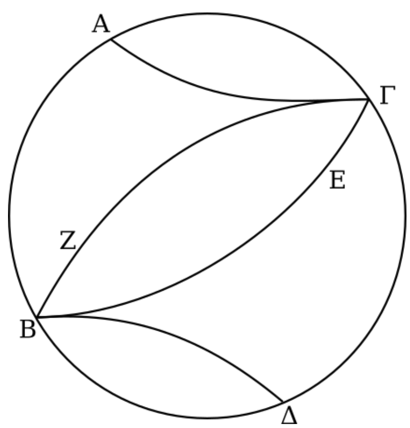
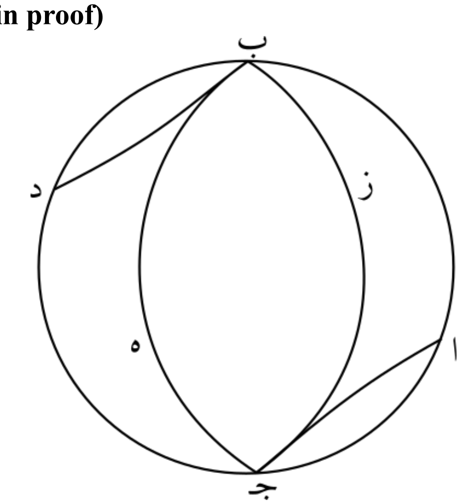
- Proposition 6: "proof of the sixth proposition in another manner" (برهان الشكل السادس على نحو آخر)[66]
- Proposition 12: "proof of the twelfth proposition in another way" (برهان الشكل الثاني عشر على جهة أخرى)[67]
- (Greek proposition 14's alternate proof is separated out into its own proposition, numbered 16)[68]
- Proposition 17 (= Greek 15): "proof of the figure in another way" (برهان الشكل على جهة أخرى)[69]

It is interesting that where propositions 6 and 12 include the proposition number in the introducing statement, proposition 17 (which disagrees with past proposition numbering schemes we have seen) omits it. There is also a small notable observation in the alternate proof to proposition 12: in the Greek, this proposition and diagram included some labels beyond the standard alphabet, including digamma (Ϛ) and sampi (ϡ). The translation in Leiden or. 1031 preserves the Greek sampi in both the Arabic text and diagram: it can be seen appearing in the same archaic pointed top form that is seen in Vat. gr. 204.[70]

Further, the translation of the *Phaenomena* attested in Leiden or. 1031 presents proposition 10 with two proofs. The Arabic text first follows the proof of the Greek recension B. When this is completed, the manuscript declares "Proof of the 10th figure according to what we found in another copy" and then

[65] Kheirandish (1999b) 34.
[66] Leiden or.1031, fol. 80b.
[67] Leiden or.1031, fol. 89a.
[68] Leiden or.1031, fol. 96a.
[69] Leiden or.1031, fol. 97b.
[70] Compare Vat. gr. 204, fol. 70v with Leiden or. 1031, fol. 91a.

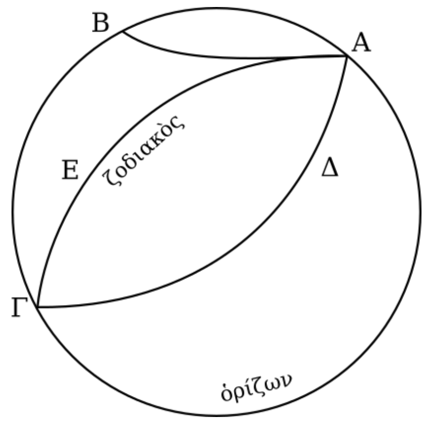
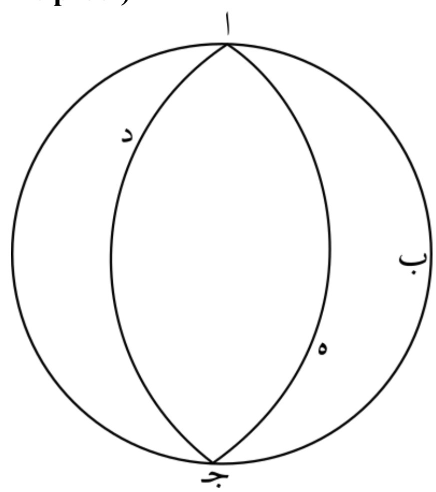
proceeds to give the proof of the proposition as it is found in the Greek recension A.[71] The following is a comparison of the Arabic and Greek texts for this proposition:

Greek *Phaenomena* proposition 10:	**Arabic *Phaenomena* proposition 10:**
(recension B) 	**(main proof)** 
If two semicircles of the circle of the zodiac rise in unequal times having some common arc, and the opposite arcs in unequal times also rise, there will also be the same difference of times in which both the semicircles rise and the opposite arcs rise, and if two semicircles of the circle of the zodiac rise in an equal time having some common arc, the opposite arcs also rise in an equal time. Let the horizon circle be ABDG, and the summer tropic AG, and the winter (tropic) BD; and the zodiac GB, and let equal arcs GE, BZ be taken. Therefore semicircles GEB, EBZ rise in unequal times. I say that arcs GE, BZ rise in unequal times. For since GEB rises in a greater time than EBZ, let the common time of rising, that of arc EB, be	If two halves of the circle of the zodiac rise in an unequal time and they have a common arc, then the opposite arcs also rise in unequal times and the difference of time in which the semi(circles) and the opposite arcs rise is one and the same. And if two halves of the circle of the zodiac rise in equal times and they have a common arc, then the opposite arcs also rise in an equal time. Example of this: we make the horizon circle ABGD and the summer tropic AG and the winter tropic BD. And we make the circle of the ecliptic GB and we define two equal arcs, which are GE, BZ, and so semicircles GEB, EBZ rise in an unequal time. I say that arcs GE, BZ also rise in an unequal time.

[71] Leiden or. 1031, fol. 86a: "برهان الشكل العاشر على ما وجدنا في نسخة أخرى"

taken away; for the arc EB always rises in an equal time to itself; then the remaining arc GE rises in a greater time than BZ, and (it is) clear that the same differences are between the times in which both semicircles GEB, EBZ rise and opposite arcs GE, BZ (rise). And it is clear that, if some semicircles rise in equal times, the opposite arcs in equal times also rise.	Proof of this: semicircle GEB rises in a time longer than the time in which semicircle EBZ rises. And the time of the rising of the common [arc] EB is subtracted, and so the rising of arc GE is maintained in a time longer than the time in which arc BZ rises. And it is clear that the difference of times in which the semicircles GEB, EBZ rise and the opposite arcs, I mean arcs GE, BZ, is the same one. And it is demonstrated also that when some semicircle rises in an equal time, so the arcs opposite to them rise in a time equal to it.
Ἐὰν τοῦ τῶν ζῳδίων κύκλου δύο ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατέλλῃ κοινήν τινα ἔχοντα περιφέρειαν, καὶ αἱ ἀπεναντίον περιφέρειαι ἐν ἀνίσοις χρόνοις ἀνατέλλουσιν, καὶ ἡ αὐτὴ διαφορὰ ἔσται τῶν χρόνων, ἐν οἷς τά τε ἡμικύκλια ἀνατέλλει καὶ αἱ ἀπεναντίον περιφέρειαι ἀνατέλλουσιν· καὶ ἐὰν τοῦ τῶν ζῳδίων κύκλου δύο ἡμικύκλια ἐν ἴσῳ χρόνῳ ἀνατέλλῃ κοινήν τινα ἔχοντα περιφέρειαν, καὶ αἱ ἀπεναντίον περιφέρειαι ἐν ἴσῳ χρόνῳ ἀνατέλλουσιν. ἔστω κύκλος ὁρίζων ὁ ΑΒΔΓ, καὶ θερινὸς μὲν τροπικὸς ὁ ΑΓ, χειμερινὸς δὲ ὁ ΒΔ, ζῳδιακὸς δὲ ὁ ΓΒ, καὶ ἀπειλήφθωσαν ἴσαι περιφέρειαι αἱ ΓΕ, ΒΖ· τὰ ἄρα ΓΕΒ, ΕΒΖ ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατέλλει· λέγω, ὅτι καὶ αἱ ΓΕ, ΒΖ περιφέρειαι ἐν ἀνίσοις χρόνοις ἀνατέλλουσιν. ἐπεὶ γὰρ τὸ ΓΕΒ τοῦ ΕΒΖ ἐν πλείονι χρόνῳ ἀνάτελλει, κοινὸς ἀφῃρήσθω ὁ τῆς ΕΒ περιφερείας ἀνατολῆς χρόνος· ἡ γὰρ ΕΒ περιφέρεια ἑαυτῇ ἀεὶ ἐν ἴσῳ χρόνῳ ἀνατέλλει· λοιπὴ ἄρα ἡ ΓΕ τῆς ΒΖ ἐν πλείονι χρόνῳ ἀνατέλλει, καὶ φανερόν, ὅτι αἱ αὐταὶ διαφοραί εἰσι	اذا طلعا نصفان من دائرة البروج في ازمان غير متساوية وكانت لهما قوس مشتركة فان القسي ايضا المتقابلة يطلع في ازمان غير متساوية ويكون اختلاف الازمان التي تطلع فيها النصفان والقسي المتقابلة واحد بعينه واذا طلعا نصفان من دائرة البروج في ازمان متساوية وكانت لهما قوس مشركة فالقسي ايضا المتقابلة تطلع في ازمان متساوية مثال ذلك ان نفرض الأفق دائرة (ا ب جـ د) و والمتقلب الصفي (ا جـ) والمتقلب الشوي (ب د) ونجعل دائرة البروج (جـ ب) ونفصل قوسين متساويتين وهما (جـ ه) (ب ز) فنصفا دائرة (جـ ه ب) (ه ب ز) تطلعان في ازمان غير متساوية فأقول أن قوسي (جـ ه) (ب ز) أيضا يطلعان في ازمان غير متساوية برهان ذلك إن نصف دائرة (جـ ه ب) تطلع في ازمان اطول من الزمان الذي يطلع فيه نصف دائرة (ه ب ز) ويسقط زمان طلوع (ه ب) المشترك فيبقي طلوع قوس (جـ ه) في زمان اطول من الزمان الذي يطلع فيه قوس (ب ز) ومن البين إن اختلاف الازمنة التي تطلع فيها نصفا دائرة (جـ ه ب) (ه ب ز) والقويتين المتقابلتين اعني قوس (جـ ه) (ب ز) واحد بعينه ويبين أيضا إنه متى طلعت نصف دائرة ما في ازمان متساوية لأن القسي المقابلة لها يطلع في ازمان متساوية لها[73]

[73] Leiden or. 1031 fol. 84b, 86a. Note that folo 85ab is skipped because it was mistakenly inserted early, and so interrupts proposition 10 with the text of propositions 11 and 12.

τῶν χρόνων, ἐν οἷς τά τε ΓΕΒ, ΕΒΖ ἡμικύκλια ἀνατέλλει καὶ αἱ ἀπεναντίον περιφέρειαι αἱ ΓΕ, ΒΖ. φανερὸν δέ, ὅτι, κἂν ἡμικύκλιά τινα ἐν ἴσοις χρόνοις ἀνατέλλῃ, καὶ αἱ ἀπεναντίον περιφέρειαι ἐν ἴσοις χρόνοις ἀνατέλλουσιν.[72]	
(recension A) 	**(alternate proof)** 
In the cosmos let the horizon be ABG, and let the circle of the zodiac have position AEGD, and let equal arcs AD, GE be taken; therefore, D is diametrically opposite to E. And let semicircles ADG, DGE in unequal times rise. I say that also the opposite arcs AD, GE in unequal times rise, and the same difference is in the times in which semicircles ADG, DGE rise and in which arcs AD, GE rise. For since the semicircles ADG, DGE in unequal times rise, let the common rising time, that of DG, be taken away; (for, the arc DG always rises in a time equal to itself); then the remaining arcs AD, GE in unequal times rise, and the same differences are between the times in which semicircles ADG, DGE rise and the opposite arcs AD, GE (rise).	Proof of the 10th figure according to what we found in another copy: we make the horizon circle ABG and let the location of the circle of the ecliptic be the position which AEGD is on, and we choose two equivalent (arcs) which are arcs AD, GE; so point D therefore is opposite to point E, and let segments ADG, DGE rise (it is found in the Syriac) in unequal times. I say that the opposite arcs, which are arcs AD, GE, rise in an unequal time, and the difference of their risings is the difference of the rising of arcs ADG, DGE, and arcs AD, DE are one and the same. Proof of this: the sections ADG, DGE rise in unequal times and we subtract the rising time of section DG in common, and this rising time of arc DG is one time, so arcs AD, GE therefore rise in

[72] Menge (1916) 54-56, lower text.

Indeed again, (suppose) the semicircles ADG, DGE in equal times rise. Let the common time of arc GD be taken away; then the remaining AD, GE in an equal time rise.	unequal times, and the difference of times in which sections ADG, DGE rise and opposite arcs to them, I mean arc AD and arc GE, are one and the same. And also sections ADG, DGE rise in one and the same time. I say that arcs AD, GE rise in one and the same time. Proof of this: sections ADG, DGE rise in one time and the rising time of arc DG in common is subtracted [from it[74]], therefore arcs AD, GE rise in one time. And this is what we wanted to prove.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΕΓΔ, καὶ ἀπειλήφθωσαν ἴσαι περιφέρειαι αἱ ΑΔ, ΓΕ· κατὰ διάμετρον ἄρα ἐστὶ τὸ Δ τῷ Ε. τὰ δὲ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατελλέτω· λέγω, ὅτι καὶ αἱ ἀπεναντίον πεμιφέρειαι αἱ ΑΔ, ΓΕ ἐν ἀνίσοις χρόνοις ἀνατέλλουσι καὶ ἡ αὐτὴ διαφορά ἐστι τῶν χρόνων, ἐν οἷς τὰ ΑΔΓ, ΔΓΕ ἡμικύκλια ἀνατέλλει καὶ ἐν οἷς αἱ ΑΔ, ΓΕ περιφέρειαι ἀνατέλλουσιν. ἐπεὶ γὰρ τὰ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατέλλει, κοινὸς ἀφῃρήσθω ὁ τῆς ΔΓ ἀνατολῆς χρόνος· (ἡ γὰρ ΔΓ περιφέρεια ἑαυτῇ ἀεὶ ἐν ἴσῳ χρόνῳ ἀνατέλλει)· λοιπαὶ ἄρα αἱ ΑΔ, ΓΕ περιφέρειαι ἐν ἀνίσῳ χρόνῳ ἀνατέλλουσι καὶ αἱ αὐταὶ διαφοραί εἴσι τῶν χρόνων, ἐν οἷς τά τε ΑΔΓ ΔΓΕ ἡμικύκλια ἀνατέλλει καὶ αἱ ἀπεναντίον περιφέρειαι αἱ ΑΔ, ΓΕ. πάλιν δὴ τὰ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἴσῳ χρόνῳ	برهان الشكل العاشر على ما وجدنا في نسخة أخرى نفرض الأفق دائرة (ا ب جـ) وليكن وضع دائرة البروج منزلة ما عليه (ا ه جـ د) ونفصل متساويتين وهما قوسا (ا د) (جـ ه) فنقطة (د) إذن مقابلة لنقطة (ه) و ليطلع قطعتا (ا د جـ) (د جـ ه)[76] وجد في السرياني في ازمان غير متساوية أقول إن القوسين المتقابلتين وهما قوسا (ا د) (جـ ه) تطلعان في ازمان غير متساوية واختلاف مطالعهما هو اختلاف مطالع قوسي (ا د جـ) (د جـ ه) وقوسا (ا د) (جـ ه) واحد بعينه برهان ذلك إن قطعي (ا د جـ) (د جـ ه) تطلعان في ازمان غير متساوية ونسقط زمان طلوع قطعة (د جـ) المشترك وذلك إن زمان طلوع قوس (د جـ) زمان واحد فقوسا (ا د) (جـ ه) إذن يطلعان في زماني غير متساو واختلاف الازمنة التي يطلع فيها قطعتا (ا د جـ) (د جـ ه) والقسي المقابلة لهما اعنى قوس (ا د) وقوس (جـ ه) واحد بعينه وأيضا فلنطلع قطعتا (ا د جـ) (د جـ ه) في زمان واحد بعينه أقول إن قوسي (ا د) (جـ ه) تطلعان في زمان واحد بعينه

[74] The word in the manuscript is unclear and may be an error for منها.

[76] An error of dittography appears to have occurred here: "فنقطة (د) إذن مقابلة لنقطة (ه) وليطلع (ا د جـ) (د جـ ه)-" is repeated. It has been removed from the above text.

ἀνατέλλει· κοινὸς ἀφῃρήσθω ὁ τῆς Γ περιφερείας χρόνος· λοιπαὶ ἄρα αἱ ΑΔ, ΓΕ ἐν ἴσῳ χρόνῳ ἀνατέλλουσιν.[75]	برهان ذلك إن قطعتي (ا د جـ) (د جـ ه) تطلعان في زمان واحدا ويسقط زمان طلوع القوس (د جـ) المشترك فبقى[؟] إذن قوسا (ا د) (جـ ه) تطلعان في زمان و)احد) وذلك ما أردنا أن نبين[77]

Table 4.14: Comparison of Greek and Arabic texts of *Phaenomena* proposition 10

So in this case, the Arabic tradition did not itself produce a new alternate proof, but rather, when the scholar responsible encountered a variant in another manuscript, he found it useful to present a proposition combining the two versions. It is unclear at what point the alternate proof was introduced into the text. While it could have occurred during the text's original translation in the ninth century, it may have been the work of the 12th century scholar Ibn al-Salah – whose manuscript copy was an ancestor of this particular manuscript – or of any unknown scholar in the transmission leading up to him. And while this is an interesting example of an added alternate proof, proposition 10 was not the only one to diverge between *Phaenomena* recensions A and B. But it is the only material from A to be included in Leiden or. 1031's text. It is unclear whether this is because of what was available in the lost Syriac source or because this proposition was of particular interest.

An alternate proof can be found added to *On Sizes and Distances* proposition 13 (= Greek 15), introduced with the same language "in another way" (على جهة أخرى).[78] A version of the argument in question can be found in Greek, appearing as a scholium on Vat. gr. 204, fol. 117r. Berggren and Sidoli point out the approach taken in the Arabic, which they attribute to Thābit, is more efficient and requires fewer steps.[79]

[75] Menge (1916) 54-56, main text.
[77] Leiden or. 1031 fol. 86a.
[78] Columbia Or. 45, fol. 46a.
[79] Berggren and Sidoli (2007) 236.

The phenomenon of alternate proofs, then, is at this stage shaped by multiple factors. Some cases of addition are clearly ones where a scholar encountered a variant proof because of the multiple versions in circulation and chose to present both versions. But in works which modern scholars see as having eventually developed multiple recensions, not all alternate proofs are presented, nor (for the *Data* and *Optics*) all proofs from what is today considered one recension. It is not clear to what extent this is the result of accidents – what sources happened to be available to translators and what form they took in the ninth century – and to what extent it is the result of deliberate choices – early scholars selecting which version of the proof they found more useful.

4.3 Addition / Suppression of Cases

Similarly as was done for alternate proofs, we can compare the presence or absence of cases in the Arabic translations with what was known from the Greek. Overall, we tend to see a decrease in cases. There are comparatively fewer instances of addition, some of which reflect the multiple Greek traditions. And then there are a couple which seem unique to the Arabic tradition. The texts considered here are the *Data*, *Sphaerica*, *Optics*, and *Phaenomena*.

In the Arabic *Data*, propositions 33, 42, and 77 (= Greek 34, 44, and 81) show an absence of cases present now in Greek. Proposition 33 has already been discussed in section 4.1 and reflects the alternate case of the Greek proof. Sidoli and Isahaya have pointed out that this proposition – specifically its second case – was used in Apollonius's *Cutting off a Ratio*. They posit that since Apollonius's text was translated by Thābit's circle, this may have motivated the preference for the second case in the Arabic text of the *Data*.[80] For proposition 42, the editors suggest either the case was not in the source manuscript, or

[80] Sidoli and Isahaya (2018) 258-259.

Thābit removed it as unnecessary.[81] But the example of proposition 77 shows both the case and the mention of it in the enunciation absent from the text.[82] They see little potential reason for its removal, and indeed the absence of it from both locations suggests that this was material added to the Greek *Data* at some point in its transmission separate from the source text used for the Arabic translation.

The *Sphaerica* II.14 (= Greek II.15), as discussed in chapter 2, has a second case added to its text, and later in one branch of the tradition a third case. This second case makes its way into Arabic; the third does not. In *Sphaerica* I.2 the Arabic also adds a brief and trivial acknowledgement of the case where the plane cuts the center of the sphere, which is not in Greek.

In the *Optics*, the Arabic translation lacks cases both from the A recension and the B recension. To summarize:

- Proposition 37 lacks an extra case
- Proposition 43 lacks the final case in A
- Proposition 45 lacks a case present in B
- Proposition 50 lacks a case present in A
- Proposition 52 lacks a case present in A
- Proposition 60 lacks cases and corresponds only to case 2 of 3 in A
- Proposition 64 lacks the final case in both A and B

So in the case of the *Optics*, we find only subtractions, no additions to the text at the level of cases. There is not an easy pattern that emerges here – cases unique to recension A sometimes do not appear, cases unique to B sometimes do not appear, and cases present in both recensions also sometimes do not appear in the Arabic text. It is possible that the Arabic was drawing on Greek texts prior to the addition of these cases. It is much less likely that they were intentionally suppressed for some reason.

[81] Sidoli and Isahaya (2018) 267.
[82] Sidoli and Isahaya (2018) 297.

The extra cases in the Greek *Phaenomena* were found in propositions 2, B11, and B12. All of these are present in Leiden or. 1031's witness of the Arabic text. The inclusion of these cases in 11 and 12 further support how this translation follows the B recension of the Greek text.

4.4 Change in Order of Propositions

Compared to what was seen in the Greek transmission in chapter 2, there is a greater frequency of propositions (and definitions) being reordered in the Arabic translations. The relevant texts are the *Data*, the *Sphaerica*, the *Optics*, and *On Days and Nights*.

Three cases of rearranged material occur in the Arabic *Data*. The third and fourth definitions are inverted between the Arabic and the Greek versions. Sidoli and Isahaya raise the suggestion that this change allows the order of the definitions to reflect the order in which they see use in the subsequent propositions of the text.[83] A potentially similar instance is seen in the case of Arabic proposition 10 (= Greek 12). This proposition is moved back and placed before Greek proposition 10. The Arabic text's modern editors suggest similar logical concerns as possible reasons for the move.[84] We also find that propositions 61 and 62 in the Arabic are inversions of Greek propositions 65 and 64, though there is not an immediately apparent reason for this.

An inversion also occurs in the *Sphaerica*, where the Arabic propositions II.12 and II.13 are inversions of Greek propositions II.14 and II.13. Similarly, in *On Days and Nights*, Arabic propositions I.6 and I.7 are Greek propositions I.7 and I.6, and Arabic propositions I.9 and I.10 are Greek propositions I.10 and I.9.

[83] Sidoli and Isahaya (2018) 222-223.

[84] Sidoli and Isahaya (2018) 235. They point out that in the Greek, this proposition's placement between propositions 11 and 13 situates it between two propositions which concern the relation greater-by-a-known-than-in-ratio, but proposition 12 itself does not concern this relation. They also note that the new order better matches the order of definitions.

Another more clearly motivated instance of rearranged propositions occurs in the *Optics*, where Arabic proposition 48 is postponed several propositions later, despite the fact that it treats a case relevant to Arabic proposition 43 (which it would have followed after, had it agreed with the Greek arrangement). Kheirandish suggests this reordering arose because of a deliberate choice to group the proposition instead with a series of later propositions on fixed objects and displaced eye conditions, for which it is also relevant.[85]

While reordered propositions are by no means common here, it will be worth remembering the frequency seen at this stage of transmission when we later discuss the phenomenon in al-Ṭūsī's edition. Reordering propositions changes the numbering of the relevant propositions and sometimes others around them. At this point, we see less interest from the Arabic scholars in perfectly maintaining a canonical numbered structure.

4.5 Fusion / Division of Propositions

Potential fusion or division of propositions can be found in the *Data*, the *Sphaerica*, the *Optics*, the *Phaenomena*, and *On Sizes and Distances*.

In the case of the *Data*, fusion of propositions can be seen in different manuscript witnesses. Three propositions in Seray Ahmet III 3464, for instance, are divided in two in the Kraus manuscript: propositions 19, 37, and 43 (= Greek 19, 39, and 45). In the first this is a second case split off as its own proposition; in the second and third it is the alternate proofs which are numbered separately.[86]

[85] Kheirandish (1999) 82, 86.
[86] Sidoli and Isahaya (2018) 320.

The *Sphaerica* has been touched on already in chapter 2 because the evidence from Pappus suggests that Greek propositions II.11 and 12 circulated as one at an early date. The Arabic tradition maintains that situation: Arabic proposition II.11 is indeed Greek II.11-12.[87]

The Arabic *Optics* tends to divide multipart propositions into separate ones. Thus we find the following:

- Arabic 6-7 = Greek A6 / B6
- Arabic 29-30 = Greek A28 / B28
- Arabic 36-37 = Greek A34, but B35-36
- Arabic 38-41 = Greek A35 / B36
- Arabic 45-47 = Greek B40

In some cases, this is again following what is actually found in the Greek manuscripts (rather than the modern editions), such as the example of Arabic propositions 38-41. Vatican gr. 204, similarly, divides Heiberg's proposition B36 into propositions 36-40.[88] Similarly to what was seen for the *Optics* in chapter 2, these divided propositions continue to lack the internal structure that would be expected of a new proposition. Several of them lack a new enunciation.

The *Phaenomena* is a similar case since, like the Greek manuscripts, Menge's proposition 14 is presented in the Arabic as propositions 14-15. In addition, proposition 16 in Leiden or. 1031 is equivalent to the Greek's alternate proof to B14.

The above are largely cases of division. The Arabic *On Sizes and Distances* shows two cases of fusion. In Aristarchus's text, Greek proposition 5 is quite short and structurally lacks much of what is typically expected from a proposition: it does not have a diagram, it does not have an exposition, and so on. In the Arabic translation preserved in MS Columbia Or. 45, it is not labeled as a separate proposition

[87] Kunitzsch and Lorch (2010b) 373 fn.5.
[88] And this in fact includes a fifth part of the proposition which did not enter into Arabic.

at all, and instead serves as a corollary at the end of Arabic proposition 4. There is no "and this is what we wanted to demonstrate" preceding it, for example – that formula does not appear in the Arabic until the end of the material that was Greek proposition 5. Greek proposition 8 is a similar case – it is comparatively short, it does not involve an exposition or a diagram, etc. It therefore ends up fused with either the preceding proposition (Greek proposition 7) or the following one (Greek proposition 9): in the Kraus manuscript we find the former occurs, and in MS Columbia Or. 45 we find the latter occurs.[89] As was the case with Greek proposition 5, here too the concluding formula ("and this is what we wanted to demonstrate") does not appear in the Arabic of MS Columbia Or. 45 until the end of the full fused proposition.

4.6 Change in Status

The deliberate alteration of a change in status largely does not occur at this stage of these works' transmissions. There are potential examples only in *On Days and Nights*, which require further study when the text is more accessible and which present somewhat of a mixed case. These examples do not quite fit under fusion of propositions since it is not propositions alone which are being fused together. Rather, the various porisms and lemmas in the Greek text are seen to lose their independent status in the Arabic translation, and they become merged with the surrounding propositions.[90]

[89] Berggren and Sidoli (2007) 216 reports on the alignment of propositions between the Greek and the Kraus manuscript.

[90] Kunitzsch and Lorch (2011). The porism after Greek proposition I.2 is merged with it in the Arabic, concluded with the usual QED ending for a proposition (see p.33 fn.7). The lemma for proposition I.4 is merged with the end of proposition 3, again concluded with the formulaic QED (p.34 fn.8). The porism after proposition I.4 is similarly merged with that proposition (p.34, fn.9). A "quasi lemma" (according to Fecht) follows Greek proposition I.8; in the Arabic it becomes merged with the enunciation of proposition 9 (p.12, 37 fn.18).

4.7 Addition / Suppression of Material

A significant amount of material extant in Greek is absent from the Arabic versions of these texts. For the *Data*, we have already noted the absence of most of the alternate proofs; it also can be seen to lack entire propositions. These are Greek propositions 21, 36, 63, and 79, all of which Sidoli and Isahaya suppose were most likely not present in the sources from which the translators worked.[91] They acknowledge that it is not impossible that Thābit, correcting the text, may have excised one or more, but see this as unlikely in light of other instances where Thābit left trivial and unnecessary propositions untouched, such as Arabic proposition 74 (= Greek 77).[92]

The Arabic *Sphaerica* lacks the final two propositions of book I which are understood to have been later additions to the Greek text. Otherwise, this text largely shows addition of material: the definitions are increased up to a count of eleven from the six currently present in the Greek. One proposition is also added to book I: proposition I.9, which joins proposition I.8 as another converse of I.7:

[91] Sidoli and Isahaya (2018) 321-333.
[92] Sidoli and Isahaya (2018) 295.

Proposition I.7	Proposition I.8	Proposition I.9
If a circle is on a sphere and what is between the center of the sphere and the center of the circle is joined by a line, then the line which is joined between them is a perpendicular on the surface of the circle.	If a circle is on a sphere and a perpendicular is drawn from the center of the sphere on it and it is produced to both sides, then it falls on the two poles of the circle.	If a circle is on a sphere and what is between one of its poles and the center is joined by a straight line, then the line is a perpendicular on the circle.
إذا كانت دائرة فى كرة وؤصل ما بين مركز الكرة وبين مركز الدائرة بخط فإن الخط الذى يصل بينهما يكون عموداً على سطح الدائرة.[93]	إذا كانت دائرة فى كرة وأخرج من مركز الكرة عمود عليها وأنفذ إلى كلتى الناحيتين فإنه يقع على قطبى الدائرة.[94]	إذا كانت دائرة فى كرة وؤصل بين أحد قطبيها وبين المركز بخط مستقيم فإن الخط عمود على الدائرة.[95]

Table 4.15: Comparison of enunciations to Arabic *Sphaerica* I.7-9

The addition of propositions to this treatise suggests an interest in adding further preliminary material; the addition of this converse proposition suggests an interest in making the text more comprehensive.

Meanwhile, multiple manuscripts of the Arabic *Sphaerica* include versions of a lemma at the end of book III (sometimes called a "proposition" ("الشكل") though not numbered with the rest). This lemma is to *Sphaerica* III.11, in which Theodosius states an inequality without proof; the lemma endeavors to fill the gap.[96] One manuscript explicitly attributes a version of this lemma to Thābit, showing that efforts in Arabic to address this gap were ongoing soon after the text's translation.[97] Earlier versions of this lemma

[93] Kunitzsch and Lorch (2010b) 34.
[94] Kunitzsch and Lorch (2010b) 38.
[95] Kunitzsch and Lorch (2010b) 42.
[96] See Kunitzsch and Lorch (2010b) 316-327 and 419-427.
[97] Kunitzsch and Lorch (2010b) 320 and 423. The manuscript in question is Paris hebr. 1101. Al-Ṭūsī also attributes a version of this lemma to Thābit in a comment on his own edition of the *Sphaerica* – for al-Ṭūsī's approach to this lemma, see Sidoli and Kusuba (2008) 22-27.

are extant from late antiquity, as Greek scholars were approaching the same gap in the commentaries and scholia they produced.[98]

In the case of *On Days and Nights*, Kunitzsch and Lorch note that the four "assumpta in sequentes propositiones" that follow after proposition II.4 do not appear in the Arabic text, and that there is no clear correspondence between the Arabic propositions II.10, 12, and 13 and those in the Greek.[99] We also see in the Arabic only three definitions to the five extant in Greek – the third and fourth, on the exchanges of arcs across the visible and invisible hemispheres, do not appear.[100] Interestingly, in Leiden or. 1031's witness of the *Phaenomena*, its preface ends earlier than the preface in the Greek. The Arabic preface ends just before the definitions on the time of revolution of the cosmos and on the exchanges of arcs across the visible and invisible hemispheres.[101] That these definitions are missing from both the Arabic *On Days and Nights* and *Phaenomena* suggests they were added to the Greek of those texts later; further, their shared subject matter suggests they may have been added by the same individual. But there may be several layers of additions occurring – the Arabic witness to *On Days and Nights* still contains the definition on the time of revolution of the cosmos.

4.8 <u>References to the Curriculum within the Texts</u>

Chapter 2 found that references within the texts under study to other texts are quite infrequent. They are even more infrequent in these translations. Only the *Phaenomena* directly cites other texts. The reference to the *Optics* seen in the Greek preface persists (وانها تكون ذلك فقط فيها حركة على الاستدارة إذا كان بعد الناظر من القوس من جمع النواحي بعدا متساويا كما بيّنا ذلك في كتاب المناظر).[102] Proposition 12 also includes a citation of

[98] See Knorr (1985), which discusses a significant number of the ancient versions of this and related lemmas.
[99] Kunitzsch and Lorch (2010a) 12 and 36 fn.17.
[100] Kunitzsch and Lorch (2010a) 15.
[101] Leiden or. 1031, fol. 76b.
[102] Leiden or. 1031, fol. 76b.

Sphaerica III.3 (ولما نوحيه البرهان على الشكل الثالث من المقالة الثالثة من كتاب الأكر).[103] We may recall that Greek recension A had a reference to *Sphaerica* III.6 in its proposition 12 – though this is a different citation, it is interesting that both appear in the same proposition.

There is also a case of *On Days and Nights* referring back to material priorly established in its own text: "That is because it has become clear in the first book that, when the sun reaches the solstice, the distance of the sunrise and the setting from the point of contact of the solstice is equal" (وذلك أنه قد تبين فى المقالة الأولى أنه إذا عملت الشمس الانقلاب يكون بُعد الشروق والغروب من نقطة مماسة الانقلاب بعداً متساوياً).[104] This occurs in the enunciation to proposition 9: which in the Arabic version this "enunciation" is a fusion of a quasi-lemma which precedes it, the original enunciation itself, and scholia material. It seems incorporation of scholia material may be the reason for this reference to the text's preceding book.

4.9 Referential Scholia

Chapters 1 and 2 discussed a category of referential scholia which appear in manuscripts of the Little Astronomy, serving as citations back to mathematical points that had been previously demonstrated in the ordered grouping. These referential scholia appear in the Middle Books as they had in the Little Astronomy. They are similarly brief and formulaic, as can be seen in the examples "[from] the twentieth [proposition] of the first [book] of the *Sphaerica* of Theodosius" (ك من ا من الاكر لثادسيوس) and "[from] the second [proposition] of the book of Autolycus *On the Moving Sphere*" (ب من كتاب اوطولوقس في الكرة المتحركة).[105] Subsequent citations of these works are further shortened by omitting the author. These scholia, however, are not present in all manuscripts of the Middle Books. For manuscripts of the

[103] Leiden or. 1031, fol. 90a.
[104] Kunitzsch and Lorch (2011) 23.
[105] Both marginal comments can be found in Leiden Or. 1031, fol. 76b.

curriculum prior to al-Ṭūsī's edition, currently they have been located in Leiden or. 1031, British Library Or. 13127, and Seray Ahmet III 3464.[106]

In the codex Leiden or. 1031, *On Risings and Settings* cites the *Phaenomena*, *On Days and Nights*, and *On the Moving Sphere*. The *Sphaerica* cites the *Elements* and itself. The *Phaenomena* cites itself, the *Sphaerica*, and *On the Moving Sphere*. While in this particular manuscript these three treatises do not follow the more standard order of the Middle Books, their referential scholia still follow the pattern of referencing only works (or internal propositions) which are earlier according to Vat. gr. 204's arrangement. The following is a listing of the referential scholia in Leiden or. 1031 which is complete for *On Risings and Settings* and the *Phaenomena*, but which omits the *Sphaerica* since that work will be compared with another source below.

Folio	Appears on	Referential Scholia[107]	Cites
3b	RS.1.prop.4	و من الطاهرات	Phaen.prop.6
4a	RS.1.prop.5	ط من الكره المىحركه	MS.prop.9
5b	RS.1.prop.6	ىو من ب من الليل والنهار	DN.2.prop.16
5b	RS.1.prop.6	ير من ب من النهار والليل	DN.2.prop.17
76a	Phaen.intro	ا من ا من الاكر لىاودسىوس	Sph.1.prop.1
76b	Phaen.intro	ك من ا من الاكر لىاودسىوس	Sph.1.prop.20
76b	Phaen.intro	ب من كتاب اوطولوقس فى الكره المتحركه ه	MS.prop.2
76b	Phaen.intro	ىب من الكره المتحركة	MS.prop.12
77b	Phaen.prop.2	حاسىه ط من ب من الاكر	Sph.2.prop.9
77b	Phaen.prop.2	حاسىه ه من ب من الاكر	Sph.2.prop.5
78a	Phaen.prop.2	ىو من ا من الاكر	Sph.1.prop.18

[106] These referential scholia do not appear in Fatih 3414, Huntington 237, Columbia Or. 45, Paris arabe 2457, Bodl. Or. 365, Leiden Or. 399, or Leiden Or. 133. As is noted below, it would be desirable to examine other texts in Seray Ahmet III 3464. It would also be desirable to examine the Kraus manuscript.

[107] The scholia in these tables are transcribed as largely undotted, according to how they are written in the manuscripts.

78a	Phaen.prop.2	ٮه من ٮ من الاكر	Sph.2.prop.15
78a	Phaen.prop.2	ر من ٮ من الاكر	Sph.2.prop.7
78b	Phaen.prop.2	ٮد من ٮ من الاكر	Sph.2.prop.14
78b	Phaen.prop.3	ر من الكره المتحركه	MS.prop.7
79a	Phaen.prop.4	ب من الكره المتحركه	MS.prop.2
80b	Phaen.prop.6	ٮط من ٮ من الاكر	Sph.2.prop.19
81b	Phaen.prop.7	و من هذا الكٮاب	Phaen.prop.6
82b	Phaen.prop.8	ٮٮ من ٮ من الاكر	Sph.2.prop.12
83a	Phaen.prop.8	ٮح من ٮ من الاكر	Sph.2.prop.18
83b	Phaen.prop.9	ٮط من ٮ من الاكر ومن صدر هدا الكٮاٮ	Sph.2.prop.19, Phaen.intro
84b	Phaen.prop.10	ط من هذا الكٮاٮ	Phaen.prop.9
85a	Phaen.prop.12[108]	ح من حـ من الاكر	Sph.3.prop.8
86b	Phaen.prop.11	و من هذا لكٮاٮ	Phaen.prop.6
87b	Phaen.prop.11	حـ من حـ من الاكر	Sph.3.prop.3
91a	Phaen.prop.11	ح من ح من الاكر	Sph.3.prop.3
91b	Phaen.prop.13	ٮٮ من ٮعد الكٮاب	Phaen.prop.12
92a	Phaen.prop.13	ٮا من هذا الكٮاب	Phaen.prop.11
92b	Phaen.prop.14	اخر سكل ٮد من ٮ من الاكر	Sph.2.prop.14
94a	Phaen.prop.15	احر شكل ٮد من ٮ من الاكر	Sph.2.prop.14
95a	Phaen.prop.15	احر ٮٮ ٮحـ من هذا الكٮاب	Phaen.prop.12, Phaen.prop.13

Table 4.16: Referential scholia on *Risings and Settings* and the *Phaenomena* in Leiden or. 1031

The scholia in the *Phaenomena* are more thorough than those for *On Risings and Settings* – in the latter, they trail off after the sixth proposition of the first book. They do however continue throughout the *Sphaerica*.

[108] Note the propositions are out of order because folio 85ab was inserted early.

Comparing the referential scholia in the *Phaenomena* between Leiden Or. 1031 and the Greek manuscripts shows that there is overlap between the two sets and that unsurprisingly the Greek manuscripts preserve a large amount of scholia which do not appear in the single Arabic manuscript. Leiden Or. 1031, however, also preserves scholia that are not attested in the extant Greek materials.[109] Some of these apparent divergences may be caused by differences in the numbering of propositions between manuscript witnesses. For example in Leiden or. 1031, fol. 78b, the Arabic *Phaenomena* proposition 2 cites *Sphaerica* II proposition 14, which corresponds to the Greek *Sphaerica* II prop.13 in at least one of the Arabic transmissions.[110] In the Greek tradition, MS Vat. gr. 204 does not have a citation for *Sphaerica* II prop.14 in this location but it does have one for prop.13.[111] The fact that proposition numbers appear to have been updated to reflect what was used in the Arabic shows that scribes were not mechanically copying these marginal comments: there was active engagement that kept these cross-textual links up to date.

Other divergences, however, are clearly distinct. *On Risings and Settings* in Leiden Or. 1031 cites *On Days and Nights*, a link which is not seen in the extant Greek scholia.[112] This instance must arise either from early Greek scholia which have not survived or from the work of Arabic scholars.

British Library Or. 13127's witness of the *Spherics*, meanwhile, contains a reference back to Theodosius's *Sphaerica*.[113] Since Menelaus's *Spherics* is no longer extant in Greek, this cannot be compared with the Greek evidence.

[109] This is based on examination of Vat. gr. 204 and the edited scholia in Menge (1916).
[110] Though this does not appear to be the case for Leiden or. 1031's witness of the *Sphaerica*.
[111] Vat. gr. 204, fol. 61v.
[112] Leiden or. 1031, fol. 5b.
[113] British Library Or. 13127, fol. 3b.

Kunitzsch and Lorch's edition of the Arabic *Sphaerica* shows that it is joined by scholia citing the *Elements* in Seray Ahmet III 3464.[114] They appear on the first eight propositions of the first book but are not continued beyond this.[115] These citations are extremely abbreviated, and in fact are similar to the very terse citations of the *Elements* found also in Leiden or. 1031's *Sphaerica*. In Leiden or. 1031, these scholia follow a format like "[proposition] 47 of [book] 1 of Euclid" (مز من ا ق),[116] sometimes with the من omitted. The letter qāf at the end should be understood as an abbreviation for Euclid (إقليدس). Meanwhile in Seray Ahmet III 3464, only the numeral of the proposition, then the numeral of the book, then the abbreviation "س" is provided, all marked with overlines. The letter "س" should be taken to refer to Euclid's *Elements* – not via its Arabic name, *al-Uṣūl* (الأصول), but via the transliteration of its Greek title, Στοιχεῖα, rendered as *Usṭuqussāt* (أسطقسات).[117] These especially abbreviated forms of cross-textual citation might be because these are citations to the *Elements*: this was plausibly a central enough text and a common enough target of citation that mathematical scholars and students were well acquainted with this terse citation style. We will see in chapter 9 that citation of the *Elements* in al-Tusi's edition is similarly condensed.

Note that in Leiden or. 1031's *Sphaerica*, the self-referential citations are also quite abbreviated, giving only the numeral of the proposition, the numeral of the book, and the letter kāf.[118] The abbreviation

[114] These are also noted in Martin (1975) 234, who records them as occurring between folios 20b and 23a and as written in a fine pen in red ink. Martin was uncertain as to the meaning of these annotations; Kunitzsch and Lorch (2010b) interpret them as citations of Euclid's *Elements* and note where they appear both in their critical apparatus and in their mathematical commentary.
[115] Since it has not yet been possible to examine Seray Ahmet III 3464 directly, it is unknown whether referential scholia appear also in its other texts. Several other editions have been published which use this manuscript, but if this category of scholia is present for those texts, the editions make no mention of it.
[116] Leiden or. 1031, fol. 23a.
[117] Note this is a slightly different transliteration than the one seen in al-Kindī's work discussed in chapter 3: Στοιχεῖα is sometimes rendered as أستقصات, sometimes as أسطقسات.
[118] E.g. Leiden or. 1031 fol. 52a: "ىر ا ك"

here may reference the *Sphaerica* specifically (الاكر), or perhaps it is better understood as indicating something like the "this book" (هذا الكتاب) seen in examples in the *Phaenomena* above.

The citations of the *Elements* in Leiden or. 1031's and Seray Ahmet III 3464's *Sphaerica* can be compared for the first eight propositions, and they show overlap between thirteen out of thirty cases. The following table presents the relevant scholia, with the ones which agree highlighted:

Appears on	**Leiden or. 1031**		**Cites**	**Seray Ahmet III 3464**	
	Folio	**Referential Scholia**		**Referential Scholia**	**K&L (2010b)** [119]
Sph.1.prop.1			El.11.prop.2[120]	ٮا ٮا س	p.16
Sph.1.prop.1	23a	مر من ا ق	El.1.prop.47	مر ا س	p.16
Sph.1.prop.2			El.3.prop.1	ا جـ س	p.18
Sph.1.prop.2	23a	ٮ من ٮا ٯ	El.11.prop.12	ٮب يا س	p.20
Sph.1.prop.2	23a	ٮا من ٮا ٯ	El.11.prop.11	ٮا ٮا س	p.20
Sph.1.prop.2	23b	ٮحـ ٮا ٯ	El.11.prop.13[121]	ٮحـ ٮا س	p.20
Sph.1.prop.3	23b	حـ ٮا ٯ	El.11.prop.3		
Sph.1.prop.3	23b	ٮ حـ ٯ	El.3.prop.2[122]	ٮ حـ س	p.24
Sph.1.prop.4	24a	حـ من ٮا ٯ	El.11.prop.3	حـ ٮا س	p.24
Sph.1.prop.4	24a	ٮر من حـ ٯ	El.3.prop.17[123]	ٮر حـ س	p.26
Sph.1.prop.4	24a	د ٮا ٯ	El.11.prop.4	د ٮا س	p.26
Sph.1.prop.5			El.11.prop.12	ٮ ٮا س	p.28
Sph.1.prop.5			El.11.prop.13	ٮحـ ٮا س	p.28
Sph.1.prop.5	24a	ٮا من ٮا ٯ	El.11.prop.11		
Sph.1.prop.5	24a	ك ٮا ٯ	El.11.prop.20		

[119] Since the manuscript was unavailable for consultation, this column gives the page numbers in Kunitzsch and Lorch's edition in which these scholia are noted in the critical apparatus.
[120] Kunizsch and Lorch (2010b) 346 fn.3 correct the numeral 12 in the citation to 2.
[121] Kunitzsch and Lorch (2010b) 347 fn.4 correct the numeral 13 in the citation to 18.
[122] Kunitzsch and Lorch (2010b) 348 fn.2 correct the numeral 12 in the citation to 2.
[123] Kunitzsch and Lorch (2010b) 348 fn.2 correct the numeral 17 in the citation to 7.

Sph.1.prop.6	24b	ص[124] ٮا ق	El.11.defs		
Sph.1.prop.6	24b	ٮا من ٮا ق	El.11.prop.11	ٮا ٮا س	p.30
Sph.1.prop.6			El.1.prop.32	لٮ ا س	p.30
Sph.1.prop.6			El.1.prop.18	ٮح ا س	p.30
Sph.1.prop.6	24b	ص ٮا ق	El.11.defs		
Sph.1.prop.6	24b	يط ا ق	El.1.prop.19		
Sph.1.prop.6	25a	مر ا ق	El.1.prop.47	مر ا س	p.32
Sph.1.prop.6			El.1.prop.47	مر ا س	p.32
Sph.1.prop.6			El.1.prop.47	مر ا س	p.34
Sph.1.prop.6	25a	ص حـ ق	El.3.defs		
Sph.1.prop.7	25b	ح ا ق	El.1.prop.8	ح ا س	p.36
Sph.1.prop.7	25b	ص ا ق	El.1.defs		
Sph.1.prop.7	25b	د ٮا ق	El.11.prop.4	د ٮا س	p.38
Sph.1.prop.8	26a	د ا ق	El.1.prop.4	د ا س	p.40
Sph.1.prop.8	26a	ص ٮا ق	El.11.defs		

Table 4.17: Agreement of referential scholia between Leiden or. 1031 and Seray Ahmet III 3464

There are only twelve scholia on these first eight propositions in the Greek manuscript Vat. gr. 204, but these also can be compared with the above. The citations which agree are:

- Sph.1.prop.4 referencing El.11.prop.4 (fol. 2v)
- Sph.1.prop.5 referencing El.11.prop.13 (fol. 2v)
- Sph.1.prop.6 referencing El.1.prop.47 (fol. 3r)
- Sph.1.prop.7 referencing El.1.defs (fol. 3v)
- Sph.1.prop.7 referencing El.11.prop.4 (fol. 3v)

[124] The abbreviation ص is for صدر, i.e. the starting-points or definitions of the text.

Heiberg's edition of the *Sphaerica* records scholia from further manuscripts as well.[125] He notes twenty such scholia on the first eight propositions. In addition to the prior list, his edition includes the following other scholia which also agree with the table of Arabic citations above:

- Sph.1.prop.1 referencing El.1.prop.47 (scholium 5)
- Sph.1.prop.2 referencing El.3.prop.1 (scholium 8)
- Sph.1.prop.2 referencing El.11.prop.12 (scholium 9)
- Sph.1.prop.3 referencing El.3.prop.2 (scholium 13)
- Sph.1.prop.6 referencing El.11.defs (scholium 22)
- Sph.1.prop.6 referencing El.1.prop.19 (scholium 26)
- Sph.1.prop.7 referencing El.1.prop.8 (scholium 34)
- Sph.1.prop.8 referencing El.11.defs (scholium 38)

Since the extant Arabic manuscripts do not go back to the ninth century, it is not immediately clear at what point these kinds of references were introduced into the Middle Books texts. But their similarity with the material found in the Greek strongly suggests that at least some of them were introduced when these texts were translated, and from Greek exemplars which themselves contained them.

5. Interpretation of Attested Alterations and References

The results of this chapter's study can be compared with those from chapter 2 – when doing so, however, we must keep in mind that the deliberate alterations in chapter 2 are not all chronologically prior to those studied here. Rather, the variations discussed here occurred in a part of the transmission that branched off partway through the Greek transmission. Thus there are points where it differs because of variations introduced into the Greek rather than into the Arabic. This is plausibly the reason for some amount of the absences we have noted in this chapter – absences, not suppression, because the material in question simply was not yet part of the sources used by the translators. There are instances of trivial

[125] Heiberg (1927) 166ff. Comparing Czinczenheim's edition of the scholia shows the same agreements for these first eight propositions: see Czinczenheim (2000) 384-389. The corresponding scholia references in Czinczenheim are the following: Heiberg scholium 5 = Czinczneheim scholium 5, H8 = C9, H9 = C10, H13 = C15, H22 = C25, H26 = C27, H34 = C37, and H38 = C44.

material left included in the Arabic texts which their modern editors suppose to be evidence that one of the correctors, Thābit, was reluctant to suppress material in the texts he worked with.

That is not to say that there were no deliberate choices by the early Arabic scholars. These still occurred and also contributed to shaping the form of this curriculum. Various material is added across several texts: propositions and definitions in the *Sphaerica* and *On Sizes and Distances*, alternate proofs and alternate cases across further texts. Arabic scholars can be seen accumulating together material they encountered in other manuscripts and will note the separate provenance of that material. Interestingly, the additions seen here follow a separate pattern from what was seen in the Greek transmission. In chapter 2, it was pointed out that where larger units of material were added to the Greek texts, they appeared either at the beginning of the work as a whole or at the end of sections: new definitions appearing after already extant ones, new propositions appearing at the ends of books. While this is sometimes the case in the Arabic, the *Sphaerica* shows an added proposition inserted into the middle of a book – perhaps unsurprisingly, since it is placed alongside the propositions it serves as a converse of.

This may be related to another pattern: two categories of deliberate alterations – changes in order and fusion / division of propositions – occur more often in the early Arabic texts than they do in the Greek transmission. In several instances, these alterations and the addition of material into the middle of a book can be seen as potentially motivated to better support the logic of the texts.

One byproduct of these particular processes – addition of material mid-section, changes in order, and fusion / division – is that the numbering of subsequent propositions is changed. At this stage there would seem, then, to be little active goal of maintaining the numbered textual structure, despite the potential usefulness a more consistent canon of propositions could feasibly have when studying or discussing these works. Though the Middle Books were already coalescing in the ninth century as preliminary reading for the *Almagest*, it seems the Middle Books themselves were still slightly unstable in form in this period.

On this topic, it would be desirable to know at what point Arabic scholars started adding referential scholia to their codices of the Middle Books. The similarity between the Arabic and Greek citations suggests that the practice was imported when these texts were translated. Early in the Arabic translations of these texts, then, there could have been multiple concurrent and sometimes conflicting scholarly practices ongoing with the Middle Books. Copyists and/or scholars were collating manuscripts and incorporating material found in other copies into their new exemplars. They were copying referential scholia into new manuscripts, or adding them in the course of their work with the text, or both. And the texts were being reshaped, even if only in select instances, in ways which seem intended to improve or make more comprehensive the logic of the works in question.

6. Appendix: Comparison of Select *Phaenomena* Expositions

The following is a comparison between the exposition portion of the proposition for Greek propositions 9, 11-16 (= Leiden or. 1031 propositions 9, 11-14, 17-18).[126] These are the propositions which differ between the two Greek recensions, and the comparison below shows the significant influence from the B recension on Leiden or. 1031's translation.

Note the following agreement between the Greek and Arabic witnesses that is found here across the lettering used for labels. To this I add the letters used in the English translation:

[126] As proposition 10 has been examined above, it is omitted here.

Greek	Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Κ	Λ	Μ	Ν	Ξ	Τ
Arabic	ا	ب	جـ	د	ه	ز	ح	ط	ك	ل	م	ن	س	*[127]
English	A	B	G	D	E	Z	H	Q	K	L	M	N	X	T

Table 4.18: Agreement in letter labelling between the Greek *Phaenomena* and its Arabic witness in Leiden or. 1031

Menge (1916) Recension A	**Menge (1916) Recension B**	**Leiden or. 1031**
Proposition 9	Proposition 9	Proposition 9
In the cosmos let the horizon be ABG, and the summer tropic DA, and the winter tropic EG, and let the circle of the zodiac have position AGZ, and let the semicircle following Cancer below the earth be AZG and let GHA be the semicircle following Capricorn and let it be above the earth.	In the cosmos let the horizon be ABG, and let the summer tropic be AD, and the winter tropic BG, and let the circle of the zodiac have position DEBZ, and let the rising side be G, D; and the setting A, B; and let DEB be the semicircle following Cancer, and BZD the semicircle following Capricorn.	Example of this: we assume the horizon is ABG, and we make the circle of the summer tropic AD, and the circle of the winter tropic GE, and the circle of the zodiac is located at position AHGZ, and let the eastern side be side A, E and the western side be side G, D. We make the semicircle AZH the semi(circle) which is after Cancer and the semicircle GHA the semi(circle) which is after Capricorn.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, θερινὸς δὲ τροπικὸς ὁ ΔΑ, χειμερινὸς δὲ τροπικὸς ὁ ΕΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΓΖ, καὶ ἔστω τὸ μετὰ τὸν Καρκίνον ἡμικύκλιον ὑπὸ γῆν τὸ ΑΖΓ τὸ δὲ ΓΗΑ ἡμικύκλιον ἔστω τὸ μετὰ τὸν Αἰγόκερων καὶ ἔστω ὑπὲρ γῆς.[130]	ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, καὶ θερινὸς μὲν τροπικὸς ἔστω ὁ ΑΔ, χειμερινὸς δὲ τροπικὸς ὁ ΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΔΕΒΖ, καὶ ἔστω ἀνατολικὰ μὲν τὰ Γ, Δ μέρη, δυτικὰ δὲ τὰ Α, Β, καὶ τὸ μὲν ΔΕΒ ἔστω τὸ μετὰ τὸν Καρκίνον ἡμικύκλιον, τὸ δὲ ΒΖΔ τὸ μετὰ	مثال ذلك ان نفرض أفق (ا ب جـ) ونجعل دائرة المتقلب الصيفي (ا د) ودائرة المتقلب شتوي (جـ ه) ووضع دائرة البروج على مثال وضع (ا جـ ح ز) وليكن النواحي المشرقية ناحي (ا) (ه) والنواحي المغربية ناحيتي (جـ) (د) نجعل نصف دائرة (ا ز ح) النصف الذي بعد السرطان ونصف دائرة (جـ ح ا) النصف الذي بعد الجدي[128]

[127] This proposition of the Arabic (proposition 18) generally agrees with recension B, but uses س where the Greek uses Τ, despite the fact that س elsewhere can be seen used where the Greek uses Ξ.
[128] Leiden or. 1031, fol. 83a.
[130] Menge (1916) 46, upper text.

Menge (1916) Recension A	**Menge (1916) Recension B**	**Leiden or. 1031**
	τὸν Αἰγόκερω·[129]	
Proposition 11	Proposition 11	Proposition 11
In the cosmos let the horizon be ABG, and let the circle of the zodiac have position AEGD, and let the semicircle below the earth be ADG, and let AD, GE be selected as both equal and opposite arcs.	Let the circle of the horizon be ABDG, and let the summer tropic be AG, and the winter tropic BD, and let the circle of the zodiac be GB, and let GE, BZ be selected from it as both equal and opposite arcs.	Example of this: we assume the horizon is ABDG, and the summer tropic AG, and the winter tropic BD, and let the circle of the zodiac be GB, and we select from it two equal opposite arcs and these are arcs GE, ZB.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΕΓΔ, καὶ ἔστω ὑπὸ γῆν τὸ ΑΔΓ ἡμικύκλιον, καὶ ἀπειλήφθωσαν ἴσαι τε καὶ ἀπεναντίον περιφέρειαι αἱ ΑΔ, ΓΕ·[133]	ἔστω ὁρίζων κύκλος ὁ ΑΒΔΓ, καὶ θερινὸς μὲν τροπικὸς ἔστω ὁ ΑΓ, χειμερινὸς δὲ ὁ ΒΔ, ζῳδιακὸς δὲ ἔστω ὁ ΓΒ, καὶ ἀπειλήφθωσαν ἐπ’ αὐτοῦ ἴσαι καὶ ἀπεναντίον περιφέρειαι αἱ ΓΕ, ΒΖ·[132]	مثال ذلك ان نفرض الافق (ا ب د جـ) والمنقلب الصيفي (ا جـ) والمتقلب الشتوي (ب د) وليكن دائرة البروج (جـ ب) ونفصل منها قوسين متساويتين متقابلتين وهما قوسا (جـ ه) (ز ب)[131]
Proposition 12	Proposition 12	Proposition 12
In the cosmos let the horizon be ABG, and the largest of the ever visible (circles) RST, and the summer tropic AE, and the winter tropic GZ, and the equator BHD, and let the circle of the zodiac have position AHG, and let the semicircle following Cancer above the earth be AHG, and let each of the quadrants AH, HG be divided into the signs at Q, K, L, M.	Let the circle of the horizon be ABGD, and the largest of the ever visible (circles) EZ, and the summer tropic BA, and the winter tropic GD, and let the semicircle following Cancer above the earth be BD, and the circle of the equator HQ, and let each of BX, DX be divided in three equal (parts) at the points K, L, M, N.	Example of this: we assume the circle of the horizon is ABDG, and the largest of the always visible circles EZ, and we make the summer tropic circle BA, and the winter tropic circle GD, and let the equator be HQ and we divide each one of the arcs XB, DX in three equal parts at points K, L, M, N.

[129] Menge (1916) 46, lower text.
[131] Leiden or. 1031, fol. 86b.
[132] Menge (1916) 58, lower text.
[133] Menge (1916) 58, upper text.

Menge (1916) Recension A	Menge (1916) Recension B	Leiden or. 1031
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, μέγιστος δὲ τῶν ἀεὶ φανερῶν ὁ ΡΣΤ, θερινὸς δὲ τροπικὸς ὁ ΑΕ, χειμερινὸς δὲ τροπικὸς ὁ ΓΖ, ἰσημερινὸς δὲ ὁ ΒΗΔ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΗΓ, καὶ ἔστω τὸ μετὰ τὸν Καρκίνον ἡμικύκλιον ὑπὲρ γῆν τὸ ΑΗΓ, καὶ διῃρήσθω ἑκάτερον τῶν ΑΗ, ΗΓ τεταρτομορίων εἰς τὰ ζῴδια κατὰ τὰ Θ, Κ, Λ, Μ·[136]	ἔστω ὁρίζων κύκλος ὁ ΑΒΓΔ, μέγιστος δὲ τῶν ἀεὶ φανερῶν ὁ ΕΖ, καὶ θερινὸς μὲν τροπικὸς ὁ ΒΑ, χειμερινὸς δὲ τροπικὸς ὁ ΓΔ, καὶ ἔστω τὸ μετὰ τὸν Καρκίνον ἡμικύκλιον τὸ ΒΔ ὑπὲρ γῆς, ἰσημερινὸς δὲ κύκλος ὁ ΗΘ, καὶ διῃρήσθω ἑκατέρα τῶν ΒΞ, ΔΞ εἰς τρία ἴσα κατὰ τὰ Κ, Λ, Μ, Ν σημεῖα·[135]	مثال ذلك أن نفرض دائرة الأفق (ا ب د جـ) واعظم الدوائر الدائمة الظهور (ه ز) ونجعل المتقلب الصيفي دائرة (ب ا) والمتقلب الشتوي دائرة (جـ د) وليكن معدل النهار (ح ط) ونقسم كل واحد من قسي (س ب) (د س) ثلثة اقسام متساوية على نقطة (ك) (ل) (م) (ن)[134]
<u>Proposition 13</u>	<u>Proposition 13</u>	<u>Proposition 13</u>
In the cosmos let the horizon be ABG, and the summer tropic AE, and the winter tropic GZ, and the equator BD, and let the circle of the zodiac have position AHGQ, and let the semicircle following Capricorn below the earth be GHA, and let each of the quadrants GH, HA below the earth be divided into the signs at K, L, M, N.	Let the circle of the horizon be ABGD, and the summer tropic BA, and the winter tropic GD, and let the semicircle following Capricorn below the earth be DHB, and the circle of the equator EQHZ, and let each of BH, HD be divided in three equal (parts) at points K, L, M, N.	Example of this: we assume the horizon is circle ABGD and the summer tropic AB and the winter tropic GD, and let the semicircle which follows Capricorn below the earth be DHB, and the circle of the equator EHQZ, and each one of the arcs BH, HD is divided in three equal parts at points K, L, M, N.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, θερινὸς δὲ τροπικὸς ὁ ΑΕ, χειμερινὸς δὲ τροπικὸς ὁ ΓΖ, ἰσημερινὸς ὁ ΒΔ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΗΓΘ, καὶ ἔστω τὸ μετὰ τὸν Αἰγόκερων ἡμικύκλιον ὑπὸ γῆν τὸ ΓΗΑ, καὶ διῃρήσθω ἑκάτερον τῶν ὑπὸ γῆν ΓΗ, ΗΑ	ἔστω ὁρίζων κύκλος ὁ ΑΒΓΔ, καὶ θερινὸς μὲν τροπικὸς ὁ ΒΑ, χειμερινὸς δὲ τροπικὸς ὁ ΓΔ, καὶ ἔστω τὸ μετὰ τὸν Αἰγόκερω ἡμικύκλιον ὑπὸ γῆν τὸ ΔΗΒ, ἰσημερινὸς δὲ κύκλος ὁ ΕΘΗΖ, καὶ διῃρήσθω ἑκατέρα τῶν ΒΗ, ΗΔ εἰς τρία ἴσα κατὰ τὰ Κ, Λ, Μ, Ν σημεῖα·[138]	مثال ذلك أن نفرض الأفق دائرة (ا ب جـ د) والمتقلب الصيفي (ا ب) والمتقلب الشتوي (جـ د) وليكن نصف الدائرة التي بعد الجدي تحت الأرض (د ح ب) ودائرة معدل النهار (ه ح ط ز) ويقسم كل واحدة من قوسي (ب ح) (ح د) بثلثة اقسام مساوية على نقطة (ك) (ل) (م) (ن)[137]

[134] Leiden or. 1031, fol. 85a.
[135] Menge (1916) 62-64, lower text.
[136] Menge (1916) 62-64, upper text.
[137] Leiden or. 1031, fol. 91a.
[138] Menge (1916) 80, lower text.

Menge (1916) Recension A	Menge (1916) Recension B	Leiden or. 1031
τεταρτημορίων εἰς τὰ ζῴδια κατὰ τὰ Κ, Λ, Μ, Ν·[139]		
Proposition 14	Proposition 14	Proposition 14
In the cosmos, let the horizon be ABG, and the largest of the ever visible (circles) AD, and (the largest) of the ever invisible (circles) ZH, and the summer tropic BGK, and the winter tropic LMN, and let the circle of the zodiac have position KXO at one time and PTR at another, and let the arc KO be cut off not longer than a semicircle, and through E let a great circle be drawn touching ADE.	Let the circle of the horizon be ABGD, and let the largest of the ever visible (circles) be EZ, and the summer tropic BA, and let the pole of ABGD be between EZ, BA; and let the circle of the zodiac have position QHK at one time and LMN at another; and let HK be cut off not longer than a semicircle; and through point K let a great circle KNZ be drawn touching EZ.	Example of this: we assume the circle of the horizon is ABGD, and the largest of the ever visible circles EZ, and the summer tropic circle BA, and let the pole of the circle ABGD be what is between circle EZ and circle BA, and we make the location of the circle of the zodiac position QHK and at another time position LMN, and we divide arc HK and we make it not larger than a semicircle, and we draw through point K a great circle touching circle EZ and it is circle KNZ.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, μέγιστος δὲ τῶν ἀεὶ φανερῶν ὁ ΑΔ, τῶν δὲ ἀεὶ ἀφανῶν ὁ ΖΗ, καὶ θερινὸς μὲν τροπικὸς ὁ ΒΓΚ, χειμερινὸς δὲ ὁ ΛΜΝ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω ὁτὲ μὲν τὴν ΚΞΟ, ὁτὲ δὲ τὴν ΠΤΡ, καὶ ἀπειλήφθω ἡ ΚΟ περιφέρεια μὴ μείζων ἡμικυκλίου, καὶ διὰ τοῦ Ε γεγράφθω μέγιστος κύκλος ἐφαπτόμενος τοῦ ΑΔΕ·[142]	ἔστω ὁρίζων κύκλος ὁ ΑΒΓΔ, μέγιστος δὲ τῶν ἀεὶ φανερῶν ἔστω ὁ ΕΖ, θερινὸς δὲ τροπικὸς ὁ ΒΑ, καὶ ἔστω ὁ τοῦ ΑΒΓΔ πόλος μεταξὺ τῶν ΕΖ, ΒΑ, ὁ δὲ τῶν ζῳδίων κύκλος ποτὲ μὲν θέσιν ἐχέτω ὡς τὴν ΘΗΚ, ποτὲ δὲ ὡς τὴν ΛΜΝ, καὶ ἀπειλήφθω ἡ ΗΚ μὴ μείζων ἡμικυκλίου, καὶ γεγράφθω διὰ τοῦ Κ σημείου μέγιστος κύκλος ὁ ΚΝΖ ἐφαπτόμενος τοῦ ΕΖ.[141]	مثال ذلك أن نفرض دائرة الأفق (ا ب جـ د) واعظم الدائرة الدائمة الظهور (ه ز) والمتقلب الصيفي دائرة (ب ا) وليكن قطب دائرة (ا ب جـ د) فما بين دائرة (ه ز) ودائرة (ب ا) ونجعل وضع دائرة البروج على مثال وضع (ط ح ك) وفي بعض الاوقات على مثال وضع (ل م ن) ونفضل قوس (ح ك) ونجعلها ليست باعظم من نصف دائرة ونرسم على نقطة (ك) دائرة عظيمة يماس دائرة (ه ز) وهي دائرة (ك ن ز)[140]

[139] Menge (1916) 78, upper text.
[140] Leiden or. 1031, fol. 92a-92b.
[141] Menge (1916) 88, lower text.
[142] Menge (1916) 88, upper text.

Menge (1916) Recension A	Menge (1916) Recension B	Leiden or. 1031
Proposition 15	Proposition 15	Proposition 17
In the cosmos let the horizon be ABG, and let the circle of the zodiac have position ZAEG, and let both equal and opposite arcs AE, GZ be cut off.	In the cosmos let the horizon be ABGD, and let the summer tropic be AD, and the winter tropic BG, and let the circle of the zodiac have position DEBZ, and let DEB be the semicircle following Cancer below the earth, and BZD the (semicircle) following Capricorn above the earth, and let the rising side be D, and the setting (side) B, and let two both equal and opposite arcs DE, BZ be cut off.	Example of this: we assume the horizon is circle ABD, and we make the summer tropic AD, and the winter tropic BG, and the circle of the zodiac is positioned at DEBZ, so let the semicircle DEB be that which follows Cancer below the earth, the semicircle BZD that which follows Capricorn above the earth, and we make the eastern side side D, and the western side side B, and we cut off two equal opposite arcs and these are arcs DE, BZ.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω ὡς ΖΑΕΓ, καὶ ἀπειλήφθωσαν ἴσαι τε καὶ ἀπεναντίον περιφέρειαι αἱ ΑΕ, ΓΖ·[145]	ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓΔ, καὶ θερινὸς μὲν τροπικὸς ἔστω ὁ ΑΔ, χειμερινὸς δὲ τροπικὸς ὁ ΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΔΕΒΖ, καὶ ἔστω τὸ μὲν ΔΕΒ ἡμικύκλιον τὸ μετὰ τὸν Καρκίνον ὑπὸ γῆν, τὸ δὲ ΒΖΔ τὸ μετὰ τὸν Αἰγόκερω ὑπὲρ γῆν, καὶ ἔστω ἀνατολικὰ μὲν τὰ Δ μέρη, δυτικὰ δὲ τὰ Β, καὶ ἀπειλήφθωσαν δύο ἴσαι τε καὶ ἀπεναντίον περιφέρειαι αἱ ΔΕ, ΒΖ·[144]	مثال ذلك أن نفرض الأفق دائرة (ا جـ د) ونجعل المتقلب الصيفي (ا د) والمتقلب الشتوي (ب جـ) ووضع دائرة البروج على مثال (د ه ب ز) فليكن نصف دائرة (د ه ب) الذي بعد السرطان تحت الأرض نصف دائرة (ب ز د) الذي بعد الجدي فوق الأرض ونجعل ناحية المشرقية ناحية (د) والناحية المغربية ناحية (ب) ونفضل قوسين متساويين متقابلين وهما قوسا (د ه) (ب ز)[143]
Proposition 16	Proposition 16	Proposition 18
In the cozmos let the horizon be ABG, and let the circle of the zodiac have position AZG, and	In the cosmos let the horizon be ABG, and let the summer tropic be AB, and let the winter tropic	Example of this: we assume horizon ABG, and we make the summer tropic AB, and the

[143] Leiden or. 1031, fol. 97a.
[144] Menge (1916) 100, lower text.
[145] Menge (1916) 100, upper text.

Menge (1916) Recension A	Menge (1916) Recension B	Leiden or. 1031
let equal arcs DE, EZ be cut off.	be GT, and let the circle of the zodiac have position AGE, and let equal arcs DE, EZ be cut off.	winter tropic GX, and the circle of the zodiac is located at AGE, and we cut off two equal arcs and these are arcs DE, EZ.
ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΖΓ, καὶ ἀπειλήφθωσαν ἴσαι περιφέρειαι αἱ ΔΕ, ΕΖ·[148]	ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, καὶ θερινὸς μὲν τροπικὸς ἔστω ὁ ΑΒ, χειμερινὸς δὲ τροπικὸς ἔστω ὁ ΓΤ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΓΕ, καὶ ἀπειλήφθωσαν ἴσαι περιφέρειαι αἱ ΔΕ, ΕΖ·[147]	مثال ذلك أن نفرض أفق (ا ب جـ) ونجعل المتقلب الصيفي (ا ب) والمتقلب الشتوي جـ س) ووضع دائرة البروج على مثال (ا جـ ه) ونفضل قوسين متساويتين ومنها قوسا (د ه) (ه ز)[146]

Table 4.19: Comparison of Greek and Arabic expositions for *Phaenomena* propositions which diverge between recensions A and B

[146] Leiden or.1031, fol. 98a.
[147] Menge (1916) 106.
[148] Menge (1916) 104.

PART III

Chapter 5

The Middle Books, Ninth to Thirteenth Centuries

1. Introduction

By the end of the ninth century, the component parts of the Greek Little Astronomy had been translated into Arabic – and already in that century, Islamicate scholars had found this cluster of works useful for introducing a student to the spherical geometry used in the *Almagest*. These works soon saw transmission far beyond Baghdad. This chapter will give an overview of engagement with the Middle Books in the Islamicate world between the treatises' translation in the ninth century and their editions by al-Ṭūsī in the thirteenth century. It will consider first the evidence from the extant manuscripts, then the engagement of scholars across the Islamicate world with the curriculum. The final section will discuss what changes the Middle Books saw during this period.

2. The Arabic Manuscripts, 9th to 13th Centuries

While most extant witnesses for the Middle Books texts date from after the thirteenth century, some examples serve as contemporary evidence for how these texts circulated between the ninth and thirteenth centuries.[1] In other cases, manuscripts which are later still preserve evidence of earlier scholars' engagement with Middle Books texts. The following section will discuss manuscripts copied before the end of the thirteenth century, or those containing important witnesses prior to al-Ṭūsī's editions.[2]

2.1 Manuscripts containing individual Middle Books

It should be noted that there are a handful of manuscripts which contain only individual Middle Books treatises, not the collection (or a significant proportion of it). Circulation within the Middle Books

[1] As can be seen from the surveys of manuscripts for these texts, as recorded in Sezgin (1974) and (1978). See Autolycus in (1974) 81, Euclid in 83, Archimedes in 121, Hypsicles in 143, Theodosius in 154, and Menelaus in 158. See also Autolycus in (1978) 73, Euclid in 74, Hypsicles in 80, and Theodosius in 80.

[2] As chapter 8 will show, al-Ṭūsī produced his editions between 651 and 663 H (1253 and 1265 CE), so several of the below were produced contemporaneously with or shortly after his work.

was not the entirety of how these works were transmitted. The following are manuscripts preserving individual Middle Books texts (or texts which were added to the collection):

- Paris, Bibliothèque nationale de France arabe 2457[3]
    - Fifty-one mathematical treatises, including:
        - *Anaphoricus*, copied in 358-60 H / 969-72 CE
- Mashhad, Riḍā 5412
        - *Anaphoricus*, copied in the 5th-6th / 11th-12th century[4]
- Leiden, Leiden University Library Or. 399[5]
    - Two mathematical treatises:
        - *Elements*, copied in Rabīʿ I 539 H / September-October 1144 CE
        - *Spherics*
- London, British Library Or. 13127[6]
    - One mathematical treatise:
        - *Spherics*, copied in 4 Rabīʿ II 548 H / 5 July 1153 CE
- Cairo, Dār al-Kutub riyāḍa 260[7]
    - Six treatises, including:
        - *Optics*, copied in 600 H / 1203-4 CE
- Istanbul, Süleymaniye Kütüphanesi, Ayasofya, 2671[8]
    - Six mathematical and astronomical treatises, including:
        - *On the Moving Sphere*, copied in Ṣafar 621 H / March 1224 CE
- Leiden, Leiden University Library Or. 133[9]
    - Six treatises, including:
        - *Optics*, copied in 692 H / 1292-3 CE
- New York, Columbia Rare Book & Manuscript Library Or. 45[10]
    - Fourteen mathematical and astronomical treatises, including:
        - *Spherics* (undated)
        - *On Sizes and Distances* (undated)

[3] Paris BnF arabe 2457 may be viewed online in the Gallica digital library. See Wœpcke (1856) 665-671 for a listing of the full manuscript contents.
[4] See Sezgin (1974) 145.
[5] Leiden Or. 399 may be viewed online in Leiden University Library Digital Collections. See Rashed and Papadopoulos (2017) 488 and 490-492.
[6] London British Library Or. 13127 may be viewed online in the Qatar Digital Library. See Rashed and Papadopoulos (2017) 489 and 492-493.
[7] See Kheirandish (1999) xxxiv.
[8] See Sezgin (1974) 82 and Nikfahm-Khubravan and Eshera (2019) 46.
[9] Leiden Or. 133 may be viewed online in Middle Eastern Manuscripts Online 1: Pioneer Orientalists. See Sezgin (1974) 117 and Kheirandish (1999) xxxvii.
[10] Columbia Or. 45 may be viewed online in the OPenn digital repository. See Rashed and Papadopoulos (2017) 399-405.

Several of these manuscripts are mathematical and astronomical miscellanies. It would be desirable in a future study to survey how the Middle Books are distributed across the extant manuscripts, and which texts they are transmitted with. This, however, requires more catalogue data than is easily accessible.

2.2 Manuscripts containing grouped Middle Books

In comparison, the manuscripts discussed below contain or discuss groupings of Middle Books texts. Many of these are from thirteenth century – although the codices themselves would be contemporary with al-Ṭūsī, they do preserve Middle Books treatises as they had circulated prior to his editions.

2.2.1 Lahore, private library M. Nabī Khān (6th/12th c.)

This manuscript is presently unavailable and largely undescribed, outside of comments on select witnesses.[11] Two of the texts which it is known to contain are Theodosius's *Sphaerica*, which occupies pages 185-281; and Theodosius's *On Habitations*, which occupies pages 282-294. Information on what other texts are contained in this codex is not available, though the fact that the *Sphaerica* starts on page 185 makes it clear enough that there was at least one other text.

Despite the limited information available for this manuscript, what is available provides valuable insights into the transmission of these mathematical texts. Kunitzsch and Lorch provide a translation of the *Sphaerica*'s colophon in this codex, which reads as follows:

> "Finished is the third chapter of Theodosius' book on the spheres, and with its ending the entire book is finished with the praise of God. It is fourteen theorems and the number of the theorems of the three chapters is 59, [in] the correction by Thābit b. Qurra al-Ḥarrānī al-Ṣābiʾ. I have copied this book from the handwriting of Qurra b. Sīnān b. Manṣūr b. Saʿīd b. Thābit b. Sinān b. Thābit b. Qurra al-Ḥarrānī al-Ṣābiʾ in the city of Mosul (God protect it!) in the Niẓāmīya Madrasa (God give it long life!), when six nights remained of Jumada I of the year 554 H [=13 June 1158] (upon its patron be the finest salam!). I found written at the end of the book: 'al-Ḥasan b. Saʿīd has finished devising the diagrams [tashkil] of this book, but the volume from which he copied the figures [ashkal] was not reliable. Moreover there was corruption in it, so it was necessary to

[11] Kunitzsch and Lorch have offered comments on xeroxes of this codex's copies of the *Sphaerica* and *On Habitations* in Kunitzsch and Lorch (2010b) 3 and Kunitzsch and Lorch (2010a) 10, respectively.

> collate it with the figures [ashkal] in another copy. That was on the eve of Tuesday, eight nights remaining of twelve [i.e. Dhu ‘l-Hijja] of the year 421 [20 December 1030]. Praise be to God richly and His blessings upon Muḥammad and all his family!”[12]

This colophon, and its report of one of its exemplar’s colophons, provides several details of interest. The scribe notes that the text was corrected by the famous Thābit ibn Qurra but furthermore that this witness has an unusually direct link to that mathematician – its exemplar was transcribed by one of his own descendants. The inclusion of this detail offers some scholarly prestige to the present codex.

Further evidence of the scribe’s emphasis on the scholarly value of this codex can be seen in his choice to transcribe one of the colophons of his exemplar. A certain al-Ḥasan ibn Saʿīd had worked on this exemplar in the 5th / 11th century. These efforts involved recognizing that the diagrams of his own exemplar were faulty and subsequently collating the figures themselves using another copy of the text available to him. The present codex additionally preserves some marginalia by al-Ḥasan ibn Saʿīd which are mathematical comments on the text or the figures. One of these reveals that al-Ḥasan ibn Saʿīd not only copied and corrected the diagrams available to him, but also provided his own when he thought it useful: “The second figure is not found in any of the copies [of the text], but it came to my mind while working on this book. So I put it [here].”[13]

The colophon also notes the location where the manuscript was produced, though in this case the copying of this manuscript in Mosul does not reflect any significant distance traveled by these texts. But the specification that the codex was copied in the Niẓāmīya Madrasa stands out, since the madrasa as an

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[12] Translation from Kunitzsch and Lorch (2010b) 3-4. See Kunitzsch and Lorch (2010b) 310-312 for the Arabic:
"تمت المقاتة الثالثة من كتاب ثاوذوسيوس في الكرات وبتمامها تم الكتاب بأسره بحمد الله ومنّه وهى أربعة عشر شكلاً وعدد أشكال المقالات الثلثة تسعة وخمسون شكلاً إصلاح ثابت بن قرة الحرانى الصابئ، نقلتُ هذا الكتاب من خط قرة بن سنان بن منصور بن سعيد بن ثابت بن سنان بن ثابت بن قرة الخرانى الصابئ بمدينة الموصل حماها لله، في المدرسة النظامية عمرها الله لست ليالٍ نقيت من جمادى الأولى لسنة أربع وخمسين وخمس مائة هجرية على صاجها أفضل السلام، ووجدت فى آخر الكتاب مكتوباً أن قد فرغ من تشكيل هذا الكتاب الحسن بن سعيد ولم يكن الأصل الذى نقل منه الأشكال موثوقاً به بل كان فيه فساد ويجب أن يقابل بالأشكال بنسخة أخرى وذلك فى ليلة الثلثاء لثمانى ليالٍ بقيت من ١٢ سنة ٤٢١، الحمد لله كثيراً وصلوته على محمد وآله أجمعين."

[13] These comments are edited in Kunitzsch and Lorch (2010b) 313-315. See p.313 for the Arabic of the quotations above: “وهذه الصورة الثانية ليس توجد فى شىء من النسخ وإنما خطرت ببالى فى وقت عملى لهذا الكتاب فأثبتها”

institution is more commonly understood to have almost exclusively supported the study of the Islamic sciences. The ancient or rational sciences were not included in this category.[14] Brentjes has pushed back against this standard understanding and sees this manuscript as an example for one of the earlier cases of the mathematical sciences being produced in a madrasa context.[15] Unfortunately, since the full codex is not presently available for further study, it is unclear what other texts it contained and whether they too might have been copied in a madrasa context. But it can be seen that this copy of the *Sphaerica* (and very probably of *On Habitations* too, which was written in the same hand) was produced in a madrasa context during the 6th / 12th century.

2.2.2 Bodleian, Thurston 11 (635 H / 1238 CE)

The manuscript Thurston 11 is a codex of Euclid's *Elements* in fifteen books. The colophon on folio 212b indicates that it was completed on 13 Jumādā al-ʾAwwal 635 H (7 January 1238 CE). It receives acknowledgement in this chapter because it contains a note mentioning "the books which are necessary to be read before the book of the *Almagest*, known as the Middle [Books]."[16] It then presents the following list:

1. *Data* (Euclid)	1 book,	95 propositions
2. *Sphaerica* (Theodosius)	3 books,	59 propositions
3. *Spherics* (Menelaus)	3 books,	91 propositions
4. *Moving Sphere* (Autolycus)	1 book,	12 propositions
5. *Optics* (Euclid)	1 book,	64 propositions
6. *Phaenomena* (Euclid)	1 book,	22 propositions
7. *Habitations* (Theodosius)	1 book,	12 propositions
8. *Risings and Settings* (Autolycus)	2 books,	37 propositions
9. *Days and Nights* (Theodosius)	2 books,	33 propositions
10. *Sizes and Distances* (Aristarchus)	1 book,	17 propositions

[14] See for example the discussion of the madrasa in EI3: Günther, "Education, general (up to 1500)." This presents the common understanding of the madrasa as an institution which most frequently saw the study of Islamic law and its ancillary subjects.

[15] Brentjes (2018a) 77; see the following discussion on 77-111 for what evidence there is for the mathematical sciences, medicine, natural philosophy, and the "occult" sciences being taught at madrasas between the twelfth and seventeenth centuries.

[16] Transcribed in Nicoll (1821) 260: "ذكر الكتب الذي يحتاج الي قرااتها قبل كتاب المجسطي وتعرف بالمتوسّطات"

11. *Anaphoricus* (Hypsicles) 1 book, 5 propositions
12. *On the Sector Figure* (Thābit ibn Qurra) 2 books

Though this codex does not itself contain these treatises, it serves as another witness to the collection prior to al-Ṭūsī's edition of the Middle Books, both in its contents and its arrangement. Chapter 4 has compared these proposition counts with what is seen in other manuscripts. It is clear that while some works had more stable proposition counts, others were more in flux throughout their Arabic translation.

2.2.3 Istanbul, Seray Ahmet III 3464 (7th/13th c.)

This manuscript is one of the most significant for the preservation of nearly all the Middle Books texts in their versions prior to al-Ṭūsī's taḥrīr. It was described by Lorch and has subsequently been used for editions and studies of multiple Middle Books texts. Since this codex was not accessible during the timeline of this study, the following description synthesizes what prior scholarship has reported about it.[17]

The first folio preserves a note that identifies one of the codex's ancient owners as Ḥusayn al-Jalabī: it then describes the contents of the manuscript as "The Middle [Books] of the *Almagest*" (متوسطات المجسطي). It provides a listing of works which is then duplicated in the form of a table of contents on the reverse of the folio. The works listed in the note and in the table of contents match except for the addition of Sharaf al-Dīn al-Ṭūsī's treatise on the linear astrolabe to the latter listing. The table of contents concludes with a note that the Middle Books preserved in this manuscript are not in the versions of Naṣīr al-Dīn al-Ṭūsī, showing that the person behind this note was aware of al-Ṭūsī's editions.[18]

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[17] See Lorch (2008) 22-23 for a short description. See also Brentjes (2018a) 232-233 for brief discussion of this codex. Editions and studies that discuss this manuscript include the following. Edition of Euclid's *Data*: Sidoli and Isahaya (2018) 27-28; editions of Theodosius's *Sphaerica*: Martin (1975) x-xv and Kunitzsch and Lorch (2010b) 3; study of Autolycus's *On the Moving Sphere*: Nikfahm-Khubravan and Eshera (2019) 44; edition of Euclid's *Optics*: Kheirandish (1999) xxvi; study and edition of Menelaus's *Spherics*: Sidoli and Kusuba (2014) 160-161 and Rashed and Papadopoulos (2017) 493-496, respectively; edition of Theodosius's *On Habitations*: Kunitzsch and Lorch (2010a) 9-10; and edition of Theodosius's *On Days and Nights*: Kunitzsch and Lorch (2011) 13.

[18] Arabic from Rashed and Papadopoulos (2017) 494: "كتاب متوسطات غير از تحريرات خواجة".

There are some differences between these two reports and the actual extant contents of the codex. The relevant data is summarized below:

Extant Contents	**Owner's Note**	**Table of Contents**
1. *Data* (Euclid) 2. *Sphaerica* (Theodosius) 3. *On the Moving Sphere* (Autolycus) 4. *Optics* (Euclid) 5. *Spherics* (Menelaus) 6. *Phaenomena* (Euclid) 7. *On Habitations* (Theodosius) 8. *On Days and Nights* (Theodosius)	● *Data* (Euclid) ● *Sphaerica* (Theodosius) ● *On the Moving Sphere* (Autolycus) ● *Optics* (Euclid) ● *Spherics* (Menelaus) ● *Phaenomena* (Euclid) ● *On Habitations* (Theodosius) ● *On Days and Nights* (Theodosius)	● *Data* (Euclid) ● *Sphaerica* (Theodosius) ● *On the Moving Sphere* (Autolycus) ● *Optics* (Euclid) ● *Spherics* (Menelaus) ● *Phaenomena* (Euclid) ● *On Habitations* (Theodosius) ● *On Days and Nights* (Theodosius)
9. *On the Linear Astrolabe*[19] (Sharaf al-Dīn al-Ṭūsī)		● *On the Linear Astrolabe* (Sharaf al-Dīn al-Ṭūsī)
10. *On Risings and Settings* (Autolycus) 11. *On the Composition of Ratios*[20] (Thābit ibn Qurra)	● *On Risings and Settings* (Autolycus) ● *On the Composition of Ratios* (Thābit ibn Qurra)	● *On Risings and Settings* (Autolycus) ● *On the Composition of Ratios* (Thābit ibn Qurra)
12. *On the Congruence of Four by Four*[21] (anonymous)		
13. *On the Sector Figure*[22] (Thābit ibn Qurra) 14. *Commentary on the Sector Figure*[23] (ʿAlī ibn Aḥmad al-Nasawī)	● *On the Sector Figure* (Thābit ibn Qurra) ● *Commentary on the Sector Figure* (ʿAlī ibn Aḥmad al-Nasawī)	● *On the Sector Figure* (Thābit ibn Qurra) ● *Commentary on the Sector Figure* (ʿAlī ibn Aḥmad al-Nasawī)
	● *Anaphoricus* (Hypsicles) ● *On Sizes and Distances* (Aristarchus)	● *Anaphoricus* (Hypsicles) ● *On Sizes and Distances* (Aristarchus) ● *Book of Enlightenment in*

[19] *Risāla fī al-asṭurlāb al-khaṭṭī.*
[20] *Kitāb al-nisba al-muʾallafa.*
[21] *Fī waqf al-arbaʿa fī al-arbaʿa.*
[22] *Fī al-shakl al-qaṭṭāʿ.*
[23] *Kitāb al-ishbāʿ fī'l-shakl al-qaṭṭāʿ.*

	• *Book of Enlightenment in Astronomy*[24] (al-Kharaqī)	*Astronomy* (al-Kharaqī)
15. botanical treatise 16. *Sufficiency in Calculation*[25] (Abū Bakr Muḥammad ibn al-Ḥusayn al-Karajī) 17. *Foundations in the Calculation of Algebra*[26] (anonymous)		

Table 5.1: Comparison of contents reported in the owner's note and the table of contents with the extant contents of Seray Ahmet III 3464

Firstly, there is a short mathematical text (*On the Congruence of Four by Four*) that appears between Thābit ibn Qurra's two works (11 and 13) above. It appears on folios 188b-189a – since the preceding work ends on folio 188a and the following work starts on folio 189b, this work was likely not a later insert into the manuscript. Secondly and more significantly for the history of this codex, the final three works reported in the owner's note and the table of contents – the *Anaphoricus*, *On Sizes and Distances*, and the *Book of Enlightenment in Astronomy* – have been lost. Instead, three different works now end MS Seray Ahmet III 3464: a botanical treatise, a mathematical work by Abū Bakr Muḥammad ibn al-Ḥusayn al-Karajī, and an anonymous work on algebra.[27]

At least six different copyists transcribed the works currently present in the codex. One hand was responsible for seven of them: the *Data*, the *Sphaerica*, *On the Moving Sphere*, the *Optics*, the *Phaenomena*, *On Risings and Settings*, and Thābit ibn Qurra's treatise on ratios (numbers 1, 2, 3, 4, 6, 10, and 11 in the list above). This hand also completes the final folio of *On Days and Nights* (number 8),

[24] *Kitāb tabṣira al-Kharaqī fī'l-hayʾa*.
[25] *Kitāb al-kafī fi'l-hisāb*.
[26] *Al-Uṣūl fī hisāb al-jabr*.
[27] Martin (1975) xi-xii. He notes there is no apparent difference in paper.

which was otherwise written in a different hand.[28] This scribe signed his name for the latter two treatises: he is Muḥammad ibn Abī Bakr al-Fārisī (d. 677 H / 1278 CE). Three of the copies he penned (6, 10, and 11) are recorded as being finished in 625 H. His completion of *On Days and Nights* (8) is recorded as occurring five years later in 630 H.

The hand that is partially responsible for *On Days and Nights* appears to be the same as the hand that penned *On Habitations* (7). Though the *On Habitations* witness is undated, 630 H could be suggested as a tentative terminus ante quem if the two shorter treatises by Theodosius are presumed to have been written together – and in the manuscript, at least, *On Habitations* immediately precedes *On Days and Nights*.

On the Sector Figure (13), however, has the earliest date of completion in this codex: its scribe Ibn al-Najāshī Muḥammad dates the transcription at 615 H, ten years earlier than most of the above.

Two hands were responsible for the transcription of Menelaus's *Spherics*, neither of which match the hand of al-Fārisī. Rashed and Papadopoulos interpret this as one copyist who changed his pen or the quality of his ink.

The copyist and dates for the lost witnesses to the *Anaphoricus*, *On Sizes and Distances*, and the astronomical treatise remain unknown. It would seem that whatever break in the codex that caused the loss and replacement of the final three listed treatises happened reasonably early in the manuscript's lifetime: the transcription of the work on algebra that was added was completed in 689 H.

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[28] There are slightly differing reports and interpretations about the hands in this text. *On Days and Nights* occupies folios 124b-151b: Martin (1975) xii claims that all folios except the last one are written in a different hand, and the last was penned by al-Fārisī. Kunitzsch and Lorch (2010a) 9-10 fn.16 states "ff. 134r-149v are in a hand different from that of the first ten folios and (probably) the De habitionibus, and ff. 150r-151r is in the hand of the Sphaerica (and of six other items)." Kunitzsch and Lorch (2011) 13 states "The text is here written in three different hands: 124v-133v in the hand that has written most of the texts in the codex (ca. 1228 AD); a second hand on 134r-149v; the third hand on 150r-151v."

The owner's note and table of contents for this codex make it clear that it was intended as a collection of the Middle Books. It originally contained all nine of the early Middle Books treatises, still in nearly the same canonical order that was demonstrated in chapter 1 for the Greek Little Astronomy – the only difference is the switched places of *On Sizes and Distances* and *On Risings and Settings*. It contained several of the works that have elsewhere been seen added to the Middle Books: the *Data*, at the head of the collection as a general geometrical treatise; the *Spherics*, situated before the *Phaenomena*'s transition towards more astronomical topics. The note on the title folio additionally claims Thābit's *On the Sector Figure*, al-Nasawī's commentary on the sector figure, and the *Book of Enlightenment in Astronomy* as included among the Middle Books.

The compiler of this manuscript included scholarly marginalia as well. The witness of the *Data* contains at least two marginalia preserving scholia by Ibn al-Ṣalāḥ; the witness of the *Spherics* includes further marginalia by this scholar.[29] This was the 6th / 12th century mathematician Najm al-Dīn Abū al-Futūḥ Aḥmad ibn Muḥammad ibn al-Sarī – he and his scholarship with the Middle Books curriculum will be discussed further below.

2.2.4 Kraus Manuscript (7th/13th c)

Similarly to MS Seray Ahmet III 3464, the Kraus manuscript is an important witness to a grouping of Middle Books texts in versions prior to al-Ṭūsī's taḥrīr. It was described by Lorch and has received discussion in editions and studies on a variety of the texts it preserves. The manuscript is presently in an anonymous and private collection – the name 'Kraus manuscript' is because it was sold by the bookseller H. P. Kraus.[30] Since this codex is not presently available, the following description synthesizes what prior scholarship has reported about it.[31]

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[29] Rashed and Papadopoulos (2017) 494.
[30] Kraus (1974) 45.
[31] See Lorch (2008) 28 for a short description. See also Kheirandish (2000) 133 for brief notes on this codex. Editions and studies that discuss this manuscript include the following. Edition of Euclid's *Data*: Sidoli and Isahaya

The name *Mutawassiṭāt* appears on the flyleaf. The manuscript's contents are:

1. Euclid's *Data*
2. Euclid's *Optics*
3. Theodosius's *Sphaerica*
4. Autolycus's *On the Moving Sphere*
5. Menelaus's *Spherics*
6. Thābit ibn Qurra's *On the Sector Figure*
7. Theodosius's *On Habitations*
8. Autolycus's *On Risings and Settings*
9. Aristarchus's *On Sizes and Distances*
10. Theodosius's *On Days and Nights*

Of the earlier corpus of the Middle Books, this codex is missing only Euclid's *Phaenomena* and Hypsicles's *Anaphoricus*. The absence of the *Anaphoricus* may perhaps be explained by its usual position at the end of this grouping – the end of the manuscript is well known as a location where it is easier for material to be lost, if loose folios go missing. The absence of the *Phaenomena* is more peculiar, and perhaps there is a connection to be drawn between this and al-Ṭūsī's complaint of finding only defective *Phaenomena* witnesses, which we will examine further in chapter 9. Otherwise, the *Data* and *On the Sector Figure* appear as common additions to the collection – other occasional additions, like those works by Archimedes, are not included in this grouping.

Kheirandish has convincingly argued that the copyist, who is named on the title page as al-Shaykh Abī ʿAlī al-Mashhūr, is to be identified with Abū ʿAlī al-Ḥasan ibn ʿAlī al-Marrākushī (ca. 680/1281-2), a famous Maghribī astronomer in Cairo.[32] The title page describes the scribe as the author of *Kitāb al-mabādīʾ wa al-ghāyāt*, which is a match for a work composed by this astronomer, the *Book of the Collected Principles and Goals of the Science of Timekeeping (Kitāb jāmiʿ al-mabādīʾ wa al-ghāyāt fī*

(2018) 29-30; Kunitzsch and Lorch (2010b) 5; study of Autolycus's *On the Moving Sphere*: Nikfahm-Khubravan and Eshera (2019) 45; edition of Euclid's *Optics*: Kheirandish (1999) xxvii and xxxiv; study and edition of Menelaus's *Spherics*: Sidoli and Kusuba (2014) 161-163 and Rashed and Papadopoulos (2017) 489, 496-498, respectively; edition of Theodosius's *On Habitations*: Kunitzsch and Lorch (2010a) 10-11; edition of Theodosius's *On Days and Nights*: Kunitzsch and Lorch (2011) 13; and study of Aristarchus's *On Sizes and Distances*: Berggren and Sidoli (2007) 235-237.

[32] Kheirandish (1999) xxvii.

ʿilm al-mīqāt).[33] From this work, it can be seen that al-Marrākushī was a well-practiced astronomer – in it, he offers detailed comments on spherical astronomy, sundials, armillary spheres, astrolabes, and various further timekeeping devices.[34]

The mathematical competency of al-Marrākushī shows in many of the witnesses in the Kraus manuscript. Several editors have noted the Kraus witnesses as possessing especially clear – and frequently mathematically preferable – text and figures.[35] But furthermore, this codex was evidently penned by a scribe willing to adapt the texts for his own goals. Scholarship on its witness of the *Spherics* has made this especially clear. For example the Kraus manuscript preserves a epitome of al-Harawī's version of the *Spherics*.[36] Many of al-Harawī's additions are removed, including the introduction (detailing the history of the translation and its difficulties) as well as the preface to the second book. Statements by al-Harawī have been restructured to be more impersonal. The enunciations of the propositions have all been stripped out, so that each proposition begins immediately with its ekthesis. Sidoli and Kusuba have suggested that these changes were intended to streamline and facilitate the mathematical study of this otherwise difficult text.[37] The removal of enunciations is quite notable, because while mathematical scholars will intervene in the proofs of a proposition, it is not common to see the enunciations significantly changed, let alone removed entirely.[38]

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[33] See Lorch (2008) 28 for the Arabic of the codex's title page: "هذه النسخة بخط الشيخ ابي علي المشهور وهو مؤلف كتاب المبادئ والغايات."

[34] See King, D.A. "al-Marrāku<u>sh</u>ī" in EI2.

[35] See Kheirandish (1999) xxvii, Sidoli and Isahaya (2018) 29, and Sidoli and Kusuba (2014) 161. Berggren and Sidoli (2007) 236-237 note copyist errors in the text but point to the diagrams as particularly well-produced.

[36] This manuscript witness is discussed in both Sidoli and Kusuba (2014) 161-163 and Rashed and Papadopoulos (2017) 489, 496-498. See the latter especially for a detailed list of differences between this epitome and al-Harawī's version.

[37] Sidoli and Kusuba (2014) 163.

[38] Rather, we instead sometimes see the opposite: codices that contain inventories of the enunciations alone, perhaps as a tool for students or other aide-mémoire. But the removal of enunciations is not unheard of: in the preface to his edition of the *Elements*, Muḥyī al-Dīn al-Maghribī criticizes Ibn Sīnā and Nīsābūrī for removing the enunciations from their recensions of the text. See Sabra (1969) 14-15.

Besides the *Spherics*, other witnesses in the Kraus manuscript also appear to show interventions by the scribe. In the case of the *Data*, Sidoli and Isahaya note that impersonal and passive constructions are frequently transformed to be personal and active ones. Since this would appear to be the opposite of what occurred in the *Spherics*, perhaps we should understand al-Marrākushī as having amassed his texts from different sources. In the case of the *Sphaerica*, Kunitzsch and Lorch identify the witness as a reworking but do not offer details on how it differs. They note it has a preface close to the preface presented by al-Ṭūsī, but with the details of the translator stripped out. Perhaps this shows a disinterest regarding the historical asides that is similar to the disinterest exhibited in the witness of the *Spherics*.

2.2.5 Leiden Or. 1031 (date unknown)

The codex Leiden Or. 1031 preserves three works of the Middle Books in versions prior to al-Ṭūsī's edition. The manuscript has no date offered by any of its colophons. Its citations of Ibn al-Ṣalāḥ, noted below, necessarily give it a terminus post quem of the twelfth century.

The mathematical diagrams have been drawn with care, with erroneous lines erased by scraping the ink away. The manuscript comprises 91 folios on Islamic paper, with 8 smaller inserts (also on Islamic paper) that have been added separately. Most of these inserts carry additional mathematical diagrams, with occasional textual comments. The count of 91 folios is in agreement with the page count written on the title page.[39] The folios are gathered together into eleven quires of varying length, and changes of hand are apparent between the quires.[40] The contents of the manuscript are as follows:

1. Autolycus's *On Risings and Settings*
2. Theodosius's *Sphaerica*
3. Euclid's *Phaenomena*

[39] A modern hand has written page numbers in European digits with pencil, including on the inserts, so that the total folio count according to this hand is 99 folios. This will be the foliation cited for references in this manuscript.
[40] On six of them, traces of original quire numbers are visible in the top left corner – only three of these are legible. What is visible does suggest that the middle quires may have been reorganized, but this requires further investigation.

Of these three texts, the *Sphaerica* and the *Phaenomena* preserve more information about how scholars had interacted with them.

The colophon to the third book of the *Sphaerica* records that this witness was copied from a copy itself copied from a manuscript in the hand of the mathematician Najm al-Dīn ibn al-Sarī – the Ibn al-Ṣalāḥ (6th/12th c), mentioned above. The lengthy colophon also mentions propositions with mathematical difficulties which were addressed in a work of Sinān ibn Thābit ibn Qurra (4th/10th c.) on tangent circles and Menelaus' *Spherics*.[41] These comments point to a nexus of works being consulted during the study of this text.

This particular witness of the *Sphaerica* offers insights into Ibn al-Ṣalāḥ's work with the treatise. The text of the witness noticeably differs from that of the ninth century translation of the *Sphaerica* – Kunitzsch and Lorch have suggested this text might be that of an edition produced by Ibn al-Ṣalāḥ himself. What can be said more definitively is that this manuscript preserves marginalia from the scholar in question, which will again be elaborated upon below.

Lastly, the colophon of the *Phaenomena* also identifies it as a copy from a copy in Ibn al-Ṣalāḥ's hand.[42] It similarly possesses numerous scholarly marginalia, several of which identify their source as the scholar in question.

3. Scholars, Teachers, and Students of Astronomy

A combination of manuscript evidence and the testimony of biobibliographers and other scholars allows for several individuals to be highlighted for their engagement with the *Elements*, Middle Books, and *Almagest* curriculum.

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[41] This colophon appears on fol. 72b of Leiden Or. 1031. It will be considered further below.
[42] Leiden Or. 1031, fol. 99b: "نسخت هذه النسخة من نسخة انتسخ من الاصل بخط الامام الاجل الاوحد نجم الدين ابو الفتوح احمد بن محمد بن السري... وفي آخره مكتوب بخطه قوبل بنسخة بخط ابي بكر الازرق وراق حنين وعليها حواشي لصاعد واصلاحات له ه ه"

3.1 Al-Nayrīzī (c. 250-310 H / 865-922 CE)

Abū al-ʿAbbās al-Faḍl ibn Ḥātim al-Nayrīzī was a well-regarded mathematician and astronomer who flourished under the reign of ʿAbbāsid Caliph al-Muʿtaḍid (r. 279-289 H / 892-902 CE). He was born in the town of Nayrīz (in modern day Iran) but spent at least part of his professional life in Baghdad. He is known to have produced works dedicated to Caliph al-Muʿtaḍid.[43]

He is noted here because he is reported to have written commentaries on the *Elements*, on the *Phaenomena*, and on the *Almagest*. Only the commentary on the *Elements* survives.[44] The commentary on the *Almagest* was known to al-Bīrūnī (d. c. 440 H / 1048 CE). The commentary on the *Phaenomena* was still extant in al-Ṭūsī's day, since al-Ṭūsī made use of it when producing his own edition of the *Phaenomena*.[45] While this of course is not the full Middle Books curriculum, the grouping is notable. Further, in al-Nayrīzī's preface to the commentary on the *Elements*, he repeats an idea which we have seen was already in circulation by the third / ninth century: that the *Elements* served as preparation for the *Almagest*.

> "Now the science in this book is preliminary to the science in the book of the great Ptolemy on the reckoning of the stars… This is what is called the *Almagest*. Whoever looks into this book and into the study of the elements that are in it, will find the study of what is in the book of the *Almagest* easy, so that he will understand it fully, if Allah is willing. But as for him who does not look into it and does not understand it, neither will he understand what is in the *Almagest* any more than if he were studying fiction, credulously, like a fool. But as for comprehensive knowledge, there is no way to that except by understanding these elements."[46]

So it is uncertain whether al-Nayrīzī's work intersected much with the Middle Books beyond the *Phaenomena*, but it is clear that part of his endeavors involved commentaries on material that was

[43] These include a treatise on meteorological phenomena and another on instruments for determining distances: see Sabra (1974).
[44] See Besthorn and Heiberg (1897, 1900, and 1905) for the edition of the text.
[45] Al-Ṭūsī reports this in the preface to his edition. This will be discussed in more detail in chapter 9.
[46] Translation from Lo Bello (2003) 86-87. See the Arabic in Besthorn and Heiberg (1897) 4-6:
"وعلم هذا الكتاب مقدمة لعلم كتاب بطلميوس الكبير فى حساب النجوم... الذى يقال له المجسطى فمن نظر فى هذا الكتاب فى علم هذه الاصول التى فيه سهل عليه العلم بما فى كتاب المجسطى حتى يحيط به علماً ان شاء الله ومن لم ينظر فيه ولم يعلمه لم يعلم ما فى المجسطى الّا علم رواية وتقليد امّعة فامّا علم احاطة فلا سبيل الى ذلك الّا بعلم هذه الاصول."

considered necessary reading before the *Almagest*. Chapter 9 will return to Nayrīzī to touch on how traces of his *Phaenomena* commentary appear in al-Ṭūsī's edition of the *Phaenomena*.

Commentaries were not al-Nayrīzī's only work. He produced various mathematical and scientific texts, including two zījes, or astronomical handbooks. The *Great Zīj* (*Kitāb al-zīj al-kabīr*) is said by Ibn al-Qifṭī to have been based on the Indian *Sindhind*; the second has been supposed to have been based on the *Almagest*.[47]

3.2 Al-Ḥasan ibn al-Haytham (c. 354-430 H / c. 965-1040 CE)

The end of the tenth century and beginning of the eleventh saw the scholarship of Abū ʿAlī al-Ḥasan ibn al-Haytham al-Baṣrī al-Miṣrī (Alhazen in medieval Europe), one of the most famous mathematicians and scientists from the medieval Islamicate world. He was born in Basra and spent the first part of his life in Iraq until he was invited to Egypt by the Fatimid caliph al-Ḥākim (386-441 H / 996-1021 CE).

A report from Ibn al-Qifṭī connects Ibn al-Haytham to the Middle Books:

> "The wise Yūsuf al-Nāshī al-Isrāʾīlī mentioned to me in Aleppo: I heard that Ibn al-Haytham would copy three books a year in the field of his interest. They were *Euclid*, the Middle Books and the *Almagest*. He would complete them in the course of a year. When he would undertake their transcription, someone would come to him giving him one hundred and fifty Egyptian dinars for them. This became the price of which there was no need of bargaining or reiteration. So he made this his provisions for the year. He did not cease this until he died in Cairo at the end of the year 430 [September 1039] or a little after. But God knows best."[48]

So Ibn al-Haytham is supposed to have output the full curricular series every year, and every year there was someone willing to pay him for these books, suggesting a certain frequency of their usage, at least in

[47] Sabra (1974).
[48] See Lippert (1903) 167 for the Arabic:
"وذكر لي يوسف الناشى الإسرائيلي الحكيم بنزيل حلب قال سمعت أن ابن الهيثم كان ينسخ في مدة سنة ثلاثة كتب في ضمن اشتغاله وهي إقليدس والمتوسطات والمجسطي ويستكملها في مدة السنة فإذا شرع فى نسخها جاءه من يعطيه فيها مائة وخمسين ديناراً مصرية وصار ذلك كالرسم الذي لا يحتاج فيه إلى مواكسة ولا معاودة قول فيجعلها مؤونته لسنته ولم يزل على ذلك إلى أن مات بالقاهرة في حدود سنة ثلاثين وأربعمائة أو بعدها بقليل والله أعلم"

Cairo during this period. Ibn al-Haytham evidently made his living in this way for the last twenty or so years of his life, after the death of Caliph Al-Ḥākim in 411/1021.[49]

This is one of several reports that stress how Ibn al-Haytham had supported himself through the transcription of mathematical texts. Ibn Abī Uṣaybiʿa adds that "Each year he was transcribing Euclid and the *Almagest* and selling them, and by this sale supported himself. He did not cease from this until he passed, may God have mercy."[50] and that "He had extremely precise handwriting, with which he transcribed the majority of the mathematical sciences."[51] There does exist an extant manuscript of Apollonius's *Conics* written in Ibn al-Haytham's hand: Ayasofya 2762, dated Ṣafar 514 H / May-June 1024 CE. While this is not one of the curricular works discussed in the passage above, it existence does lend support to the reports that Ibn al-Haytham made a living through copying mathematical texts.

Ibn al-Haytham was well-known for his research in various topics in mathematics and for his work with the texts of the ancients.[52] He famously wrote *Doubts on Ptolemy* (*al-Shukūk ʿalā Batlamyūs*), in which he took a very critical stance on Ptolemy's mathematical astronomy.[53] His work with the *Almagest* can also be seen in a work titled *Resolution of Doubts about the Work "Almagest" which are Difficult for Some People of Science* (*Ḥall shukūk fī kitāb al-Majisṭī yashukku fīhā baʿḍ ahl al-ʿilm*).

Ibn al-Haytham's approach to these mathematical treatises is well illustrated in his *Commentary on the premises of Euclid's Elements* (*Maqāla fī sharḥ muṣādarāt Kitāb Uqlīdis*).[54] Despite the title identifying this work as a "commentary" (شرح), the scholar did not intend this work as a didactic tool, or at least not one for students early in their studies. The commentary sets out to expand on the postulates in

[49] Lippert (1903) 167.
[50] Ibn Abī Uṣaybiʿa (2000) 14.22.2: "وكان يكتب في كل سنة إقليدس والمجسطي ويبيعهما ويقتات من ذلك الثمن ولم تزل هذه حاله إلى أن توفي رحمه الله"
[51] Ibn Abī Uṣaybiʿa (2000) 14.22.3.1: "وكان له خط قاعد في غاية الصحة كتب به الكثير من علوم الرياضة"
[52] Rosenfeld and Ihsanoglu (2003) 131-138 list fifty two works in mathematics and thirty in astronomy.
[53] See Sabra (1978).
[54] Edition in Sude (1974).

Euclid's *Elements*, which Ibn al-Haytham notes were left unexplained by Euclid *because* Euclid was writing "for beginners" (للمبتدئين). Ibn al-Haytham, conversely, writes for those who are past this stage and who are interested in further discussion of the topic:

> "He [Euclid] only postulates them [the five postulates] and does not explicate them not because he is unable to explain them and not because it is impossible to explain them, but he refrains from explaining them only in order to use them as starting points, since it is complicated to explain them or [even] some of them. This book of his on the 'Elements' is an introduction to the sciences of mathematics and is designed for beginners in this science; one does not use complex syllogisms in introductions made for beginners. Euclid avoided explaining them because of the complexity which would have occurred in their syllogisms. He did not hesitate to postulate them, however, since they are true and possible, not impossible, propositions. The reason why Euclid began with these concepts but did not explain them has now been demonstrated. Because our intention in this book is to comment on what Euclid presented in his preliminary remarks and to clear up what is obscure in them and explain it, it is necessary for us to demonstrate these concepts in a way clear to the understanding and in which no confusion will occur afterwards."[55]

Overall, the scholarship demonstrated by Ibn al-Haytham is not that of a didactic type – there is no suggestion that he was teaching particular texts. His oeuvre is oriented more towards research than towards didactic tools like introductions, summaries, and (student-oriented) commentaries. But he contributed to the circulation of these texts by producing manuscripts, and was clearly extremely experienced with the ancient sciences.

3.3 Muḥammad ibn al-Haytham (4th-5th c. H / 10th-11th c. CE)

The Ibn al-Haytham discussed above should not be confused with Abū ʿAlī Muḥammad ibn al-Ḥasan ibn al-Haytham al-Baṣrī – also 10-11th century, also born in Basra – who worked in Baghdad. Care must be taken because there is confusion in the sources which has led to the conflation of the two

[55] Sude (1974) 75. See p. ٣٢-٣١ for the Arabic:
"وانما صادر عليها ولك يبينها لا بعجز منه عن تبيينها ولا لأنها لا يمكن تتبين وانما عدل عن تبيينها الى المصادرة عليها لأن في تبيينها أو تبيين بعضها تعلفا. وكتابه هذا الذى هو في الاصول هو مدخل الى صناعة التعاليم وهو موضوع للمبتديين بهذه الصناعة، والمداخل الموضوعة للمبتديين لا تستعمل فيها المقاييس المتعسفة. وانما عدل اقليدس عن تبيينها للتعسف الذى يعرض في مقاييسها. ولم يتحاش أن يصادر عليها لأنها قضايا صادقة وممكنة غير متعذرة. فقد تبينت العلة التي من اجلها صادر اقليدس على هذا المعاني ولم يبينها. ولأن قصدنا في هذا الكتاب شرح ما قدمه اقليدس في صدور اقاويله وكشف ما غمض منها وتبيينه، وجب أن نبين هذه المعاني بيانا يتضح للفهم ولا يعرض فيها من بعده التباس."

scholars, which has persisted in scholarship on Ibn al-Haytham until recently.[56] Where al-Ḥasan ibn al-Haytham, above, was a scholar involved with research into various mathematical sciences, Muḥammad ibn al-Haytham was more of a philosopher who taught mathematics and astronomy as part of a philosophical curriculum.

Ibn Abī Uṣaybiʿa, who had access to a scholarly autobiography of Muḥammad ibn al-Haytham written in his own hand, transcribes much of this in his *History of Medicine* and preserves insights into the philosopher's works and goals. These include thoughts by Muḥammad ibn al-Haytham on the placement and role of mathematics:

> "Thus, Aristotle has judiciously set out the guidelines along which one may travel toward the truth, and so attain to its nature and substance, and find its essence and nature. When I – Ibn al-Haytham – realized that, I devoted all my efforts to studying the philosophical disciplines, which comprise three branches of learning: mathematics, physics, and metaphysics. I therefore concentrated on the fundamentals and principles which govern these three fields and their consequences, and I arrived at a good understanding of them in all their depths and heights… From these three fundamental subjects (mathematics, physics, metaphysics) I explained in detail, summarized and condensed in an orderly way what I was able to understand and discern. I have drawn upon their assorted contents to compose works that clarify and reveal the obscurities of these three fundamental domains right up to the present time, which is the month of Dhū l-Ḥijjah in the year 417 [January/February 1027] of the migration (Hijra) of the Prophet, God bless him and keep him. As long as I live, I will devote all my energy and all my strength to such endeavors with three aims in mind: first, to benefit the person seeking truth and influence him during my lifetime and after my death; second, as an exercise for myself in these matters to confirm what my reflection on these disciplines has formulated and organized; and, third, to create for myself a treasure-house and provision for the time of old age and period of senility. In doing this I have followed what Galen says in the seventh book of his treatise On the Method of Healing: 'In all my writings, it has been and remains my intention to do one of two things: either to benefit someone through something useful and profitable, or to benefit myself through mental exercise, by which I enjoy myself at the time of my writing it and [at the same time] make a store-house for the time of old age'."[57]

[56] Rashed (1993) 8-19 gives an overview of the problem and the contradictions and shows how the two scholars must be distinguished. See also Rashed (2013) 11-25 for an English translation. The problem of whether "Ibn al-Haytham" should be identified as one person or distinguished as two separate individuals saw disagreement between Sabra, who argued for the former, and Rashed, who argued for the latter. See Thomann (2017) 931-932 for an overview of this scholarship, as well as for some further evidence in support of Rashed's position.

[57] Translation from Ibn Abī Uṣaybiʿa (2000) 14.22.4.1. Note, as mentioned above, the confusion in the sources: Ibn Abī Uṣaybiʿa erroneously conflates al-Ḥasan ibn al-Haytham and Muḥammad ibn al-Haytham, reporting on both

An Aristotelian mindset comes across very clearly here. Mathematical sciences serve as one of several preliminary subjects to prepare one for the study of physics and metaphysics. Meanwhile, Muḥammad ibn al-Haytham's goals are multiple, but among them are didactic ones: through commentaries and summaries of fundamental subjects (including mathematics), he aims to provide a benefit to the person who seeks truth through study of these subjects.

Al-Ḥasan ibn al-Haytham's reported involvement with the Middle Books was through the copies he produced to support himself later in life. Conversely, Muḥammad ibn al-Haytham seems to have used the Middle Books – or works from the grouping, at least – to support his didactic goals in the commentaries he wrote. His commentary on the *Almagest*, for example, is expressly oriented towards students:

> "I found the main intention of the majority of those who have given their commentary on the Almagest was to describe the chapters on calculation and to expand on them, revealing aspects other than those revealed by Ptolemy, without clarifying those chapters containing ideas too obscure for the beginner… I had the idea of setting out a proposition in the commentary of this book, the Almagest, where my principal objective would be to elucidate subtle ideas for the benefit of students."[58]

The text of this commentary cites from multiple authors both original to and added to the Middle Books: Euclid, Archimedes, Autolycus, Hypsicles, the Banū Mūsā, and Thābit ibn Qurra; it also draws from al-Nayrīzī, who has been seen to have written commentaries on some relevant works. It does draw from

under the same entry. This text is reported to have come from "Muḥammad ibn al-Ḥasan ibn al-Haytham." See also the Arabic text from the edition Ibn Abī Uṣaybiʿa (2000) 14.22.4.1:

"فأحكم الأصول التي فيها يسلك إلى الحق فيدرك طبيعته وجوهره ويوجد ذاته وماهيته. فلما تبينت ذلك أفرغت وسعي في طلب علوم الفلسفة وهي ثلاثة علوم رياضية وطبيعية وإلهية فتعلقت من هذه الأمور الثلاثة بالأصول والمبادئ التي ملكت بها فروعها وتوقلت بأحكامها رعانها وعلوها... فشرحت ولخصت واختصرت من هذه الأصول الثلاثة ما أحاط فكري بتصوره ووقف تمييزي على تدبره وصنفت من فروعها ما جرى مجرى الإيضاح والإفصاح عن غوامض هذه الأصول الثلاثة إلى وقت قولي هذا وهو ذو الحجة سنة سبع عشرة وأربعمائة لهجرة النبي صلى الله عليه وسلم وأنا ما مدت لي الحياة باذل جهدي ومستفرغ قوتي في مثل ذلك توخياً به أموراً ثلاثة أحدها إفادة من يطلب الحق ويؤثره في حياتي وبعد وفاتي الآخر أني جعلت ذلك ارتياضاً لي بهذه الأمور في إثبات ما تصوره وأتقنه فكري من تلك العلوم والثالث أني صيرته ذخيرة وعدة لزمان الشيخوخة وأوان الهرم فكنت في ذلك كما قال جالينوس في المقالة السابعة من كتابه في حيلة البرء إنما قصدت وأقصد في وضع ما وضعته وأضعه من الكتب إلى أحد أمرين إما إلى نفع رجل أفيده إياه وأما أن أتعجل أنا في ذلك رياضة أروض بها نفسي في وقت وضعي إياه وأجعله ذخيرة لوقت الشيخوخة."

[58] Translation from Thomann (2017) 928.

further sources beyond the Middle Books: Apollonius, Ibn Sinān, and Galen are cited as well.[59] But the inclusion of Autolycus and especially Hypsicles, both of whom receive very limited study outside of Little Astronomy or Middle Books contexts, strongly suggests that the Middle Books were one source that this commentary drew upon.

3.4 Al-Nasawī (5th c. H / 11th c. CE)

Abū al-Ḥasan ʿAlī ibn Aḥmad al-Nasawī was an eleventh century mathematician and astronomer from Rayy (in Iran). Several reports identify al-Nasawī as a teacher in particular. His student Shahmardān Rāzī and, later, Naṣīr al-Dīn al-Ṭūsī both refer to the scholar with the respectful title "distinguished teacher" (al-ustādh al-mukhtaṣṣ) There is furthermore a report from the Iranian poet Nāṣir-i Khusraw (1003-1088) that al-Nasawī was teaching Euclid's *Elements*, medicine, and arithmetic in Simnān (Iran) in 1046.[60]

Al-Nasawī's work on the *Elements* is further seen in his *Abstract of Euclid* (*Tajrīd Uqlīdis*), which summarizes books I-IV and XI. The scholar writes that this work provides both an introduction to the *Elements* and all the necessary geometry required for the *Almagest*.[61] So subjects preliminary to the *Almagest* was clearly an area of work of interest to al-Nasawī.

His familiarity with the Middle Books is seen in a comment preserved by al-Ṭūsī. Al-Nasawī writes that "the moderns added [the *Lemmata*] to the collection of the Middle [Books] which are to be

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[59] On authorities cited in the commentary, see Rashed (2013) 23.

[60] Thackston (1986) 2-3. Granted, this particular report is not especially impressed by this meeting with al-Nasawī, stressing how he took pains to namedrop his teacher Ibn Sina and how he was engaged in teaching arithmetic when he himself was not yet familiar with the subject.

[61] See Sezgin (1974) 347: "Der Verfasser sagt nämlich in seinem Vorwort, daß er aus den Elementen des Euklid und anderen Werken solche Figuren und Lehrsätze ausgezogen und zu einem Werk verarbeitet habe, die als geometrisches Material für die astronomische Wissenschaft, besonders für das Verständnis des Almagest des Ptolemäus, notwendig sind; Hds.: Haidarabad, Salar Junk 3142." Note this work is sometimes confused with a similarly titled work, *al-Tajrīd fī uṣūl al-ḥandasa* – Sezgin finds these to be two separate works.

read between the book of Euclid and the *Almagest*."[62] Al-Nasawī was not only passingly familiar with this curriculum – he was able to comment on how his contemporaries had updated it.

This is far from al-Nasawī's only comment on the *Lemmata* – extant signs of his scholarship are most evident today in the many comments he made on the text. Al-Ṭūsī preserved many, if not all, of these comments in his own edition of the *Lemmata*. In al-Ṭūsī's text, each of al-Nasawī's comments are clearly cited as coming from "the distinguished teacher." Of the comments preserved in al-Ṭūsī's edition, they are all mathematical ones, often discussing particular cases of the proposition. They do not discuss, for example, the history of the text or its transmission.[63]

There is additionally surviving evidence of al-Nasawī's work with Euclid's *Data* that has been preserved in marginalia, again via al-Ṭūsī. In several manuscripts[64] of his edition of the *Data* a note on its 64th[65] proposition reads as follows: "I found, in a manuscript that Abū Naṣr Aḥmad ibn Ibrāhīm ibn Muḥammad al-Sizjī read to the distinguished master ʿAlī ibn Aḥmad al-Nasawī, Proposition 64 in this way…"[66] It then proceeds to provide an alternate proof. The note points both to al-Nasawī's work with the *Data* and to the fact that this instance occurred in a teaching context. The expression "read to" (قرأ على) in this context points to the practice of a student reciting a copied and/or memorized text back to a teacher.

Al-Nasawī's also produced a *Commentary on the Sector Figure*. As noted above, evidence from the manuscript Seray Ahmet III 3464 shows that this commentary text itself came to be grouped with the

[62] As reported by al-Ṭūsī. See Hyderabad (1939-40) "Kitāb Makhūdhāt" 2: "قد اضافها المحدثون الى جملة المتوسطات التى يلزم قراءتها فيما بين كتاب اقليدس والمجسطى"
[63] This text has been printed in Hyderabad (1939-40) *Kitāb al-maʾkhūdhāt*: see, for example, p.3, 4, 7, 10, etc.
[64] Teheran Kitābkhānayi Madrasayi ʿĀlī Shahīd Muṭahharī 4727, p. 105; Istanbul Topkapı Sarayı Library Ahmet III 3453, f. 69b; Teheran Kitābkhānayi Madrasayi ʿĀlī Shahīd Muṭahharī 597, f. 10b.
[65] This is the 64th proposition in al-Ṭūsī's count, which corresponds with the 60th proposition in the earlier Arabic version and the 62nd proposition in Greek.
[66] Translation from Sidoli and Isahaya (2019) 97. See p.96-97 for their edition of the Arabic:
"وجدت في نسخة قرأها نصر أحمد ابن إبراهيم بن محمد السجزي على الأستاذ المختصّ علي بن أحمد النسوي شكل سد على هذا الوجه. ونجعل نسبة (ا ب) إلى (جـ د) معلومة، ونقيم على (ا ب) شكل (ا ه ب) معلوم الصورة، وعلى (جـ د) متوازي الأضلاع (ا جـ د ب). وأقول: إنّه معلوم. لأنّه قد أقيم على (ا ب) شكلان كيف ما اتّفقا (ا ه ب) (ا جـ د ب)، فنسبة (ا ه ب) إلى (ا جـ د ب) معلومة. و(ا ه ب) معلوم الصورة، فـ(ا جـ د ب) معلوم الصورة."

Middle Books later in their history, alongside Thābit's work *On the Sector Figure* and Menelaus's *Spherics*, where the sector theorem appears. As this theorem sees use in Ptolemy's *Almagest*, it is not surprising to see several of the Arabic treatments of it being studied with the Middle Books.

3.5 Ibn Hūd (5th c. H /11th c. CE)

In the eleventh century, Yūsuf al-Muʾtaman ibn Hūd – the king of Saragossa between 1081 and 1085, patron of the sciences and a scholar himself – authored the *Book of Perfection* (*Kitāb al-Istikmāl*). This was a mathematical encyclopedia, albeit an unfinished one.[67] The completed first genus of the work is divided into five "species" (أنواع): the first on number; the second on the properties of lines, angles, and plane figures; the third on lines, angles, and plane figures in combination; the fourth on solid figures; and the fifth on the combination of solid figures with plane surfaces.[68]

The sources which Ibn Hūd's *Book of Perfection* draws upon reveal what mathematical works were available in eleventh century al-Andalus. They are the following:

- *Elements* (Euclid)
- *Data* (Euclid)
- *Almagest* (Ptolemy)
- *Conics* (Apollonius)
- *Measurement of the Circle* (Archimedes)
- *On the Sphere and Cylinder* (Archimedes)
- *Commentary on the Sphere and Cylinder* (Eutocius)
- *Sphaerica* (Theodosius)
- *Spherics* (Menelaus)
- *Book of Knowledge* (Banū Mūsā)
- *On the Transversal Theorem* (Thābit ibn Qurra)
- *On the Sections of the Cylinder* (Thābit ibn Qurra)
- *Measurement of the Parabola* (Ibrāhīm ibn Sinān)
- *On Analysis and Synthesis* (Ibn al-Haytham)
- *On Known Things* (Ibn al-Haytham)
- *Optics* (Ibn al-Haytham)

[67] Hogendijk (1986) 43.
[68] Hogendijk (1991) 210-213.

A significant segment of these works, of course, overlaps with the Middle Books and the works which were sometimes appended to the collection. The circulation of Middle Books works in al-Andalus will be further seen in the Latin and Hebrew translations to be discussed in chapter 6.

Some manuscript titles describe the extant five species as part of the "first genus of the two genera of the mathematical sciences" – it appears, however, that ibn Hūd never wrote the second genus, and so it is unclear what it might have contained. Hogendijk suggests that the inclusion of the *Almagest* might indicate that the second genus would have included astronomy.[69]

Though ibn Hūd does not make direct reference to the Middle Books, their wide circulation and availability likely had some influence on his selection of sources. The ordered presentation of mathematical topics in his *Book of Perfection* seems intended to serve similar didactic goals as those of the Middle Books. The later scholar Ibn Aknīn (ca. 1160-1226) recommends the *Book of Perfection* as the culmination of works which should be read by students of geometry, and the list leading up to this work includes the *Elements*, the *Sphaerica*, the *Spherics*, the *Sphere and Cylinder*, and the *Conics*.[70]

3.6 Al-Kharaqī (6th c. H / 12th c. CE)

Bahāʾ al-Dīn Abū Bakr Muḥammad ibn Aḥmad al-Kharaqī (d. 553 H / 1158-9 CE) was a mathematician and astronomer presumed to have been from Kharaq near Marw (Iranian city in modern day Turkmenistan). He worked for some time in Marw under the reigning Shāh of the Khwarazmian Empire, either either Quṭb al-Dīn Muḥammad (r. 490-521 / 1097-1127) or his son Atsiz (r. 521-551 / 1127-1156).

His *Enlightenment in Astronomy* once appeared at the end of Seray Ahmet III 3464 – the preface to this treatise explains that it is briefer presentation of material that sees further elaboration in his

[69] Hogendijk (1991) 214-215. Hogendijk comments on the terms he translates as 'genus' and 'species' as Aristotelian ones, suggesting that this was another instance of a didactic arrangement by Aristotelian-inspired principles.

[70] See the German translation in Güdemann (1873) 86-88.

Ultimate Attainment in the Division of the Spheres (*Muntahā l-idrāk fī taqāsīm al-aflāk*).[71] Al-Kharaqī's didactic efforts in the subject of astronomy can be seen to be operating in the same sphere as the Middle Books. As he concludes the preface to his *Ultimate Attainment*, he writes:

> "Then it occurred to me to assemble for my friends a book on this subject... for it to free its reader from mere imitation, ascend him through what he ponders to the level of the Middle [Books], and awaken a desire for the utmost of what is possible to achieve in this art."[72]

Both the *Ultimate Attainment* and its briefer version *Enlightenment in Astronomy*, then, were intentionally conceived of as didactic astronomical works that could function alongside the Middle Books. This is very likely the reason for the *Enlightenment in Astronomy*'s inclusion in Seray Ahmet III 3464's listing of the Middle Books. This work was apparently frequently copied and is reported to appear in a number of manuscript copies, including transliterations and translations into Hebrew.[73]

<u>3.7 Ibn al-Ṣalāḥ (6th c. H / 12th c. CE)</u>

Najm al-Dīn Abū al-Futūḥ Aḥmad ibn Muḥammad ibn al-Sarī was a mathematician from Hamadan (Iran) who spent his career in Baghdad, Mārdīn, and Damascus. As section 2 touched upon, there is clear evidence that he worked with the *Data*, the *Sphaerica*, the *Spherics*, and the *Phaenomena*. His scholarship included work with the *Almagest* and the *Elements* as well.

3.7.1 Work with the Elements and Almagest

To first note Ibn al-Ṣalāḥ's work with the *Elements* and *Almagest*, multiple treatises of his in mathematics and astronomy show research with these texts and efforts to address difficulties with them.[74] See, for example, several titles addressing components of the *Elements*:

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[71] Wiedemann and Kohl (1970) 634-636.
[72] See Ghalandari (2012) 150 for the Arabic: "فوقع لي أن أجمع لأصحابي في هذا الشأن كتاباً... ليخرج الناظر فيه عن التقليد المحض ويرتقي بما يتصوره إلى درجة المتوسطين ويتشوّق إلى كنه ما يمكن إدراكه من هذا الفن"
[73] See Langermann "al-Kharaqī, Abū Bakr" in EI3.
[74] See the inventory of Ibn al-Ṣalāḥ's known mathematical and scientific works in Rosenfeld and İhsanoğlu (2003) 177-178.

- *Answer on Proof of a Problem Attributed to the Seventh Book of Euclid's Work "Elements" and Related Discussions*[75]
- *Reasoning on Proof of what was Meant by Abū ʿAlī ibn al-Haytham in His Book on Doubts in Euclid*[76]
- *Reasoning on Explanation of the Error of Abū ʿAlī ibn al-Haytham on the First Proposition of the Tenth Book of Euclid's Work "Elements"*[77]
- *Book Revealing the Doubts of those who study Mathematical Sciences by Euclid in the Fourteenth Proposition of the Twelfth Book of the Work "Elements"*[78]

The above are still unpublished and largely unstudied. Their titles do show that Ibn al-Ṣalāḥ was quite involved with ongoing research and conversations around this text and its challenges. The Ibn al-Haytham named in these titles is likely al-Ḥasan ibn al-Haytham the mathematician and researcher rather than Muḥammad ibn al-Haytham the philosopher and teacher – al-Ḥasan ibn al-Haytham is the one responsible for a work titled *On the resolution of doubts about Book I of Euclid's treatise* (*fī ḥall shukūk al-maqālah al-ūlā min Kitāb Uqlīdis*). Ibn al-Ṣalāḥ's surviving works relevant to the *Almagest* show similar patterns of engagement:

- *Reasoning on Establishment on an Error and a Fault in Tables of the Seventh and Eighth Books of the Work "Almagest" and their Possible Correction*[79]
- *Reasoning on Proof of the Error made by Abū Naṣr al-Fārābī in his Commentary on the Seventeenth Section of the Fifth Book of "Almagest" and the Explanation of this Section*[80]

[75] *Jawāb ʿan burhān masʾala muḍāfa ilā al-maqāla al-sābiʿa min kitāb Uqlīdis fi al-Uṣul wa sāʾir mā jarrahu al-kalām fīhi*.
[76] *Qawl fī bayān mā wahama fīhi Abū ʿAlī ibn al-Haytham fī kitābihi fī al-Shukūk ʾalā Uqlīdis*.
[77] *Qawl fī īḍāḥ ghalaṭ Abī ʿAlī ibn al-Haytham fī al-shakl al-awwal min al-maqāla al-ʿāshira min kitāb Uqlīdis fī al-Uṣūl*.
[78] *Maqāla fī kashf al-shubha allatī ʿaraḍat li-jamāʿa miman yansubu nafsahu ilā ʿulūm al-taʿālīm ʿalā Uqlīdis fī shakl al-rābiʿ ʿashar min al-maqāla al-thāniya ʿashara min Kitāb al-Uṣūl*.
[79] *Qawl fī thabt al-khaṭa wa al-taṣḥīf al-ʿāriḍayn fī jadāwil al-maqālatayn al-sābiʿa wa al-thāmina min kitab al-Majisṭī wa taṣḥīḥ mā amkana taṣḥīḥuhi min hadhā*. See Kunitzsch (1975) for an edition and German translation of this text.
[80] *Qawl fī bayān mā wahama fīhi Abū Naṣr al-Fārābī ʿinda sharḥihī al-faṣl al-sābiʿ ʿashar min al-maqāla al-khāmisa min al-Majisṭī wa sharḥ hadhā al-faṣl*.

- *On what Ptolemy Mentioned in the Second Chapter of the Twelfth Book on Defining the Magnitude of the Retrograde Movement of Saturn and in the following four chapters on retrograde Movement of Remaining Planets*[81]

These are, again, works focused on particular problems or engaging with the scholarship of others working with these texts. The first of these concerns the star catalogue in Ptolemy's *Almagest* and is of historical interest for its report on the transmission of the *Almagest* in Syriac and Arabic – Ibn al-Ṣalāḥ describes, in order, five versions of the *Almagest* that were produced in Islamicate times:

1. An early translation into Syriac
2. The early 3rd / 9th century translation made for Caliph al-Maʾmūn by al-Ḥasan ibn Quraysh
3. The 212 H / 827-8 CE translation made for Caliph al-Maʾmūn by al-Ḥajjāj and Sarjūn ibn Hilīyā
4. The ca. 265-277 H / ca. 879-890 CE translation made by Isḥāq ibn Ḥunayn
5. The revision of Isḥāq's translation by Thābit ibn Qurra (d. 288 H / 901 CE)

Ibn al-Ṣalāḥ reports that he made use of all of these versions in his work on Ptolemy's star catalogue.[82] This is valuable insight into the scholar's own practices: for his scholarship he will delve into the multitude of different versions and manuscript copies that are available to him. He shows attention both to mathematical arguments and to the historical transmission of these texts.

3.7.2 Work with the Data

The remnants of Ibn al-Ṣalāḥ's engagement with the Middle Books are to be found in marginalia and colophons attached to some manuscripts of these treatises. There are two marginalia citing him by name (Aḥmad ibn al-Sarī) that are extant in witnesses of the *Data*, in the version revised by Thābit ibn Qurra.[83] The first of these appears on proposition 55:

> "Aḥmad ibn al-Sarī said: I found this proposition with no condition in the available copies, but we need to stipulate that it is a parallelogram."[84]

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[81] *[Mā] dhakarahu Baṭlamyūs fī al-bāb al-thānī min al-maqāla al-thāniyya ʿashar fī marifat miqdār rujūʿ Zuḥal wa fī al-abwāb al-arbaʿa allatī baʿdahu lī rujūʿ bāqī al-kawākib*.

[82] Kunitzsch (1975) 155 (Arabic), 40 (German translation).

[83] These appear in Istanbul Topkapi Saray Ahmet III 3464. The Arabic texts of these marginalia are edited in Sidoli and Isahaya (2018) 125 and 179, respectively.

[84] See Sidoli and Isahaya (2018) 125 for the Arabic: "قال أحمد بن السري: وجدت هذا الشكل بغير شريطة في سائر النسخ، ونحتاج إلى أن نشترط فيه متوازي الأضلاع"

The phrase "in the available copies" (في سائر النسخ) suggests that for his work with the *Data*, too, Ibn al-Ṣalāḥ consulted a variety of manuscripts. The second marginal comment appears on proposition 82 and reads as follows:

> "Aḥmad ibn al-Sarī said: I am not prematurely of the opinion that AB by BG is surface AG, because this is [true] only if angle ABG is right. And if it is not, then surface AB by BG is greater than surface AG. And rather, the ratio AB by BG to AB by BD becomes known, because AB by BG is known, because its ratio to known AG is known. And this is that their sides are similar, so their ratios are known. The angles AB by BG are right, so they are known. The angles of surface AG by supposition are known, so surface AB by BG and AG, their angles are different (and) known. And the ratio of their sides, each of them to each, is known, so the ratio of one of them to the other is known, from Proposition 67. So AB by BG is known."[85]

The two comments range from a brief comment identifying a condition required by the argument (present in the Greek but missing in the Arabic version) to presenting a fuller mathematical argument, complete with reference to a previous proposition. Sidoli and Isahaya see in these comments evidence that the scholar was probably teaching Thābit's revision of the *Data*.[86]

3.7.3 Work with the Spaherica

Evidence of Ibn al-Ṣalāḥ's work with Theodosius's *Sphaerica* is to be found in the manuscript Leiden Or. 1031. As noted, the colophon of this witness identifies it as copied from a copy itself copied from a manuscript in the hand of Najm al-Dīn ibn al-Sarī – i.e., Ibn al-Ṣalāḥ.[87] The colophon points out propositions from the third book of the *Sphaerica* which have received objections – first, proposition

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[85] See Sidoli and Isahaya (2018) 179 for the Arabic:
"قال أحمد بن السري: لا يسبقني إلى الظن أنّ (ا ب) في (ب جـ) هو سطح (ا جـ)، لأنّ هذا إنّما يكون إذا كانت زاوية (ا ب جـ) قايمة. وإذا لم تكن، فسطح (ا ب) في (ب جـ) أعظم من سطح (ا جـ). وإنّما صارت نسبة (ا ب) في "ب ج" إلى (ا ب) في (ب د) معلومة، لأنّ (ا ب) في (ب جـ) معلوم، لأنّ نسبته إلى (ا جـ) المعلوم معلومة. وذلك أنّ أضلاعهما متساوية، فنسبتها معلومة. وزاويا (ا ب) في (ب جـ) قايمة، فهي معلومة. وزوايا سطح (ا جـ) بالفرض معلومة، فسطحا (ا ب) في (ب جـ) و(ا جـ) زواياهما مختلفة معلومة، ونسبة أضلاعهما بعضها إلى بعض معلومة، فنسبة أحدهما إلى الآخر معلومة، من شكل سز. فـ(ا ب) في (ب جـ) معلوم."

[86] Sidoli and Isahaya (2018) 26.

[87] Leiden Or. 1031, fol. 72b: "تمت المقالة الثالثة من كتاب ثودوسيوس وتم الكتاب... نسخ هذا الكتاب من نسخة انتسخ من نسخة بخط الاجل الامام نجم الدين ابن السري وفي آخرها بخطه هذا الفصل بلغت تصفيحا لهذا الكتاب جميعه وفي هذه المقالة الثانية منه اشكال معترضة مزيفة الحكم وهي شكل "ح" قد اعترضه ابرهيم بن سنان بن ثابت في المقالة الرابعة من كتابه في الدوائر المماسة وشكل "يا" قد اعترضه منالاوس في اواخر المقالة الثانية من كتابه في الكريات ونحن قد ذكرنا ذلك بكلام ابسط في هذه الاشكال وما يبنى في قول لنا مفرد سى(؟) عن كيفية التحليل بالعكس ه ه"

III.8, which the colophon says was addressed in the fourth chapter of Ibrāhīm ibn Sinān ibn Thābit's work on tangent circles. In MS Leiden Or. 1031's transcription of proposition III.8 there are four short and unattributed marginalia on fol. 66b. Three of these are marked as corrections with the abbreviation صح for "correct" (صحيح). One of them indicates a step in the argument is because two circles are great circles.

The second proposition mentioned is *Sphaerica* III.11, which the colophon states was addressed by the end of the third book of Menelaus's *Spherics*. In this instance, the manuscript preserves more on the matter – fol. 70a has a marginal note on *Sphaerica* III.11 that is attributed to Ibn al-Ṣalāḥ. This is his take on the lemmas that appear on III.11 in multiple manuscripts and which aim to prove an inequality Theodosius had stated in the proposition without proof.[88] It is the one marginal note in this witness of the *Sphaerica* that is explicitly attributed to Ibn al-Ṣalāḥ. It provides a mathematical argument, similarly to the second of the marginalia in the *Data* discussed above, complete with a marginal diagram. There are multiple other marginalia to this witness – their source is unnamed, but it may not be unreasonable to suggest some of these come from Ibn al-Ṣalāḥ.

3.7.4 Work with the Spherics

Ibn al-Ṣalāḥ's work with with Menelaus's *Spherics*, meanwhile, is evidenced by the manuscript London British Library Or. 13127, completed in Damascus in 4 Rabīʿ II 548 / 5 July 1153. In the colophon, the copyist Ismaʿīl reports that he transcribed this manuscript from a copy in the hand of Ibn al-Ṣalāḥ.[89] This Ismaʿīl may have been a student of Ibn al-Ṣalāḥ, to whom he refers with the title "our master" (سيدنا), among other honorifics. The mathematician is reported to have ultimately settled in

[88] See Kunitzsch and Lorch (2010b) 316-327 for multiple examples of these lemmas and 419-427 for their mathematical commentary on them.

[89] Colophon in London British Library Or. 13127, fol. 51a: "تمت المقالة الثالثة من كتاب منالاوس في الأشكال الكرية وتم الكتاب بأسره وذلك في يوم الأثنين رابع ربيع الآخر سنة ثمان وأربعين وخمس مائة للهجرة النبوية الحمد لله رب العالمين وصلواته على سيدنا محمد النبي وآله الطاهرين وسلم تسليمًا حسبنا الله ونعم الوكيل قال إسمعيل نسخت هذه النسخة بدمشق من نسخة سيدنا الشيخ الأجل الإمام العالم الفاضل الزاهد نجم الدين سيد الحكماء بديع الزمان أبي الفتوح أحمد بن السري أطال الله بقاه وحرس نعماه"

Damascus (where he lived until his death at the end of the year 548/1153-4[90]) – this manuscript would have been copied shortly before he died.

There are marginalia in this manuscript, though multiple are faded to the point of illegibility. Some of these are attested also in the manuscript Topkapi Seray Ahmet III 3464.[91] Three of these can be seen to be attributed by name to Ibn al-Ṣalāḥ, and a fourth to "our master," paralleling what is found in the colophon. The one attributed to "our master" appears first, attached to *Spherics* proposition I.14. It begins as follows:

> "Our master – may God perpetuate his days – said: it is possible to demonstrate this proposition with a proof that is better than what is in the original and with a decrease in its conditions, and it is said this way…"[92]

This marginal note then proceeds into an alternate proof, again accompanied by marginal diagrams. It has the full structure of enunciation, exposition (مثاله), specification (فأقول إن), demonstration (برهان ذلك), and QED (ذلك ما أردنا أن نبين).

The notes attributed to Ibn al-Ṣalāḥ by name appear on propositions *Spherics* I.37, I.41, and III.5 (= II.71, as numbered in this particular witness). The first of these begins as follows:

> "Najm al-Dīn – may God perpetuate his existence – <said:> it is possible to demonstrate this last statement of Proposition 37 without a reductio ad absurdum by a method similar to the method of Menelaus in brevity…"[93]

Proposition I.41's marginal comment reads as follows:

> "Najm al-Dīn Abū al-Futūḥ Aḥmad ibn al-Sarī – may God prolong his existence – said: this is a huge mistake because the point G is not imagined to fall between the two points A and C, but rather outside between the two points A and E, as the angle A is obtuse and the angle G is right, so GL, GA are greater than a semicircle, but both were smaller than a semicircle; this is absurd."[94]

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[90] Rashed and Papadopoulos (2017) 243.
[91] Rashed and Papadopoulos (2017) 492-496.
[92] London British Library Or. 13127, fol. 7b. The Arabic has been edited in Rashed and Papadopoulos (2017) 535:
"قال سيدنا أدام الله أيامه يمكن أن يبرهن هذا الشكل ببرهان هو أحسن من الذي في الأصل ومع أنقص شرائطه وهو أن يقال هذا..."
[93] London British Library Or. 13127, fol. 21b. See Rashed and Papadopoulos (2017) 615:
"نجم الدين أطال الله بقائه قد قد يمكن أن نبرهن هذا القول الأخير من شكل "لز" بغير الخلف وبطريق تشابه طريق مانالاوس في الإيجاز..."
[94] London British Library Or. 13127, fol. 23b. See Rashed and Papadopoulos (2017) 629:

Proposition III.5's marginal comment begins as follows in the London manuscript:

> "Our master – may God be pleased with him – said: the proof of this is evident from the converse of proposition 30 of book I...."[95]

In MS Topkapi Seray Ahmet III 3464, "Our master – may God be pleased with him" is replaced by the name Ahmad ibn al-Sari.[96]

There are further marginalia on the *Spherics* which, though they neither name Ibn al-Ṣalāḥ nor make reference to a title like "our master," still plausibly come from his hand. An example can be seen in the following comment to *Spherics* I.24:

> "In the ancient translation: and if it is greater than the two remaining angles, then the arc drawn will be smaller than half the base; and if it is smaller than the two remaining angles, then the arc drawn will be greater than half of the base."[97]

This section has already discussed above how Ibn al-Ṣalāḥ drew upon a multitude of translations and editions of the *Almagest* for his efforts to correct the star catalogue. Consulting a variety of versions was clearly part of his scholarly procedure, and it might be suggested that this marginal comment coming from his work as well.[98]

3.7.5 Work with the Phaenomena

Ibn al-Ṣalāḥ's scholarship with the *Phaenomena* follows the same pattern as what has been demonstrated: in the manuscript Leiden Or. 1031 the colophon reports that the witness was transcribed from a copy in the hand of Ibn al-Ṣalāḥ.[99] Furthermore, two marginalia reference the scholar. The first

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"قال نجم الدين ابن الفتوح أحمد بن السري أطال الله بقاءه: هذا خطأ فاحش لأن نقطة (ز) لا تتصور أن تقع بين نقطتي (ا) (جـ) بل خارجاً فيما بين
نقطتي (ا) (ه)، لأن زاوية (ا) منفرجة وزاوية (ز) قائمة، فتكون (ز ل) (ز ا) أعظم من نصف دائرة، وهما قد كانا أصغر منها؛ هذا خلف"

[95] London British Library Or. 13127, fol. 42a. See Rashed and Papadopoulos (2017) 725:
"قال سيدنا رضي الله عنه: بيان ذلك ظاهر من عكس شكل ل من ا"

[96] Rashed and Papadopoulos (2017) 725.

[97] London British Library Or. 13127, fol. 13a. See Rashed and Papadopoulos (2017) 565:
"في النقل القديم وإن كان أعظم من زاويتيه الباقيتين، فإن القوس المخرجة أصغر من نصف القاعدة؛ وإن كانت أصغر من زاويتيه الباقيتين، فإن القوس المخرجة تكون أعظم من نصف القاعدة."

[98] In Seray Ahmet III 3464, this marginal comment becomes incorporated into the main text of the manuscript.

[99] Leiden Or. 1031, fol. 99b:

appears alongside the introduction to the *Phaenomena*, commenting on Euclid's discussion of why the shape of the cosmos cannot be a cone:

> "Aḥmad ibn al-Sarī said: it is necessary to stipulate that this cone is right angled. Concerning an oblique cone, it is possible to be cut by a surface parallel to its base and the section will be a circle, as is demonstrated in proposition 5 of book 1 of Apollonius's *Conics*."[100]

The second marginal comment appears beside the figure to proposition 12 and offers a simple critique:

> "The imam Najm al-Dīn said the figure of the proposition is bad. We drew it in the appended [page]."[101]

An alternate figure does then indeed appear on one of the inserted folios.

The main text of this witness also shows attention to different copies used as part of its composition sometime in its transmission. As chapter 4 has discussed, the text of Leiden Or. 1031's *Phaenomena* largely translates the B recension of the Greek *Phaenomena*, but after its version of proposition 10, the text continues into an alternate proof of the proposition: that of *Phaenomena* recension A. This is introduced as follows:

> "Proof of the 10th figure according to what we found in another copy. We make the horizon circle ABG and let the location of the circle of the ecliptic be AEHD and we choose two similar (arcs) which are arcs AD, GE. So point D therefore is opposite to point E. And let segments ADG, DHE rise — So point D therefore is opposite to point E. And let ADG, DHE rise (it is found in the Syriac) — in unequal times."[102]

This is the only location in the text of Leiden Or. 1031's *Phaenomena* that references another copy (نسخة أخرى) and the only location that draws from recension A, though it does elsewhere offer other alternate

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"نسخت هذه النسخة من نسخة انتسخ من الأصل بخط الامام الاجل الاوحد نجم الدين ابو الفتوح أحمد بن محمد بن السري"

[100] Leiden Or. 1031, fol. 76a: "قال احمد بن السرى ينبغي أن نشرط في هذا المخروط أنه قائم الزاوية والاتي كان مخروطا مائلة امكن ان يقطع بسطح مواز لقاعدته وكون القطع دائرة كما يبين في شكل ه من مقالة ا من كتاب ابلسيوس في المخروطات ه" The above section has already noted the nexus of works used by Ibn al-Ṣalāḥ in his work with Middle Books treatises – here, Apollonius's Conics appears as another example.

[101] Leiden Or. 1031, fol. 90a: "قال الإمام نجم الدين صورة الشكل رديئة وقد صورناها في الملحقة ه"

[102] Leiden Or. 1031, fol. 86a:
"برهان الشكل العاشر على ما وجدنا في نسخة أخرى نفرض الأفق دائرة (ابح) وليكن وضع دائرة البروج بمنزله ما عليه (اهحد) ونفصل متساويتين وهما قوسا (اد) (حـ ه) فنقطة (د) إذن مقابلة لنقطة (ه) وليطلع قطعتا (ادحـ) (دحه) فنقطة (د) إذن مقابلة لنقطة (ه) وليطلع (ادحـ) (دحه) وجد في السرياني في ازمان غير متساوية"

proofs. It is especially striking that the aside "it is found in the Syriac" (وجد في السريانية) appears shortly after. There is what appears to be an error of dittography before this aside, but what might have happened here is a marginal note that has become incorporated into the main text. (In the quotation above, this is set apart by dashes.) If this was originally a marginal note, it was one which presented a very slightly different reading according to a copy from the Syriac, since it omits the word "segments" (قطعتا) from the phrase. While the main text does not reference Ibn al-Ṣalāḥ by either name or title, the examination of multiple copies (including Syriac ones or translations from them) has already been seen to be part of his process. So it is not surprising to find such material in a witness descended from one of his.

In his attested and surviving works, Ibn al-Ṣalāḥ appears to be involved with ongoing conversations and research around mathematical problems. In the case of the Middle Books, he clearly copied several of the texts and these copies saw further dissemination. He had access to multiple copies and translations that he referenced in his comments on these texts. The possibility that Ibn al-Ṣalāḥ actively taught these texts (and perhaps the Middle Books as a whole) comes from the nature of his comments – they range from trivial stipulations to full alternative proofs, both of which would serve students. The title "our master" (سيدنا) also hints at a teacher-pupil relationship.

4. Expanding the Curriculum

Chapter 3 has already alluded to the fact that a variety of works came to be added to the Middle Books between the ninth and thirteenth centuries. This can be seen both through manuscript evidence – examining what works are grouped together with the Middle Books – and through reports about the collection.

Al-Nasawī, as has been seen, offers the most direct testimony about works being added to the collection: (Pseudo-)Archimedes's *Lemmata*, he says, was added to the Middle Books because the moderns (محدثون) found in it propositions they considered useful. The *Lemmata* appears again edited as part of al-Ṭūsī's edition of the Middle Books. But it is clearly not a consistent addition to the collection – the Kraus manuscript, for example, lacks it, as does Seray Ahmet III 3464. Nor does it appear in Seray Ahmet III 3464's note about the works included in the "Middle Books of the *Almagest*," nor in Bodleian Thurston 11's note on the same matter. The additions to the Middle Book were not necessarily consistent – inclusion by one authority did not imply inclusion by others.

4.1 Originally Greek Works

The other Greek texts that can be seen added to the Middle Books at various times are Euclid's *Data*, Menelaus's *Spherics*, Archimedes's *Measurement of the Circle*, his *On the Sphere and Cylinder*, and Eutocius's *Commentary on the Sphere and Cylinder*.

Euclid's Data

The *Data* is perhaps the most consistent Greek addition to the Middle Books. It repeatedly appears at the head of the collection: in the listing in Thurston 11, in the Kraus manuscript, in Seray Ahmet III 3464, in the manuscripts of al-Ṭūsī's edition. This work receives a consistent position in the ordering of the collection, to the extent that its placement can be inferred from a description of *On the Moving Sphere* as the fifth book of the Middle Books:[103] it is the fifth because the three books of Theodosius's *Sphaerica* are the second, third, and fourth, and the book of Euclid's *Data* is the first.

Menelaus's Spherics

The *Spherics* similarly appears often, in both the above-mentioned manuscripts and the edition by al-Ṭūsī. Even so, it is a work which received significant engagement outside the Middle Books, as the various revisions by scholars otherwise uninvolved with the curriculum shows. In his preface to the

[103] Nicoll (1821) 260 records MS Bodleian Huntington 237 as containing this statement. Carmody (1960) 21 mentions it appearing in multiple manuscripts but does not elaborate. It can be found at the start of Autolycus's text in Bodl. Hunt. 237, fol. 76a.

Spherics, for example, al-Ṭūsī names three correctors: Māhānī (3rd / 9th century), Abū al-Faḍl Aḥmad ibn Abī Saʿd al-Harawī (4th / 10th century), al-Amīr Abū Naṣr Manṣūr ibn ʿIrāq (5th / 11th century).

Archimedes Measurement of the Circle, On the Sphere and Cylinder, and Eutocius's Commentary

These treatises can be found added to the Middle Books in al-Ṭūsī's edition. It is unclear if Archimedean works besides the *Lemmata* were often grouped with the Middle Books before al-Ṭūsī. It would not be surprising to see them read as useful works of the 'ancients' more generally, and so they potentially featured in the reading of many of these mathematical scholars regardless. Ibn al-Ṣalāḥ, after all, was clearly familiar with other works like the *Conics* and would leverage them in his comments. But this does not mean they often featured among the Middle Books themselves.

4.2 Originally Arabic Works

Several original Arabic works appear grouped with some instances of the Middle Books as well, whether in the manuscripts discussed above or in al-Ṭūsī's editions. They include the following:

- the Banū Mūsā's *Book of Knowledge of Plane and Spherical Figures*[104]
- Thābit's *Assumptions*[105]
- Thābit's *On the Composition of Ratios*[106]
- Thābit *On the Sector Figure*[107]
- al-Nasawī's *Commentary on the Sector Figure*[108]
- al-Kharaqī's *Book of Enlightenment in Astronomy*[109]

Most of these are early works by ninth century (and one tenth century) authors. The exception to this is the twelfth century al-Kharaqī, who as we have seen above intentionally set out to produce an astronomical work that served a similar didactic function as the Middle Books. Otherwise, the authors represented are Thābit (who translated and corrected multiple Middle Books), the Banū Mūsā (who had

[104] See discussion in Clagett (1964) 223ff.
[105] See the study in Dold-Samplonius (1996).
[106] See the study and edition in Lorch (2008), especially 167-326.
[107] See the study and edition in Lorch (2008), especially 41-166.
[108] See Lorch (2008) 355ff.
[109] See Ghalandari (2012) for the edition of the *Ultimate Attainment in the Division of the Spheres*.

significant and continuing involvement in Thābit's training and subsequent work), and al-Nasawī (who wrote a commentary on one of Thābit's texts and who also saw other involvement with the Middle Books).

The works themselves are variously relevant. The *Book of Knowledge of Plane and Spherical Figures* deals with matters such as the areas and volumes of figures like circles and spheres. The *Assumptions* deal with plane geometry – triangles, circles, and chords. Thābit's *On the Composition of Ratios* was relevant to the problem of the sector figure, since the theorem involved the composition of ratios. *On the Sector Figure* deals more directly with the theorem and al-Nasawī's commentary is a natural addition. As noted above, the relevance of this problem to the *Almagest* – where Menelaus's sector figure theorem was used to determine various astronomical arcs and angles – would have motivated these works' inclusion. Al-Kharaqī's work, of course, is generally relevant astronomy.

5. Conclusion

This chapter has taken us up to the seventh / thirteenth century in the Islamicate world, and chapter 8 will return to this point in the timeline to set the stage for al-Ṭūsī's edition in that century. But before that, Part III will proceed onwards to the Byzantine world to explore to what extent the Little Astronomy continues to see circulation and use in the Greek tradition after the ninth century.

Chapter 6

The Little Astronomy, Ninth to Thirteenth Centuries?

1. Introduction

Chapter 1 has considered the problem of what evidence is available for the existence and usage of the Little Astronomy as a curriculum before the ninth century, and chapter 3 has shown that this grouping of texts was translated into Arabic in the ninth century and in that same century began to see didactic use under the name the Middle Books. The previous chapter explored the continuing study of these Middle Books in the Arabic tradition.

What of the Little Astronomy in the Greek tradition after the ninth century? Like the previous chapter, this chapter will consider the extant manuscript evidence (which is unfortunately scarce prior to the thirteenth century), and it will consider the scholars, teachers, and students who engaged with the study of astronomy in this period, noting where their efforts may have intersected with the Little Astronomy.

In comparison with the prior chapter, this present chapter will be seen to largely discuss an absence. While the treatises of the Little Astronomy were not entirely lost, it will be seen that there is little surviving evidence that points to these works' usage as a group in Greek during this period.

2. The Greek Manuscripts, 9th to 13th Centuries

While there may have been more manuscripts of Little Astronomy texts written in the intervening centuries, what is extant leaves a large gap between the ninth and thirteenth centuries.

Many of the manuscripts to be discussed below have been judged by their modern editors to lack extant ancestors: in the proposed stemmata, they are the oldest surviving witnesses to their respective branches. These are the manuscripts Vaticanus graecus 204, 203, 202, 192, and 191 and Paris grec 2390 and 2448. Vat. gr. 204, though the oldest extant witness, is not the ancestor of the others.[1]

Mogenet, in his detailed examination of the variants in the tradition of Autolycus, found no signs in these manuscripts to suggest how the stemmata converged towards a common ancestor. Rather, he supposed their parent manuscripts still represented separate, parallel traditions that only converged at a more ancient point in time.[2]

Czinczenheim, delving into the variants of the *Sphaerica*'s transmission history, was able to bring forth a larger sample of data to the question and with this Theodosian material argues that the manuscripts discussed by Mogenet can be grouped in three families. She sees Vat. gr. 202, Vat. gr. 203, and Paris gr. 2390 as descending from a common ancestor; Vat. gr. 191 as one branch removed and Paris gr. 2448 as two branches removed. Tracing the lines further back, all five of these manuscripts descend from an exemplar which shared a parent with Vat. gr. 204.[3]

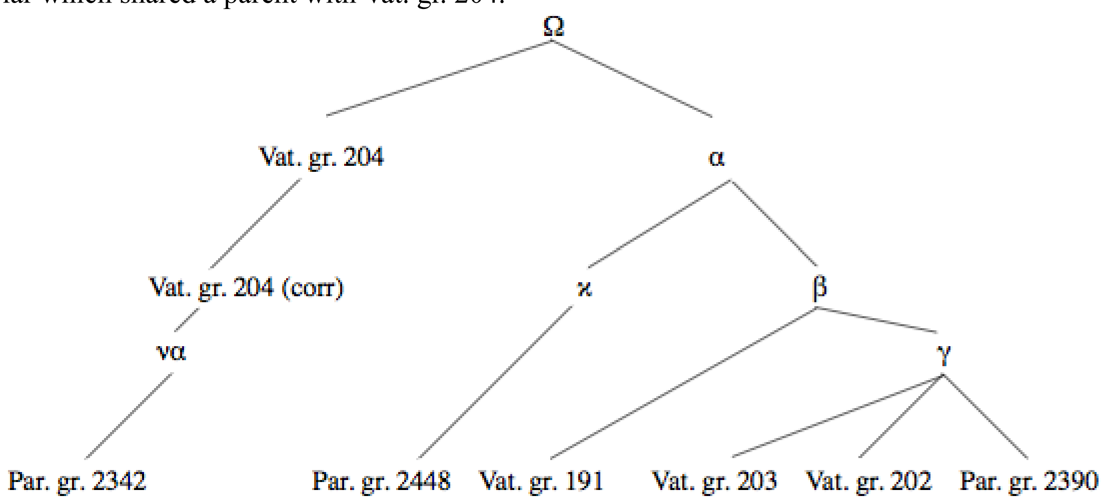

Figure 1: The stemma according to Czinczenheim.[4]

[1] See e.g. Heiberg (1927) v, Czinczenheim (2000) 180, Mogenet (1950) 156, Heiberg (1895) vii-viii, Menge (1916) v, Noack (1992) 336-344.
[2] Mogenet (1950) 145-151.
[3] Czinczenheim (2000) 372-373.
[4] Czinczenheim (2000) 373.

Noack, studying the witnesses for Aristarchus, supposes that the archetype for the presently extant manuscripts was itself written in minuscule and so had already been produced by the Byzantine transliteration. Since Vat. gr. 204 is itself a codex of the ninth century, Noack suggests its antigraph was this lost transliteration.[5] She also identifies Vat. gr. 192 as the head of another family of manuscripts, separate from the ones discussed by Mogenet and Czinczenheim.[6]

The following will note some details of the existing Little Astronomy manuscripts.

Vat. gr. 204 (9th century)[7]

The ninth century is represented only by Vat. gr. 204, the oldest witness to any of the texts in the collection. This is the only parchment manuscript of the Little Astronomy. It would have been produced shortly after the ninth century Byzantine transliteration that saw texts copied out of older majuscule witnesses into newer minuscule versions. Chapter 1 has already surveyed its contents and the arrangement of these texts from works on spherical geometry more generally to works on astronomy more particularly. It comprises the full grouping of the Little Astronomy, followed by works which appear to have been intended as supplementary or commentary. It is a manuscript containing many contemporary scholia, including a large number of referential scholia, as discussed.

Vat. gr. 204 was the ancestor of several subsequent manuscripts including Paris gr. 2342, which was one codex of a two-part personal encyclopedia transcribed by the fourteenth century copist Malachias, alias Anonymous Aristotelicus. This latter manuscript, however, drew from sources besides Vat. gr. 204, since it presents the A recensions of both the *Optics* and the *Phaenomena*.[8]

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[5] Noack (1992) 89-90.

[6] Noack (1992) 143-150. Vat. gr. 192 does not include either Theodosius's or Autolycus's treatises and so was not examined by Mogenet or Czinczenheim.

[7] Vat. gr. 204 may be viewed online in the DigiVatLib repository. See descriptions in Mercati and Franchi de' Cavalieri (1923) 246-248, Mogenet (1950) 70-72, Acerbi (2012) 150-155, and Vitrac (2021) 138.

[8] Paris gr. 2342 may be viewed online in the Gallica repository. This manuscript included the *Elements*, the *Data* and Marinus's commentary, the full corpus of the Little Astronomy, with the additions of Damianus of Larissa, Geminus,

Vat. gr. 192 (11th - 12th century)[9]

For witnesses to the Little Astronomy, the interval between the ninth and thirteenth centuries is represented only by Vat. gr. 192. Heiberg, Menge, and De Falco presumed this manuscript to date from the thirteenth or fourteenth century, but more recently Noack has argued for a composition around 1100 based on palaeographic considerations of *On Sizes and Distances*'s text.[10]

Vat. gr. 192 is a codex of several geometrical, astronomical, and musical treatises and scholia, comprising fourteen works. It is begun by the *Elements*, and interspersed among the subsequent treatises are works from the Little Astronomy: the *Data*, the commentary on the *Data*, the *Optics*, the *Catoptrics*, the *Anaphoricus*, *On Sizes and Distances*, and the *Phaenomena*. In this instance the texts have departed from the old order, but it is unclear what motivated the new order.

While the manuscripts Laur. Plut. 28.3 (10th c.)[11] and ÖNB phil. gr. 31 (12th c.)[12] additionally appear in the interval between the ninth and thirteenth centuries, they present Euclid's *Optics* and *Phaenomena* both in recensions A. They therefore represent a separate transmission of these texts, not the transmission connected with the Little Astronomy. They do not include any other Little Astronomy treatises.

Moving onwards: the thirteenth century preserves six manuscripts containing either the full or partial groupings of Little Astronomy texts. These manuscripts are written on paper. There are also several thirteenth century manuscripts of recensions A of the *Optics* and *Phaenomena* that will not be

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Apollonius and Eutocius, and Serenus. Its second part, Vat. gr. 198, contained Nicomachus's *Arithmetic* and then various works on music and astronomy, including of course the *Almagest* and commentaries upon it.

[9] Vat. gr. 192 may be viewed online in the DigiVatLib repository. See descriptions in Mercati and Franchi de' Cavalieri (1923) 227-229 and Vitrac (2021) 148.

[10] Heiberg (1895) xvi, Menge (1916) vi, and De Falco and Krause (1966) 24; Noack (1992) 150-151. Vitrac's list also records this as a thirteenth century manuscript.

[11] Laur. Plut. 28.3 may be viewed online in the Biblioteca Medicea Laurenziana digital repository. See description in Vitrac (2021) 138-139.

[12] ÖNB phil. gr. 31 may be viewed online in the Österreichische Nationalbibliothek digital repository. See description in Vitrac (2021) 144-146.

covered here: Bodleian Auct. F. 6. 23, Vat. gr. 1038, and Biblioteca Medicea Laurenziana Plut. 28.6. The codices containing Little Astronomy treatises follow below.

Vat. gr. 191 (13th century)[13]

Vat. gr. 191 is a codex of various mathematical and astronomical treatises and scholia, comprising over thirty works. It includes additions from an unknown scholar of the thirteenth or fourteenth century.[14]

This is a large grouping of texts, but it is started by the following: the *Catoptrics*, *Phaenomena*, *Optics*, *Data*, *Commentary on the Data*, *Sphaerica*, *On Habitations*, *On Days and Nights*, *On Sizes and Distances*, *On Risings and Settings*, the *Anaphoricus*, and *On the Moving Sphere*.

The arrangement here is largely by author: Euclid, Theodosius, Aristarchus, one treatise by Autolycus, Hypsicles, and the other treatise by Autolycus, though the span from *On Habitations* to the *Anaphoricus* does follow the relative order in Vat. gr. 204. There is furthermore the noteworthy fact that on folio 74r, after the conclusion of *On the Moving Sphere*, the scribe has transcribed again the start of the *Optics*. That this was quickly recognized to be an error is evidenced by that text being crossed out and not continued on the next folio. But it does suggest that the scribe's exemplar for these Little Astronomy texts was arranged not by author but rather in the older order, where the *Optics* did indeed follow *On the Moving Sphere*.[15]

[13] Vat. gr. 191 may be viewed online in the DigiVatLib repository. See descriptions in Mercati and Franchi de' Cavalieri (1923) 220-227 and Vitrac (2021) 147-148.
[14] Mercati and Franchi De' Cavalieri list 35 items, grouping some works under their shared author, and the Pinakes digital catalogue lists 42 items.
[15] This exemplar is unknown – where editors of the relevant texts have reconstructed manuscript stemmata, Vat. gr. 191 is understood to have descended from a codex other than Vat. gr. 204, and there are no other extant candidates. See e.g. Mogenet (1950) 156.

Vat. gr. 202 (13th century)[16]

Vat. gr. 202 is a codex of Little Astronomy treatises with scholia. Its arrangement matches Vat. gr. 204's, though it lacks *On Habitations*, *On Days and Nights*, the *Catoptrica*, the commentary by Eutocius, and the Euclidean scholia. It is understood, however, to have descended from a separate codex than Vat. gr. 204, and so shows a preservation of this order from a source besides that one.

Vat. gr. 203 (13th century)

Vat. gr. 203 is a thirteenth century codex of geometrical and astronomical treatises with scholia. The Little Astronomy treatises are arranged by author: Theodosius, then Autolycus, then Hypsicles and Aristarchus. They are joined by the *Conica* and its commentary, along with Serenus's works on the sections of a cylinder and a cone.

Paris gr. 2390 (13th century)[17]

Paris gr. 2390 is a manuscript of Ptolemaic treatises and commentaries on them, whose conclusion comprises the first three Little Astronomy treatises: the *Sphaerica*, *On the Moving Sphere*, and the *Optics*. The beginning of the *Sphaerica* has been lost and the extant text starts on folio 236r with the enunciation of prop. I.3.

Although there are only very sparse scholia on these final three treatises, this codex notably contains autograph scholia on Ptolemy's *Almagest* by the scholar Manuel Bryennios (ca. 1275 - ca. 1340) – this scholar, the astronomy teacher of Theodore Metochites, will receive mention at the end of this chapter. The scholia in question concern the sector theorem and have been studied by Acerbi and Pérez Martín.[18]

[16] Vat. gr. 202 may be viewed online in the DigiVatLib repository: part 1 and part 2. See descriptions in Mercati and Franchi de' Cavalieri (1923) 244-245 and Vitrac (2021) 148.
[17] Paris gr. 2390 may be viewed online in the Gallica repository. See description in Vitrac (2021) 159.
[18] Acerbi and Pérez Martín (2015).

Paris gr. 2448 (13th ex - 14th in century)[19]

Paris gr. 2448 is a geometrical miscellany of the end of the thirteenth or beginning of the fourteenth century which includes several Little Astronomy treatises interspersed among works by other authors such as Archimedes, Hero of Alexandria, and Domninus of Larissa. It includes Euclid's *Data* and *Catoptrics*, Autolycus's *On the Moving Sphere*, and Theodosius's *Sphaerica*. This codex is begun by the mathematical chapters of the Anonymous Heiberg quadrivium, which will be discussed below.

3. Scholars, Teachers, and Students of Astronomy

Chapter 5 has discussed the individuals who were involved in teaching and studying the Middle Books between the ninth and thirteenth century in the Islamicate world. A comparison with the Little Astronomy in the Byzantine world can be made with this section. The following will consider a selection of Byzantine individuals that are attested to have studied or taught astronomy during these centuries. It will be seen that the Little Astronomy left a less clear impact on Byzantine astronomical education than the Middle Books did for Islamicate astronomical education.

3.1 Leo "the Mathematician" (ca. 790 - after 869)

Leo the Mathematician (ὁ Μαθηματικός), or the Philosopher (ὁ Φιλόσοφος) or the Grammarian (ὁ Γραμματικός), was a Byzantine scholar who flourished during the Macedonian renaissance. His library has been reconstructed to include Ptolemy, Archimedes, Euclid, Plato, the *Mechanics* of Quirinus and Marcellus, Paul of Alexandria, Theon of Alexandria, Proclus, Porphyrius, Apollonius, and perhaps Thucydides.[20] While showcasing an interesting range of mathematical and astronomical authors, this list of course bears little connection to the Little Astronomy. Nevertheless, past scholarship on the Little

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[19] Paris gr. 2448 may be viewed online in the Gallica repository. See description in Vitrac (2021) 159.

[20] Browning (1964) 8. Browning refers to copies of Leo's colophons in mss, but does not specify which these are. He points to an ownership statement, "τοῦ ἀστρονομικωτάτου Λέοντος ἡ βίβλος" in the Ptolemaic codex Vat. gr. 1594 and to a marginal comment in the Euclidean manuscript Bodleian D'Orville 301 copied from Leo's copy of the *Elements*.

Astronomy has sometimes put forth Leo the Mathematician as having some involvement in arranging or teaching the curriculum in the Byzantine world.[21]

Acerbi, conversely, sees little direct evidence for full-fledged scientific activity on Leo's part, and argues that this idea has rather resulted from scholars' tendency to assign activities by unknown actors to the only available known individual.[22] Leo "ὁ Μαθηματικός" receives credit for a variety of renewed mathematical teachings and efforts in ninth century Constantinople, but in the extant reports about him he is called "μαθηματικός" only once,[23] and this term can simply mean a "man of study" more generally, not specifically a mathematical one.

Overall, there is no ninth century scholarship on the Little Astronomy that can be clearly attributed to Leo, and nothing else that directly links him together with the curriculum. If the collection was still in didactic use during that century it is conceivable that Leo may have studied those texts as a student, but there is no relevant evidence on the matter.

3.2 The Author of the "Anonymous Heiberg" Quadrivium (1007)[24]

While the author of a text authored at the end of the eleventh century is unknown, the text itself deserves attention as an example of a written "quadrivium" – a comprehensive work on the four mathematical subjects: arithmetic, music, geometry, and astronomy. This particular work is five chapters in total, with the first chapter being on logic. The chapters on the four mathematical sciences also circulate together separately, with attributions to Michael Psellos or to a Euthymios or a Gregory.[25]

[21] Pingree (1968) 16.
[22] Acerbi (2014) 125ff.
[23] Theophanes continuatus 4.197.4: Featherstone and Signes-Codoñer (2015) 280.
[24] The Anonymous Heiberg is preserved in the eleventh century manuscript Universitätsbibliothek Heidelberg, Cod. Pal. graec. 281. A digitization of this manuscript is available online in Universitätsbibliothek Heidelberg's digital library: https://digi.ub.uni-heidelberg.de/diglit/cpgraec281/0203. Other witnesses to the text date between the fourteenth and seventeenth centuries: see for example Paris grec 1931, 2062, 2136, 2465, 3067; suppl. grec 541 and 677; and Vat. gr. 111.
[25] Pérez Martín and Manolova (2020) 85.

However Michael Psellos, who will be discussed in the next section, cannot have been the author of this work: the Anonymous Heiberg was produced in 1007, approximately ten years before his birth.[26] Nor is there evidence of him having edited the quadrivium, whether as a whole or only these four mathematical chapters.

The astronomical chapter opens with statements that present (without citation, as usual) the first three definitions of Theodosius's *Sphaerica*, largely word-for-word:

	Anonymous Heiberg[27]	Theodosius *Sphaerica*[28]
Def. 1	A sphere is a solid figure contained by one surface, in which all straight lines drawn from one point lying inside the sphere are equal to each other.	A sphere is a solid figure contained by one surface, in which all straight lines drawn from one point lying inside the figure are equal to each other.
	Σφαῖρά ἐστι σχῆμα στερεὸν ὑπὸ μιᾶς ἐπιφανείας περιεχόμενον, πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τῆς σφαίρας κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι ἴσαι ἀλλήλαις εἰσί.	Σφαῖρά ἐστι σχῆμα στερεὸν ὑπὸ μιᾶς ἐπιφανείας περιεχόμενον, πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι ἴσαι ἀλλήλαις εἰσίν.
Def. 2	The center of the sphere is that point.	The center of the sphere is that point.
	κέντρον δὲ τῆς σφαίρας τὸ σημεῖόν ἐστι,	κέντρον δὲ τῆς σφαίρας τὸ σημεῖόν ἐστι.
Def. 3	The diameter is some straight line passing through the center and terminating at both sides; if the sphere rotates on it, it is called the axis of the sphere.	The axis of the sphere is some straight line passing through the center and terminating at both sides of the surface of the sphere, around which stationary line the sphere rotates.
	διάμετρος δὲ εὐθεῖά τις διὰ τοῦ κέντρου ἠγμένη καὶ ἐφ' ἑκάτερα τὰ μέρη	Ἄξων δὲ τῆς σφαίρας ἐστὶν εὐθεῖά τις διὰ τοῦ κέντρου ἠγμένη καὶ περατουμένη ἐφ'

[26] See Taisbak (1981) on dating the astronomical section of the Anonymous Heiberg. The astronomical chapter works through a problem using "the present, that is the 6516 year" (τοῦ δεῦρο ἤγουν τοῦ ͵ςφις′ ἔτους) in the Byzantine calendar, which is the year 1007 or 1008 in the Julian calendar. Taisbak further narrows the date for this chapter down to between September 1st and December 14th, 1007. Heiberg, who edited the anonymous quadrivium, expressed uncertainty as to where the attribution to Psellos originated. He indicates two editions from the sixteenth century that print the chapters under Psellos's name – see Heiberg (1929) 108.
[27] Heiberg (1929) 104.
[28] Czinczenheim (2000) 52.

	περατουμένη· εἰ δὲ περὶ αὐτὴν ἡ σφαῖρα στρέφοιτο, ἄξων τῆς σφαίρας καλεῖται.	ἑκάτερα τὰ μέρη ὑπὸ τῆς ἐπιφανείας τῆς σφαίρας περὶ ἣν μένουσαν εὐθεῖαν ἡ σφαῖρα στρέφεται.

Table 6.1: Comparison of the opening of the Anonymous Heiberg's astronomical chapter with *Sphaerica* book I defs. 1-3

This section on astronomy is otherwise largely dependent on Ptolemy's *Almagest*. It does not reference other texts from the Little Astronomy or any more of Theodosius's *Sphaerica* besides these introductory definitions on spheres. The preceding geometrical chapter also does not draw from relevant Little Astronomy texts. Besides the above Theodosian definitions, the authors who are referenced in these two chapters are instead Euclid, Proclus, Nicomachus, Plato, Theon, Pappus, Hero, Eutocius, and Archimedes (in the geometrical chapter) and Ptolemy, Euclid, and Pappus (in the astronomical one).[29]

While this anonymous quadrivium filled a similar didactic niche as the Little Astronomy once did, its author did not use the latter as a source for his work. This quadrivium rather reveals a different cluster of authors who could be relied upon for geometrical matters, and the significant dependence on Ptolemy for astronomical ones.

3.3 Michael Psellos (1017 or 1018 - 1078 or 1096)

Michael Psellos was a Byzantine monk, scholar, and political advisor in the eleventh century. His *Chronographia* notably contains autobiographical portions which are illuminating for the scholarship of his day and for his own intellectual biography. An often-cited passage for the state of the Byzantine astral sciences in the eleventh century is section 5.19. After noting in the prior section how Emperor Michael V (r. 1041-1042) relied on the counsel of those studied in the astral sciences (ἀστρονομούντων), he explains:

> "At that time there was a group of distinguished men engaged in the study of that science, men with whom I myself had dealings. These gentlemen were not specially concerned with the position or movements of stars in the celestial sphere (actually they had no training in the proof of

[29] See, e.g., Heiberg (1929) 72, 80, 83, 88, 92, 101, 107, and 143.

> such things by the laws of geometry and certainly this power of demonstration was not acquired by them before they studied astrology)..."[30]

So the individuals Psellos speaks of were practiced ones, but they had no background in geometrical necessities (γεωμετρικαῖς ἀνάγκαις). They were not trained in proof (ἀπόδειξιν). They had not studied these topics before (οὔτε… προέγνωσαν). Psellos continues with an explanation of what these individuals *had* studied, making it clear that their focus was astrological:

> "…they confined themselves rather to the setting up of astrological centres, the examination of the rise and fall of the zodiacal signs above or below the horizon. Other phenomena connected with these movements also became the object of their study – the ruling planets, the relative positions and limits of the planets, together with those aspects considered favourable and those which were not propitious. Certain predictions were then offered to persons who asked for advice and their questions were answered."[31]

Psellos's criticisms of these astrologers would imply that there was a path to studying the astral sciences through the means of geometrical proof. These individuals took a different path. Granted, Psellos does not offer details of what the study of astronomy through geometrical proof should look like – by this he may simply mean the study of the *Almagest*. The notion of astronomy via geometrical proofs as a separate educational path, however, is still worth acknowledging.

In book six of the *Chronographia*, Psellos takes a moment to describe his own education. The study of mathematics followed after some of his earlier studies and was intended to support philosophical study:

> "From Proclus I intended to proceed to more advanced studies – metaphysics, with an introduction to pure science, – so I began with an examination of abstract conceptions in the so-called mathematics, which hold a position midway between the science of corporeal nature,

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[30] Translation by Sewter (1953) 95. The Greek text is in Reinsch (2014) 89-90: "ὑπῆρχε δὲ τηνικαῦτα μοῖρα οὐκ ἀγενὴς τῆς περὶ ταῦτα μαθήσεως, ἄνδρες οἷς κἀγὼ συνωμίλησα, τῶν μὲν περὶ τὴν σφαῖραν τάξεων καὶ κινήσεων ἔλαττον πεφροντικότες τὸν νοῦν (οὔτε γὰρ γεωμετρικαῖς ἀνάγκαις τὴν περὶ ταῦτα ἀπόδειξιν προειλήφεισαν οὔτε μὴν προέγνωσαν)..."

[31] Translation by Sewter (1953) 95. The Greek text is in Reinsch (2014) 90: "ἀλλ' ἁπλῶς οὕτως τὰ κέντρα ἱστῶντες· εἶτα δὴ τὰς ἀναφοράς τε καὶ ἀποκλίσεις τοῦ ζωηφόρου κύκλου καταμανθάνοντες· καὶ τὰ ἄλλα ὅσα τούτοις ἕπεται (οἰκοδεσπότας φημὶ· καὶ σχημάτων τόπους· καὶ ὅρια· καὶ ὁπόσα μὲν τούτων κρείττω· ὁπόσα δὲ χείρω), προὔλεγόν τι τοῖς πυθομένοις, περὶ ὧν ἐπηρωτήκεσαν."

> with the external apprehension of these bodies, and the ideas themselves, the object of pure thought. I hoped from this study to apprehend something that was beyond the reach of mind, something that was not subject to the limitations of substance."[32]

He goes on to elaborate somewhat on what mathematics involved, and on what subjects were related to it. The study of astronomy accompanied the study of mathematics, as did the study of music:

> "It was therefore consonant with this plan that I should pay especial attention to systems of number and examine geometrical proofs, which some call 'logical necessities'. Moreover, I devoted time to the study of music and astronomy, as well as to their various subsidiary arts. First I would concentrate on each study by itself, then synthesize my knowledge, in the belief that the several branches of learning would by their individual contributions lead me to one simple goal, according to the teaching of Plato's *Epinomis*. So, thanks to these sciences, I was able to launch out into the more advanced studies."[33]

These reports from Michael Psellos serve as examples of the general scarcity of detail available for mathematical and astronomical education in the Byzantine Empire during this period. Accounts of education will cover the topics of study, but the particular sources used by students for those topics go unnamed, except for select works like the *Elements* and the *Almagest*. What comes across is a broad picture of quadrivial studies, but with the details left unspoken. The study of astronomy usually appears in the context of more advanced studies, often in the service of the study of philosophy.

Michael Psellos's critiques of the astrologers above can be compared with his description of his education. The details of what kinds of astronomy and astrology they had or had not studied reveals a distinction between the training of the astrologers (who were practiced in techniques that might be found in strictly astrological handbooks) and Psellos's own, philosophically-directed education. It was in the

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[32] *Chronographia* 6.38, translation by Sewter (1953) 128. The Greek text is in Reinsch (2014) 122: "μέλλων δὲ μετὰ ταῦτα ἐπὶ τὴν πρώτην ἀναβαίνειν φιλοσοφίαν· καὶ τὴν καθαρὰν ἐπιστήμην μυεῖσθαι, τὴν περὶ τῶν ἀσωμάτων θεωρίαν προὔλαβον ἐν τοῖς λεγομένοις μαθήμασιν (ἃ δὴ μέσην τινὰ τάξιν τετάχαται, τῆς τε περὶ τὰ σώματα φύσεως· καὶ τῆς ἀσχέτου πρὸς ταῦτα νοήσεως· καὶ αὐτῶν δὴ τῶν οὐσιῶν, αἷς ἡ καθαρὰ συμβαίνει νόησις), ἵν' ἐντεῦθεν εἴ τι καὶ ὑπὲρ ταῦτα ὑπέρνουν ἢ ὑπερούσιον καταλήψομαι."

[33] *Chronographia* 6.39, translation by Sewter (1953) 128. The Greek text is in Reinsch (2014) 122-123: "διὰ ταῦτα, ἀριθμῶν τε μεθόδοις ἑαυτὸν ἐντείνας· καὶ γεωμετρικὰς ἀποδείξεις ἀναλαμβάνων, ἃς ἀνάγκας τινὲς ὀνομάζουσιν· ἔτι τε μουσικοῖς καὶ ἀστρονομικοῖς ἐνδιδοὺς λόγοις· καὶ εἴ τινες ἄλλαι μαθήσεις ταύταις ὑπόκειται, οὐδὲ τούτων οὐδεμίαν ἀπολείπων· καὶ πρῶτα μὲν κατὰ μίαν ἑκάστην διεξιὼν· εἶθ' ἁπάσας συνάψας, ὡς δι' ἀλλήλων ἡκούσας εἰς ἕν, ὡς ἡ Ἐπινομὶς βούλεται, οὕτω διὰ τούτων τοῖς ὑψηλοτέροις ἐπέβαλλον."

latter that astronomical studies via deductive mathematics and proofs were to be found. The Little Astronomy may have served as a vehicle of this kind of education, but from the vagueness of the description Psellos's education could have easily comprised only the "major" texts, the *Elements* and the *Almagest*, or have drawn from the variety of other geometrical works on astronomy that were available in the Byzantine world – the anonymous quadrivium or works by authors like Theon, Cleomedes, etc.

3.4 Studying Mathematics in the Thirteenth Century Nicaean Empire

In 1204, Constantinople fell to the Fourth Crusade and the Byzantine Empire existed in exile until Michael VIII Palaiologos reconquered Constantinople in 1261. This interim period saw three successor states, the Empire of Nicaea, the Empire of Trebizond, and the Despotate of Epirus, where Byzantine culture persisted. A number of scholars are known from this period and from the Palaiologan Renaissance that flourished after the recapture of Constantinople. Several of these individuals are recognized for their study of the astral sciences.

The below will offer an overview of select relevant figures but, as will become clear, the surviving attestations of their work in the astral sciences have little to connect them with the Little Astronomy.

3.5 Nikephoros Blemmydes (d. 1272)

Nikephoros Blemmydes was Byzantine scholar, teacher, and monk of the thirteenth century. He received part of his education in the Empire of Nicaea and also travelled within Asia Minor.

He provides a report of his education in his work, *A Partial Account*. The course of his education as he describes it starts with grammar, then Homer and poetry. Around age sixteen he studied logic, and he took up the study of medicine for about seven years. He writes that he found a teacher, Prodromos, in the Scamader region, where he had traveled despite it being under Latin control at the time. With

Prodromos, in his twenties at the earliest, Blemmydes took up the study of mathematics. After this he returned to logic, then progressed to physics.[34]

On his mathematical studies, Blemmydes has the following to say:

> "So it was that I undertook the study of mathematics according to the textbook of Nikomachos, and also the sort of 'divination' developed by Diophantos (not the full science, but that for which my master claimed to be competent); geometry, both plane and solid, and with particular attention to the *Data* and to spherical geometry, but also elementary optics and the theory of reflexion. I then gave myself completely to astronomy."[35]

Arithmetic was learned via Nicomachus; algebra via Diophantus. Geometry receives further details: the *Data* was one of the works studied, along with works of spherics (σφαιρικοῖς), optics (ὀπτικοῖς), and catoptrics (κατοπτρικοῖς). It is not impossible that the Greek here refers not just to subjects but to titles: the *Sphaerica* of Theodosius, and the *Optics* and *Catoptrics* of Euclid. This is made more plausible by Blemmydes's statement that the subject of astronomy followed these studies – the arrangement is reminiscent of the several general and spherical geometrical works that headed the Little Astronomy before its turn (with the *Phaenomena*) into astronomy proper.

Blemmydes continues by providing further details on his astronomical studies:

> "I do not refer to the astronomy which is despised by sensible men (and which makes despicable those who are not sensible), which deals with events and happenings and births and the foretelling of special, appropriate occasions, and other such stupidities and follies. Rather, I mean the astronomy that is lofty and elevating, that explains heaven's cycle, both in the whole of heaven and in its parts, and clarifies the movements of the stars, both those that are swept along having the same motion as the whole, and only as parts of it, and those that have in addition a different motion from the whole, one proper to themselves; this astronomy demonstrates the stars' different and peculiar rising and falling, their constant presence or their disappearance, their relations and disagreements, their real and apparent changes, as also the growth and diminishing of night and day (which are not the same everywhere) and the equinoxes. Astronomy explains also the corresponding changes of the seasons, their coincidences or rather similarities, and many other

[34] Heisenberg (1896) 4ff.

[35] Munitiz (1988) 46-47. See Heisenberg (1896) 4-5 for the Greek: "Ὢ πῶς ἡμῖν καταπραΰνει τὸ ἄρχον ἔθνος ὁ πάντα μετασκευάζων βουλήματι καὶ τῶν συνήθων μηδὲν λειπόμενον εἰς ἡμερότητα διατίθησι. καὶ δὴ τῆς ἀριθμητικῆς ἀκροασάμενος Νικομάχου κἀκ τῆς Διοφάντου τῆς οἱονεὶ χρησμολογικῆς – οὐ γὰρ πάσης, ἀλλ' ὅσης ἦν εἰδήμων ὁ ἐκδιδάσκων, ὡς ἔλεγε – τῆς τε γεωμετρίας τῆς ἐν ἐπιπέδοις καὶ στερεοῖς, οὐ μὴν ἀλλὰ καὶ δεδομένοις ἐμμελετήσας καὶ σφαιρικοῖς καὶ τοῖς ὀλιγομόχθοις ὀπτικοῖς καὶ κατοπτρικοῖς, ὅλος ἔχομαι τῆς ἀστρονομίας."

> such things. It brings to light the causes of all these and shows how they can all be exposed before the naked eye, with linear proofs that require neither rhetoric nor grammar."[36]

The scholar takes pains to distinguish his astronomical studies from the more astrologically-inclined subjects, which he speaks of disdainfully. In doing so, Blemmydes speaks quite clearly of which his astronomical studies did and did not entail even if he does not indicate particular works.

The comments Blemmydes offers on his education also help to suggest what topics or works he may have covered with his own students. Blemmydes is known to have taught George Akropolites, Theodore II Laskaris, and Gregory of Cyrus (temporarily).

<u>**3.6** George Pachymeres (d. c. 1310)</u>

In the early Palaiologan period, the Byzantine scholar George Pachymeres authored another instance of a quadrivium, titled the *Treatise on The Four Mathematical Sciences: Arithmetic, Music, Geometry, and Astronomy*. Compared to the Anonymous Heiberg, its section on astronomy draws upon a significantly wider range of authors: Euclid, Ptolemy, Plato, Aristotle, Cleomedes, Archimedes, Homer, Theodosius, Aratus, Posidonius, Hipparchus, and Menelaus. Pachymeres frequently cites these authors by name, sometimes indicating a particular work by title as well.[37]

More Little Astronomy works receive citation in Pachymeres' quadrivium. He references Euclid's *Optics* three times,[38] his *Phaenomena* twice,[39] and Theodosius's *On Habitations* once.[40] However, these

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[36] Munitiz (1988) 47. See Heisenberg (1896) 5 for the Greek: "οὐ τῆς χαμαὶ ῥιπτουμένης παρὰ τῶν νοῦν ἐχόντων καὶ ῥιπτούσης τοὺς νοῦν οὐκ ἔχοντας, εἰς ἐκτελεσμοὺς ἀπαγούσης καὶ ἀποβάσεις καὶ γενέσεις καὶ περισκοπήσεις καιρῶν καὶ λήρους ἄλλους καὶ βάραθρα, τῆς ὑψιβάμονος δὲ καὶ ἄνω φερούσης, ἣ τὴν <τοῦ> οὐρανοῦ περιφορὰν καθ' ὅλον αὐτὸν καὶ κατὰ μέρη διασαφεῖ καὶ τὰς τῶν ἀστέρων κινήσεις, ὅσοι τε σὺν τῷ παντὶ μόνως ὡς αὐτοῦ μέρη φέρονται τὴν αὐτὴν ὅσοι τε πρὸς ταύτῃ καὶ ἰδιαιτάτην ἔχουσιν ἄλλην ἀντιφοράν, δήλας τίθησιν, ἐπιτολάς τε καὶ ἀφανείας παρίστησι, σχέσεις τε καὶ ἀποστάσεις καὶ πάθη τὰ μὲν ὄντα τὰ δὲ δοκοῦντα καὶ ἡμερῶν καὶ νυκτῶν αὐξήσεις καὶ μειώσεις, οὐ τὰς αὐτὰς ἁπανταχῇ, καὶ ἰσότητας, ἔτι δ' ἐναλλαγὰς ὡρῶν ἑτέρας ἑτέρωθι καὶ ταυτότητας ἢ γοῦν ὁμοιότητας καὶ ἄλλα πολλὰ παραπλήσια παραδίδωσι, καὶ τὰς ἁπάντων αἰτίας διατρανοῖ καὶ ὑπ' ὄψιν αὐτὴν τὰ πράγματα κεῖσθαι παρασκευάζει διὰ τῆς γραμμικῆς ἀποδείξεως, μὴ ῥητορικῆς δεομένη μηδὲ γραμματικῆς."

[37] The *Elements* is frequently cited by title: see for example Tannery (1940) 352, 360, 389. Other works by Euclid are also denoted by their title, as below.

[38] *Optics* propositions 3 and 36, in Tannery (1940) 366, and proposition 5 in Tannery (1940) 389.

[39] *Phaenomena* propositions 1 and 9, in Tannery (1940) 371 and 380 respectively.

[40] *On Habitations* proposition 8, in Tannery (1940) 382.

instances represent only six Little Astronomy citations across a group of forty-six named references to astronomical authors or works. Despite the increase from what was found in the Anonymous Heiberg, it cannot at all be said that Pachymeres' quadrivium was based upon the curriculum that the Little Astronomy offered. Rather, the reappearance of references to these works, among other references to authors like Cleomedes, Posidonios, Aratus, and Hipparchus, suggests instead the re-availability and resumption of use of a wide variety of astronomical texts in this period. The usage of Little Astronomy texts here is a small part of this larger trend.

4. Conclusion

The evidence for the study of the astral sciences in the Byzantine Empire during this period is already limited, and tends to concern matters separate from the type of astronomy considered in this study – namely, that dependent on proposition-based spherical geometry There is also the problem that what sources are available do not offer details of exactly which works they studied. Even so, cases where source works can be identified – like the two quadrivia – show essentially no dependence on the Little Astronomy.

The difference in what was available between the Islamicate and Byzantine worlds may have played a role here. Byzantine scholars pursuing the subject of astronomy and spherical geometry could draw on a greater range of ancient geometrical authors beyond those whose works were included in the Little Astronomy, not all of whom were translated or copied in Arabic. Islamicate scholars, when they wished to study Greek geometry (be it preparation for the *Almagest* or otherwise), largely had the Middle Books available to them, plus works by authors like Apollonius and Archimedes. The Middle Books were not the entirety of Greek geometry available in Arabic, but they were a larger proportion of it – further, we do see works outside it, like the Archimedean treatises, were sometimes added to it anyway.

5. Coda: Arabic and Persian Connections

Shortly after the end of the timespan explored in this chapter, Theodore Metochites (1270 - 1332) set out to write an epitome of the *Almagest*. This, however, was a daunting project, and one which required sufficient preparatory studies in geometry and astronomy. Metochites himself admits as much, writing in his *Abridgement of the Elements of Astronomy* (Ἀστρονομικῆς κατ’ ἐπιτομὴν στοιχειώσεως) that, beyond thoroughly reading the *Almagest* cover to cover several times, he benefited from the study of several mathematical texts. He refers to or names Euclid’s *Elements* (ὅση τε ἐν ἐπιπέδοις Εὐκλείδῃ στοιχειοῦται καὶ ὅση ἐν στερεοῖς) and his *Optics*, *Catoptrics*, *Data*, and *Phaenomena* (καὶ μὴν ἔτι καὶ ἄττα τῷ ἀνδρὶ προσεξείργασται Ὀπτικά τε καὶ Κατοπτρικὰ καὶ Δεδομένα καὶ περὶ τῶν κατ' οὐρανὸν φαινομένων); Theodosius’s *Sphaerica* (Θεοδοσίου τε Σφαιρικὰ) and his *On Habitations* and *On Nights and Days* (καὶ ὅσα περὶ διακρίσεων, οἰκήσεών τε καὶ νυκτῶν καὶ ἡμερῶν); and <Autolycus’s> *On Risings and Settings* (ἄστρων ἐπιτολῶν τε καὶ δύσεων). Several other works follow later in the section: <Nicomachus’s> *Introduction to Arithmetic* and perhaps works on algebra (ἀριθμητικῶν εἰσαγωγαὶ καὶ βαθύτεραι θεωρίαι), Apollonius’s *Conics* (Ἀπολλωνίοου τοῦ Περγαίου Κωνικά), and Serenus’s *On the Section of a Cylinder* (Σερηνου Κυλινδρικά).[41]

[41] Metochites I.1.32: see Bydén (2003) 436-437: “ἐδόκει δὴ λοιπὸν οὕτω, καὶ ὅλως ἐν τούτοις εἶχον, τὰ μὲν κατὰ ταὐτὸν μετιὼν σὺν τοῖς Πτολεμαίου – χρῆναι γὰρ οὕτω καὶ οὐκ ἦν ἄλλως τοῖς ἐκείνου χρῆσθαι – τὰ δὲ καὶ μεθύστερον τούτου μοι τοῦ σκοποῦ καὶ τοῦ πόνου ἡ τῆς γεωμετρικῆς θεωρίας εἰς τέλος ἔρευνα, ὅση τε ἐν ἐπιπέδοις Εὐκλείδῃ στοιχειοῦται καὶ ὅση ἐν στερεοῖς, καὶ μὴν ἔτι καὶ ἄττα τῷ ἀνδρὶ προσεξείργασται Ὀπτικά τε καὶ Κατοπτρικὰ καὶ Δεδομένα καὶ περὶ τῶν κατ' οὐρανὸν φαινομένων, ὡσπερεὶ πρόθυρά τινα ταῦτα καὶ προαύλια τῶν ἐντὸς ἀπορρήτων τε καὶ ἀδύτων ἀστρονομίας καὶ τῆς ὅλης αὐτῇ κατασκευῆς, Θεοδοσίου τε Σφαιρικὰ καὶ ὅσα περὶ διακρίσεων, οἰκήσεών τε καὶ νυκτῶν καὶ ἡμερῶν, ἄλλοτ' ἄλλων, πᾶσα ἀνάγκη, καὶ ἄστρων ἐπιτολῶν τε καὶ δύσεων παντοίων τε καὶ ἀλλοίων ἐν ἄλλοις καὶ οὐ πάντα παραπλησίων, καὶ περὶ τῶν τοῦ ζῳδιακοῦ τμημάτων καὶ τῶν διαφόρων αὐτοῦ σχηματισμῶν ἐν τῇ περιστροφῇ τοῦ παντός – ταῦτα μέν γε καὶ φανερῶς εἰς ἀστρονομίαν ἤδη φέρει, καὶ τὰς κατ' αὐτὴν ὑποθέσις ἐξεργάζεται, καὶ ταῦτ' ἐν ἁπλαῖς αἶστισιν ἄρ' ἐπαγωγαῖς ἀνὴρ διέξεισι, καὶ γραμμικαῖς ἔστιν οὗ κρατύνει δείξεσι – καὶ μὴν ἔτι περὶ ταὐτὰ καὶ ἄλλων ὡντινωνοῦν φιλοπονίαι καὶ ἀριθμητικῶν εἰσαγωγαὶ καὶ βαθύτεραι θεωρίαι, καὶ οὐκ οἶδ' ὅτι δεῖ τὰ ὀνόματα καταριθμεῖσθαι, πλήν γε ὅτι καὶ πρὸς ταῦτα κατὰ τῆν ἐξ ἀρχῆς πρόθεσιν ἐπιμελῶς ἤνυτον· ἃ δὲ δῆτ' εἴρηταί μοι πρότερον Ἀπολλωνίοου τοῦ Περγαίου Κωνικά, θαυμαστῆς ὄντως γεωμετρικῆς ἕξεως καὶ κράτους ἐν ταύτῃ τοῦ ἀνδρὸς δείγματα, καὶ Σερηνου Κυλινδρικά, μάλιστ' ἐπονήθη μοι, δυσδιεξίτητα ταῖς καταγραφαῖς ἐντυχεῖν, καὶ κομιδῇ πως ἐργώδη συσχεῖν παντάπασιν, ὅσα γ' ἐμὲ εἰδέναι, διὰ τὴν ἐπίπεδον ἐπίσκεψιν καὶ ἔστιν ὁτῳοῦν χρῆσθαι καὶ πειρᾶσθαι εἰ ἀληθὴς ὁ λόγος.”

In this account we can find a report mirroring reports that have already been seen many times in the Arabic tradition: particular texts being read as preparation for work with the *Almagest*. This list lacks references to Autolycus's *On the Moving Sphere* or to Hypsicles's or Aristarchus's works, but otherwise comprises a significant proportion of Little Astronomy or Middle Books texts. So it is in Metochites that these works as a curriculum most noticeably resurface.[42]

Metochites indicates his teacher in mathematics was Manuel Bryennios (ca. 1275 - ca. 1340) – he writes that Bryennios himself had learned astronomy from a relative of his who was knowledgeable in the subject, and elsewhere writes that Bryennios had learned the subject from a man who had been to Persia.[43] If both these reports are accurate, then Bryennios had a relative who had learned astronomy in Persia. This would have been approximately around the time when Gregory Chioniades (ca. 1240 - ca. 1320) and unknown others had traveled and brought back Persian astronomy to the Byzantine Empire.

The late thirteenth century Ilkhānate and its astronomical scholars and teachers – exemplified by Naṣīr al-Dīn al-Ṭūsī and Muḥyī al-Dīn al-Maghribī – will be discussed in chapter 7. But by this point it is clear enough that a student of astronomy in the Islamicate world often would have, in addition to originally Arabic (and Persian) texts, encountered translations of, abridgements of, and commentaries on originally Greek treatises. Gregory Chioniades is remembered for bringing Persian astronomy to the

[42] Metochites complains about the lack of mathematical study available in the Byzantine Empire in his time in I.1.6: see Bydén (2003) 420: "τῶν γὰρ ἐν τῷ δεκάτῳ τῆς Στοιχειώσεως ῥητῶν τε καὶ ἀλόγων γραμμῶν τε καὶ εἰδῶν καὶ τῶν ποικίλων ἀποτομῶν ἀνίδεος, ὡς εἰπεῖν, ἦν, ἄρρητός τε καὶ ἄλογος, σφίσιν ἡ ἐποπτεία, καὶ οὐκ εἶχον ὁπῃοῦν ἐνταῦθα χρῆσθαι, οὔτ' οἴκοθεν τῆς αὐτὸς αὑτοῦ φύσεως καὶ δυνάμεως ἕκαστος, οὔτ' ἄλλου του μεταλαμβάνειν ὅπως ἄρα. καὶ λοιπὸν ἔπειτα προσεπικτᾶσθαι καὶ προστιθέναι τήνδε περὶ τὰ στερεὰ τῆς ἐπιστήμης πολυπραγμοσύνην, καὶ μάλιστα τὴν τῶν περὶ τὰ κωνικὰ θαυμάτων τὴς μαθηματικῆς, ἄρρητον παντάπασι καὶ ἀνεννόητον πρὶν ἢ ἐντυχεῖν ὁντιναοῦν καὶ προσσχεῖν εὖ μάλα εὕρεσιν καὶ ὑποτύπωσιν Ἀπολλωνίου τοῦ ἐκ Πέργης, ἀνδρὸς ὡς ἀληθῶς θαυμαστοῦ τῶν ἐξ ἀρχῆς ἀνθρώπων, ὅσα ἐμὲ εἰδέναι, περὶ τὴν γεωμετρικὴν ἐπιστήμην, αὐτοῦ τε τὴν περὶ τὰ κυλινδρικά, καὶ Σερήνου, κατ' αὐτὸν σχεδὸν ἀνδρὸς ἢ ὅτι ἔγγιστα, ἔτι δὲ τὰ περὶ σφαῖραν κινουμένην τε καὶ μή, θατέρου δὴ μέρους τῆς μαθηματικῆς, σχήματα καὶ εἴδη καὶ συμπτώματα, οἷς ἄρα καὶ συνεισάγονται αἱ τῆς ἀστρονομικῆς ἕξεως ὑποθέσεις Θεοδοσίου καὶ ὧντινων ἄλλων, καὶ οὐχ ἥκιστ' Εὐκλείδου ἁρμονικούς τε λόγους καὶ κατατομὰς συμφύτους τῇ τοῦ παντὸς κατασκευῇ, καὶ ὄντως ἀπόδειξιν ἄσειστον καὶ ἀκλόνητον τῆς τῶν ὄντων ἀλληλουχίας, καὶ τῆς θείας προνοίας καὶ καλλιτεχνίας, ὡς εἰπεῖν, ὑπόμνησιν παμπλείστων πανσόφων ἀνδρῶν."

[43] The first report appears in Metochites, *Abridgement* I.1.26; see Bydén (2003) 432. The second report appears in Metochites, *Poem* 1, 633-50; see Bydén (2003) 249.

Byzantine Empire and translating several of these works into Greek. Byzantine students would have seen little need of retranslating originally Greek works, but it is possible that exposure in Persia to the many works on the *Almagest* and to works preliminary to its study influenced how these individuals engaged with and taught astronomy when they returned to the Byzantine Empire.

Chapter 7

Translations into Latin and Hebrew

1. Introduction

The circulation of the works which comprised the Little Astronomy and the Middle Books was extensive enough that many of them received further translations between the ninth and thirteenth centuries. These translations were into Latin and Hebrew. While there is not space in this study to examine these translations in great detail, their historical occurrence and the contexts in which they were produced are examined in this chapter as further evidence of how widely this astronomical curriculum was circulated and used in the time leading up to the thirteenth century.

2. Translations into Latin

2.1 Overview

The following is an overview of the known translations into Latin from Greek and Arabic of Little Astronomy / Middle Books treatises and treatises sometimes appended to these collections. Works which are not known to be translated are included in the table to offer an impression of what proportion of the collection was translated versus left untranslated. Attested translations are indicated with 'x,' translations of uncertain provenance are indicated with '?' and will be expanded upon below. Further description and references for all these translations are offered in the following section.

The Arabo-Latin translators known to be involved are Plato of Tivoli (fl. 1116 - 1138), Gerard of Cremona (ca. 1114 - 1187), and Campanus of Novara (ca. 1220 - 1296). The Graeco-Latin translator known to be involved is William of Moerbeke (1215-35 - ca. 1286).[1]

[1] For more on these figures, see Haskins (1924). For Gerard of Cremona and his school, see also Burnett (2001). For Campanus of Novara, see the introduction in Benjamin and Toomer (1971). For William of Moerbeke, see Clagett (1982).

	Arabo-Latin				Graeco-Latin	
	Plato of Tivoli	Gerard of Cremona	Campanus of Novara	Anon.	William of Moerbeke	Anon.
Data (Euclid)		x				x
Sphaerica (Theodosius)	?	x	?			
Spherics (Menelaus)		x				
On the Moving Sphere (Autolycus)		x				
Optics (Euclid)		x		x		x
Catoptrics (Euclid)						x
Phaenomena (Euclid)						
On Habitations (Theodosius)		x				
On Days and Nights (Theodosius)						
On Sizes and Distances (Aristarchus)						
On Risings and Settings (Autolycus)						
Anaphoricus (Hypsicles)		x				
Lemmata (pseudo-Archimedes)						
Measurement of the Circle (Archimedes)	?	x			x	
On the Sphere and Cylinder (Archimedes)		?			x	
Commentary on the Sphere and Cylinder (Eutocius)					x	
On the Sector Figure (Thābit)		x		x x		

	Arabo-Latin				**Graeco-Latin**	
	Plato of Tivoli	Gerard of Cremona	Campanus of Novara	Anon.	William of Moerbeke	Anon.
Commentary on the Sector Figure (al-Nasawī)						
Book of Knowledge (Banū Mūsā)		x				
Assumptions (Thābit)						
On the Composition of Ratios (Thābit)						
Book of Enlightenment in Astronomy (al-Kharaqī)						

Table 7.1: Latin translations and translators of Little Astronomy / Middle Books texts

2.2 Details of the Translations

The *Elements* and the *Almagest*

The translations of the *Elements* and *Almagest* into Latin through the thirteenth century are summarized here so that they might be compared with what is found for treatises of the Little Astronomy and the Middle Books. Euclid's and Ptolemy's treatises see translation out of both Arabic and Greek.

In the case of the *Elements*, Adelard of Bath produced an Arabo-Latin translation ca. 1120. A second Arabo-Latin translation is commonly attributed to Gerard of Cremona's efforts in Toledo – the work is listed among the bibliography of Gerard's translations by his students, and the translation style matches his. Another may be the work of Hermann of Carinthia, ca. 1140. The twelfth century also saw a translation of the *Elements* out of Greek.[2] These translations subsequently led to further adaptions and editions in Latin.[3]

The *Almagest*, meanwhile, saw a translation produced out of Greek in Sicily ca. 1150. There were additionally two Arabo-Latin translations before the end of the thirteenth century: Gerard of Cremona's translation, produced in Toledo between ca. 1140 and 1187, and a translation produced by an otherwise unknown ʿAbd al-Masīḥ of Winchester – the single extant witness of this text was copied in the mid 13th century.[4]

<u>Data</u>

A translation from Arabic by Gerard of Cremona is attested in the list of his translations – "Liber datorum Euclidis tractatus .i." – however, this translation does not appear to have survived.[5] There was also a translation from the Greek which has been edited by Ito.[6] The earliest extant manuscript of this text, Oxford Bodleian Auct. F.5.28, dates on paleographic grounds from the middle of the thirteenth century; Ito suggests that the production of this text should be situated among the Graeco-Latin translations of the twelfth century. He argues its translator was also responsible for the Graeco-Latin translations of the *Optics* and *Catoptrics*, and that it is possible this figure was the same as the Graeco-Latin translator of the

[2] Adelard of Bath's translation of the *Elements* is edited in Busard (1983), Gerard of Cremona's in Busard (1984), Hermann of Carinthia's(?) in Busard (1968), and the Graeco-Latin translation in Busard (1987). For more details on the Gerard translation, see de Young (2004). The Graeco-Latin translation is first discussed in Murdoch (1967).

[3] Adaptions and editions which were produced before the end of the thirteenth century include an adaption of Adelard's translation attributed to Robert of Chester (12th c.), edited in Busard (1992); another adaption attributed to John of Tynemouth (13th c.), edited in Busard (2001); and an edition by Campanus of Novara (13th c.), edited in Busard (2005).

[4] On the Graeco-Latin translation, see i.e. Haskins and Lockwood (1910), Haskins (1912), and Angold (2020) 153-154. On Gerard's translation, see Kunitzsch (1974) 83-112 and Kunitzsch (2004). On the translation by ʿAbd al-Masīḥ of Winchester, see Heiberg (1911), Haskins (1927) 108-110, and Burnett (1999).

[5] See the list of translations in Burnett (2001) 277. Hultsch in Pauly and Wissowa (1907) 1043 believed the Latin *Data* in Dresden Db 86 to be Gerard's translation, but Ito (1980) 16 has found its text to be a match for the Graeco-Latin translation.

[6] Ito (1980).

Almagest.[7] The latter suggestion has however received pushback.[8] More detailed textual studies to address this question have yet to be done.

Sphaerica

The *Sphaerica* had two Arabo-Latin versions by the thirteenth century. It was translated by Gerard of Cremona, and this translation has been edited by Kunitzsch and Lorch.[9] The ascription to Gerard is supported both by its match for his style and by its inclusion in the list of his translations compiled by his students. It appears as the second entry under *De geometria*: "Liber Theodosii de speris tractatus .iii."[10] The second version's translator is uncertain; it has been attributed to either Plato of Tivoli or to Campanus of Novara.[11]

Spherics

Gerard of Cremona additionally translated Menelaus's *Spherics* out of Arabic.[12] This treatise is listed as the fifth entry under *De geometria* by his students: "Liber Milei tractatus .iii."[13] A comparison of the Latin text with the versions available in Arabic shows that the first two books seem to come from some version of al-Māhānī's revision before al-Harawī's revisions, but the third book shows more similarity with the edition of Ibn ʿIrāq and presumably draws from its source.[14]

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[7] On Oxford Bodleian Auct. F.5.28, see Ito (1980) 39. On the proposed context of the translation, see Ito (1980) 23-41.

[8] Clagett (1982) 358.

[9] Kunitzsch and Lorch (2010b).

[10] See the list edited in Burnett (2001) 276.

[11] Pena (1558) first attributed this version to Plato of Tivoli, and was followed by Boncompagni (1851) 251-252 and Heiberg (1927) viii. Lorch (1996) conversely argued that this version was the work of Campanus.

[12] See Bjørnbo (1902) 10ff.

[13] Burnett (2001) 276.

[14] See Krause (1936) 11ff and 85-86 for comparison of this translation with the Arabic versions. Hogendijk (1996) demonstrates that the parts of Ibn Hūd's *Book of Perfection* which depend on Menelaus show that Ibn Hūd had access to an Arabic version of the *Spherics* with these characteristics: the first part showing similarity with al-Māhānī's revision, and the second part with the source of Ibn ʿIrāq's edition. The evidence put forth by Rashed and Papadopoulos (2017) 26-71 confirm these findings.

On the Moving Sphere

On the Moving Sphere was translated from Arabic by Gerard of Cremona; it is the ninth entry under *De astrologia* in his students' list: "Liber Autolici de spera mota tractatus .i."[15] It has been edited by Mogenet.[16]

Optics

The *Optics* saw multiple transmissions into Latin, via both Greek and Arabic. The version which circulated in Latin under the title *De visu* was translated out of Greek – scholarship tends to agree in pointing to twelfth century Sicily as the context of this translation.[17] It has been suggested that it, together with the Graeco-Latin translations of the *Data* and *Catoptrics*, were all the work of one translator, though this is not certain and has received pushback.[18] This translation of the *Optics* has been edited by Theisen.[19] There are also four other versions extant in Latin manuscripts which appear to be adaptions of *De visu*.[20]

There are multiple Arabo-Latin translations of the *Optics*. One of these circulates with the title *Liber de aspectibus*. It may have been translated by Gerard or a member of his school – while Euclid's *Optics* is not included in the list of Gerard's translations, this version does appear in the manuscript Paris lat. 9335, a codex containing a multitude of translations by Gerard and his circle.[21] This translation has also been edited by Theisen.[22]

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[15] Burnett (2001) 278.
[16] Mogenet (1948).
[17] See for example, Björnbo (1909), Haskins (1912), and Steinschneider (1956). There are multiple thirteenth century manuscript witnesses for the text, and one potentially twelfth century one – Oxford, Bodleian Corpus Christi College 283 – on these, see Theisen (1972) 52-55.
[18] See Ito (1980) for the argument in favor of this hypothesis, and Murdoch (1967) for the argument against it.
[19] See the editions in Theisen (1972) 66-320 and Theisen (1979).
[20] Theisen (1972) 12.
[21] Theisen 327-328.
[22] See the edition in Theisen (1972) 336-384.

A second is extant under the title *Liber de radius visualibus*. The translator is uncertain and the earliest surviving manuscripts date to the thirteenth century.[23] It has additionally been edited by Theisen.[24]

Another version, under the title *Euclidis de aspectuum diversitate*, may also be a translation from the Arabic rather than an adaption. The context of its production is unclear. It survives in a fifteenth century manuscript and has yet to be edited.[25]

<u>Catoptrics</u>

The *Catoptrics* was translated from Greek, and there are extant two later adaptions of this translation as well. All three versions have been edited and discussed by Takahashi.[26] The translators and adaptors are not specified in the manuscripts, but the earliest extant manuscript of the translation, Oxford Bodleian Corpus Christi College 283, dates from the 12th-13th century.[27] The earliest extant manuscripts of the two adaptions date from the thirteenth and twelfth centuries.[28]

<u>Phaenomena</u>

There is no extant medieval Latin translation of the *Phaenomena*, nor attestation of one.

<u>On Habitations</u>

On Habitations was translated from the Arabic by Gerard of Cremona, appearing as the sixth entry under *De astrologia* in the list of his translations: "Liber Theodosii de locis habitabilibus tractatus .i." It has been edited by Kunitzsch and Lorch.[29]

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[23] The earliest extant manuscripts appear to be the thirteenth century Seville, Biblioteca Capitular Colombina MS 7.6.2 and Florence, Biblioteca Nazionale MS conv. soppr. J. I. 32. See Theisen (1972) 334-335.

[24] See the edition in Theisen (1972) 403-422.

[25] The manuscript in question is Vatican urb. lat. 1329. See Theisen (1972) 324, fn.10.

[26] Takahashi (1992). For the edition of the Graeco-Arabic translation *De speculis*, see p.114-211; for the adaption *Liber de speculis*, see p.212-291; for the adaption *De speculis*, see p.292-318.

[27] Takahashi (1992) 77.

[28] For the adaption *Liber de speculis*, these are Oxford Bodleian Auct.F.5.28 and Venice Biblioteca Nazionale Marciana, Zanetti Lat. 332: see Takahashi (1992) 78-79. The adaption *De speculis* is extant in only one manuscript, the twelfth century British Library Add. 17368: see Takahashi (1992) 79.

[29] See the list of translations in Burnett (2001) 278. The edition is Kunitzsch and Lorch (2010a).

On Days and Nights

There is no extant medieval Latin translation of *On Days and Nights*.

On Sizes and Distances

There is no extant medieval Latin translation of *On Sizes and Distances*.[30]

On Risings and Settings

There is no extant medieval Latin translation of *On Risings and Settings*.

Anaphoricus

Gerard of Cremona produced a translation of this text from the Arabic, and this is noted in the list of his translations as the sixth entry under *De astrologia*: "Liber Esculegii tractatus .i." The Latin has been printed by Manitius.[31]

Lemmata

There is no extant medieval Latin translation of the *Lemmata*.

Measurement of the Circle

There are extant two Arabo-Latin translations of this short Archimedean treatise. The more frequently copied was that of Gerard of Cremona – it is possibly the *Measurement of the Circle* which is indicated in the list of his translations by the entry "Liber Archimedis tractatus .i."[32] There was also a translation tentatively attributed to Plato of Tivoli, since it follows a different translation of his in Paris lat. 11246 and uses similar terminology. Both translations have been discussed, edited, and translated by

[30] Noack (1992) 45 notes it there is a small possibility that a Latin *On Sizes and Distances* by Gerard of Cremona remains to be discovered. This possibility is raised largely by the text's inclusion in a note in Paris lat. 9335, to be discussed below. However, the treatise does not appear in the list of Gerard's translations.
[31] See the entry in the list of translations in Burnett (2001) 278. The edition is Manitius (1888).
[32] Burnett (2001) 276.

Clagett.[33] William of Moerbeke produced a translation of this text from the Greek in 1269, and this translation has been edited by Clagett as well.[34]

On the Sphere and Cylinder

On the Sphere and Cylinder was translated from the Greek by William of Moerbeke in 1269. This translation has been edited by Clagett.[35] There additionally exists a fragment of what appears to be an Arabo-Latin translation of *On the Sphere and Cylinder* on f. 121r of the manuscript Oxford Digby 168. Clagett suggests that this fragment might come from the work of Gerard of Cremona, as in the manuscript it appears alongside several works translated by him.[36]

Commentary on the Sphere and Cylinder

Eutocius's commentary on the above work was also translated from the Greek by William of Moerbeke in 1269, and this translation has been edited by Clagett.[37]

On the Sector Figure (Thābit ibn Qurra)

There are three Latin versions of *On the Sector Figure* extant. One of them was produced by Gerard of Cremona, and this is supported by its entry in the list of his translations: "Liber Thebit de figura alkata tractatus .i." The second appears to be another translation, with interpretations of the Arabic text not seen in Gerard's version. The third in some places appears to be a rewriting, but in others includes material from the Arabic not seen in the other two versions. These translations have been edited by Lorch.[38]

[33] Clagett (1964): on Gerard's translation, see p. 30-58; on Plato(?)'s translation, see p.16-29.
[34] Clagett (1976) 157-160.
[35] Clagett (1976) 161-220. On William of Moerbeke as translator, see p. 28-53. Clagett (1982) 363-365 argues that the manuscript Vat. Ottob. Lat. 1850 is an autograph of his.
[36] Clagett (1952) 36-37.
[37] Clagett (1976) 221-286.
[38] See the list of Gerard's translations in Burnett (2001) 276. The three Latin translations are discussed in Lorch (2008) 30-36. The "Grecising" translation is edited in p.124-141, and the "inter universas" translation in p.142-153.

Commentary on the Sector Figure (al-Nasawī)

There is no extant medieval Latin translation of al-Nasawī's *Commentary on the Sector Figure*.

Book of Knowledge (Banū Mūsā)

The *Book of Knowledge of Plane and Spherical Figures* was translated by Gerard of Cremona and appears in the list of his translations under the entry "Liber Trium Fratrum tractatus .i." It has been edited by Clagett.[39]

Assumptions (Thābit ibn Qurra)

There is no extant medieval Latin translation of Thābit's *Assumptions*.

On the Composition of Ratios (Thābit ibn Qurra)

There is no extant medieval Latin translation of Thābit's *On the Composition of Ratios*.

Book of Enlightenment in Astronomy (al-Kharaqī)

There is no extant medieval Latin translation of al-Kharaqī's *Book of Enlightenment in Astronomy*.

2.3 Translations of Other Preliminary Astronomical Works

Thābit's *Simplification of the Almagest* (*Tashīl al-Majisṭī*)[40] was translated by Gerard of Cremona. In the manuscripts it receives the title *Liber quem edidit Tebit filius Chore de his que indigent expositione antequam legatur Almagesti*; in the listing of Gerard's translations, its entry is "Liber Thebith de expositione nominum Almagesti tractatus .i." It has been edited by Carmody.[41] There is another version appearing in two manuscripts – Burnett supposed it to be a distinct translation on the basis of one of these

[39] See the list of translations in Burnett (2001) 277, and the discussion and edition in Clagett (1964) 223-367.
[40] Note this text also is found transliterated into Hebrew as well – see Langermann (1996) 158.
[41] See the list in Burnett (2001) 278. The text is discussed in Carmody (1960) 117-118 and edited in 131-139.

manuscripts, potentially by Hugo of Santalla (12th c.), but the more complete manuscript shows matches with the Gerard translation in several places and might indicate that the alternate version was a draft.[42]

3. Traces of Influence in Latin from the Curricula

3.1 The Graeco-Latin Translator of the *Almagest*

This translator's identity is unclear. A later gloss in Vatican Pal. lat. 1371[43] claims the translator's name as Hermann, but there are several issues with interpreting this as Hermann of Carinthia (12th century).[44]

The scholar does provide an interesting account of the path he embarked upon in order to access and translate the *Almagest* in the preface to his translation:

> "Certainly on the part of this which concerns the movement of the stars, Claudius Ptolemy, polisher of the ancients and model for the moderns, most skilled in the science of the stars, wrote thirteen books. These are called by the Greeks the *Mathematical* or *Great Syntaxis*; by the Arabs a name "*Almagest*" which is a corruption. When I was laboring in medicine in Salerno, hearing that a messenger of the king of Sicily, by name Aristippus, whom [the king] himself had sent to Constantinope with imperial generosity, had carried these through Palermo, inspired with the hope of my long-lived desire, I did not fear the barks of Scylla, I passed through Charybdis, I went around the flowing fires of Etna, seeking him from whom I hoped for the end of my desire. Finally finding him near the font of Pergusa, investigating the wonders of Etna with some danger, hidden matters and [the fact that] clearly [my] mind was lacking experience in knowledge of the stars prohibited him from handing over the aforementioned work to me. I, already attentively learned in Greek letters, indeed first used as a prelude Euclid's *Data*, *Optics*, and *Catoptrics*, and Proclus's *Elements of Physics*. Then, undertaking the aforementioned work of Ptolemy, kind Grace providing me with a favorable expositor in Eugenius, a man as much learned in Greek as in the Arabic language and not ignorant of Latin, (against the will of an ill-tempered man) I translated this [work] into Latin."[45]

[42] The manuscripts in question are Dijon, Bibliothèque Municipale, 449 and Milan, Biblioteca Ambrosiana, A. 183 inf. See Burnett (2007) 33-35.
[43] Vatican Pal. lat. 1371 may be viewed online in the DigiVatLib repository.
[44] See Haskins (1912) 157 and more recently Angold (2020) 153-154.
[45] Latin edited in Haskins and Lockwood (1910) 99-100: "Huius vero eam partem que siderum motus specculatur, veterum lima, specculum modernorum, Claudius Ptolomeus astrorum scientie peritissimus .XIII. perscripsit libris. Qui a grecis quidem mathematica seu meguisti sintaxis, a saracenis vero elmeguisti corrupto nomine appellantur. Hos autem cum Salerni medicine insudassem audiens quendam ex nuntiis regis Scicilie quos ipse Constantinopolim miserrat nomine Aristipum largicione susceptos imperatoria Panormum transvexisse, rei diu desiderate spe succensus, Scilleos lactractus non exhorui, Caripdim permeavi, ignea Ethene fluenta circuivi, eum queritans a quo mei finem sperabam desiderii. Quem tandem inventum Perguse prope fontem Ethnea miracula satis cum perriculo perscrutantem, cum occulte quidem alia, manifeste vero mens scientie siderum expers prefatum michi transferre

The translator's mention of a preliminary course in Euclid's *Data*, *Optics*, and *Catoptrics* (along with Proclus's *Elements of Physics*) stands out in this narrative.[46] While this is hardly the full corpus of the *Little Astronomy*, the three Euclidean works do appear as part of it later in its transmission. The translator's choice to study them prior to his attempt at translating the *Almagest* echoes the Arabic use of the Middle Books as preliminaries to the *Almagest*. Perhaps this reflects some transmission of this didactic strategy to Sicily by the twelfth century. Certainly some Arabic influence can already be seen in the translator's comments about the Arabic title of the *Almagest*.

3.2 The Manuscript Paris lat. 9335

The manuscript Paris lat. 9335 is a codex from the twelfth century. It contains a large variety of Latin translations from Arabic – its contents are as follows:[47]

1. 1r-19r: Theodosius *Sphaerica*
2. 19r-21v: Autolycus *On the Moving Sphere*
3. 22r-23r: Hypsicles *Anaphoricus*
4. 23v-25r: Thābit ibn Qurra *Introduction to the Almagest*
5. 25r-28v: Theodosius *On Habitations*
6. 28v-30r: Archimedes *Measurement of the Circle*
7. 30r-31v: Aḥmad ibn Yūsuf *Epistola abuiafar ameti filii josephi de arcubus simibilus*
8. 31v-32v: Al-Kindī *De quinque essentiis*
9. 32v-54v: Menelaus *Spherics*
10. 55r-53v: Banū Mūsā *Book of Knowledge*
11. 64r-75r: Aḥmad ibn Yūsuf *De proportione et proportionalite*
12. 75r-82r: Al-Kindī *De aspectibus*
13. 82r-83v: Pseudo-Euclid *De aspectibus*
14. 84r-88v: Tideus *De speculis*
15. 88v-92r: Euclid *Optics* (*liber de aspectibus euclidis*)
16. 92v-110v: Al-Nayrīzī *Commentary on Euclid Elements X*

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opus prohiberent, grecis ego litteris diligentissime preinstructus, primo quidem in Euclidis Dedomenis, Opticis, et Catoptricis, Phisicaque Procli Elementatione prelusi. Dehinc vero prefatum Ptolomei opus aggressus, expositorem propitium divina michi gratia providente Eugenium, virum tam grece quam arabice lingue peritissimum, latine quoque non ignarum, illud contra viri discoli voluntatem latine dedi orationi."

[46] See Haskins (1912) 158 for more discussion on the availability of these texts in Latin translations and the possibility of their translation by the present translator.

[47] A digital facsimile is available online in the Gallica repository. This manuscript has been described in Bjørnbo (1902b) 67-75.

17. 110v-115v: Al-Khwārizmī *Algebra*
18. 116v-125v: Abu Bakr *Liber mensurationis*
19. 125v-126r: Saʿīd Abū ʿUthmān *Liber Saydi abuothmi*
20. 126r-126v: ʿAbd al-Raḥmān(?) *Liber aderameti*
21. 126v-133v: Abraham(?) *Liber augmenti et diminutionis*
22. 135r-139v: Al-Kindī *De gradibus*
23. 140r-141r: *Capitulum cognitionis mansionis luna*
24. 141r-143r: Thābit ibn Qurra *De motu octavae sphaerae*
25. 143v-151v: Al-Fārābī *de scientiis*
26. 151v-160v: ʻArīb Ibn Saʻd al-Kātib al-Qurṭubī *Liber Anoe*

Most notably for the purposes of this investigation, however, is the note which has already received some discussion in chapter 3. On folio 28v there is a passage after the conclusion of *On Habitations* which reads as follows:

> The order which follows after the book of Euclid, which is found in the writing of Johanicus: ¶ Euclid's *Optics*, one book. ¶ Theodosius's *Sphaerica*, three books. ¶ Autolycus's *On the Moving Sphere*, one book. ¶ Euclid's *Phaenomena*, one book. ¶ Theodosius's *On Habitations*, one book. ¶ Autolycus's *On Risings and Settings*, two books. ¶ Theodosius's *On Nights and Days*, two books. ¶ Hypsicles's *Anaphoricus*, one book. ¶ Aristarchus's *On the Sizes and Distances of the Sun and the Moon*, one book.
>
> Ordo qui est post librum Euclidis secundum quod invenitur in scriptis Iohanicii. ¶ Euclidis de aspectibus. tractatus unus. ¶ Theodosii de speris. tractatus tres. ¶ Autolici de spera mota. tractatus unus. ¶ Euclidis de apparentibus. tractatus unus. ¶ Theodosii de locis habitabilibus. tractatus unus. ¶ Autholici de ortu et occasu. duo tractatus. ¶ Theodosii de die et nocte. duo tractatus. ¶ Esculei de ascensionibus. tractatus unus. ¶ Arsodochii de elongationibus planetarum et earum magnitudinibus. tractatus unus.

As noted, scholars have previously identified this Johanicus as Ḥunayn ibn Isḥāq, but comparison of the Latin version of this report with its Arabic counterpart in Beirut MS St. Joseph University, BO 223A makes it clear that the son Isḥāq ibn Ḥunayn was probably mistaken for his more famous father Ḥunayn at some point in the report's transmission.

In any case, it is striking that the report translated into Latin appears to have been a very early one – a list of the relevant treatises, copied and then transmitted from the writings of the ninth century Isḥāq. This report in the manuscript Paris lat. 9335, then, shows awareness in Latin of the core grouping of the

Middle Books. It is not followed up with any acknowledgement of the texts that variously came to be included later. The below lists the texts and their orders in the manuscripts Vat. gr. 204 and Seray Ahmet III 3464 as examples of the Little Astronomy and Middle Books, respectively, to compare these with what the note in Paris lat. 9335 presents:

Vat. gr. 204	**Seray Ahmet III 3464**	**“Ordo qui est post librum Euclidis…”**
1. *Sphaerica* 2. *On the Moving Sphere* 3. *Optics* 4. *Phaenomena* 5. *On Habitations* 6. *On Days and Nights* 7. *On Sizes and Distances* 8. *On Risings and Settings* 9. *Anaphoricus* 10. *Catoptrics* 11. *Data*	1. *Data* 2. *Sphaerica* 3. *On the Moving Sphere* 4. *Optics* 5. *Spherics* 6. *Phaenomena* 7. *On Habitations* 8. *On Days and Nights* 9. *On the Linear Astrolabe* 10. *On Risings and Settings* 11. *On the Composition of Ratios* 12. *On the Sector Figure* 13. *Commentary on the Sector Figure* 14. *Anaphoricus* 15. *On Sizes and Distances* 16. *Book of Enlightenment in Astronomy*	1. *Optics* 2. *Sphaerica* 3. *On the Moving Sphere* 4. *Phaenomena* 5. *On Habitations* 6. *On Risings and Settings* 7. *On Days and Nights* 8. *Anaphoricus* 9. *On Sizes and Distances*

Table 7.2: Comparison of manuscript content orders in Vat. gr. 204 and Seray Ahmet III 3464 with the order presented in the report in Paris lat. 9335

In broad strokes, the orders of these lists largely match up. Paris lat. 9335’s report omits additions that were made later in the Arabic transmission – it contains neither originally Arabic works nor the ancient Greek mathematical treatises that saw later inclusion. The works of Archimedes do not appear here. Euclid’s *Data* is not included either, though the *Optics* has been placed at the head of the list instead, perhaps because of its role as background geometry that was useful for astronomical geometry but which did not properly fit in that subject. The relative order of *On Risings and Settings* and *On Days and Nights* is flipped as well, though this change has little significance for the subject.

So by the twelfth century, awareness of the Middle Books grouping passes into Latin, though via a relatively old description of the collection. The note in Paris lat. 9335 does lack the name or description 'middle' or 'intermediate', and the grouping of works is not overtly described as being intended for preparation for the *Almagest*. Only the preceding work, the *Elements*, is indicated.

3.3 Gerard of Cremona

The Latin scholar responsible for transmitting the above note is unknown, but scholars often point to Gerard of Cremona or his students – the manuscript in question is full of translations by the well-known Arabo-Latin translator.

We are unusually well-informed about the translations of Gerard because of the *Vita* and listing of translations written by his students. The narrative in the *Vita* notably highlights the *Almagest* as the text that set Gerard on the path to becoming such a prolific translator:

> "He was educated from this cradle of childhood in the bosom of philosophy and he had arrived at a study of all parts of what was known to the Latins. Yet for love of the *Almagest*, which he scarcely discovered among the Latins, he went to Toledo. There, seeing the abundance of books on all subjects in Arabic and pitying the poverty of the Latins in the things he studied, he learned the Arabic language for the purpose of translation. Thus, relying on both science and language… he passed on Arabic literature. Until the end of his life he did not cease to transmit to the Latins (as if to his dear heir), as plainly and clearly as was possible for him, books from many subjects, whichever prevailed in elegance."[48]

A few details stand out in this narrative. First, we see the *Almagest* presented as a recognizably prestigious scientific text. We also see that interest in astronomical knowledge, and specifically in the

[48] The Latin has been edited in Burnett (2001) 275-276: "Et cum ab istis infantie cunabulis in gremiis phylosophie educatus esset et ad cuiuslibet partis ipsius notitiam secundum Latinorum studium pervenisset, amore tamen Almagesti, quem apud Latinos minime reperit, Toletum per<r>exit, ubi librorum cuiuslibet facultatis habundantiam in Arabico cernens et Latinorum penurie de ipsis quam noverat miserans, amore transferendi linguam <e>didicit Arabicam, et sic de utroque – de scientia videlicet et ydiomate – confisus… scripturam revolvit Arabicam, de qua plurium facultatum libros quoscunque valuit elegantiores Latinitati tamquam dilecte heredi, planius ac intelligibilius quo ei possibile fuit, usque ad finem vite sue transmittere non cessavit."

Almagest, is being presented in the narrative as an important motivator for Gerard. Further, we see that upon failing to gain access to the *Almagest* in Latin, his next course was to seek out the text in Arabic.

Despite an awareness of an Arabic grouping of works on spherical geometry and the translation of several of them by Gerard and his school, the listing provided in the *Vita* does not suggest a concerted effort to translate the whole of this corpus into Latin. The *Vita* groups works under several headings: De dialectica (three works), De geometria (seventeen works), De astrologia (twelve works), De phylosophia (eleven works), De fisica (twenty-one works), De alchimia (three works), and De geomantia (four works). Works relevant to our study appear scattered under De geometria and De astrologia.

There seems to be little logic to the ordering of works within these subcategories beyond the most basic. The *Elements* heads De geometria – it is followed immediately after by the *Sphaerica*, but from there works vary. (Thabit's *On the Sector Figure* also follows Menelaus's *Spherics*). De astrologia is headed by al-Farghani's *Rudiments*, an introductory astronomical text, before listing the more complex *Almagest*. But a text by Thabit on what should be read before the *Almagest* appears six entries later.

As an aside, the section De Fisica makes it clear that the ordering of this list is showing no influence from curricular reading orders. The first nine works are Galenic works, but their arrangement has nothing to do with the reading order suggested by Galen, nor the curriculums recorded by sources like Hunayn ibn Ishaq, Ibn al-Nadīm, or Ibn Riḍwān.[49]

Of the Middle Books not listed above in the *Vita*, some made their way into Latin, others did not. As discussed above, there are several works not included in Gerard's students' list which modern scholars still suppose to have been his work.

[49] See Bergsträsser (1925) ٤-١٧، ٣٩ for the curriculum reported by Ḥunayn ibn Isḥāq, Flūgel (1872) 289-290 for what is reported by Ibn al-Nadīm, and Iskandar (1976) 249-252 for what is reported by Ibn Riḍwān.

Thus the wide circulation these texts enjoyed as members of the Middle Books brought them to Toledo's school of translation, but the didactic role which they served in the Arabic world does not appear to have motivated the translation of the curriculum in full.

4. Translations and Transliterations into Hebrew

4.1 Overview

The following is an overview of the known translations into Hebrew or transliterations into Judeo-Arabic of Middle Books treatises and treatises sometimes appended to the collection. The *Elements* and *Almagest* are also included at the start and end of this list, and works which are not known to be translated are still included in the table to offer an impression of what proportion of the collection was translated versus left untranslated.

The translators known to be involved are Jacob Anatoli (ca. 1194 - ca. 1256), Moses ibn Tibbon (fl. 1240-1283), Jacob ben Makhir (ca. 1236 - ca. 1304), and Qalonymos ben Qalonymos (1286 - after 1328).[50] The final column acknowledges where Judeo-Arabic transliterations of the relevant treatises exist. Further information and references for the below translations and transliterations will be expanded upon in the footnotes rather than elaborated on in the text.

[50] On these figures see Lévy (1997b) 440-447. See also the relevant entries in the *Jewish Encyclopedia* (1901-1906). For Jacob ben Makhir see also Mercier "Jacob ben Makhir ibn Tibbon" in the *The Biographical Encyclopedia of Astronomers*.

	Jacob Anatoli	Moses ibn Tibbon	Jacob ben Makhir	Qalonymos ben Qalonymos	Anon.	Judeo-Arabic
Elements (Euclid)[51]	?	x	x		x	x
Data (Euclid)[52]			x			
Sphaerica (Theodosius)[53]		x	x			x
Spherics (Menelaus)[54]			x			
On the Moving Sphere (Autolycus)[55]			x			
Optics (Euclid)[56]			?			
Catoptrics (Euclid)						
Phaenomena (Euclid)						
On Habitations (Theodosius)						
On Days and Nights (Theodosius)						
On Sizes and Distances (Aristarchus)						
On Risings and Settings (Autolycus)						

[51] On the four translations of the *Elements*, see Lévy (1997a). Moses ibn Tibbon produced his in 1270 and Jacob ben Makhir in 1289. There is also an anonymous version that presents Book I and beginning of Book II. A fourth is attributed to a "Jacob" – if this is Jacob Anatoli, it would be the oldest version – see Lévy (1997c). On this translation, see also Elior (2018). Books I and II of Moses ibn Tibbon's and "Jacob"'s translations have been edited in Elior (2021). There is also a Judeo-Arabic version of the *Elements* preserved in Paris BNF héb. 1381.

[52] On the translation by Jacob ben Makhir in 1272, see Steinschneider (1956) 510.

[53] On the translation by Moses ibn Tibbon in 1271, see Steinschneider (1956) 542. On the translation by Jacob ben Makhir (begun around the same time, but the initial copy was stolen and he returned to the task 20 years later), see Knorr (1986) 232-35. On the Judeo-Arabic transliteration, see Lorch (2014); he suggests the fourteenth century for the two extant manuscripts (Florence Laur. Med. 124 and Cambridge University Library add. 1220).

[54] On the translation by Jacob ben Makhir in 1271, see Steinschneider (1956) 516. Part of the text has been published in Ginsburg (1943).

[55] On the translation by Jacob ben Makhir in 1273, see Steinschneider (1956) 503.

[56] The translator is unknown – see Lévy (1997b) 433. The suggestion of Jacob ben Makhir is made by Mortara (1878).

Anaphoricus (Hypsicles)						
Lemmata (pseudo-Archimedes)						
Measurement of the Circle (Archimedes)[57]				?	x	
On the Sphere and Cylinder (Archimedes)[58]				x x		
Commentary on the Sphere and Cylinder (Eutocius)[59]				?		
On the Sector Figure (Thābit)[60]				x		
Commentary on the Sector Figure (al-Nasawī)						
Book of Knowledge (Banū Mūsā)[61]					x	
Assumptions (Thābit)						
On the Composition of Ratios (Thābit)						
Book of Enlightenment in Astronomy (al-Kharaqī)[62]					x	x
Almagest (Ptolemy)[63]	x					

Table 7.3: Hebrew translations and translators of Middle Books texts

[57] There are two known versions, both with unknown translators. Steinschneider (1965) 502 suggested Qalonymos for the first, but Lévy (1997b) 437 disagrees. The two versions have been edited in Lévy (2011).
[58] This text was translated twice by Qalonymos, per a copyist's testimony in Oxford Bodleian Laud. or. 93, fol. 28b – see the colophon translated in Lévy (1997c) 436 fn. 12.
[59] The translator is unknown but the suggestion of Qalonymos has been raised – see Lévy (1997c) 437.
[60] On the translation by Qalonymos ben Qalonymos in 1311, see Steinschneider (1964) 126 and Lorch (2008) 37.
[61] There is a fragment of an anonymous translation in Paris BNF Zotenberg Heb. 1026 – see Lévy (1997c) 437.
[62] See Langermann (1996) 150 for a listing of three manuscripts containing the Arabic text in Hebrew characters. The Hebrew translation appears in the fourteenth century manuscript Vat. ebr. 389, fol. 61a-123a.
[63] On the translation by Jacob Anatoli between 1231 and 1236, see Steinschneider (1956) 523 and Zonta (1993).

5. Traces of Influence in Hebrew from the Curricula

A full examination of the Arabic Middle Books' influence on texts transmitted and studied in Hebrew is beyond the scope of this present dissertation and deserves further study. Present scholarship has found only limited Hebrew references to the relevant texts, but they are not nonexistent.[64]

In the twelfth century, Abraham bar Ḥiyya of Barcelona (ca. 1065-1145) composed a scientific encyclopedia, *The Foundations of Science and the Tower of Faith* (*Yesodey ha-tevuna u-migdal ha-emuna*). Only the mathematical sections are known to survive today, but in them he recommends the study of Theodosius, Menelaus, and Autolycus.[65]

Chapter 5 already made reference to Ibn Aknīn, a Jewish scholar of the twelfth to thirteenth century. Ibn Aknīn's list of books recommended after the *Elements* included Theodosius's *Sphaerica*, Menelaus's *Spherics*, Archimedes's *On the Sphere and Cylinder*, and Apollonius's *Conics*, and this list of recommendations was translated into Hebrew. Of course, this particular list shows only partial overlap with the Middle Books. But the idea of books to be read after the *Elements* persists.

A notable potential reference to the Middle Books appears in the incipit of the Hebrew translation of Euclid's *Optics*. As noted in the table above, the translator is unknown, although Jacob ben Makhir has been put forth. The source of the incipit itself is unclear, however – it reads as follows: "The translator of this book said: his author said: After I completed the book which bears my name – and which includes thirteen books – as an introduction to what would be necessary for [the study of] the book Almagest – I undertook to compose this book."[66] The reference is undoubtedly a curious one, since it seems to be putting into the author (Euclid's) mouth a claim that he *composed* the *Elements* and the *Optics* as

[64] The following three references have been noted in Lévy (1997b) 444 fn.34.
[65] Millás Vallicrossa (1952) 41 and 78.
[66] Translation from Lévy (1997b) 444 fn.34. See for example Paris hebr. 1021, fol. 49a. This manuscript dates to 1507 CE and is available online in the Gallica repository. See also Paris hebr. 1011, fol. 65b, from the fourteenth century – this manuscript is also available online in the Gallica repository. Lévy notes that not all manuscripts contain the first few words ("the translator of this book said: his author said:").

preparation for the *Almagest*. For the purposes of this chapter, it is noteworthy that this report offers evidence of awareness in Hebrew circles of the *Optics*' intermediary use between the *Elements* and the *Almagest*.

These are instances of evidence from the twelfth and thirteenth centuries, but future study should examine also historical materials beyond this, to seek out potential further ripples of influence from the Middle Books. In the fourteenth or fifteenth century, for example, the *Hebrew Geometrical Compendium* was composed, and among its sources were Euclid, Theodosius, Menelaus, Autolycus, Hypsicles, Archimedes, and Thābit ibn Qurra.[67] In a recent study, Glasner argues that this work was composed in Hebrew (rather than translated from an Arabic composition) for Jewish audiences and that it relied on Hebrew translations of the sources in question.[68]

Overall, the transmissions seen in this chapter further emphasize what was being found in chapters 5 and 6 on the continuing traditions of the Middle Books and the Little Astronomy in Arabic and Greek, respectively. The large majority of translations discussed in this chapter stem from the Arabic and speak to the widespread use of the Middle Books. Translations from the Greek are far fewer, and there is little to suggest they were inspired by any kind of ongoing study of the Little Astronomy.

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[67] Glasner (2019) 201.
[68] Glasner (2019) 201, 204.

PART IV

Chapter 8

The Middle Books in the Thirteenth Century

1. Introduction

The purpose of this chapter is to lay out the context behind al-Ṭūsī's editions of the Middle Books, in preparation for the examination of those editions in the following chapter. While there were several editions of Middle Books texts and the *Elements* and *Almagest* in this period, al-Ṭūsī is the figure who will receive focus in these two chapters because of the significant influence his editions held in the subsequent centuries. This chapter will briefly sketch out the historical backdrop to the changing Islamicate world of the thirteenth century before it delves into al-Ṭūsī, relevant portions of his scholarly biography, and his editions. The following will furthermore note the editions of Muḥyī al-Dīn al-Maghribī, a colleague of al-Ṭūsī's at the Maragha observatory. Al-Maghribī's editions serve as an example of how scholarly engagement with these texts was not limited to the efforts of al-Ṭūsī, influential though the latter's efforts subsequently were. Rather, these activities can be fit into a broader pattern of the production of editions (taḥrīr) in the thirteenth century. In the case of al-Ṭūsī and al-Maghribī, it can be seen that their work with these texts clearly served teaching purposes. This will be considered from the student's side as well – this chapter will argue that Gregory Bar Hebraeus can be identified as a student who engaged with the *Elements*, the Middle Books, and the *Almagest* through the editions of both al-Ṭūsī and al-Maghribī.

The historical biographical sources that are relevant for the figures in this chapter are the following. The *Chronography* and *Ecclesiastical Chronicle* of Bar Hebraeus (d. 685/1286) both serve as contemporary sources, as does the *History of Physicians* (*ʿUyūn al-anbāʾ fī ṭabaqāt al-aṭibbāʾ*) of Ibn Abī Uṣaybiʿa (d. 668/1270) and the *Obituaries of Eminent Men* (*Wafayāt al-aʿyān wa-anbāʾ abnāʾ al-zamān*) of Ibn Khallikān (d. 681/1282). The fourteenth century offers biographical dictionaries both in the

Omissions of the Obituaries (*Fawāt al-wafayāt*) of al-Kutubī (d. 764/1363) and the *Completion of the Obituaries* (*Al-Wāfī bi 'l-wafayāt*) of al-Ṣafadī (d. 764/1363). A later source is Kâtip Çelebi's encyclopedia, *The Removal of Doubt from the Names of Books and the Arts* (*Kashf al-zunūn ʿan asāmī al-kutub wa al-funūn*), completed around 1062/1652. While this is a late work, it preserves bibliographical data from sources no longer extant today.

2. The Islamicate World in the Thirteenth Century

Chapters 3 and 4 examined the translation and early study of Middle Books texts in the third / ninth century in the Abbasid Caliphate, a power centered in Baghdad. Chapter 5's span of centuries saw a changing landscape: it touched for example on scholarship in the Taifa of Saragossa, one of the Muslim kingdoms that arose after the decline of the Umayyads in al-Andalus, as well as in the Khwarazmian Empire which had succeeded the Seljuk Empire. The power of the Abbasid Caliphate had fractured already before the lifetimes of many of the scholars discussed.

Significant changes were also occurring over the course of al-Ṭūsī's, al-Maghribī's, and other thirteenth century scholars' lives and careers. The major figures in this chapter traveled and studied in a world of multiple Islamic political entities. Baghdad remained the center of the fractured Abbasid Caliphate, while the Ayyubids held power in Syria. Other locations were ruled over by a variety of smaller states, such as the Nizari Ismāʿīlī state.

This landscape was overturned with the arrival of the Ilkhānids, who captured the Ismāʿīlī citadel of Alamut in 654/1256, Abbasid Baghdad in 656/1258, and Ayyubid Damascus in 658/1260. The Ilkhānids united these and other Muslim states of the period into one empire. Hülagü Khan established the new Ilkhānid capital at Maragha in northwest Iran in 654/1256. Both al-Ṭūsī and al-Maghribī, as will be seen below, weathered the turbulence of war and politics and established themselves as respected scholars in the new Ilkhānate.

3. Naṣīr al-Dīn al-Ṭūsī

3.1 His Scholarly Career

The purpose of this section is to lay out the broad strokes of Nasir al-Din al-Ṭūsī's career and intellectual activities as they pertain to mathematics and astronomy.[1] The scholar was a renowned polymath – while this section will note some of his other works, its focus will be on his work with those two subjects.

Al-Ṭūsī was born in 597/1201 and spent his youth in Tus in northeast Iran, where he received his early education in Imāmī (Twelver) Shiʿism from his father, his uncle, and his father's uncle. In his spiritual autobiography written in Persian, *Contemplation and Action* (*Sayr wa-sulūk*), al-Ṭūsī offers some details of his early education. He writes how his father encouraged him to study widely:

> "...But my father, a man of the world who had heard the opinions of different kinds of people and had [received] his education from his maternal uncle, who was one of the attendants and students of the chief dāʿī, Tāj al-Dīn Shahrastāna, was less enthusiastic about following these regulations. He used to encourage me to study [all] the branches of knowledge, and to listen to the opinions of the followers of [various] sects and doctrines."[2]

He furthermore records one of his early teachers in mathematics:

> "Then it happened that one of the students of Afḍal al-Dīn Kāshī – may God have mercy on him – came to the region. His name was Kamāl al-Dīn Muḥammad Ḥāsib, who had acquired a first-rate knowledge in a variety of philosophical subjects, especially in the art of mathematics; he had previously been a friend and acquaintance of my father. My father suggested that I should learn from him and frequent his company; so I began to study mathematics with him."[3]

Unfortunately this is the extent of the information this autobiographical work provides on al-Ṭūsī's secular education. The Kamāl al-Dīn Muḥammad Ḥāsib named here is otherwise unknown, though it is clear enough from his name (Ḥāsib) that he was a mathematician.

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[1] For a recent overview on al-Ṭūsī, see Ragep (1993) 3-23.
[2] Translation of the Persian from Badakhchani (1999) 26.
[3] Translation of the Persian from Badakhchani (1999) 26.

Further details of al-Ṭūsī's education can be gleaned from other sources. He relocated to Nisabur in Khorasan to study with the physician Quṭb al-Dīn al-Miṣrī and with the polymath Farīd al-Dīn Dāmādh. As Ragep notes, al-Ṭūsī's time in Nisabur likely spanned some period between 610/1213 (when he would have been twelve or thirteen years old) and 618/1221 (when Genghis Khan's armies attacked the region of Khorasan).[4]

Al-Ṭūsī subsequently traveled to Mosul, where he became a student of the Shīʿite legal scholar Muʿīn al-Dīn Sālim ibn Badrān al-Miṣrī and the famed Kamāl al-Dīn ibn Yūnus (d. 639/1242).[5] The latter was well-known for his expertise in mathematics and astronomy. Ibn Abī Uṣaybiʿa reports on Ibn Yūnus's expertise in an unspecified variety of disciplines, and later several fields are mentioned: the religious sciences, jurisprudence, philosophy, grammar, medicine, magic, mathematics, and alchemy.[6] Ibn Khallikān, who studied with Ibn Yūnus in Mosul, offers more information on the scholar's expertise with astronomy and mathematics:

> "[Kamāl al-Dīn ibn Yūnus] was acquainted with all parts of mathematical science explained by Euclid, astronomy, conic sections, mean proportionals (*mutawassita*), the Almagest, the different modes of calculation both numerical, and algebraic, arithmetic, the system of double false position, music and mensuration. In all these sciences he was without a rival."[7]

The word "mutawassita," especially located after the *Elements* and immediately before the *Almagest*, is worth attention. De Slane translates "المتوسطات" as "mean proportionals," but in this context it seems much more likely that Ibn Khallikān means instead the middle subjects, namely those of the Middle Books. And

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[4] Ragep (1993) 6. Ibn Abī Uṣaybiʿa (2000) 2.30 reports that Quṭb al-Dīn al-Miṣrī perished in the attack on Nīsābūr in 618/1221.

[5] Al-Ṭūsī's studies with Kamāl al-Dīn ibn Yūnus are reported by Ṣafadī and Kutubī, on the authority of al-Shams, son of Muʾayyad al-Dīn al-ʿUrḍī.

[6] See Ibn Abī Uṣaybiʿa (2000) 10.83 and 15.40.

[7] Trans. de Slane (1868) 468. See Abbas (1977) 312 for the Arabic:
"ويعرف فنون الرياضة من اقليدس والهيئة والمخروطات والمتوسطات والمجسطي المجسطي لفظة يونانية معناها بالعربي الترتيب ذكر ذلك الوكري في كتابه وأنواع الحساب المفتوح منه والجبر والمقابلة والأرثماطيقي وطريق الخطأين والموسيقى والمساحة."

while the extant sources may not speak to precisely what Nasir al-Din al-Ṭūsī studied with Ibn Yūnus,[8] but it can be presumed that the former's education involved many of the subdisciplines and texts that Ibn Yūnus was noted to be experienced with. So it was plausibly with this scholar that al-Ṭūsī worked through the curriculum that comprised the *Elements*, Middle Books, and *Almagest*.

Ibn Khallikān also preserves a further report on Ibn Yūnus's practice with mathematics from Abū al-Barakāt ibn al-Mustawfī:

> "[Kamāl al-Dīn ibn Yūnus] was… a most learned man, well versed in every science and particularly distinguished by his acquaintance with those of the Ancients (the Greeks), such as geometry and logic. He got over the difficulties of Euclid and of the Almagest under the tuition of the shaikh Sharaf ad-Dîn al-Muzaffar Ibn Muhammad Ibn al-Muzaffar at-Tûsi…"[9]

A short scholarly genealogy becomes apparent from these reports, one in which mathematical and astronomical study using Euclid, Ptolemy, and perhaps other "ancients" (الأوائل) featured strongly.

After his studies, al-Ṭūsī's career took him to the Nizari Ismāʿīlī state, where his first patron was the Ismāʿīlī governor of Qūhistān. Al-Ṭūsī dedicated a text – the *Risālah Muʿīniyya* – to this patron in 632/1235.[10] He spent some time in Qūhistān before moving (or being relocated) to the Ismāʿīlī capital at Alamūt at an unspecified date.[11] As noted above, Alamūt fell to Hülagü Khan in 654/1256. Al-Ṭūsī, who was located at this capital at the time, successfully transitioned from the Ismāʿīlī court to Hülagü Khan's court.

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[8] Al-Kutubī and al-Ṣafadī note only that al-Ṭūsī studied with Kamāl al-Dīn. See al-Kutubī (2000) 253: "وقال شمس الدين بن المؤيد العرضي أخذ النصير العلم عن كمال الدين بن يونس الموصلي ومعين الدين سالم بن بدران المصري المعتزلي" and al-Ṣafadī (2000) 148: "وقال الشمس ابن المؤيد العرضي أخذ النصير العلم عن الشيخ كمال الدين ابن يونس الموصلي ومعين الدين سالم بن بدران المصري المعتزلي".

[9] Trans. de Slane (1868) 468. See Abbas (1977) 312 for the Arabic:
"هو عالم مقدم ضرب في كل علم وهو في علم الأوائل كالهندسة والمنطق وغيرهما ممن يشار إليه حل اقليدس والمجسطي على الشيخ شرف الدين المظفر بن محمد بن المظفر الطوسي."
In addition, Ibn Abī Uṣaybiʿa (2000) 15.24 and 15.33 reports further on Sharaf al-Dīn al-Ṭūsī's renown as a scholar of the geometrical and mathematical sciences.

[10] ʿAbd al-Raḥīm b. Abī Manṣūr Nāṣir al-Dīn Muḥtasham (d. 655/1257). See Ragep (1993) 10.

[11] Ragep (1993) 11.

The famous Maragha Observatory was founded in 657/1259 by the order of the Ilkhānid ruler Hülagü Khan. In the preface to his *Zīj-i Ilkhānī* (*Ilkhanan Zīj*; the zīj is a genre of tabular astronomical texts), al-Ṭūsī reports that Hülagü Khan had ordered him to observe the stars and that he and the other astronomers chose Maragha as the site for the observatory:

> "At the time that [Hulagu Khan] seized the dominions of the heretics, I Naṣīr al-Dīn who am of Ṭūs and had fallen into the power of the heretics – me he brought forth from that place and ordered to observe the stars. He sought philosophers having knowledge of observation, such as Muʾaiyid al-Dīn ʿUrḍi who was in Damascus, Fakhr al-Dīn Khilāṭi of Tiflīs, Fakhr al-Dīn Marāghī of Mauṣil and Najm al-Dīn Dabīrān of Qazvīn. They chose Marāgha as the place for the observations to be made, and applied themselves to this task, making instruments and erecting buildings suitable for this purpose. He also ordered them to bring books from Baghdad, Syria, Mauṣil and Khurāsān and to put them in the place where they would make observations, so that the whole affair went forward in excellent order."[12]

Other biographical sources report that al-Ṭūsī had been the one to come to the khan with the request to construct an observatory in Maragha. This request was granted, and the observatory even secured waqf funds for its continuing activities.[13] Research at this observatory led to new astronomical tables, mathematical models, and planetary theory. Al-Ṭūsī served as its first director until his death in 672/1274.

Al-Ṭūsī was a prolific writer, and evidence of his engagement with the mathematical and astronomical sciences can be found at all stages of his career.[14] See for example his early Persian treatise on astronomy, the *Risālah Muʿīniyya*, dedicated to the first of his Ismāʿīlī patrons in 632/1235; his appendix to the previous work, the *Ḥall-i Mushkilāt-i Muʿīnīye*, for that same patron in 643/1245; his influential Arabic treatise, the *al-Tadhkira fī ʿilm al-hayʾa*, completed in 659/1260-1 during his first few

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[12] Translation of the Persian from Arberry (1958) 259-260.
[13] For the report on al-Ṭūsī, see al-Ṣafadī (2000) 146. The waqf fund is often pointed to as one of the factors which contributed to the length and success of the observatory, since with this fund it was able to persist after the death of its original patron Hülagü Khan in 663/1265 and that of its original director al-Ṭūsī, nine years later. Naturally, the continuing favor of the khans remained an important factor as well: on their continuing patronage, see Yang (2019) 394.
[14] Inventories of al-Ṭūsī's works can be consulted in a variety of sources, including Brockelmann (for his Arabic works) and Storey (for his Persian works). A more recent partial listing can be found in Rosenthal and Ihsanoğlu (2003) 211-219.

years at Maragha; and his *Zīj-i Ilkhānī*, completed in 670/1272 and presenting the results of his work with one of the observation programs at Maragha.

3.2 Timeline of al-Ṭūsī's Editions

Al-Ṭūsī produced editions of all three units of the curriculum: the *Elements*, the Middle Books, and the *Almagest*. Manuscript colophons preserve dates for many of these editions and reveal that he completed the majority of these works during his time with the Ismāʿīlīs, before he joined Hülagü Khan and began work at Maragha. The following is a summary of the information that can be gleaned across the manuscripts:

Edition	**Hijri Date**	**Gregorian Date**
Almagest[15]	5 Shawwāl 644 H	13 February 1247 CE
Elements[16]	22 Shaʿbān 646 H	10 December 1248 CE
Sphaerica[17]	Jumādā al-ʾŪlā 651 H	July/August 1253 CE
On the Moving Sphere[18]	Jumādā al-ʾŪlā 651 H / Jumādā al-ʾĀkhirah 651 H[19]	June/July/August 1253 CE
Optics[20]	end of Shawwāl 651 H	December 1253 CE

[15] Saliba (1987) 5.
[16] See for example the colophon in British Library Add MS 23387, fol. 216v:
“وإذ قد وفقني الله تعالى في تحرير هذا \ الكتاب حسب ما قصد به فلأختم الكلام \ بحمد الله خير موفق ومعين وفرغ المصنف \ من تحريره قدس الله روحه ونور ضريحه \ في 22 شعبان المبارك سنة 646 هجرية \ والحمد لله رب العالمين”
This manuscript is available online in the Qatar Digital Library.
[17] See Hyderabad (1939-40) *Kitāb al-Ukar* 52: “فرغ المصنف من تحريره في شهور سنة خنا”
Compare Ayasofya 2758, fol. 90a: “ز ج ى جمادى الاولى من سنة ٦٥١”
Compare Ayasofya 2759, fol. 49b: “فرغ المصنف من تحريره في جمادى الاولى سنة خنا”
[18] See Hyderabad (1939-40) *Kitāb al-kurah al-mutaḥarrikah* 10: “فرغ المصنف رحمه الله عليه من تحريره في جمادى الاولى من سنة خنا”
Compare Bodleian Marsh 709, fol. 5a: “فرغ المصنف من تحريره يوم الجمعة ٧ من شهر جمادى الاول سنة ٦٥١”
[19] See Ayasofya 2758, fol. 92b: “ز و ز جمادى الاخرة سنة ٦٥١”
[20] See Hyderabad (1939-40) *Kitāb al-manāẓir* 24: “فرغ المحرر من تحريره رحمة الله عليه في اواخر شوال من سنة خنا”
Compare the colophon for BnF arabe 5974, fol. 76b: “وفرغ المحرر رحمه الله من تحريره (روحـ) شوال سنة خنا”
Note Krause (1936) 500 indicates the text was completed on 13 Shawwāl 651 H, but it is unclear what his source was for this date.

Phaenomena[21]	20 Rabīʿ al-ʾĀkhir 653 H	20 May 1255 CE
Days and Nights[22]	7 / 9 Jumādā al-ʾŪlā 653 H	14 / 16 June 1255 CE
Sizes and Distances[23]	653 H	1255/6 CE
Risings and Settings[24]	653 H	1255/6 CE
Anaphoricus[25]	653 H	1255/6 CE
Thābit *Assumptions*[26]	653 H	1255/6 CE
Archimedes *Lemmata*[27]	653 H	1255/6 CE
Archimedes *Sphere and Cylinder* and *Measurement of the Circle*[28]	661 H	1262/3 CE
Menelaus *Spherics*[29]	Shaʿbān 663 H	May/June 1265 CE

Table 8.1: Dates for the editions by al-Ṭūsī

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[21] See Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 33: "فرغ المصنف من تحريره في (ز ج ى) ربيع الاخر - خنج"
Compare the colophons recorded in Sulaymān (1996) 120, which give the date as well: "(ز ج ى) ربيع الآخر سنة خنج"
Compare the colophon for BnF arabe 5974, fol. 86a: "وفرغ المصنف رحمه الله من تحريره فى (ز ج ى) ربيع الآخر سنة خنج"
Note Krause (1936) 500 indicates the text was completed on 10 Rabīʿ al-ʾĀkhir 653 H, but it is unclear what his source was for this date.

[22] See Sezgin (1974) 156 and Hyderabad (1939-40) *Kitāb al-ayyām wa-al-layālī* 30: "فرغ المصنف سابع جمادى الاولى سنة ثلث وخمسين"
Compare the colophon for BnF arabe 5974, fol. 94a: "وفرغ المصنف من تحريره تاسع جمادى الاولى سنة ثلث وخمسين وستمائة"

[23] See BnF arabe 5974, fol. 107a: "وفرغ المصنف طاب مثواه من - ز ب ى ه - خنج"
Krause (1936) 503 apparently encountered a different report, and indicates the completion year as 658 H.

[24] See Hyderabad (1939-40) *Kitāb al-ṭulūʿ wa-al-ghurūb* 28: "فرغ المصنف رحمة الله عليه من تحريره في - ز ب - و ى ح - سنة خنج"
Compare the colophon for BnF arabe 5974, fol. 101a: "وفرغ المصنف رحمة الله من تحريره في - ز ب - ى ح - سنة خنج"

[25] See Hyderabad (1939-40) *Kitāb fī al-maṭāliʿ* 6: "فرغ المحرر رحمة الله عليه من تحريره (ز د ى ه) - سنة - خنج"
Compare the colophon for BnF arabe 5974, fol. 102a: "وفرغ المحرر رحمة الله من تحريره (ز د ى ه) سنة خنج"

[26] See Hyderabad (1939-40) *Kitāb al-mafrūḍāt* 14: "فرغ المصنف رحمه الله منه في - ز د ح - - خنج"

[27] See Hyderabad (1939-40) *Kitāb al-maʾkhūdhāt* 17: "فرغ المصنف رحمه الله منه (ز ز ك ه) خنج"

[28] These two text were edited together as one, with *Measurement of the Circle* being added to *Sphere and Cylidner* as an appendix. Van Lit (2012) 4 notes that "Tūsī wrote his revision of *The measurement of the circle* at around 661/1262-63," and Krause (1936) 501 indicates the text was completed on 661 H, but neither provide the source for this date.

[29] See Kâtip Çelebi, ed. Flügel (1835) 391: "وفرغ من تحريره : في شعبان ، سنة ثلاث وستين وستمائة"

The year component of these dates are also recorded in Sezgin, with little variation.[30] These colophons are not present in all manuscripts: Tabriz National Library 3484, for example, lacks them for all works.

These dates of completion reveal an order to al-Ṭūsī's work: the scholar dedicated himself to the task of editing the *Almagest* first. With Ptolemy's treatise completed, he then went back to produce his own edition of Euclid's *Elements*. He approached the Middle Books last.[31]

Within that final collection it is possible to identify three groupings: the works edited in 651 H, those edited in 653 H, and those additional works edited later. This rough division does broadly agree with the order of study that has been discussed for the Little Astronomy. The *Sphaerica*, *On the Moving Sphere*, and the *Optics* were completed in 651 H. Some manuscript colophons for these texts offer the relevant months, which show that al-Ṭūsī's editing project proceeded through these texts in the standard order.[32] Based on this evidence, either the *Sphaerica* and *On the Moving Sphere* were both completed in Jumādā al-ʾŪlā (June/July), with the *Optics* following in Shawwāl (December); or there is a clear

[30] The data in the table agrees with Sezgin's data for the *Almagest* (Sezgin (1978) 93), the *Elements* (Sezgin (1974) 111), the *Sphaerica* (Sezgin (1974) 155), *On the Moving Sphere* (Sezgin (1974) 82), the *Optics* (Sezgin (1974) 117), the *Phaenomena* (Sezgin (1974) 119), and *Risings and Settings* (Sezgin (1978) 74). Sezgin records the year 654 H for the *Anaphoricus* (Sezgin (1974) 145), which disagrees with the year 653 H above.

[31] Note that there is also an edition of Apollonius's *Conics* attributed to al-Ṭūsī. According to the colophon of the manuscript Leiden or. 14 on p.163, this was completed in 645 H (خمس وأربعين وستمائة) and so would have been produced between the editions of the *Almagest* and the *Elements*. There has been no evidence to suggest the *Conics* ever numbered among the Middle Books, and here too, while al-Ṭūsī does produce an edition of this text, it is separate from his later project to edit the Middle Books.

[32] In practice, the order of al-Ṭūsī's Middle Books as they were subsequently copied in manuscripts seems to have varied significantly. The order of his "Taḥrīr al-Handasiyāt" as it is presented in Kâtip Çelebi is consistent with the standard order up until Euclid's *Phaenomena*, but the treatises afterwards are listed differently. Granted, Kâtip Çelebi or his source may not have intended this list to be arranged in agreement with an order of study: he does list the *Elements* and the *Almagest* first before proceeding to the Middle Books. The order of his list is: *Elements*, *Almagest*, Euclid *Data*, Theodosius *Sphaerica*, Menelaus *Spherics*, Autolycus *Moving Sphere*, Euclid *Optics*, Euclid *Phaenomena*, Theodosius *Days and Nights*, Autolycus *Risings and Settings*, Hypsicles *Anaphoricus*, Aristarchus *Sizes and Distances*, Archimedes *Lemmata*, Thābit *Assumptions*, Banū Mūsā *Book of Knowledge*, Archimedes *Sphere and Cylinder*, and Theodosius *Habitations*. See Flügel (1837) 213.

progression starting with the *Sphaerica* in Jumādā al-ʾŪlā (June/July), then *On the Moving Sphere* in Jumādā al-ʾĀkhirah (July/August), and lastly the *Optics* in Shawwāl (December).

The more particular astronomical treatises – the *Phaenomena*, *On Risings and Settings*, *On Days and Nights*, and the *Anaphoricus* – were completed in 653 H.[33] As above, their manuscript colophons sometimes indicate the month of completion. The *Phaenomena*, completed in Rabīʿ al-ʾĀkhir (May), is situated earliest among those works with this information; it is followed by *Days and Nights*, completed in Jumādā al-ʾŪlā (June/July).

653 H was also the year in which two of the Arabic additions to the Middle Books – Thābit's *Assumptions* and Archimedes' *Lemmata* – are recorded to have been completed. But other additions to the Middle Books received new editions by al-Ṭūsī much later, during his time at Maragha. The *Measurement of the Circle* may have been completed in 661 H, while Kâtip Çelebi records the date of the edition of Menelaus's *Spherics* to be 663 H. Clearly al-Ṭūsī's editing project was interrupted by the siege of Alamut and his transition to the Ilkhānid court.

The *Spherics* seems to have been the final edition of al-Ṭūsī's project. It is in his introduction to this text that he comments on his specific intention to edit the Middle Books: "I wanted to edit the books characterized as the Middle [Books] – I mean the books whose subjects are in the middle of the educational arrangement between the *Elements* by Euclid and the *Almagest* by Ptolemy."[34] The term "educational arrangement" (*al-tartīb al-taʿlīmī*) here reinforces that al-Ṭūsī was thinking about these texts in a curricular context.

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[33] Sezgin (1974) 145 has 654 H as the completion date for the edition of the *Anaphoricus*, which would provide a more evident agreement with the order. This, however, disagrees with the manuscripts cited above.

[34] Hyderabad (1939-40) *Kitāb Mānalāʾus* 2: "انى كنت اريد أن احرر الكتب الموسومة بالمتوسطات اعنى الكتب التى من شأنها ان تتوسط في الترتيب التعليمى بين كتاب الاصول لأقليدس وبين كتاب المجسطى لبطلميوس"

3.3 Al-Ṭūsī on his *Almagest* and *Elements*

While al-Ṭūsī does not elaborate on his editorial intentions for the Middle Books in the text he presents, it is possible to look to his related editions, those of the *Almagest* and the *Elements*, for insights into his rationale and process.

In his edition of the *Almagest*, the scholar lays out his goals very clearly in the introduction to the text:

> "One does not omit the theoretical goals nor the practical methods of this book down to the arrangement of the chapters, the computational sections, the tables and the arrangement of the diagrams. Nothing outside of it corrupts it except what it needs to present for the facilitation of difficulties or resolution of doubts. I mention some of what the moderns invented or used, with which the theories are increased in beauty and brilliance or the processes are decreased in toil and hardship, with the stipulation that there is a preference for brevity and abbreviation and caution against elaboration and repetition."

> لا يفوته مقاصد ذلك الكتاب النظريّة ومناهجه العمليّة حتّى ترتيب الفصول وأبواب الحساب ورسوم الجداول وأوضاع الأشكال ولا يشوبه شيء خارج منه غير ما يحتاج إلى تقديمه في تيسير عسير أو حلّ إشكال وأشير إلى بعض ما استنبطه المحدّثون أو ذهب إليه المتأخّرون ممّا زادت النظريّات به حسنًا وبهاءً أو نقصت العمليّات منه كدًا وعناء بشرط إيثار الإيجاز والاختصار والاحتراز عن الإسهاب والتكرار.[35]

So al-Ṭūsī intends to maintain all parts of the structure of the original *Almagest*, and to limit himself from adding to the text except in cases where, in his judgment, such additions would improve clarity or address doubts. Stylistically, however, his text aims for brevity.

Furthermore, in his edition of the *Almagest* he shows care in distinguishing between Ptolemy's original material and the later material he himself had brought to the new edition. He offers a description and rationale for his efforts:

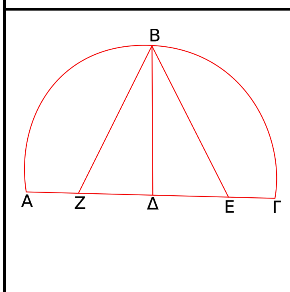
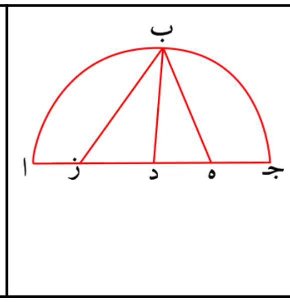
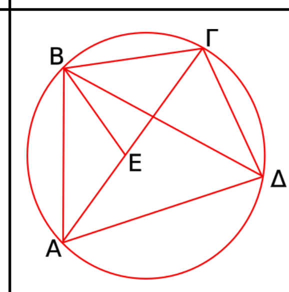
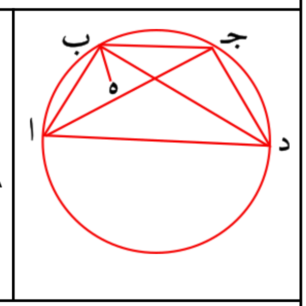
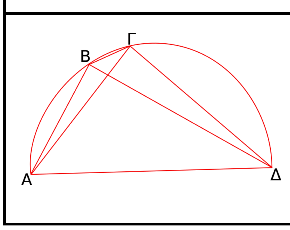
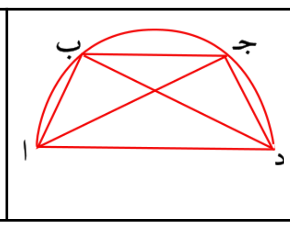
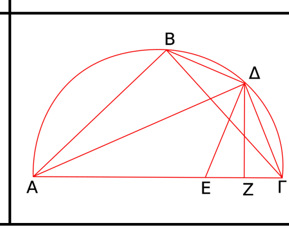
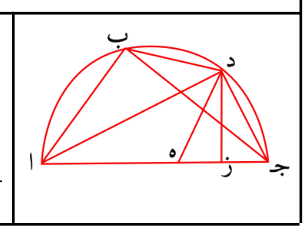
> "I noted clearly what was outside the original [text] of the book, which I added so that understanding might be acquired with minimal exertion. I differentiated between diagrams of the main [text] and the rest of what I added via the color of the lines and the numerals so they might be distinguished at first glance, without need of increased concern. But I took on this burden

[35] Al-Ṭūsī's edition of the *Almagest* has not been edited. The Arabic text is from the Ptolemaeus Arabus et Latinus project's transcription of Istanbul Nuruosmaniye Kütüphanesi 2941, fol. 1b. The witness was completed in late Shaʿbān 684/late October 1285 and is one of the earliest extant witnesses to the text.

because the book is known among scholars who specify it in their discussions and they cite the location of its matters from the sections and the diagrams in their notes."

ونبهّت صريحًا على ما هو خارج من أصل الكتاب ممّا أضفت إليه ليحصل الوقوف بأيسر السعي عليه وخالفت بين أشكال المتن وغيرها ممّا أوردته بلون الخطوط والأرقام ليتميّزا في بادئ النظر من غير احتياج إلى زيادة اهتمام وإنّما تكلّفت ذلك لكون الكتاب علمًا بين أهل العلم ينصّون عليه في محاوراتهم ويشيرون إلى مواضع مسائله من الفصول والأشكال في حوالاتهم.[36]

Extant manuscripts of al-Ṭūsī's *Almagest* do indeed show these editorial choices. An example of the use of different ink colors to immediately distinguish diagrams he added from ones original to the *Almagest* can be seen in the manuscript Istanbul, Nuruosmaniye Kütüphanesi 2941. Below are the first eight diagrams of the manuscript, spanning folios 5a to 6a, in comparison with the the first seven figures from a Greek manuscript of the *Almagest*, Paris gr. 2389.[37]

<table>
<tr><td colspan="2">Greek and Arabic Diagram 1</td><td></td><td colspan="2">Greek and Arabic Diagram 2</td></tr>
<tr><td>
</td><td>
</td><td></td><td>
</td><td>
</td></tr>
<tr><td colspan="2">Greek and Arabic Diagram 3</td><td></td><td colspan="2">Greek and Arabic Diagram 4</td></tr>
<tr><td>
</td><td>
</td><td></td><td>
</td><td>
</td></tr>
</table>

[36] Istanbul Nuruosmaniye Kütüphanesi 2941, fol. 1b.

[37] For the Greek diagrams, see folios 12v-16r of this manuscript. Compare also Heiberg (1898) 32-45 for diagrams to these propositions.

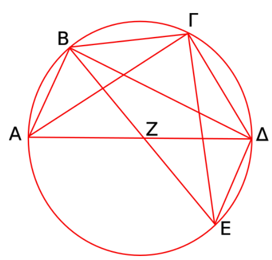
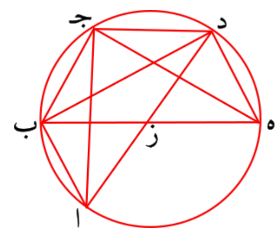
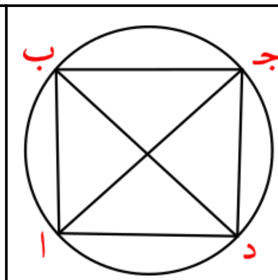
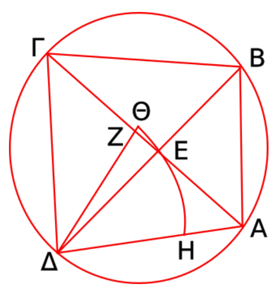
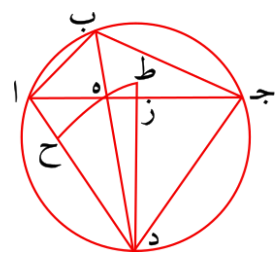
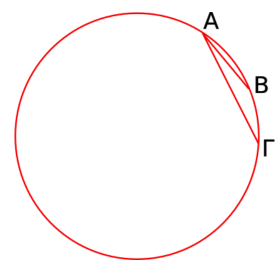
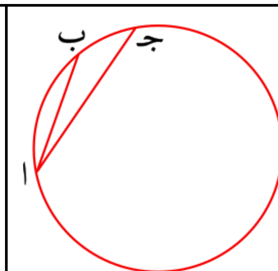
Greek and Arabic Diagram 5			Arabic Diagram 6	
			N/A	
Greek Diagram 6 and Arabic Diagram 7			Greek Diagram 7 and Arabic Diagram 8	
				

Table 8.2: Use of different ink colors in the diagrams of al-Ṭūsī's edition of the *Almagest*

In this witness to al-Ṭūsī's edition of the *Almagest*, diagrams original to the *Almagest* are drawn with red lines and black labels. Diagrams which al-Ṭūsī adds to the text are drawn conversely with black lines and red labels. In the example diagrams above, the sixth is an addition by al-Ṭūsī, lacking a counterpart in the Greek text, and is indeed drawn with the appropriate color scheme.

After completing his edition of the *Almagest*, al-Ṭūsī proceeded onwards to the *Elements*. He explains as much in the introduction to his edition of the latter text:

> "After I completed the edition of the *Almagest*, I thought to edit the book, *Elements of Geometry and Arithmetic*, attributed to Euclid of Tyre, briefly without fault. And it inquires into the proofs of its goals without tediousness. I added to it what was suitable from what benefitted me in the books of this science's scholars and what I discovered with my talent. And I distinguished what is found in the original book in the two copies of al-Ḥajjāj and Thābit from the additions to it, either with indications to these or with a difference of colors and their numerals."[38]

[38] Translated from p.2 of the Istanbul (1801-2) printing of the Arabic text:

As he did for the *Almagest*, in the introduction to the *Elements* the scholar expresses a desire for brevity and removing repetition, along with an allowance for similar kinds of additions to Euclid's text as those permitted for his edition of the *Almagest*. He notes again his use of different colors and numbers to distinguish between original material and later additions.

Studies on al-Ṭūsī's *Almagest* and *Elements* have acknowledged that these features are indeed present in the editions. Al-Ṭūsī does not take away from the broader structure of the original works – propositions and similar units of the works are maintained. But the texts themselves have been streamlined and condensed.

The extent to which his editions of the Middle Books follow this pattern will be discussed more thoroughly in the following chapter, but al-Ṭūsī's efforts with the *Almagest* and the *Elements* establish similar expectations for the third and final part of his curricular editing project.

4. Muḥyī al-Dīn al-Maghribī

4.1 His Scholarly Career

Muḥyī al-Dīn al-Maghribī (d. 682/1283) came from al-Andalus and previously worked in Syria under the Ayyūbids.[39] Compared to al-Ṭūsī, much less is known about al-Maghribī's education and early career. Extant sources on al-Maghribī do not report on these topics.

"وبعد فلما فرغت عن تحرير المجسطى رأيت ان احرّر كتاب اصول الهندسة والحساب المنسوب الى اقليدس الصوري بايجاز غير مخل واستقصى في ثبت مقاصده استقصاء غير ممل، واضيف اليه ما يليق به مما استفدته من كتب اهل هذا العلم واستنبطته بقريحتي، وافرز ما يوجد من اصل الكتاب في نسختي الحجاج وثابت عن المزيد عليه اما بالاشارة الى ذلك او باختلاف الوان الاشكال وارقامها."

Note that while there is a printed edition of the *Elements* attributed to al-Ṭūsī in *Kitāb Uṣūl li-Uqlīdis min taʾlif Khawājah Naṣīr al-Dīn al-Ṭūsī* (Rome: Typographia Medicea: 1594), this has been recognized to be a misattribution. That text was the work of a different and anonymous editor, also in the second half of the thirteenth century. The date of completion in the colophon, 698 / 1298, postdates al-Ṭūsī's death, as Sabra (1969) 18 points out. Al-Ṭūsī's own edition of the *Elements* has not yet been critically edited. As de Young (2008-9) 3 notes, the misattribution of the published treatise to Tusi has led to some incorrect claims in the secondary scholarship about al-Ṭūsī's work on the *Elements*.

[39] For more on Muḥyī al-Milla wa al-Dīn Yaḥyā Abū ʿAbdallāh ibn Muḥammad ibn Abī al-Shukr al-Maghribī al-Andalusī, see e.g. al-Fuwaṭī (1955) 115, Saliba (1983) 391-392, and Comes (2014).

Al-Maghribī was captured during Hülagü Khan's campaigns in Syria in 658/1260 and it was his expertise with the astral sciences which saved his life, according to the account which he himself told to Bar Hebraeus. Upon hearing about al-Maghribī's skill, Hülagü Khan sent him to the Maragha Observatory to take part in the work there.[40] Like al-Ṭūsī, al-Maghribī's work at Maragha involved making astronomical observations, writing texts, and teaching.

This can be seen in some of the texts which have come down to us. The *Adwār al-anwār* (675/1276–7), for example, is the zīj he produced out of his observation program at Maragha, which seems to have been run separately from al-Ṭūsī's. The *ʿUmdat al-ḥāsib wa-ghunyat al-ṭālib*, meanwhile, is a zīj which speaks to his teaching activities, as it was a work for students that was put together by one of al-Maghribī's pupils during study under the astronomer.

4.2 Rough Timeline of al-Maghribī's Editions

Al-Maghribī too was responsible for new editions of the *Elements* and the *Almagest*.[41] It is unclear whether al-Maghribī, like his colleague, edited the whole of the Middle Books as well, but modern scholarship has long been aware of his editions of two of the relevant texts, Theodosius's *Sphaerica* and Menelaus's *Spherics*.[42] He is more recently recognized to have produced an edition of Autolycus's *On the Moving Sphere*.[43] In addition, al-Maghribī produced an edition of Apollonius's *Conics* – while this treatise does not seem to have been part of the tradition of the Middle Books themselves, it speaks to his work editing ancient Greek mathematical treatises.

[40] Bar Hebraeus (1958) 280-281.
[41] For the *Elements* see discussion in Sabra (1969) 13ff. For the report of the *Almagest* see Flügel (1850) 387, 389.
[42] See Carra de Vaux (1891) for more on al-Maghribī's edition of Theodosius's *Sphaerica*. Rashed and Papadopoulos (2017) 15 note his edition of Menelaus's *Spherics* but it has not been edited.
[43] This had been misidentified in manuscript catalogues as the edition of al-Ṭūsī. See for example the witness in Chester Beatty Ar. 3035, whose catalogue entry can be found in Arberry (1955) 13. The copy of *On the Moving Sphere* in this manuscript does not name its editor, but shows stylistic similarity with other editions by al-Maghribī. See Nikfahm-Khubravan and Eshera (2019) 10-11 for some comments on al-Maghribī's edition.

Consideration of the manuscript evidence similarly allows for a rough dating of al-Maghribī's activities editing these works. In the case of his edition of the *Elements*, the oldest manuscript extant today is MS Bodleian Library Or 448, which was completed in Maragha in 659 / 1260-1.[44] As noted above, the year 1260 is also the year in which al-Maghribī was captured by Hülagü's forces, after which he was sent to the Maragha Observatory. Either his works had preceded him in making their way to the observatory, or his relocation prompted their use and copying. Either way, al-Maghribī must have first produced his edition of the *Elements* during his time with the Ayyubids, not during his time at Maragha. Similarly to al-Ṭūsī, he was a scholar who brought his already extant experience and scholarly productions with him to the observatory.

Al-Maghribī's edition of the *Almagest* is known from a report about one which he produced for Bar Hebraeus. Kâtip Çelebi does not give a date in which Bar Hebraeus requested an edition of the *Almagest* from al-Maghribī, nor a date for this edition's completion. However, such a request would necessarily have been made during al-Maghribī's career in Maragha, as this is where Bar Hebraeus interacted with him. The Syriac scholar may have sought an edition of the *Almagest* during his 1273 visit to Maragha in which he studied that particular text, but his first visit was as early as 1268 and he made subsequent visits as well.

For al-Maghribī's editions of the Middle Books texts, there is some information to be found in the manuscripts. For *On the Moving Sphere*, the colophon in Chester Beatty Ar. 3035 states that its witness was completed in 26 Rabīʿ I 669 (11 November 1270). For the *Sphaerica*, an ownership note in Mashhad Kitābkhāna-yi Markazī Astān-i Quds 5232 records the date as 680/1281. While these reports provide termini ante quem, they unfortunately reveal little about the order of editions, or at what stage in his career al-Maghribī worked on them.

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[44] Sabra (1969) 21.

4.3 Al-Maghribī on his *Elements*

Like al-Ṭūsī, al-Maghribī was well-acquainted with prior scholarship on the *Elements*. In the introduction to his own edition, he comments on past editions by Ibn Sīnā (d. 427/1037), Nīsābūrī, and Abū Jaʿfar al-Khāzin (d. 361/971). He expresses similar sentiments about how prior editions did not meet the expectations he would set for his own work, and hence he was setting out to produce something that would achieve those goals. Al-Maghribī specifies what these goals are as part of his introduction. His edition would clarify anything needing explanation, aim for brevity and cut out repetition, provide answers for doubts, and add whatever lemmas that are required by the propositions.[45]

In the case of the *Almagest*, al-Maghribī's edition is not known to be extant today. He is reported to have produced an edition on the request of Bar Hebraeus. His text comprised ten books, clearly an abbreviation of the original Greek's thirteen.[46] Further study of al-Maghribī's extant editions would be required to judge whether the scholar was, as a rule, more willing than al-Ṭūsī to change the structure of the original text, or whether this abbreviation rather might have been to address needs particular to Bar Hebraeus's request. His edition of *On the Moving Sphere* suggests it may be the former, since in it al-Maghribī adds two propositions between propositions 11 and 12.[47]

5. Teaching at Maragha: Al-Ṭūsī and al-Maghribī

While al-Ṭūsī and al-Maghribī presumably taught at various points in their careers, it is during their time at the Maragha Observatory that their teaching activities are indicated in the surviving sources. In general, sources speak of numerous students at the Maragha Observatory. Many of these were attached

[45] This introduction is discussed in Sabra (1969) 14-15.
[46] Flügel (1850) 387, 389. There exists a *Talkhīṣ al-majisṭī* by al-Maghribī that is extant: see Saliba (1983). In the preface to this al-Maghribī also mentions a summary of the *Almagest* he had produced titled *Khulāṣat al-majisṭī*, which has not been discovered. It is not clear if either of these treatises are the edition produced for Bar Hebraeus.
[47] Nikfahm-Khubravan and Eshera (2019).

to Nasir al-Din al-Ṭūsī, though their studies persisted after his death, as the report about Abaqa Khan funding them afterwards shows.[48]

Several of al-Ṭūsī's students are known by name, including the famous Quṭb al-Dīn al-Shīrāzī (d. 710/1311), who studied astronomy under him.[49] An impression of some of the texts al-Shīrāzī must have read under al-Ṭūsī can be gleaned from his subsequent works as well as manuscript evidence. Al-Ṭūsī's edition of the *Almagest* was evidently one of these works studied, and the multiple early manuscript copies in al-Shīrāzī's hand or copied from his hand show that al-Shīrāzī contributed to the broader circulation of this edition after he left Maragha.[50] Al-Shīrāzī also studied the *Tadhkira* under his teacher: a colophon to a manuscript of the *Tadhkira* copied from al-Shīrāzī's own copy reports that the scholar had read it back to al-Ṭūsī. The *Tadhkira* proved to be an important influence on al-Shīrāzī's subsequent astronomical works.[51]

Manuscript notes also survive to indicate that al-Ṭūsī was teaching several of his other mathematical texts. Notes and the colophon on one witness of his *Tadhkira*, for instance, show that he was teaching this text at Maragha alongside his other scholarly activity.[52] Since the codex in question also contains al-Ṭūsī's edition of the *Data*, it is not impossible that he was actively teaching this text at the time as well.[53]

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[48] Sayılı (1960) 219. Bar Hebraeus's *Chronography* reports al-Ṭūsī allotting stipends to the teachers and students under him during his lifetime: see Bar Hebraeus (1932) 451.
[49] Al-Fuwaṭī (1955) 440-441. While in Maragha, al-Shīrāzī also benefited from studies with the philosopher Najm al-Dīn al-Kātibī and the astronomer Muʾayyad al-Dīn al-ʿUrḍī.
[50] E.g. Chester Beatty Library, Ar. 3637 (691/1292), Nuruosmaniye Kütüphanesi 2941 (684/1285), and Bibliothèque nationale de France ar. 2485 (9th / 15th century).
[51] On al-Shīrāzī's authorized copy of the *Tadhkira*, see Ragep (1993) 72-73 and 78. On the influence of the *Tadhkira* on his works, see *ibid*., p. 57. On al-Ṭūsī's intentions for the *Tadhkira*'s usefulness to students and nonspecialists, see *ibid.*, p. 37-38 and 56.
[52] Sidoli and Isahaya (2019) 91. The manuscript is Tehran, Sipahsalar 4727, completed in 671/1272.
[53] Sidoli and Isahaya (2019) have put forth the hypothesis that the version of the *Data* in this particular manuscript was an earlier draft compared to a more polished one that can be found in codices related to Haci Selim Ağa Library 743 (671/1272). The hypothesized earlier draft contains material in the marginalia that appear worked more thoroughly into the text of the hypothesized later draft. This suggests that the processes of teaching and improving

5.1 Studying at Maragha: Bar Hebraeus

The Syriac scholar Bar Hebraeus (d. 1286 CE) writes about several visits he made to Maragha. This section will look at him in more detail as an example of an individual who plausibly interacted with a range of subjects in the astral sciences at Maragha: spherical geometry, *hayʾa* (treatises on cosmographical subjects, especially geometrical models and configuration of the universe), and *zījes.*

Bar Hebraeus pursued the astral sciences in several ways during his time in Maragha. Part of this involved work with the curriculum of the *Elements*, the Middle Books, and the *Almagest*. Bar Hebraeus writes that his first visit to Maragha in 1268 included work with Euclid and his second in 1273 with the *Almagest*.[54] Scholars have previously interpreted the Syriac verb used in connection with these texts to mean that Bar Hebraeus was involved in teaching or commenting on the *Elements* and the *Almagest* at the observatory. Takahashi has more recently put forth the suggestion that the verb in question should be interpreted as "studied."[55]

Bar Hebraeus's writings furthermore demonstrate familiarity with al-Ṭūsī, though it is not certain whether he studied with him directly.[56] Regardless, the Syriac scholar worked with the Middle Books astronomical curriculum through al-Ṭūsī's editions of many of the texts. This is suggested by two manuscripts from this curriculum, one of which is plausibly connected with Bar Hebraeus and the other of which names him in an ownership note.

this text were intertwined – al-Ṭūsī was plausibly collecting and evaluating new material in the course of teaching the text.

54 In his *Ecclesiastical Chronicle*: see Abbeloos and Lamy (1877) 441-443.

55 Takahashi (2005) 84. The verb is the Syriac "*šrā*" – past scholars have interpreted this to mean Bar Hebraeus taught or orally explained Euclid and the *Almagest*. Takahashi compares its use to that of its Arabic equivalent "*ḥalla*" which Bar Hebraeus uses to mean "study". For an overview of how these passages in Bar Hebraeus have been interpreted, see Borbone (2017) 125-126.

56 See Bar Hebraeus's comments on al-Ṭūsī in Budge (1932) 451-452. See also the similarities between Bar Hebraeus's *Ascent of the Mind* and al-Ṭūsī's *hayʾa* treatises, discussed below.

The first of these is London, British Library, Add. 23387, which contains al-Ṭūsī's edition of the *Elements*. Syriac and Garshuni notes written in a thirteenth-century Western *serto* appear in this manuscript. A recent study has compared the handwriting of these notes with a sample of Bar Hebraeus's own writing and argues it is plausible these notes come from the Syriac scholar.[57] The colophon on folio 216b declares that the manuscript in question was completed on 15 Rabīʿ II 656 (21 April 1258). It is not, therefore, a manuscript which was written during Bar Hebraeus's study of the *Elements* in Maragha, since this occurred ten years later. If the Syriac scholar used it during his time at the observatory, he acquired an existing codex for his studies.[58]

The second of these manuscripts is Istanbul, Hacı Selim Ağa 743, which contains al-Ṭūsī's edition of the Middle Books. A Syriac ownership note written in a Western *serto* states that the codex belonged to "Gregory, the lowly maphrian" with a year that corresponds to 1280–1 CE.[59] Several of the texts in the manuscript have dates of completion, ranging from 671-678 / 1272-1279.[60] It is unclear precisely when this manuscript came into Bar Hebraeus's possession. He may have acquired several initial treatises during his second visit to Maragha, and added to the compilation manuscript over time. He may have acquired it during one of his later visits to Maragha, such as the one in 1279. Or the manuscript may have come into his possession elsewhere. He certainly owned it by 1281 at the latest.[61]

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[57] Borbone (2017) 129-131.
[58] The manuscript additionally would have had to have been copied outside of any circles associated with al-Ṭūsī, since in the colophon the scribe seems to have erroneously believed that al-Ṭūsī had already died.
[59] The ownership note appears on f. 136r; see Takahashi (2014) 322. The contents of the manuscript are the fifteen treatises which comprised al-Ṭūsī's edition of the Middle Books: Theodosius's *Sphaerica*, Theodosius's *Nights and Days*, Autolycus's *Risings and Settings*, Hypsicles's *Anaphoricus*, Aristarchus's *Sizes and Distances*, pseudo-Archimedes's *Lemmata*, Thābit's *Assumptions*, the Banū Mūsā's *Book of Knowledge*, Archimedes's *Sphere and Cylinder*, Menelaus's *Spherics*, Autolycus's *On the Moving Sphere*, Euclid's *Data*, Theodosius's *On Habitations*, Euclid's *Optics*, and Euclid's *Phaenomena*.
[60] Euclid's *Data*: 14 Rabīʿ II 671 H, Euclid's *Optics*: Rabīʿ II 671 H, Autolycus's *Moving Sphere*: 4 Muḥarram 672 H, Menelaus's *Spherics*: 9 Jumadā 678 H, Theodosius *On Habitations*: 671 H. See Krause (1936) 499-504.
[61] For his visit in 1279, see Abbeloos and Lamy (1877) 447-450. For the suggestion that Bar Hebraeus personally transcribed the Arabic of the editions of Archimedes in this manuscript, see Sayılı (1956) 11 and Borbone (2017) 130-131.

Bar Hebraeus's study of this astronomical curriculum can also be seen in a third manuscript: Mashhad, Kitābkhāna-yi Markazī Astān-i Quds 5232. The codex's first folio is marked by an ownership statement that matches the ownership statement in the prior manuscript. Unlike the prior manuscript, however, the two Middle Books texts contained in this codex (the *Sphaerica* and *Moving Sphere*) are the editions of al-Maghribī.[62]

There is further evidence for Bar Hebraeus having studied the *Almagest* at Maragha, though not in the recension by al-Ṭūsī. As noted above, Kâtip Çelebi reports that the Syriac scholar requested a new edition of the *Almagest* from al-Maghribī.[63]

Outside of this astronomical curriculum, hints of other texts Bar Hebraeus may have encountered at Maragha are offered by the Syriac scholar's own works. His *Ascent of the Mind*, completed in 1279, is a handbook of astronomy that bears similarity to al-Ṭūsī's *hayʾa* treatises. Some scholars have pointed to al-Ṭūsī's *Tadhkira* as its model based on its agreements in structure and values. More recently, al-Ṭūsī's *Zubdat al-idrāk fī hayʾat al-aflāk* (undated) has been raised as a possible model for Bar Hebraeus's text.[64] The *Zubdat al-idrāk* is a short and simplified *hayʾa* treatise that is intended to epitomize works on the subject.[65] While this shorter work appears to have had little lasting influence compared to al-Ṭūsī's other treatises, it is possible that it was being used as an elementary teaching text during Bar Hebraeus's time in Maragha. Lastly, Bar Hebraeus reports that he wrote a book on *zījes* for beginners in Syriac.[66] This work has not been found, so it is not certain which *zījes* Bar Hebraeus may have drawn upon for it. In any case,

[62] For a description of this manuscript, see Nikfahm-Khubravan and Eshera (2019) 48.
[63] Flügel (1850) 387, 389.
[64] For its relation to the *Tadhkira*, see Nau (1899) vii. For the possible connection with the *Zubdat*, see Takahashi (2011), p. 486-487.
[65] Al-Ṭūsī may have intended the *Zubdat al-idrāk* as an abridgement of his *Tadhkira*, but the former text has received very little study and so its relationship to the scholar's other treatises is unclear: see Ragep (1993) 66-67. One difference between the *Zubdat al-idrāk* and the *Tadhkira* is the former's avoidance of criticisms of the Ptolemaic system. The idea that Bar Hebraeus preferred it as a model because of this closer adherence to Ptolemy has been raised by Takahashi (2011) 487.
[66] Budge (1932) xxxiii.

the (former) existence of this treatise raises the possibility that his astronomical studies at Maragha included various *zīj* texts.

Chapter 9

The Edition of Naṣīr al-Dīn al-Ṭūsī

1. Introduction

The Taḥrīr al-Mutawassiṭāt of al-Ṭūsī presents a form of the Middle Books where it is possible to say with certainty that a single editor set out to shape the whole of the curriculum as a unit. Chapter 2, in addressing the Greek manuscript tradition of the Little Astronomy, ascribed many of the deliberate alterations to editors with didactic motivations, but these individuals were multiple and anonymous.[1] The alterations discussed in chapter 4 present a similar scenario, where alterations could have been introduced by any of the multiple translators and correctors;[2] further, they may diverge from the Greek because of the particular state of the texts in Greek at the time of their translation.

The case of al-Ṭūsī's edition is less obscured. The edition was produced in the thirteenth century, and it is possible to have an idea of what several of the texts preceding his edition looked like because the thirteenth century is the period from which several important Arabic manuscripts survive. The educational context of these texts and al-Ṭūsī's work within that sphere are clear from his own comment, brief though it is, about the Middle Books as an educational arrangement between the *Elements* and the *Almagest*. And even as the editor rewrites each proposition in his own contemporary mathematical style, he takes care to separate out most of the additions he himself makes to the text.

[1] While it might be tempting to ascribe editorial efforts with the curriculum to a historical individual like Theon of Alexandria, ultimately there is little evidence for such suggestions beyond his being a known mathematical and astronomical scholar of the period.
[2] And later editors of the texts, endeavor though we may to access the text in a form close to its ninth century version.

2. Overview of Evidence

While al-Ṭūsī's edition of the Middle Books is available for study in printed form, it should be recognized that it has largely not been critically edited. The only critical editions are for his renditions of Euclid's *Phaenomena*, in Sulaymān (1996), and of *On the Measurement of the Circle*, in van Lit (2012). Instead the full edition of the curriculum is printed in Hyderabad (1939-40), as previously noted; this is based on at least three manuscripts but lacks a critical apparatus. There is also a facsimile printing of the manuscript Tabriz 3484 (late 7th-early 8th / late 13th-early 14th c), which serves as a useful comparison to the Hyderabad edition.[3]

Before delving into the major sections of this chapter, it should be noted that while al-Ṭūsī's Taḥrīr al-Mutawassiṭāt has not been studied as a unit, several studies hae been done on works within the grouping.[4] These studies, taken together, present a (nearly) consistent picture of al-Ṭūsī as an editor.[5] The scholar after all presents not just his own copy (نسخة) of the Middle Books, but a deliberately produced edition (تحرير). As part of his project, he does intervene liberally in the text. Where many of the original Arabic translations followed the Greek quite closely, echoing its phrasing and following all the mathematical Greek tendencies towards features like repetition, this is not the aim of al-Ṭūsī. Rather, the texts which have been studied all show his tendency towards conciseness and the elimination of repetition. Since al-Ṭūsī has expressed such preferences in his other editions, as the last chapter discussed,

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[3] For the facsimile, see Aghayanī-Chavoshī (2005).
[4] On the edition of the *Data*, see Thaer (1942) and Sidoli and Isahaya (2019). On the edition of the *Sphaerica*, see Sidoli and Kusuba (2008). On the edition of *Sizes and Distances*, see Berggren and Sidoli (2007). On the edition of Thābit's *Assumptions*, see Dold-Samplonius (1996). On the edition of the *Sphere and Cylinder* and *Measurement of the Circle*, see Lorch (1989). On latter, see also van Lit (2012). On the edition of the *Book of Knowledge*, see Rashed (1996).
[5] The two exceptions are the edition of Thābit's *Assumptions* and *On Measurement of the Circle*, which will be discussed.

to find similar efforts in the edition of the Middle Books is only to be expected. Examples of al-Ṭūsī's style will be seen in the section below on substitution of proof.

While al-Ṭūsī is willing to intervene in the texts, he does so only in particular ways. He will rewrite the text at the local and stylistic level, but he strives to remain faithful to his source texts on the global and structural level. As will be seen below, he refrains from rearranging propositions – there are perhaps only two exceptions, in the case of Thābit's *Assumptions* and Archimedes' *On the Measurement of the Circle*.[6] He also generally refrains from adding or suppressing propositions, maintaining the larger structure of the sources available to him. Al-Ṭūsī takes care to note the number of propositions in each text and where he found that number differed in different copies. The sections below will look further at how the curriculum's texts referenced each other and how a consistent numbering scheme would facilitate this.

In his edition, al-Ṭūsī also clearly indicates what portions were his own addition versus what was originally the content of these works. The editor most frequently demarcates his own contributions with the opening "I say…" (أقول). In cases where he is drawing from another scholar's commentary, as in Archimedes's *Lemmata*, he indicates this as well – in the *Lemmata* multiple additions start with "the teacher said…" (قال الأستاذ). These comments consistently appear after the ending QED statement of the proposition ("وذلك ما أردناه" in al-Ṭūsī's editions) and do not interrupt the flow or logic of the proposition itself.

Overall, then, the secondary scholarship agrees in depicting al-Ṭūsī as an editor who balanced streamlining the texts with maintaining their larger structure and also as an editor who carefully delimited

[6] These texts serve as a significant enough break from al-Ṭūsī's usual editing practice that they will be discussed in the section below on change in order of propositions.

his own comments from what he saw as original to the works. Al-Ṭūsī's additions vary in character, from providing information about what is found in other copies of the text to offering mathematical comments that provide background or address gaps in the text. These comments will be discussed further below.

3. Summary of Deliberate Alterations in al-Ṭūsī's Edition

3.1 Al-Ṭūsī on his Editing Project

As part of his project, al-Ṭūsī gave each of the works in the Middle Books a (usually short) preface. At minimum, this preface declares the number of books and propositions in the work. Often it notes which translator and/or corrector al-Ṭūsī understood to have been responsible for the manuscripts he had in hand. And, in several cases, the preface offers further details on the materials al-Ṭūsī had available to him and on how he sought to craft a coherent edition from imperfect sources.

While Euclid's *Data* appears at the head of the grouping, its preface is lacking in detail. It is instead the final work which al-Ṭūsī edited, Menelaus's *Spherics*, whose preface mentions the scholar's decision to edit the whole of the Middle Books. This has already been discussed, but the full preface speaks also to the difficulties al-Ṭūsī encountered when working with this text, and to the varied engagement it had from different scholars:

> I say (praise be to God and praise on him in what is proper for him and prayers upon Muhammad and his family) that I was wanting to edit the books called the Middle (Books), I mean the books whose matters are in the middle in the educational arrangement between the book of *Elements* by Euclid and the book of the *Almagest* by Ptolemy. So when I arrived at the book of Menelaus on *Spherical Figures*, I found for it many different copies without reception of the issues and failed corrections for them like the correction of Māhānī and Abū al-Faḍl Aḥmad ibn Abī Saʿd al-Harawī and others, some of them not complete and some of them not correct. So I remained baffled in the explanation of some of the issues of the book until I came across the correction of al-Amīr Abū Naṣr Manṣūr ibn ʿIrāq, God have mercy on him. So it became clear to me from what was expected in it. So I edited the book to the extent of my capability and my success is only through God...[7]

[7] Hyderabad (1939-40) *Kitāb Mānalāʾus* 2:

The *Spherics* is, itself, a mathematically challenging text, and errors introduced from misunderstandings or poor copying only exacerbate this. But it was not the only text which al-Ṭūsī highlights as a challenge in the course of his project. Another one of the works added to the Middle Books, Archimedes's *On the Sphere and Cylinder*, also caused him difficulties:

> That I have been in search of the study of the mentioned issues in the *Book of the Sphere and Cylinder* by Archimedes for a long time, for the many needs of it in the noble subject of geometry, until I arrived at the famous copy of the book which Thābit ibn Qurra corrected. And this lacks some axioms from insufficient understanding by its transmitters to Arabic concerning its comprehension. And its weakness is because of this from the transmission. So I examined it and the notebook was faulty due to ignorance of its copyist, so I remedied it to the extent possible. I endeavored in investigation of the mentioned affairs in it until I completed the second book, and I discovered what Archimedes neglected from the introduction based on some of his demands in it. So I was confused by it and my desire for its acquisition increased. Then I succeeded with an old notebook in which was the commentary of Eutocius of Ascalon on the problems of this book, which Isḥāq ibn Ḥunayn translated to Arabic in a discerning translation. And in that notebook also was the text of the book from its beginning to the end of the fourteenth figure in the first book, also from the translation of Isḥāq, and what Eutocius mentioned in the course of his commentary on the text of the book was in accordance with this copy, so I found from this notebook what I had required. And I decided to edit the book according to the arrangement and to summarize its meanings and to explain its axioms which rather become clear through the geometrical principles and to present the principles (which are) needed by these in it. And I mention what figures in it (are) from what the commentary of Eutocius presented or (from what) I made use of from other books of the people of this craft. And I distinguished between what is in the main text of the book and what is not in it via indication of this. And I established the number of the figures according to their collection in the two accounts: that the figures of the first book in the copy of Thābit are 48 and in the copy of Isḥāq are 43. So I did this and I attached to its end the book of Archimedes on the *Measurement of the Circle*, for (that work) was based on some of the axioms mentioned in this book.[8]

"اقول بعد حمد الله والثناء عليه بما يليق به والصلوة على محمد وآله اني كنت اريد ان احرر الكتب الموسومة بالمتوسطات اعنى الكتب التى من شأنها ان تتوسط في الترتيب التعليمي بين كتاب الاصول لأقليدس وبين كتاب المجسطى لبطلميوس فلما وصلت الى كتاب مانالاؤس في الاشكال الكرية وجدت له نسخا كثيرة مختلفة غير محصلة المسائل واصلاحات لها نحبطة كاصلاح الماهانى وابى الفضل احمد بن ابي سعد الهروي وغيرهما بعضها غير تام وبعضها غير صحيح فبقيت متحيرا في ايضاح بعض مسائل الكتاب الى ان عثرت على اصلاح الامير ابي نصر منصور بن عراق رحمة الله عليه فاتضح لي منه ما كنت متوقعا فيه فحررت الكتاب بقدر استطاعتي وما توفيقي الا بالله عليه أتوكل واليه انيب"

[8] Hyderabad (1939-40) *Kitāb al-kurah wa-al-usṭuwānah* 2-3:

"رب انعمت فزد اقول بعد تحميد الله وتمجيده والصلاة على محمد وآله المصطفين من عبيده اني كنت في طلب الوقوف على بعض المسائل المذكورة في كتاب الكرة والاسطوانة لارشميدس زمانا طويلا لكثرة الاحتياج اليه في المطالب الشريفة الهندسية الى ان وقعت الى النسخة المشهورة من الكتاب التى اصلحها ثابت بن قرة وهي التي سقط عنها بعض المصادرات لقصور فهم ناقله الى العربية عن ادراكه وعجزه بسبب ذلك عن النقل فطالعتها وكان الدفتر سقيما لجهل ناسخه فسددته بقدر الامكان وجهدت في تحقيق المسائل المذكورة فيه الى ان انتهيت الى المقالة الثانية وعثرت على ما اهمله ارشميدس من

It is perhaps surprising to read that al-Ṭūsī encountered so many difficulties working with texts that were – at least sometimes – used as part of a widely-transmitted curriculum. The cases of the *Spherics* and *On the Sphere and Cylinder* may possibly be excused because these texts did not number among those which formed the core of the curriculum, but rather they were sometimes added to it when scholars deemed them useful. But in the preface to the *Phaenomena*, a text solidly numbering among the Middle Books, we read the following:

> The editor of this book says… nothing of this book came to me except a highly defective copy, most of it (full of) misspellings and distortions. Since it was not possible to study anything from it except with the utmost strain, and the commentary on it by al-Tabrīzī [sic][9] was also very poor, so I redoubled the study of both of them and I edited anything from the book that was contradictory to me according to what I envisioned. So if it is not in agreement with the book, this is the reason for it. And it is my intention that I repair its faults if I come across a correct copy, inshallah, and he is the guardian of success.[10]

So despite the Middle Books being a recognized grouping in astronomical education, it should not be taken for granted that this circumstance encouraged the transmission of all these texts equally as well or as widely. For whatever reason, al-Ṭūsī found copies of these texts in varying levels of coherence. Furthermore, some texts which were more tangentially connected to the grouping – as al-Nayrīzī's

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المقدمات مع بناء بعض مطالبه عليه فتحيرت فيه وزاد حرصي على تحصيله فظفرت بدفتر عتيق فيه شرح اوطوقيوس للعسقلانى لمشكلات هذا الكتاب الذى نقله اسحق بن حنين الى العربية نقلا على بصيرة وكان في ذلك الدفتر ايضا متن الكتاب من صدره الى آخر الشكل الرابع عشر من المقالة الاولى ايضا من نقل اسحق وكان ما يذكره اوطوقيوس في اثناء شرحه من متن الكتاب مطابقا لتلك النسخة فوجدت من ذلك الدفتر ما كنت اطلبه ورأيت ان احرر الكتاب على الترتيب والخص معانيه وابين مصادراته التى انما تتبين بالاصول الهندسية واورد المقدمات المحتاج اليها فيه واذكر شرح ما اشكل منه مما اورده الشارح اوطوقيوس او استفدته من سائر كتب اهل هذه الصناعة واميز بين ما هو من متن الكتب وبين ما ليس منه بالاشارة الى ذلك واثبت اعداد الاشكال على حاشتها بالروايتين فان اشكال المقالة الاولى فى نسخة ثابت ثمانية واربعون وفي نسخة اسحاق ثلاثة واربعون ففعلت ذلك والحقت بآخرها مقالة ارشميدس في تكسير الدائرة فانها كانت مبنية على بعض المصادرات المذكورة في هذا الكتاب"

[9] Note that the "al-Tabrīzī" above and also cited elsewhere in the *Phaenomena* should be understood to be an error for the famous mathematician al-Nayrīzī (d. 310/922). The error arises because the names are indistinguishable if the Arabic is left undotted (النيريزي vs. التبريزي), as often happens in the manuscripts.

[10] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 2:
"يقول محرر هذا الكتاب... لم يقع الي من الكتاب غير نسخة في غاية السقم اكثرها من التصحيف والتحريف بحيث لم يكن يمكن الوقوف على شيء منه الا بجهد كثير وشرح له التبريزى سقيم ايضا جدا فأكثرت النظر فيهما وحررت ما ترا آى لى من الكتاب على ما تصورته فان لم يكن مطابقا الكتاب فالسبب فيه ذلك وفي نيتى ان اصلح خلله اذا عثرت على نسخة صحيحة ان شاء الله وهوولى التوفيق."

commentary on the *Phaenomena* may have been – ultimately did not survive past al-Ṭūsī's new edition of the curriculum, which circulated widely and likely superseded the earlier texts.[11]

These prefaces also make clear that – despite the imperfect witnesses some of his manuscripts contained – al-Ṭūsī did have for many of the Middle Books multiple manuscript copies at his disposal. Several other texts contain references to work with multiple copies. In the comment to his *Data* 64, he writes that the proposition "found in the copies is thus" (الموجود فى النسخ هكذا).[12] Several times in the *Sphaerica* al-Ṭūsī notes differences that are found "in some copies" (في بعض النسخ).[13] In *Phaenomena* proposition 8, he mentions material found "in a copy" (في بعض النسخة).[14] The preface of *Nights and Days* acknowledges how the title differs between "On Days and Nights" (في الأيام والليالي) and "On the Night and Day" (في الليل والنهار) "in some copies" (في بعض نسخ).[15] And for Thābit's *Assumptions*, the editor notes how proposition counts differed "in some copies."[16] So for seven of the Middle Books which al-Ṭūsī edited, there is clear evidence that he had more than one exemplar from which to work. While there are no other such hints for the other seven Middle Books he edited, we can assume that if multiple copies were available to him, he certainly used them.

Further, the prefaces show that al-Ṭūsī's editorial process involved not just consulting multiple witnesses for these texts, but also available commentaries on them. The above passages already show how

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[11] Granted, there are still extant manuscripts of Eutocius's commentary, though none are complete; see Lorch (1989) 106.
[12] Hyderabad (1939-40) *Kitāb al-Muʿṭīyāt* 27.
[13] See Hyderabad (1939-40) *Kitāb al-ukar* 2, 19, and 48 for the following. The *Sphaerica* preface notes that "in some copies" (في بعض النسخ) a proposition is missing. The comments to II.prop.12 and III.prop.11 also reference material found "in some copies."
[14] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 12. Admittedly, it is not immediately clear if al-Ṭūsī added this material or if he found it already added in his source. He does not head the material with "I say" as he does for the majority of his other additions.
[15] Hyderabad (1939-40) *Kitāb al-ayyām wa-al-layālī* 2.
[16] Hyderabad (1939-40) *Kitāb al-mafrūḍāt* 2.

he drew upon al-Nayrīzī's commentary on the *Phaenomena* and the translated commentary of Eutocius on the *Sphere and Cylinder*. In Archimedes's *Lemmata* the scholar relies on the work of "the distinguished teacher" al-Nasawi, to the extent that where in his other MB editions al-Ṭūsī includes his own statements with "I say," in the *Lemmata* instead we find "the teacher said."

3.2 Concordances of Propositions

As was done for chapter 4, the following will present concordances of the propositions according to al-Ṭūsī versus the earlier Arabic versions (and the Greek ones before them). Proposition numbers used in this chapter will be those from al-Ṭūsī's edition, except where indicated otherwise.

Euclid's *Data*

The proposition arrangement of al-Ṭūsī's edition of the Data agrees with that of the Kraus MS. It has a total of 95 propositions, an increase of four from Seray Ahmet III 3464's 91 propositions because of the divisions of multi-part propositions into separate ones.

Theodosius's *Sphaerica*

Al-Ṭūsī's edition of the *Sphaerica* is similarly quite close structurally to its predecessors. The one difference is in Book II, where the manuscripts Seray Ahmet III 3464 and Paris hebr. 1101 have 22 propositions and al-Ṭūsī has 23 because he has divided their proposition 11 into two separate propositions. Al-Ṭūsī's proposition count agrees with that in Priv. lib. M. Nabī Khān, but this is because though the latter agrees with the other two manuscripts in not dividing proposition 11, it still numbers the following proposition as 13 and skips the numeral 12 entirely.

Autolycus's *On the Moving Sphere*

Comparing al-Ṭūsī's edition of *On the Moving Sphere* with the earlier version in Bodleian Hunt. 237 and with the original Greek shows full structural agreement between the three versions of the text.

Euclid's *Optics*

A comparison of the proposition arrangement of al-Ṭūsī's *Optics* with the earlier version of the *Optics* according to Kheirandish shows full structural agreement in the case of the propositions. Al-Ṭūsī does however rearrange and add some preliminary material in the definitions.

Euclid's *Phaenomena*

There is more significant structural disagreement between al-Ṭūsī's edition of the *Phaenomena*, the version presented in Leiden Or. 1031, and the Greek. Where the Greek recension B had a total of 18 propositions (according to Menge) and Leiden Or. 1031 had 20 propositions, al-Ṭūsī's edition comes to a total of 23. Furthermore, he notes in the preface that some manuscripts he encountered had up to 25.[17] Al-Ṭūsī's edition is increased by three from Leiden or. 1031's version through division of propositions: he presents Leiden or. 1031's proposition 2 as his propositions 2, 3, and 4. He also presents Leiden or. 1031's proposition 20 as his propositions 22 and 23. But there are other structural differences between the two versions which cancel out in the proposition count. The material which Leiden or. 1031 labels as proposition 16 (which is an alternate proof of the preceding proposition in the Greek transmission) does not make it into al-Ṭūsī's text. Meanwhile, al-Ṭūsī's proposition 17 is not present in Leiden or. 1031's text. It is not, however, an original contribution by the editor – it can be recognized as having its source in the Greek tradition. In recension A, this material was a lemma in the text; in recension B, it was a scholium.

[17] Since al-Ṭūsī presents the text with 23 propositions, it is not immediately clear what would have resulted in some manuscripts having 25, whether this was through division of other propositions or through the addition of material. Perhaps this was caused by the two Greek recensions intermingling in the Arabic tradition – we have already seen material from the A recension being incorporated into a version that otherwise is largely based on the B recension.

𝕋	**L**	𝔾		𝕋	**L**	𝔾		𝕋	**L**	𝔾
intro	intro	intro		9	7	7		17		(*)[18]
1	1	1		10	8	8		18	15	14[19]
2	2	2		11	9	9			16	14[20]
3	2	2[21]			10	10[22]		19	17	15
4[23]	2	2[24]		12	10	10[25]		20	18	16
5	3	3		13	11	11		21	19	17
6	4	4		14	12	12		22	20	18
7	5	5		15	13	13		23	20	18
8	6	6		16	14	14				

Table 9.1: Concordance of propositions for the *Phaenomena*.
𝕋 = al-Ṭūsī's edition, L = Leiden or. 1031, and 𝔾 = the Greek according to Menge (1916).

<u>Theodosius's *On Habitations*</u>

There are no significant structural differences between al-Ṭūsī's edition of *On Habitations* and the earlier Arabic or Greek versions.

<u>Theodosius's *On Days and Nights*</u>

The propositions of al-Ṭūsī's edition of *On Days and Nights* agree structurally with the earlier Arabic translation and the Greek text in its first book. The definitions agree with the earlier Arabic version as well. It is in the second book, where the earlier Arabic and Greek texts saw more structural disagreement, that this also happens between al-Ṭūsī's edition and its extant predecessors. Like the earlier

[18] This material is a scholium in the B recension and a lemma in the A recension.
[19] This is part two of the proposition.
[20] This is the alternate proof to the proposition.
[21] This is part three of the proposition.
[22] The first part of Leiden or. 1031's proposition 10 corresponds to the proof in the B recension.
[23] This "proposition" is the fourth case of the Greek proposition 2. Note that Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 8 and Sulaymān (1996) 74 do not number this "proposition" and instead silently skip the numeral 4 and continue on to label the next proposition as 5. However the manuscript Tabriz 3484, p. 126, shows that this material did sometimes receive the numeral 4.
[24] This is part four of the proposition.
[25] The second part of Leiden or. 1031's proposition 10, and al-Ṭūsī's edition, corresponds to the proof in the A recension.

Arabic translation partially edited by Kunitzsch and Lorch, book 2 of al-Ṭūsī's text is in 21 propositions. But a comparison of enunciations between the two shows that it seems to lack the earlier translation's proposition II.16 and that it seems to add a proposition II.17 that does not correspond with the earlier material. Future study of the Arabic *On Days and Nights* when its full text is available would be worthwhile.

𝕋	**K&L**	𝔾		𝕋	**K&L**	𝔾		𝕋	**K&L**	𝔾
Book II										
1	1	1		9	9	9		15	15	14
2	2	2				10			16	
3	3	3		10	10			16	17	15
4	4	4		11	11	11		17		
5	5	5				12		18	18	16
6	6	6		12	12			19	19	17
7	7	7		13	13			20	20	18
8	8	8		14	14			21	21	19

Table 9.2: Concordance of propositions for *On Days and Nights* Book II.
𝕋 = al-Ṭūsī's edition, K&L = Kunitzsch and Lorch (2011), and 𝔾 = the Greek according to Fecht (1927).

<u>Aristarchus's *On Sizes and Distances*</u>

There are no significant structural differences between al-Ṭūsī's edition of *On Sizes and Distances* and the earlier Arabic translation – it maintains the seventeenth proposition of the Arabic which is not present in the Greek. Where the Kraus MS and Columbia Or. 45 disagree on whether the Greek proposition 8 should be merged with the proposition before or after it, al-Ṭūsī follows the Kraus MS in merging it with the proposition before.

<u>Autolycus's *On Risings and Settings*</u>

Al-Ṭūsī's edition of *On Risings and Settings* has some structural disagreements with the earlier translation preserved in Leiden or. 1031 because of how al-Ṭūsī's edition divides some of the propositions. In Book I he presents 15 propositions (two more because of two divisions), and in book II he presents 20 (one more because of one division). His total of 36 propositions is similarly higher than the total of 33 recorded in Bodleian Thurston 11's list. All three versions, however, contain the same definitions. The concordance is laid out below:

𝕋	**L**	𝔾		𝕋	**L**	𝔾
Book I						
1	1	1		9	7	7
2	2	2		10	8	8
3	3	3		11	9	9
4	4	4		12	10	10
5	5	4[26]		13	11	11
6	5	4[27]		14	12	12
7	(*)[28]	5		15	13	13
8	6	6				

𝕋	**L**	𝔾		𝕋	**L**	𝔾
Book II						
1	1	1		12	11	
2	2	2		13	12	11
3	3	3		14	13	12
4	4	4		15	14	
5	5	5		16	15	13
6	6	6		17	16	14
7	7	7		18	17	15
8	8	8		19	18	16
9	9	9		20	19	17
10	10	10		21	20	18
11	10[29]					

Table 9.3: Concordance of propositions for *On Risings and Settings*.
𝕋 = al-Ṭūsī's edition, L = Leiden or. 1031, and 𝔾 = the Greek according to Mogenet (1950).

<u>Hypsicles's *Anaphoricus*</u>

Al-Ṭūsī presents overall the same material for the *Anaphoricus*, but conceives of it differently than the earlier Arabic translation did. Rather than claiming the parts of the text as five propositions, as was seen in chapter 4, he writes that the text "comprises three parts: lemmas and starting-point and two propositions" (يشتمل على ثلاث مقدمات وصدر وشكلين).[30] Al-Ṭūsī's lemmas are three in number and correspond

[26] This is case two of the proposition.
[27] This is case three of the proposition.
[28] This is not numbered as a separate proposition in Leiden or. 1031, but it is preceded by the proposition before it ending with the usual QED.
[29] This is case two of the proposition.
[30] Hyderabad (1939-40) *Kitāb fī al-maṭāliʿ* 2.

to the first three propositions according to the Arabic translation; his two propositions in the latter part correspond with the final two propositions.

3.3 Other Deliberate Alterations

As has already been indicated, in his editions al-Ṭūsī took care to separate out his own comments from the texts of the Middle Books. The following table gives a general overview of the deliberate alterations that can be found within what is presented as the main text. Alterations which occur within al-Ṭūsī's own comments follow in the table after. For both tables, propositions are numbered according to al-Ṭūsī's edition. The alterations are identified in comparison with the earlier Arabic versions.

Data				Sph. I		II	III	MS	Opt.			Ph.	Hab.	D&N I	II	S&D	R&S I	II	Ana.
d.1	16	46	76	d.1	20	d.1	1	d.1	intro	20	50	intro	1	d.1	1	d.1	d.1	1	lem.1
d.2	17	47	77	d.2	21	1	2	d.2	d.1	21	51	1	2	d.2	2	d.2	d.2	2	lem.2
d.3	18	48	78	d.3	22	2	3	d.3	d.2	22	52	2	3	d.3	3	d.3	d.3	3	lem.3
d.4	19	49	79	d.4		3	4	d.4	d.3	23	53	3	4	1	4	d.4	d.4	4	1
d.5	20	50	80	d.5		4	5	1	d.4-1	24	54	4	5	2	5	d.5	d.5	5	2
d.6	21	51	81	d.6		5	6	2	d.4-2	25	55	5	6	3	6	d.6	d.6	6	
d.7	22	52	82	d.7		6	7	3	d.4-3	26	56	6	7	4	7	1	d.7	7	
d.8	23	53	83	d.8		7	8	4	d.4-4	27	57	7	8	5	8	2	d.8	8	
d.9	24	54	84	d.9		8	9	5	d.5	28	58	8	9	6	9	3	d.9	9	
d.10	25	55	85	d.10		9	10	6	d.6	29	59	9	10	7	10	4	1	10	
d.11	26	56	86	d.11		10	11	7	d.7	30	60	10	11	8	11	5	2	11	
d.12	27	57	87	1		11	12	8	1	31	61	11	12	9	12	6	3	12	
d.13	28	58	88	2		12	13	9	2	32	62	[-] 12		10	13	7	4	13	
d.14	29	59	89	3		13	14	10	3	33	63	13		11	14	8	5	14	
d.15	30	60	90	4		14		11	4	34	64	14		12	15	9	6	15	
1	31	61	91	5		15		12	5	35		15			[-p]	10	7	16	
2	32	62	92	6		16			6	36		16			16	11	8	17	
3	33	63	93	7		17			7	37		17			17	12	9	18	
4	34	64	94	8		18			8	38		18			18	13	10	19	
5	35	65	95	9		19			9	39		[-p]			19	14	11	20	
6	36	66		10		20			10	40		19			20	15	12	21	
7	37	67		11		21			11	41		20			21	16	13		
8	38	68		12		22			12	42		21				17	14		
9	39	69		13		23			13	43		22					15		
10	40	70		14					14	44		23							
11	41	71		15					15	45									
12	42	72		16					16	46									
13	43	73		17					17	47									
14	44	74		18					18	48									
15	45	75		19					19	49									

Rearranged propositions	Addition / suppression of cases
Fusion / division of propositions	Addition / suppression of material
Addition / suppression of alternate proofs	Change in status

Table 9.4: Overview of deliberate alterations in the main body of core Middle Books works. Entries indicated with "[-]" represent material from the Arabic which does not appear in al-Ṭūsī's edition. [-] = cases or proofs, [-p] = propositions

When the comments al-Ṭūsī identifies as his own are separated out, the alterations in the remaining text are comparatively few. Of these, the alterations which arose as al-Ṭūsī's personal choice are likely even fewer. The concordances above have already noted how for the *Data*, the new edition arranged its propositions in the same way as the Kraus manuscript did: both total 95 propositions rather than Seray Ahmet III 3464's 91 because some propositions have been split and separately numbered as two. In producing his *Data*, the editor most likely worked with exemplars that already presented 95 propositions. This will be further expanded upon below; it is possible that such a scenario was the case for several of the divided propositions seen in the survey.[31]

Addition or suppression of material within the main content of these works is uncommon. Outside of the preliminary material to the *Optics*, it can be seen occurring in the *Phaenomena* and in *On Days and Nights*, both texts which had complicated transmissions.

In comparison, the following table shows the material which al-Ṭūsī added in the form of comments. Some of these contribute alternate proofs or additional cases to the text, and are indicated as such. Others – textual or mathematical comments, or new lemmas and other such supporting material – are indicated as the more general 'addition of material.'

[31] Examination of the Kraus manuscript's witness of *On Risings and Settings* would be worthwhile to see whether it, like that codex's witness of the *Data*, divides the propositions in the same way al-Ṭūsī does.

Data				Sph. I		II	III	MS	Opt.			Ph.	Hab.	D&N I	II	S&D	R&S I	II	Ana.
d.1	16	46	76	d.1	19	d.1	1	d.1	intro	20	50	intro	1	d.1	1	d.1	d.1	1	lem.1
d.2	17	47	77	d.2	20	1	2	d.2	d.1	21	51	1	2	d.2	2	d.2	d.2	2	lem.2
d.3	18	48	78	d.3	21	2	3	d.3	d.2	22	52	2	3	d.3	3	d.3	d.3	3	lem.3
d.4	19	49	79	d.4	22	3	4	d.4	d.3	23	53	3	4	1	4	d.4	d.4	4	1
d.5	20	50	80	d.5		4	5	1	d.4-1	24	54	4	5	2	5	d.5	d.5	5	2
d.6	21	51	81	d.6		5	6	2	d.4-2	25	55	5	6	3	6	d.6	d.6	6	
d.7	22	52	82	d.7		6	7	3	d.4-3	26	56	6	7	4	7	1	d.7	7	
d.8	23	53	83	d.8		7	8	4	d.4-4	27	57	7	8	5	8	2	d.8	8	
d.9	24	54	84	d.9		8	9	5	d.5	28	58	8	9	6	9	3	d.9	9	
d.10	25	55	85	d.10		9	10	6	d.6	29	59	9	10	7	10	4	1	10	
d.11	26	56	86	d.11		10	11	7	d.7	30	60	10	11	8	11	5	2	11	
d.12	27	57	87	X		11	12	8	1	31	61	11	12	9	12	6	3	12	
d.13	28	58	88	1		12	13	9	2	32	62	12		10	13	7	4	13	
d.14	29	59	89	2		13	14	10	3	33	63	13		11	14	8	5	14	
d.15	30	60	90	3		14		11	4	34	64	14		12	15	9	6	15	
1	31	61	91	4		15		12	5	35		15			16	10	7	16	
2	32	62	92	5		16			6	36		16			17	11	8	17	
3	33	63	93	6		17			7	37		17			18	12	9	18	
4	34	64	94	7		18			8	38		18			19	13	10	19	
5	35	65	95	8		19			9	39		19			20	14	11	20	
6	36	66		9		20			10	40		20			21	15	12	21	
7	37	67		10		21			11	41		21				16	13		
8	38	68		11		22			12	42		22				17	14		
9	39	69		12		23			13	43		23					15		
10	40	70		13					14	44									
11	41	71		14					15	45									
12	42	72		15					16	46									
13	43	73		16					17	47									
14	44	74		17					18	48									
15	45	75		18					19	49									

Addition of alternate proofs | Addition of cases | Addition of material

Table 9.5: Overview of deliberate alterations in al-Ṭūsī's comments to core Middle Books works. The "X" in *Sphaerica* Book I indicates preliminary material al-Ṭūsī adds after the main definitions.

The general impression that this table offers is how widespread al-Ṭūsī's contributions are throughout these Middle Books. Of the ten works examined here, only *On the Moving Sphere* and the very brief *Anaphoricus* lack any comment from the editor. In a few other texts they appear sparsely, but the *Data*, the *Sphaerica*, the *Optics*, the *Phaenomena*, and *On Days and Nights* show a multitude of comments. The purposes of al-Ṭūsī's comments vary: some present alternate cases or proofs, as has been indicated. Some note differences in the manuscripts he consulted. Some clarify the requirements of the proposition, or present lemmas that will be used later in the text, or otherwise comment on the mathematics involved. They will be discussed in more detail below.

3.4 Al-Ṭūsī's Edition and the Greek Recensions of the *Phaenomena*

The witness to the *Phaenomena* in Leiden or. 1031 was identified in chapter 4 as being a translation of recension B of the Greek text (with select small instances of material from the A recension). Al-Ṭūsī, conversely, seems to have had as an exemplar a manuscript which at least partly drew from the A recension. This can be seen through a few details to be discussed below. But its proposition 12 (= Greek proposition 10) clearly presents the A recension's version of the proof. Its proposition 17 corresponds to material which in the B recension appeared as a scholium but in the A recension as a lemma within the text – furthermore, in some Greek manuscripts of the A recension this lemma is numbered as its own proposition.[32] And al-Ṭūsī's edition lacks the alternate proofs which appear in the Greek B recension and in Leiden or. 1031.

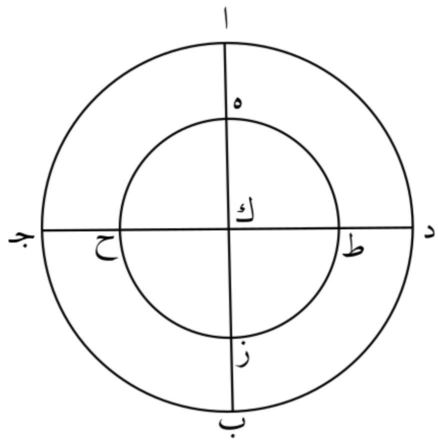
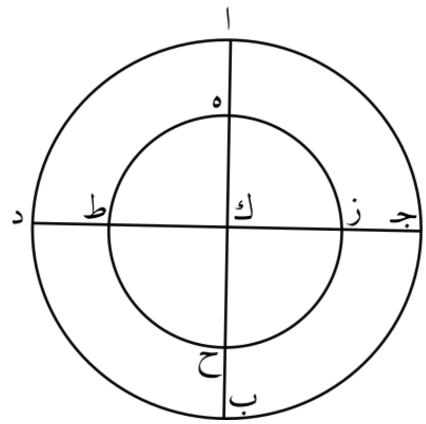
It seems then that some form of the A recension of the *Phaenomena* was translated into Arabic, and not only was a small fragment of it incorporated into Leiden or. 1031's witness or one of its ancestors, but a manuscript of it was still extant in al-Ṭūsī's day, even if in poor condition.

4. Deliberate Alterations and References in Detail

4.1 Substitution of Proof

The editor thoroughly rewrites the propositions of each of the texts in question. Since chapter 4 has already discussed *On Habitations* proposition 2, which in the Arabic was a quite close rendition of the Greek, this can be used as a comparison to show a general example of al-Ṭūsī's style.

[32] See Menge (1916) 84.

Earlier Arabic Version	**Al-Ṭūsī's Version**
	
For those whose places of habitation are under the circuit of the equator, all the fixed stars rise above them and set from them, and the time of their movement above their horizon is equal to the time of their movement below it. Example of this: we assume for those whose places of habitation are under the equator a line of the meridian: in the sphere of the cosmos circle ABGD and in the sphere of the earth circle EZHT, the diameter of the circuit of the equator line AB, and we assume some place of habitation at point E. So the zenith of the place of habitation E is point A. I say: for those whose places of habitation are at point E, all the fixed stars rise above them and set from them and the time of their movement above their horizon is equal to the time of their movement below it. Proof of this: we assume as center of the earth point K, so point K is indeed the center of the cosmos. And we draw through point K a line standing on line AB, and it is line GKD. So it is clear that line GKD is the axis of the sphere, and that the circle drawn on diameter GD standing on	For those whose places of habitation are under the circle of the equator, all of the stars and the points rise above them and are absent from them except for the two poles. And the time of visibility and invisibility for each one of them is equal. So let one of the semicircles of their day be on the sphere of the cosmos AGBD and on the earth EZHT, and let AB be on the surface of the circle of the equator, and (let) the habitation (be) E. I impose its zenith as A, (and) the center of the cosmos as K. And let GKD pass through it as a perpendicular on AB, so it is the axis of the sphere. The circle on which GD is a diameter and on which AB is perpendicular is a horizon of habitation E. And let A be a pole on it, it and circle AGBD and the circle of the equator, the three are intersecting at right angles. Similarly the (horizon of) habitation E is passing through the poles of the equator cutting all the parallel (circles) in half. Therefore the portions of the orbits, I mean the visible and invisible, are equal, and similarly the times of the progressions of all the points and stars above the earth are equal to the times of their progressions below it, and this is what we wanted.

line AB is the horizon of place of habitation E, and the circle drawn on diameter GD which stands on circle ABGD passes through the two poles of the sphere. So the horizon of place of habitation E passes through the two poles of the sphere. Since the fixed stars travel on parallel circuits parallel to the circuit of the equator, and the circle passing through the two poles of the sphere cuts the parallel circles in half, and the horizon of place of habitation E passes through the two poles of the sphere, then the horizon of place of habitation E cuts in half the parallel circuits on which the fixed stars move. So the time of the movement of the fixed stars above the horizon of place of habitation E is equal to the time of their movement below it, because each of them in place of habitation E travels a semicircle above the earth and a semicircle below the earth. And this is what we wanted to demonstrate.	
الذين مساكنهم تحت فلك معدل النهار الكواكب الثابتة كلها تطلع عليهم وتغرب عنهم ويكون زمان مسيرها فوق أفقهم مساوياً لزمان مسيرها تحته، مثل ذلك أن نفرض الذين مساكنهم تحت معدل النهار خط نصف النهار أما من كرة الكل فدائرة (ا ب جـ د) وأما من كرة الأرض فدائرة (ه ز ح ط) ونفرض قطر فلك معدل النهار خط (ا ب) ونفرض مسكناً ما على نقطة (ه) فيكون سمت الرأس لمسكن (ه) نقطة (ا)، فأقول إن الذين مساكنهم على نقطة (ه) الكواكب الثابتة كلها تطلع عليهم وتغرب عنهم ويكون زمان مسيرها فوق أفقهم مساوياً لزمان مسيرها تحته، برهان ذلك أن نفرض مركز الأرض نقطة (ك) فنقطة (ك) مركز لكرة الكل ونخرج على نقطة (ك) خطاً قائماً على خط (ا ب) وهو خط (جـ ك د) فظاهر أن خط (جـ ك د) هو محور الكرة وأن الدائرة المرسومة على قطر (جـ د) القائم على خط (ا ب) هى أفق لمسكن (ه) والدائرة المرسومة على قطر (جـ د) هى قائمة على دائرة (ا ب جـ د) تجوز على قطبى الكرة فإذاً أفق مسكن (ه) يجوز	الذين مساكنهم تحت دائرة معدل النهار فجميع الكواكب والنقط يطلع عليهم ويغيب عنهم ما خلا القطبين ويكون زمانا الظهور والخفاء لكل واحد منهما متساويين، فلتكن احدى دائر انصاف نهارهم على كرة الكل (ا ج ب د) وعلى الارض (ه ز ح ط) وليكن (ا ب) فى سطح دائرة معدل النهار والمسكن (ه) وسمت رأسه (ا) مركز العالم (ك) وليمر به (ج ك د) عمودا على (ا ب) فهو محور الكرة الدائرة التى تكون (ج د) قطرا لها و(ا ب) قائما عليها هى افق مسكن (ه) وليكون (ا) قطبا لها تكون هى ودائرة (ا ج ب د) ودائرة معدل النهار الثلثة متقاطعة على قوائم وكذلك يكون مسكن (ه) مارة بقطبى معدل النهار قاطعة لجميع الموازية لها منصفة اياها فاذا القسمان من المدارات اعنى الظاهر والخفى متساويان وكذلك تكون ازمنة مسيرات جميع القطب والكواكب فوق الارض مساوية لازمنة مسيراتها تحتها وذلك ما اردناه.[34]

[34] Hyderabad (1939-40) *Kitāb al-masākin* 2-3.

على قطبى الكرة ومن أجل أن الكواكب الثابتة تسير على أفلاك متوازية موازية لفلك معدل النهار والدائرة التى تجوز على قطبى الكرة تقطع الدوائر المتوازية على أنصافها وأفق مسكن (ه) يجوز على قطبى الكرة فإن أفق مسكن (ه) يقطع الأفلاك المتوازية التى تسير عليها الكواكب الثابتة على أنصافها فيكون زمان مسير الكواكب الثابتة فوق أفق مسكن (ه) مساوياً لزمان مسيرها تحته إذ كان كل واحد منها فى مسكن (ه) يسير نصف دائرة فوق الأرض ونصف دائرة تحت الأرض، وذلك ما أردنا أن نبين.[33]	

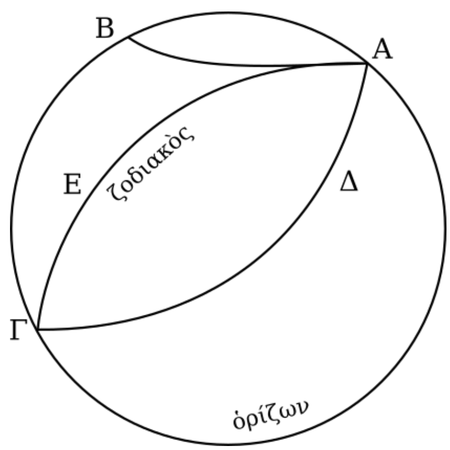
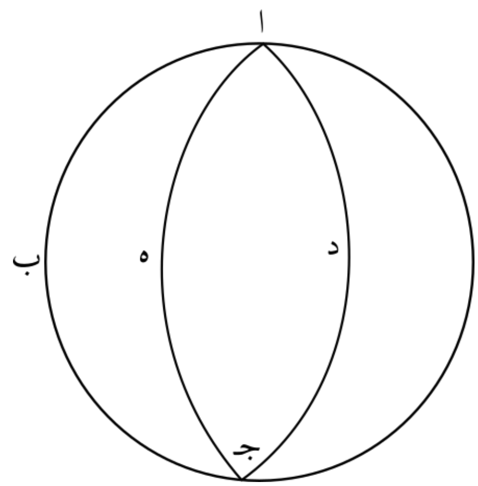
Table 9.6: Comparison of Arabic versions of *On Habitations* proposition 2

Here we see small a mathematical addition to al-Ṭūsī's text – he clarifies in the enunciation that the poles of the celestial sphere of course do not rise and set – but overall the text is significantly condensed. The editor presents the proposition in a bit over half the words of the earlier Arabic rendition. One feature which he removes, a feature which has its origins in the original Greek structure of the proposition, is the specification (the section which the earlier Arabic opens with "I say…"). The specification would restate the enunciation in terms of the labels the start of the proof had set out; it is by nature a repetitive element of the original Greek mathematical proof. Al-Ṭūsī consistently excises these from his text. The editor's overall proof is streamlined as well. Where the earlier Arabic text, following the Greek, methodically walks through a series of logical steps and invokes the different parts of the figure by their labels again and again, al-Ṭūsī much more quickly conveys to his reader that the circle of E's horizon, the circle of the equator, and the circles of the fixed stars are all three perpendicular to each other, and as such the horizon bisects the circles of the fixed stars.

An instance of substitution of proof which is of note for what it suggests about the transmission of a text appears in al-Ṭūsī's edition of the *Phaenomena*. Chapter 4 already discussed how the witness to the *Phaenomena* in Leiden or. 1031 followed the B recension except in its proposition 10 where, after

[33] Kunitzsch and Lorch (2010a) 20-22.

presenting the B recension's proof, it proceeded to present an alternate proof found in an unspecified version via Syriac, which was shown to be the A recension's proof for that proposition. Al-Ṭūsī's edition does not present a double proof for this proposition (proposition 12 in al-Ṭūsī's count). But the single proof al-Ṭūsī presents is clearly the proof from the A recension. It lacks any reference to the summer and winter tropics in the exposition, the usage of labels follow what is seen in the A recension, and its (brief) summation of the case where the two arcs rise in equal times is closer to A's rendition than B's.

Greek Recension A	**Al-Ṭūsī's Version**
	
If two semicircles of the circle of the zodiac rise in unequal times having some common arc, and the opposite arcs in unequal times also rise, there will also be the same difference of times in which both the semicircles rise and the opposite arcs rise, and if two semicircles of the circle of the zodiac rise in an equal time having some common arc, the opposite arcs also rise in an equal time. In the cosmos let the horizon be ABG, and let the circle of the zodiac have position AEGD, and let equal arcs AD, GE be taken; therefore, D is diametrically opposite to E. And let semicircles ADG, DGE in unequal times rise.	Each two semi(circles) of the circuit of the zodiac that share in an arc, the two times of (their) risings are different, after the common part sinks down the two remainders of them also have different times of rising. And the remainder between them is like the remainder between the two times of rising of the semi(circles). And if the two times of their rising are equal, the two remainders are also like this. So let the horizon be ABG, and the circuit of the zodiac ADGE, and let semi(circles) ADG, DGE share from it in arc DG. So the two risings of semi(circles) ADG, DGE are different. We

I say that also the opposite arcs AD, GE in unequal times rise, and the same difference is in the times in which semicircles ADG, DGE rise and in which arcs AD, GE rise. For since the semicircles ADG, DGE in unequal times rise, let the common rising time, that of DG, be taken away; (for, the arc DG always rises in a time equal to itself); then the remaining arcs AD, GE in unequal times rise, and the same differences are between the times in which semicircles ADG, DGE rise and the opposite arcs AD, GE (rise). Indeed again, (suppose) the semicircles ADG, DGE in equal times rise. Let the common time of arc GD be taken away; then the remaining AD, GE in an equal time rise.	remove arc DG; the remaining two risings of arcs AD, GE are also different because the rising of arc DG was removed from them, and it (DG) is the same thing, and the difference between the two risings of ADG, DGE is like the difference between the two risings of AD, GE. If the the two risings of semi(circles) ADG, DGE are equal, the remaining two risings of AD, GE are also equal for the same (reason) as this. And this is apparent, and this is what we wanted.
Ἐὰν τοῦ τῶν ζῳδίων κύκλου δύο ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατέλλῃ κοινήν τινα ἔχοντα περιφέρειαν, καὶ αἱ ἀπεναντίον περιφέρειαι ἐν ἀνίσοις χρόνοις ἀνατέλλουσιν, καὶ ἡ αὐτὴ διαφορὰ ἔσται τῶν χρόνων, ἐν οἷς τά τε ἡμικύκλια ἀνατέλλει καὶ αἱ ἀπεναντίον περιφέρειαι ἀνατέλλουσιν· καὶ ἐὰν τοῦ τῶν ζῳδίων κύκλου δύο ἡμικύκλια ἐν ἴσῳ χρόνῳ ἀνατέλλῃ κοινήν τινα ἔχοντα περιφέρειαν, καὶ αἱ ἀπεναντίον περιφέρειαι ἐν ἴσῳ χρόνῳ ἀνατέλλουσιν. ἔστω ἐν κόσμῳ ὁρίζων ὁ ΑΒΓ, ὁ δὲ τῶν ζῳδίων κύκλος θέσιν ἐχέτω τὴν ΑΕΓΔ, καὶ ἀπειλήφθωσαν ἴσαι περιφέρειαι αἱ ΑΔ, ΓΕ· κατὰ διάμετρον ἄρα ἐστὶ τὸ Δ τῷ Ε. τὰ δὲ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατελλέτω· λέγω, ὅτι καὶ αἱ ἀπεναντίον πεμιφέρειαι αἱ ΑΔ, ΓΕ ἐν ἀνίσοις χρόνοις ἀνατέλλουσι καὶ ἡ αὐτὴ διαφορά ἐστι τῶν χρόνων, ἐν οἷς τὰ ΑΔΓ, ΔΓΕ	كل نصفين من فلك البروج يشتركان في قوس فان كانا مختلفى زمانى الطلوع كان الباقيان منهما بعد اسقاط المشتركة ايضا مختلفى زمانى الطلوع وكان الفضل بينهما كالفضل بين زمانى طلوع النصفين وان كانا متساوى زمانى الطلوع كان الباقيان ايضا كذلك، فليكن الأفق (ا ب جـ)، وفلك البروج (ا د جـ ه)، وتشترك نصفا (ا د جـ) (د جـ ه) منه فى قوس (د جـ). فإن كان مطالعا نصفى (ا د جـ) (د جـ ه) مختلفين؛ وأسقطنا قوس (د جـ)، بقى مطالعا قوسى (ا د) (جـ ه) أيضاً مختلفتين؛ لأن مطالع قوس (د جـ) يسقط عنها وهى شئ واحد، ويكون التفاضل بين مطالعى (ا د جـ) (د جـ ه) كالتفاضل بين مطالعى (ا د) (جـ ه). وإن كانت مطالعا نصفى (ا د جـ) (د جـ ه) متساويتين، بقيت مطالعا (ا د) (جـ ه) أيضا متساويتين لمثل ذلك. وذلك ظاهر، وذلك ما أردناه.[36]

[36] Sulaymān (1996) 94-95; compare Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 19.

ἡμικύκλια ἀνατέλλει καὶ ἐν οἷς αἱ ΑΔ, ΓΕ περιφέρειαι ἀνατέλλουσιν. ἐπεὶ γὰρ τὰ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἀνίσοις χρόνοις ἀνατέλλει, κοινὸς ἀφῃρήσθω ὁ τῆς ΔΓ ἀνατολῆς χρόνος· (ἡ γὰρ ΔΓ περιφέρεια ἑαυτῇ ἀεὶ ἐν ἴσῳ χρόνῳ ἀνατέλλει)· λοιπαὶ ἄρα αἱ ΑΔ, ΓΕ περιφέρειαι ἐν ἀνίσῳ χρόνῳ ἀνατέλλουσι καὶ αἱ αὐταὶ διαφοραί εἴσι τῶν χρόνων, ἐν οἷς τά τε ΑΔΓ ΔΓΕ ἡμικύκλια ἀνατέλλει καὶ αἱ ἀπεναντίον περιφέρειαι αἱ ΑΔ, ΓΕ. πάλιν δὴ τὰ ΑΔΓ, ΔΓΕ ἡμικύκλια ἐν ἴσῳ χρόνῳ ἀνατέλλει· κοινὸς ἀφῃρήσθω ὁ τῆς Γ περιφερείας χρόνος· λοιπαὶ ἄρα αἱ ΑΔ, ΓΕ ἐν ἴσῳ χρόνῳ ἀνατέλλουσιν.[35]	

Table 9.7: Comparison of *Phaenomena* proposition 10 in Greek recension A and al-Ṭūsī’s edition

The usual features of al-Ṭūsī’s style occur in this proof as well: the specification is eliminated as repetitive and the entire proof is streamlined – compared to the version which was preserved as an alternate proof in Leiden or. 1031, al-Ṭūsī has condensed the text by approximately a quarter.

4.2 Alternate Proof

The alternate proofs present in al-Ṭūsī’s edition occur in his *Data*’s proposition 64 (= Greek 62); his *On the Moving Sphere* proposition 2; his *Optics* propositions 3, 10, and 26 (= Greek 3, 9, and 25); his *Phaenomena* proposition 8 (= Greek 6); and his *On Sizes and Distances* proposition 13 (= Greek 15).

These are presented in several different ways. In the comment to *Data* 64, he heads the alternate proof with “I say the [text] found in the copies is as such: and we work this proposition in another way”

[35] Menge (1916) 54-56, main text.

(أقول الموجود فى النسخ هكذا ونعمل هذا الشكل على جهة أخرى).[37] In *On the Moving Sphere*, it is introduced with "and this proposition is found in another copy as such" (ووجد هذا الشكل فى نسخة اخرى هكذا).[38] The alternate proofs to the three propositions in the *Optics* are clearly included among al-Ṭūsī's comments – they are headed with "I say" (اقول) but without further comment.[39] In the *Phaenomena* the language is simply "and in a copy" (وفي نسخة).[40] And in *On Sizes and Distances* the language is "and by another way" (وعلى جهة اخرى).[41]

Comparison with the earlier Arabic translations of *On the Moving Sphere* and *On Sizes and Distances* shows that these two alternate proofs were already incorporated into the Arabic texts – the doubling of proofs was not a deliberate addition of al-Ṭūsī's, and as such they are incorporated into the main proposition rather than set apart within one of his own indicated comments. The fact that the *Phaenomena*'s alternate proof is similarly set apart by "and in a copy" alone, without the editor's usual "I say," implies that al-Ṭūsī may have found this material already incorporated in the sources he consulted like he would have for *On the Moving Sphere* and *On Sizes and Distances*.

Overall, al-Ṭūsī maintains the alternate proofs which are found in the earlier Arabic versions. Some instances – like the doubled proof seen in the earlier version of the *Data*'s proposition 37 – are not listed above because in al-Ṭūsī's edition they are presented as two separately numbered proofs (in this example, his propositions 38 and 39), but they are still present.

The *Phaenomena* is the work in which the editor does not preserve alternate proofs found elsewhere in the extant manuscripts, but this may be caused more by those proofs not being transmitted in any of the manuscripts al-Ṭūsī had available to him than by a conscious choice to excise it. Proposition 12

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[37] Hyderabad (1939-40) *Kitāb al-Muʿtīyāt* 27.
[38] Hyderabad (1939-40) *Kitāb al-kurah al-mutaḥarrikah* 3.
[39] Hyderabad (1939-40) *Kitāb al-manāẓir* 4, 7, and 11.
[40] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 13.
[41] Hyderabad (1939-40) *Kitāb fī jirmay al-nayyirayn wa-al-buʿdayhimā* 18.

of his *Phaenomena* (= Greek 10), for example presents only the one proof, which as noted is the one descending from the Greek recension A.[42] Comparison with more witnesses would be worthwhile to see whether this version of the proof appears elsewhere, as it did in Leiden or. 1031. Considering al-Ṭūsī's tendency to preserve content rather than omit it, it is quite possible that the witnesses he had at hand only preserved propositions without alternate proofs.

The case of *Data* proposition 64 (= Greek 62) especially shows al-Ṭūsī's efforts to have his editions faithfully present what he found to be in circulation. Sidoli and Isahaya have argued that this alternate proof was not originally included in al-Ṭūsī's draft of the text. Rather, early manuscripts show it being added as a marginal comment, and it was moved into the main text only afterwards, in what Sidoli and Isahaya believe to be a later draft of the text.[43] If this is correct, al-Ṭūsī did not find this proof in the manuscripts he initially had on hand, but noted it down later and only subsequently incorporated it into his edition. Besides speaking to al-Ṭūsī's continuing efforts to improve his editions, this instance is notable because the proof that he introduced to comment on was an erroneous one, and al-Ṭūsī was aware of this. In the manuscripts where it is present as a comment, he introduces the proof with the aforementioned "I say the [text] found in the copies is as such: and we work this proposition in another way…" After laying out the details of the proof and his issues with it, he concludes with "so let it be seen in it that this proposition is a mess" (فلينظر فيه فأن هذا الشكل مخبط).[44] Evidently faithfully preserving what was in his sources – material which his contemporary and predecessor scholars were well acquainted with – was important enough to al-Ṭūsī that in a later draft of his *Data* he added in a false proof to comment on its failings. It may be also that the authorities attached to this proof were notable enough that al-Ṭūsī

[42] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 19.
[43] Sidoli and Isahaya (2019) 96-98.
[44] Hyderabad (1939-40) *Kitāb al-Muʿṭīyāt* 28.

found it important to comment on it – the marginal comment version of this proof reports that it had been read back to al-Nasawī by al-Sizjī.[45] Interestingly, al-Ṭūsī removes the mention of these authorities in the version that he presents in his comment.

Meanwhile, proposition 3 of the *Optics* is another case of al-Ṭūsī incorporating supplementary material available in the sources into his text, since this proof originally appears in the margins of a manuscript of the early version of the text.[46] Whether al-Ṭūsī encountered the alternate proofs to propositions 10 and 26 in his sources as well or whether he introduced them himself is unclear. They certainly could have been al-Ṭūsī's own work.

4.3 Addition / Suppression of Cases

Added cases are seen in al-Ṭūsī's *Data* 14, 15, 44, and 80 (same in Greek); his *Sphaerica* II.15 (same as Greek); his *Optics* 34 and 56 (= Greek 32 and A49); and his *On Days and Nights* II.15 (= Greek 14).

In the *Sphaerica* and the *Optics*, these occur within the main body of the proofs themselves. These additions are located at the end of the proof and serve as brief acknowledgements of other cases; none are especially long.[47] In the *Data* and *On Days and Nights*, they instead appear within his comments, after the main proof.[48]

The example of *Data* 80 is worth further note. Sidoli and Isahaya have argued that it provides further hints towards al-Ṭūsī's continuing editorial process across multiple drafts of his edition. The

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[45] This marginal comment was briefly noted also in chapter 5.
[46] It appears in Seray Ahmet III 3464. See Kheirandish (1999) II 35.
[47] See Hyderabad (1939-40) *Kitāb al-ukar* 23 and *Kitāb al-manāẓir* 13 and 21. See also discussion in Sidoli and Kusuba (2008) 31 and Kheirandish (1999) II 73 and 92.
[48] See Hyderabad (1939-40) *Kitāb al-muʿṭayāt* 7-8, 19 and 36 and *Kitāb al-ayyām wa-al-layālī* 27 See also discussion in Sidoli and Isahaya (2019) 92-93, 95-96, and 101-102.

comment in question does not appear in all the manuscripts; rather, it occurs in the family which they hypothesize to represent the earlier draft of al-Ṭūsī's text. The comment claims that, though the enunciation of the proposition is general, the demonstration of the proposition holds true only for the case where a particular angle is acute, and so a different demonstration must be set forward for the other cases. Sidoli and Isahaya point out that this is not correct, the proposition as written does actually hold for any angle, not just the acute case. Since this erroneous comment does not appear in the hypothesized later draft of the text, they argue that al-Ṭūsī most likely noticed his error and excised it.[49]

4.4 Change in Order of Propositions

Overall, al-Ṭūsī does not change how propositions in his editions are ordered compared to the earlier Arabic versions.

While Thābit's *Assumptions* is not one of the main texts examined in this study, it provides a significant enough divergence from this overall pattern that it deserves note here. The following aligns the propositions in al-Ṭūsī with those in the one extant manuscript for what has been supposed to be Thābit's version:

al-Ṭūsī	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Thābit	5	6	8	33	9	10	17	19	29	30	34	35	11	36	1	2	3	4

al-Ṭūsī	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Thābit	7	26	31	12	24	13	16	23	14	15	18	20	21	22	25	27	28	32

Table 9.8: Comparison of proposition orders in the *Assumptions* between al-Ṭūsī's and Thābit's versions

[49] Sidoli and Isahaya (2019) 101.

Dold-Samplonius suggests that al-Ṭūsī was the one responsible for rearranging the propositions of the treatise.[50] However, we only have one extant witness for "Thābit's" version, MS Ayasofya 4832 (4th / 11th century). It is very difficult to say at this time whether that manuscript indeed preserves the text as it was originally arranged, or whether it may itself present a separate and reordered version. It would be a unusual choice for al-Ṭūsī to restructure the text, let alone so dramatically. This is especially so considering his note at the beginning: "and it is thirty-six propositions and in some copies thirty-four according to the fixed arrangement of black numerals in the margins, and proposition four and proposition twenty-three are not in it" (وهى ستة وثلاثون شكلا وفى بعض النسخ اربعة وثلاثون شكلا على الترتيب المثبت بالارقام السود على الحاشية ولم يكن فيه شكل د ولا شكل كج).[51] If he fully reordered the propositions, the numbers he provides for the missing propositions have meaning within his own edition, but lose all meaning when consulting the text prior to his edition.

Similarly, *On Measurement of the Circle* is not one of the main texts under study, but receives brief comment here since al-Ṭūsī moves Archimedes's third proposition forwards to serve as the second instead. Van Lit supposes that this reordering is because Archimedes's second proposition relies on the result of his third proposition.[52] There would then be motivation for an editor to reorder the text, even if it departs from al-Ṭūsī's usual pattern. But the edition of *On the Measurement of the Circle* is already an unusual case. The editor writes that he chose to add it as an appendix to the larger *On the Sphere and Cylinder*, and this is indeed how it is transmitted in subsequent manuscripts, appended to the end of the longer text. It is not presented as a standalone work – its beginning lacks the standard bismillah, and the work is described as the "chapter of Archimedes on the measurement of the circle" (مقالة ارشميدس فى تكسير

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[50] Dold-Samplonius (1996) 211.
[51] Hyderabad (1939-40) *Kitāb al-mafrūḍāt* 2.
[52] Van Lit (2012) 32.

الدائرة) rather than as an edition (تحرير) of that text. Perhaps since the short work as a whole was already, in some ways, an addition by al-Ṭūsī to *On the Sphere and Cylinder*, he allowed himself more flexibility in rearranging it. Or he encountered a copy which had already taken the liberty of doing so, but if so such a version has not yet been found.

4.5 Fusion / Division of Propositions

There are what appear to be instances of division in al-Ṭūsī's text, but the example of the *Data* raises the possibility that al-Ṭūsī's sources themselves were already responsible for these divided propositions. For the earlier Arabic version of that text, the witness in Seray Ahmet III 3464 has 91 propositions, but the proposition count rises to 95 in the Kraus manuscript. This is because content which the former codex presents as one proposition with doubled proofs, the latter presents as two separate propositions. So Seray Ahmet III 3464's propositions 19, 37, 43, and 60 become the Kraus manuscript's propositions 19-20, 38-39, 45-46, and 63-64 respectively. Al-Ṭūsī's proposition arrangement agrees with the Kraus manuscript, and he very likely relied on a version like it.

Evidence from the *Sphaerica* offers further support. This is a work which the editor did encounter in differing counts of propositions, and he acknowledges this at the very start of his edition: "It is three books and fifty nine propositions, and in some of the copies with an omission of a proposition in the number" (هو ثلاث مقالات وتسعة وخمسون شكلا وفي بعض النسخ بنقصان شكل في العدد).[53] The phrasing of this conveys that the proposition in question is not entirely absent from the text, but rather is not numbered. Later on, the editor writes in his comment to proposition II.12: "and in some of the copies this proposition is not counted separately but rather is counted within the reckoning of the preceding proposition" (وفي بعض النسخ

[53] Hyderabad (1939-40) *Kitāb al-ukar* 2.

(لا يعد هذا شكلا مفردا بل يعد من حساب الشكل المتقدم).[54] This is the proposition missing in number which was referenced in the preface – the discrepancy is because some manuscripts fuse it with its preceding proposition, but al-Ṭūsī follows other sources and leaves them divided.

There is also the evidence from the preface of Thābit's *Assumptions*, discussed above, which indicated a disagreement between proposition counts in the sources and provided numbers for which propositions al-Ṭūsī found to be missing. While not a case of fusion or division of propositions, this evidence together with the *Sphaerica*'s evidence shows that al-Ṭūsī had a practice of telling his reader where structural disagreements appeared in his sources, and that he would clearly indicate within his own edition which propositions were affected by these disagreements. Since the divisions receive no remark in his edition of the *Data* and since there is precedence for propositions to have already been divided in versions of the text prior to al-Ṭūsī, such versions were very likely the ones which he used. Division of propositions also appear to be occurring in al-Ṭūsī's editions of the *Phaenomena* and *On Risings and Settings* – it is quite possible that here, too, these divisions were not actively introduced by al-Ṭūsī but were already present in his sources.

In the edition of the *Phaenomena*, Greek proposition 2 gets split into three propositions as different cases are numbered separately. The last proposition of the work is also divided in two in al-Ṭūsī's edition. These propositions were indeed probably already divided in the editor's sources because in the preface he writes that he found the *Phaenomena* in 23 propositions, and even up to 25 in some copies.[55] This is already higher than Leiden Or. 1031's 20 propositions – the number 23 is reached

[54] Hyderabad (1939-40) *Kitāb al-ukar* 19.
[55] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 2.

through the divisions that have been mentioned.[56] There is however a difference between the examples of al-Ṭūsī's *Phaenomena* and *Sphaerica*, however – though al-Ṭūsī notes in the preface encountering the text in different numbers of propositions, he does not specify in his comments anything like what he did for *Sphaerica* II.12. Perhaps this is because al-Ṭūsī considered his work with the Phaenomena incomplete, as he indicates in the preface with his intention to return to it should he encounter a better source for the text. But the difference is worth noting.

4.6 Change in Status

For change in status, the possible instances are his *Phaenomena* proposition 17, which corresponds to Greek material that circulated as a scholion or a lemma, and his *Anaphoricus*, where al-Ṭūsī makes clear that the parts of the text are not all traditional propositions.

The instance in the *Phaenomena*, however, was probably not a change in status introduced by al-Ṭūsī himself – in the A recension of the Greek text, this lemma was sometimes numbered as its own proposition.[57] The editor probably had a source text that did the same.

The example of the *Anaphoricus* is because this was not actually a traditional proposition-based text, though its earlier version in the Arabic did number its five parts as five propositions. Al-Ṭūsī instead indicates the first three sections as lemmas, then the start of the astronomical section as providing starting-points, then that there are two figures which follow after.

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[56] It may be that the proposition count of 25 was reached also through division of propositions. In the witness to his *Phaenomena* in the manuscript Columbia or. 306, for example, the alternate proof to al-Ṭūsī's proposition 8 is numbered separately, as is the end of his proposition 11. See Columbia or. 306, fol. 76a and 78a. From fol. 76a onwards, there are two sets of proposition numerals: the original continues in red ink and the count with the added "propositions" is presented in black ink.

[57] See Menge (1916) 84.

4.7 Addition / Suppression of Material

The comments which al-Ṭūsī adds to several of these Middle Books have been discussed in several studies.[58] Both the comments to the *Data* and the *Sphaerica* show a wide mixture of interests, from adding further preliminary material or connecting it more explicitly to the propositions which rely on it, to comments on the history and copies of the text, to more complex mathematical material addressing gaps in the text.

An interesting example of the latter is his lemma to *Sphaerica* III.12, which is based on an earlier proposition II.11. Al-Ṭūsī prepares his reader for the future lemma already in his comment to II.14 (= Greek II.13), where he gives a short lemma that he notes will be required in material to come.[59]

The *Optics*, meanwhile, shows suppression of material by al-Ṭūsī. We have already discussed how within the texts, his style is to condense and streamline. In two places in the *Optics*, propositions 15 and 17, the editor suppresses the demonstration of the proposition because the demonstration is the same as the one which precedes it. Al-Ṭūsī does not remove the entire proposition – it remains structurally a part of the text – but after setting out the enunciation and exposition, he cuts the proposition short: "the demonstration is like what passed in the preceding proposition" (والبيان كما مر فى الشكل المتقدم).[60]

4.8 References to the Curriculum within the Texts

In his preface to the *Almagest*, the editor expressed a preference for maintaining the proposition structure he received because scholars were accustomed to referencing particular propositions and

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[58] For the *Data*, see Sidoli and Isahaya (2019); for the *Sphaerica*, Sidoli and Kusuba (2008); for *On Sizes and Distances*, see Berggren and Sidoli (2007).
[59] Sidoli and Kusuba (2008) 17-18.
[60] Hyderabad (1939-40) *Kitāb al-manāẓir* 8.

diagrams in their own lessons and works.[61] He seems to have been motivated similarly in his edition of the Middle Book, in whose main text can be found multiple cross references to other curricular propositions. While references within the main text are not so frequent as referential scholia, which will be discussed in the next section, they do appear within al-Ṭūsī's edition of the curriculum and as intentional pieces of the text, not inadvertently incorporated marginalia. They appear both within the main propositions and al-Ṭūsī's separate comments.

Both can be seen in the *Sphaerica*. Proposition III.9 references material to be found after the tenth proposition (يوجد ذلك بعد الشكل العاشر); later, al-Ṭūsī's comment on III.12 refers back to II.14 (الشكل الرابع عشر من المقالة الثانية) and II.11 (الشكل الحاذى عشر من المقالة الثانية).[62]

There are also two citations in his comments to the *Optics*, as propositions 43 and 48 reference each other. At the end of proposition 43, al-Ṭūsī's comment mentions a related case and says, "and we will mention this proof in proposition 48" (وسنذكر هذا الحكم فى الشكل الثامن والاربعين). Proposition 48 then ends with "I say: and this is what we mentioned at the end of proposition 43" (اقول وهذا ما ذكرناه بعينه في آخر الشكل الثالث والاربعين).[63] Al-Ṭūsī says nothing to suggest he is aware that his proposition 48 originally followed immediately after his proposition 43 in their Greek versions – this detail of textual history does not seem to have been preserved in the Arabic translation, which reordered the propositions. But the relation of the propositions is clear enough to al-Ṭūsī that he draws the link, making it clear to his reader that the separate case one could conceive of in proposition 43 would be addressed at a later point.

[61] Istanbul Nuruosmaniye Kütüphanesi 2941, fol. 1b: "But I took on this burden because the book is known among scholars who specify it in their discussions and they cite the location of its matters from the sections and diagrams in their notes…"
[62] Hyderabad (1939-40) *Kitāb al-ukar* 43, 50, and 51 respectively.
[63] Hyderabad (1939-40) *Kitāb al-manāẓir* 17 and 19.

The *Phaenomena* is particularly scattered with references to other texts. Its preface cites the *Optics*, as it has since its Greek transmission.[64] In the propositions the citations are the following:

- Proposition 2:
  - "As for the first case, it is evident from what Autolycus mentioned in the tenth proposition of his treatise on the *Moving Sphere*" (أما الحكم الاول فظاهر مما ذكره أوطولوقس فى الشكل العاشر من مقالته فى الكرة المتحركة)[65]

- Proposition 7:
  - "according to what was proved in proposition 5 of this treatise" (على ما تبين فى شكل - ه - من هذه المقالة)[66]

- Proposition 8:
  - "as for what was presented in proposition 5" (ولما تقدم فى شكل - ه)[67]
  - "and concerning the definition of it Autolycus [in] his book" (ولما صادر به أوطولوقس كتابه)[68]
  - "and with the definitions of Autolycus" (وبمصادرة أوطولوقس)[69]

- Proposition 9:
  - "as for what was proved in proposition 11 of the book of Autolycus" (فلما تبين فى شكل - يا - من كتاب أوطولوقس)[70]

- Proposition 10:
  - "for what was established in proposition 7 of book 3 of the *Sphaerica* of Theodosius" (لما ثبت فى شكل - ز - من مقالة - ج - من أكر ثاوذوسيوس)[71]
  - "for what was established in proposition 5 of book 3 of the *Sphaerica* of Theodosius" (بما ثبت فى شكل - ه - من مقالة - ج - من أكر ثاوذوسيوس)[72]

- Comment to proposition 15:
  - "so according to what Menelaus demonstrated in his book on *Spherical Figures*" (فعلى ما بين مانالاوس فى كتابه فى الاشكال الكرية)[73]

- Comment to proposition 18:

[64] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 2.
[65] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 7.
[66] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 11.
[67] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 12.
[68] Ibid.
[69] Ibid.
[70] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 14.
[71] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 16.
[72] Ibid.
[73] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 24.

- “what I presented in the sixteenth proposition” (ما اورده فى الشكل السادس عشر)[74]

This is an unusual degree of density for references, and nearly all of them appear in the main text of the *Phaenomena* rather than in al-Ṭūsī’s comments. There are perhaps two relevant details to consider here: (1) al-Ṭūsī found it necessary, because of the poor state of his exemplars, to rework much of the *Phaenomena* and (2) there are several manuscripts where the *Phaenomena* has a notably high density of referential scholia. We will see below that it has the highest count in MS Tabriz 3484, despite not being one of the longest texts. Perhaps this has origins in al-Nayrīzī’s commentary – while his commentary on the *Phaenomena* is not extant, it can be seen in his commentary to the *Elements* that he regularly referred back to earlier propositions in the course of his explanation.[75] So al-Ṭūsī’s efforts to work with al-Nayrīzī’s commentary (unsatisfactory though he found that witness, as well) to address his deficient *Phaenomena* source may have introduced more cross textual material than is otherwise seen in his editions of the Middle Books.

There are several other works which contain references to other texts, but none as numerous as the *Phaenomena*. *On Days and Nights* has two comments on propositions II.3 and II.4 referring back to II.1 and II.2 respectively.[76] It also includes the reference to the preceding book that appears in proposition

[74] Hyderabad (1939-40) *Kitāb al-ẓāhirāt al-falak* 28.

[75] See Besthorn and Heiberg (1897), (1900), and (1905) for the edition of al-Nayrīzī’s commentary. Books II-IV have been translated into English in Lo Bello (2003) and (2009). Besthorn and Heiberg’s edition is available on the Digital Corpus for Graeco-Arabic Studies (https://www.graeco-arabic-studies.org/). Querying that digital text reveals that al-Nayrīzī cites other propositions approximately 418 times in the course of his commentary. 401 of these instances follow the abbreviated formula “[proposition numeral] من [book numeral]” – see for example his comment to proposition I.15 (p.82), which cites “so by the proof of I.13” (فببرهان يجـ من ا).

[76] Hyderabad (1939-40) *Kitāb al-ayyām wa-al-layālī* 17: “وتبين بمثل ما مر فى الشكل الأول”; and 18: “وتبين بمثل ما مر فى الشكل الثانى”

II.9's enunciation, and which was seen also in chapter 4.[77] The *Lemmata* attributed to Archimedes has a citation to *Elements* XIII.13 in the comment to proposition 15.[78]

Citations also appear in the edition of *Sphere and Cylinder*, for example propositions I.48 cites propositions I.35 and I.36.[79] We should note also that there are a multitude of references to Euclid, Eutocius, Apollonius, and citations of the *Elements* and the *Conics* – these would seem to be the result of al-Ṭūsī's use of the translated commentary of Eutocius. Furthermore, appended to the end of the edition of *Sphere and Cylinder* is the edition of *Measurement of the Circle*. In his comment to his second proposition, al-Ṭūsī specifically invokes the *Almagest* when discussing principles behind the alternate proof he presents: "I say the astronomers have another way and it is that they obtain a chord of a small arc which is a part of the circumference of the circle with the principles which are demonstrated in the book of the *Almagest* and the rest of their books of demonstrations."[80]

4.9 <u>Referential Scholia</u>

The past chapters have already shown that scholia referring back to earlier propositions in the curriculum can be found in manuscripts both of the Little Astronomy and the pre-Tusi Middle Books. It should be no surprise, then, that these brief citations continue to appear in manuscripts of al-Ṭūsī's edition as well.

In the manuscript Tabriz 3464, these referential scholia appear on most of the Middle Books works. The citation style is again abbreviated, and especially so for citations of the *Elements*. For citations

[77] Hyderabad (1939-40) *Kitāb al-ayyām wa-al-layālī* 21: "وذلك لانا بينا فى المقالة الاولى انها اذا طلعت او غربت فى موازية واحدة بعينها فهى تواقى الانقلاب فى انتصاف النهار"

[78] Hyderabad (1939-40) *Kitāb al-maʾkhūdhāt* 18: "وقد تبين ذلك فى الشكل الثالث عشر من المقالة الثالثة عشر من الاصول"

[79] Lorch (1989) 99.

[80] Hyderabad (1939-40) *Kitāb al-kurah wa-al-usṭuwānah* 131: "اقول وللمنجمين طريق آخر وهو انهم يحصلون وتر قوس صغيرة يكون جزءا من محيط الدائرة بالاصول التي تبينت في كتاب المجسطي وغيره من كتبهم البرهانية"

to the *Elements*, Tabriz 3464 follows the style seen in Leiden or. 1031, where the *Elements* is indicated with the abbreviation "ق" for Euclid. Other works cited in Tabriz 3484 do not have their titles abbreviated, and so *Sphaerica* I.21 for example is cited as "كا ا من الاكر."

A survey of the referential scholia in Tabriz 3484 shows that the previously established pattern holds: only earlier propositions from within the curriculum receive citation.

- the *Data*: at least 39 scholia,[81] all to the *Elements*
- the *Sphaerica*: at least 51 scholia,[82] all to the *Elements*
- *On the Moving Sphere*: 31 scholia, all to the *Sphaerica*
- the *Optics*: 28 scholia, all to the *Elements*
- the *Phaenomena*: 82 scholia: 52 on the *Sphaerica*, 15 on the *Moving Sphere*, 15 on itself
- *On Habitations*: none
- *On Risings and Settings*: 1 scholium, on itself
- *On Days and Nights*: 9 scholia: 6 on the *Sphaerica*, 2 on the *Phaenomena*, and 1 on itself
- *On Sizes and Distances*: none
- *Anaphoricus*: none

We can compare the citations in Tabriz 3484 on *Sphaerica* book I propositions 1-8 with what was found in chapter 4 for the manuscripts Leiden or. 1031 and Seray Ahmet III 3464. The referential scholia in those manuscripts were more numerous – in al-Ṭūsī's edition in Tabriz 3484, there are only eight such scholia on those propositions. They do, however, fully agree with the earlier manuscripts:[83]

- Sph.1.prop.2 references El.11.prop.13 (p.28)
- Sph.1.prop.3 references El.3.prop.2 (p.28)
- Sph.1.prop.4 references El.3.prop.17 (p.28)
- Sph.1.prop.4 references El.11.prop.4 (p.28)
- Sph.1.prop.5 references El.11.prop.13[84] (p.25)
- Sph.1.prop.7 references El.1.prop.8 (p.25)
- Sph.1.prop.7 references El.11.prop.4 (p.25)
- Sph.1.prop.8 references El.1.prop.4 (p.25)

[81] Some folios of the *Data* in Tabriz 3484 are in disarray or missing.
[82] Some folios of the *Sphaerica* in Tabriz 3484 are in disarray or missing.
[83] Note the pagination for Tabriz 3484 is out of order because the folios were bound and paginated incorrectly.
[84] This agrees only with Seray Ahmet III 3464, not Leiden or. 1031.

We can also compare the citations in Tabriz 3484 to those in Leiden or. 1031 for the *Phaenomena* and *On Risings and Settings*. For the latter text, we see no overlap,[85] but there is some for the former. It is striking, however, how different the referential scholia look for the *Phaenomena* between Leiden or. 1031 and al-Ṭūsī's edition. Only twelve overlap between the two versions, even when accounting for different proposition numbering schemes between al-Ṭūsī's editions and the earlier texts. They are highlighted below.

Appears on[86]	**Leiden or. 1031**		**Cites**	**Tabriz 3484**	
	Folio	**Referential Scholia**		**Referential Scholia**	**Pages**
Phaen.intro	76a	ا من ا من الاكر لىاودسىوس	Sph.1.prop.1		
Phaen.intro	76b	ك من ا من الاكر لىاودسىوس	Sph.1.prop.20		
Phaen.intro	76b	ٮ من كتاب اوطولوفس فى الكره المتحركه ه	MS.prop.2	ب من الكرة المتحركة	p.124
Phaen.intro	76b	ٮٮ من الكره المتحركة	MS.prop.12		
			Sph.1.prop.21	كا ا (؟) من الاكر	p.125
			Sph.1.prop.21	د ب من الاكر	p.125
Phaen.prop.2	77b	حاسىه ط من ٮ من الاكر	Sph.2.prop.9	ط ب من الاكر	p.125
			Sph.2.prop.9	يط ب من الاكر	p.125
Phaen.prop.2	77b	حاسىه ه من ٮ من الاكر	Sph.2.prop.5	ه ب من الاكر	p.126
Phaen.prop.2	78a	ٮو من ا من الاكر	Sph.1.prop.18		
Phaen.prop.2[87]			Sph.1.prop.14	يد ا من الاكر	p.126
Phaen.prop.2[88]			Sph.1.prop.16	يو ا من الاكر	p.126
Phaen.prop.2	78a	ٮه من ٮ من الاكر	Sph.2.prop.15	يه ب من الاكر	p.127
Phaen.prop.2			Sph.1.prop.16	يو ا من الاكر	p.127

[85] Tabriz 3484 shows only *On Risings and Settings* 1.prop.12 citing its own earlier proposition 1.prop.3. In Leiden or. 1031, referential scholia in this text cut off after 1.prop.6.
[86] The proposition numbering used here is the one which appears in Leiden or. 1031. Disagreements with al-Ṭūsī's proposition numbering will be noted in the footnotes.
[87] This is the start of al-Ṭūsī proposition 3.
[88] This is the start of al-Ṭūsī proposition 4.

Phaen.prop.2	78a	ر من ب من الاكر	Sph.2.prop.7	ز ب من الاكر	p.127
Phaen.prop.2	78b	ىد من ب من الاكر	Sph.2.prop.14	يد ب من الاكر	p.127
Phaen.prop.2			Sph.2.prop.14	يد ب من الاكر	p.127
Phaen.prop.3[89]			MS.prop.1	ا من الكرة المتحركة (؟)	p.127
Phaen.prop.3	78b	ر من الكره المتحركه	MS.prop.7	ز من الكرة المتحركة	p.127
Phaen.prop.4[90]			Sph.2.prop.15	يه ب من الاكر	p.128
Phaen.prop.4			Sph.2.prop.15	يه ب من الاكر	p.128
Phaen.prop.4	79a	ب من الكره المتحركه	MS.prop.2	ب من الكرة المتحركة	p.128
Phaen.prop.4			Sph.2.prop.15	يه ب من الاكر	p.128
Phaen.prop.4			Sph.2.prop.14	يد ب من الاكر	p.128
Phaen.prop.4			MS.prop.2	ب من الكرة المتحركة	p.128
Phaen.prop.5[91]			Sph.2.prop.15	يه ب من الاكر	p.128
Phaen.prop.5			Sph.2.prop.14	يد ب من الاكر	p.128
Phaen.prop.5			MS.prop.2	ب من الكرة المتحركة	p.128
Phaen.prop.5			Sph.2.prop.15	يه ب من الاكر	p.128
Phaen.prop.5			Sph.2.prop.14	يد ب من الاكر	p.128
Phaen.prop.5			MS.prop.2	ب من الكرة المتحركة	p.128
Phaen.prop.6[92]			Sph.1.prop.21	كا ا من الاكر	p.129
Phaen.prop.6			Sph.2.prop.4, Sph.2.prop.5	د ه من ب من الاكر	p.129
Phaen.prop.6			Sph.2.prop.5	ه من ب من الاكر	p.129
Phaen.prop.6			Sph.2.prop.9	ط ب من الاكر	p.129
Phaen.prop.6			Sph.2.prop.17	يز ب من الاكر	p.129
Phaen.prop.6	80b	ىط من ب من الاكر	Sph.2.prop.19		
Phaen.prop.7[93]			Sph.2.prop.14	يد ب من الاكر	p.131

[89] This is the start of al-Ṭūsī proposition 5.
[90] This is the start of al-Ṭūsī proposition 6.
[91] This is the start of al-Ṭūsī proposition 7.
[92] This is the start of al-Ṭūsī proposition 8.
[93] This is the start of al-Ṭūsī proposition 9.

Phaen.prop.7	81b	و من هذا الكتاب	Phaen.prop.6		
Phaen.prop.8[94]			Sph.2.prop.14	يد ب من الاكر	p.132
			Sph.2.prop.17	يز ب من الاكر	p.132
Phaen.prop.8	82b	يب من ب من الاكر	Sph.2.prop.12		
Phaen.prop.8	83a	يح من ب من الاكر	Sph.2.prop.18	يح ب من الاكر	p.132
Phaen.prop.9[95]			Sph.2.prop.16	يو ب من الاكر	p.133
Phaen.prop.9	83b	يط من ب من الاكر…	Sph.2.prop.19	يط ب من الاكر	p.133
Phaen.prop.9	83b	…ومن صدر هذا الكتاب	Phaen.intro		
Phaen.prop.9			MS.prop.2	ب من الكرة المتحركة	p.133
Phaen.prop.9			Sph.2.prop.19, MS.defs	يط ب من الاكر ومصادرة[96] الاكر المتحركة	p.133
Phaen.prop.10[97]	84b	ط من هذا الكتاب	Phaen.prop.9		
Phaen.prop.11[98]	86b	و من هذا لكتاب	Phaen.prop.6		
Phaen.prop.11	87b	حـ من حـ من الاكر	Sph.3.prop.3		
Phaen.prop.11	91a	ح من ح من الاكر	Sph.3.prop.3		
Phaen.prop.12[99]			Sph.2.prop.15	يه ب من الاكر	p.135
Phaen.prop.12			Sph.2.prop.14	يد ب من الاكر	p.135
Phaen.prop.12			MS.prop.2	ب من الكرة المتحركة	p.135
Phaen.prop.12			Phaen.prop.7, Phaen.prop.6	ط ثم ح من هذا الكتاب[100]	p.136
Phaen.prop.12			Sph.2.prop.14	يد ب من الاكر	p.136
Phaen.prop.12			Sph.2.prop.17	يز ب من الاكر	p.136
Phaen.prop.12			Sph.2.prop.18	يح ب من الاكر	p.136

94 This is the start of al-Ṭūsī proposition 10.
95 This is the start of al-Ṭūsī proposition 11.
96 In al-Ṭūsī's edition, Autolycus's definitions appear under the header "صدر" – these are starting-points of the text.
97 This would be al-Ṭūsī proposition 12, though al-Ṭūsī follows a different recension than what is seen in Leiden or. 1031.
98 This is the start of al-Ṭūsī proposition 13.
99 This is the start of al-Ṭūsī proposition 14.
100 Note that al-Ṭūsī's *Phaenomena* propositions 8 and 9 would align with Greek propositions 6 and 7.

Phaen.prop.12			MS.prop.2	ب من الكرة المتحركة	p.136
Phaen.prop.12			Sph.2.prop.14	يد ب من الاكر	p.136
Phaen.prop.12	85a	ح من حـ من الاكر	Sph.3.prop.8		
Phaen.prop.13[101]	91b	ىب من بعد الكتاب	Phaen.prop.12		
Phaen.prop.13	92a	يا من هذا الكتاب	Phaen.prop.11		
Phaen.prop.14[102]			Sph.2.prop.15	يه ب من الاكر	p.138
Phaen.prop.14			Sph.2.prop.7	ز ب من الاكر	p.138
Phaen.prop.14			Sph.1.prop.16	يو ا من الاكر	p.138
Phaen.prop.14			Sph.3.prop.1	ا ج من الاكر	p.138
Phaen.prop.14			Sph.2.prop.5	ه ب من الاكر	p.138
Phaen.prop.14			Sph.1.prop.17	يز ا من الاكر	p.138
Phaen.prop.14			Sph.1.prop.16	يو ا من الاكر	p.138
Phaen.prop.14			Sph.3.prop.1	ا ج من الاكر	p.138
Phaen.prop.14			MS.prop.1	ا من الكرة المتحركة	p.138
Phaen.prop.14			Sph.2.prop.14		
Phaen.prop.14	92b	اخر سكل ىد من ب من الاكر	Sph.2.prop.14	يد ب من الاكر	p.139
Phaen.prop.14			MS.prop.2	ب من الكرة المتحركة	p.139
Phaen.prop.14			Phaen.prop.11	يج من هذا الكتاب[103]	p.140
Phaen.prop.15[104]			MS.prop.1	ا من الكرة المتحركة	p.140
Phaen.prop.15			MS.defs	صدر الكرة المتحركة	p.141
Phaen.prop.15	94a	احر شكل ىد من ب من الاكر	Sph.2.prop.14		
Phaen.prop.15	95a	احر ىب ىحـ من هذا الكتاب	Phaen.prop.12, Phaen.prop.13	يد من الكتاب[105]	p.141
Phaen.prop.15			Sph.2.prop.20	ك ب من الاكر	p.141

[101] This is the start of al-Ṭūsī proposition 15.
[102] This is the start of al-Ṭūsī proposition 16.
[103] Note here that al-Ṭūsī's *Phaenomena* proposition 13 would align with Greek proposition 11.
[104] This is the start of al-Ṭūsī proposition 18. Al-Ṭūsī proposition 17 is not in Leiden or. 1031, nor does it have any referential scholia in Tabriz 3484.
[105] Note here that al-Ṭūsī's *Phaenomena* proposition 14 would align with Greek proposition 12.

Phaen.prop.17[106]			Sph.2.prop.17	يز ب من الاكر	p.141
Phaen.prop.17			Sph.2.prop.19	يط ب من الاكر	p.141
Phaen.prop.17			MS.prop.2	ب من الكرة المتحركة	p.141
Phaen.prop.18[107]			Phaen.prop.14	يو من الكتاب[108]	p.144
Phaen.prop.18			Phaen.prop.17	يط من الكتاب[109]	p.144
Phaen.prop.18			Sph.2.prop.14	يد ب من الاكر	p.144
Phaen.prop.18			Phaen.prop.14	يح من الكتاب[110]	p.144
Phaen.prop.18			Phaen.prop.15	يط من الكتاب[111]	p.144
Phaen.prop.19[112]			Phaen.prop.16	ك من الكتاب[113]	p.144
Phaen.prop.19			Phaen.prop.15	يط من الكتاب[114]	p.144
Phaen.prop.20[115]			Phaen.prop.14	يو من الكتاب[116]	p.145
Phaen.prop.20			Phaen.prop.15	يط من الكتاب[117]	p.145
Phaen.prop.20			Phaen.prop.16	ك من الكتاب[118]	p.145

Table 9.9: Agreement of referential scholia between Leiden or. 1031 and Tabriz 3484

Since al-Ṭūsī has been shown to have worked with a different Arabic version of the *Phaenomena* than the one presented in Leiden or. 1031, this would suggest either that the referential scholia on these two different versions also diverged significantly or that the ones on al-Ṭūsī's edition were produced separately for the new edition. The al-Ṭūsī scholia do follow his proposition numbering system, though

[106] This is the start of al-Ṭūsī proposition 19.
[107] This is the start of al-Ṭūsī proposition 20.
[108] Note here that al-Ṭūsī's *Phaenomena* proposition 16 would align with Greek proposition 14.
[109] Note here that al-Ṭūsī's Phaenomena proposition 19 would align with Greek proposition 15.
[110] Note here that al-Ṭūsī's *Phaenomena* proposition 18 would align with Greek proposition 14 alternate.
[111] Note here that al-Ṭūsī's *Phaenomena* proposition 19 would align with Greek proposition 15.
[112] This is the start of al-Ṭūsī proposition 21.
[113] Note here that al-Ṭūsī's *Phaenomena* proposition 20 would align with Greek proposition 16.
[114] Note here that al-Ṭūsī's *Phaenomena* proposition 19 would align with Greek proposition 15.
[115] This is the start of al-Ṭūsī proposition 22.
[116] Note here that al-Ṭūsī's *Phaenomena* proposition 16 would align with Greek proposition 14.
[117] Note here that al-Ṭūsī's *Phaenomena* proposition 19 would align with Greek proposition 15.
[118] Note here that al-Ṭūsī's *Phaenomena* proposition 20 would align with Greek proposition 16.

since the editor mentioned finding copies of the text with 23 and 25 propositions they may still have their origins in the sources he used rather than in his own edition.

It should be noted, however, that several of these referential scholia appear not on the main proposition itself, but on al-Ṭūsī's comment to that proposition, showing that at least some of them were the result of active study with al-Ṭūsī's new edition. Such is the case, for example, for al-Ṭūsī's first comment on his proposition 16 of the *Phaenomena* – a scholion on it refers back to proposition 13 (يجـ من هذا الكتاب).[119]

The manuscript Tabriz 3484 has been examined here as a witness relatively close to al-Ṭūsī's original texts, but these referential scholia do persist in other codices of the edition. In the Bodleian manuscript Arch. Seld. A. 45, for instance, we find the *Sphaerica* citing the *Elements* with the same formula "[proposition numeral] [book numeral] ق."[120] *On the Moving Sphere* cites the *Sphaerica*;[121] the *Data* cites the *Elements*;[122] the *Optics* also cites the *Elements*;[123] *On Risings and Settings* cites itself;[124] the *Phaenomena* cites the *Sphaerica*,[125] *On the Moving Sphere*,[126] and itself;[127] *On Days and Nights* cites the *Sphaerica*[128] and the *Phaenomena*.[129] The manuscripts Bodleian Arch. Seld. A 46 and Bodleian Marsh 709 have these citations as well, and presumably they continue in many other witnesses to al-Ṭūsī's edition.

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[119] Tabriz 3484, p.140.
[120] See for example "د با ق" on fol. 3b.
[121] See for example "ب ب من الاكر" on fol. 21a.
[122] See for example "كب ا ق" on fol. 28a.
[123] See for example "مر ا ق" on fol. 87a.
[124] See for example "حـ من الكتاب" on fol. 106a. This is in fact the same reference as Tabriz 3484: RS.1.prop.12 citing RS.1.prop.3.
[125] See for example "د ب من الاكر" on fol. 117b.
[126] See for example "ب من الكره المتحركه" on fol. 117a.
[127] See for example "ح من الكتاب" on fol. 122b.
[128] See for example "ط ب اكر" on fol. 136b.
[129] See for example "يح من الظاهرات" on fol. 129b.

5. Interpretation of Attested Alterations and References

Al-Ṭūsī does not set out a reason for his choice to edit the Middle Books anywhere within his editions. As noted, in the preface to the final text he mentions only that he had decided to set upon editing the educational arrangement. The editor does not take time to extol the value or uses of the Middle Books – rather, he seems to expect that his intended audience would already understand the use of the curriculum.

Al-Ṭūsī was very conscious, however, of how his work on these texts was the latest in a long history. He takes care to note the translators, correctors, and other editors who had a hand in the sources he used for his edition; further, in several works he notes past commentators as well and incorporates their comments into his new text. Al-Ṭūsī, then, in some cases, follows by adding his own contribution to the conversation. The story of the Middle Books that emerges from al-Ṭūsī's historical scholarship identifies its translators into Arabic as Isḥāq ibn Ḥunayn, Qusṭā ibn Lūqā, and Thābit ibn Qurra. One patron is named, and this occurs in the *Sphaerica*, near the head of the cycle of texts: Abū l-ʿAbbās Aḥmad ibn Muʿtaṣim bi-llāh. Several correctors are named: Thābit ibn Qurra (again), al-Kindī, Māhānī, al-Harawī, al-Amīr Abū Naṣr Manṣūr ibn ʿIrāq. Several commentators and other individuals also appear throughout the curriculum: al-Nasawi, al-Sizjī, "al-Tabrīzī" (al-Nayrīzī), Abū Sahl al-Qūhī, and so on.

The long history of this curriculum as an ordered one, in which earlier propositions were considered to support deductions in later ones, also left its mark on al-Ṭūsī's edition. Intra-corpus citations of Middle Books works occur more frequently in this edition than what was seen in the earlier Arabic translations or Greek texts. Certainly referential scholia abounded through different stages of the curricula's transmissions, but it is in al-Ṭūsī's edition where such citations are more often woven into the main texts. As discussed, this occurs most frequently in al-Ṭūsī's edition of the *Phaenomena*, perhaps because he made use of an exterior commentary as an aid to approaching the problematic text.

Chapter 8 discussed how many of al-Ṭūsī's editions circulate with dates in the colophon indicating when the edition was produced. This chapter, meanwhile, has introduced evidence that shows al-Ṭūsī's editorial procedure did not necessarily have a finite end. Rather, in the preface to the *Phaenomena* for example he expressly admits his dissatisfaction with the sources available to him and states his intention to improve the text should he acquire better manuscripts in the future. Meanwhile, two separate families of manuscripts seem to show two different drafts of his *Data*, where the earlier draft accumulated potential material to be added as marginalia, and the later draft shows this worked into the text. So al-Ṭūsī approached the creation of his editions as more of an ongoing process, and it is quite possible that this continuing work intersected with teaching or other scholarship he devoted his time to.

For al-Ṭūsī's editorial process in the Middle Books, we see that it indeed takes much the same form as what he described for his *Elements* and *Almagest*. Al-Ṭūsī's editions are not aimed at preserving and maintaining the original words of these texts' authors (or at presenting a reasonably faithful rendition of them in Arabic). The exact words of the Middle Books were not what al-Ṭūsī saw as the important substance of these works. Rather, the editor liberally rewrites the text. He speaks to this in his prefaces to the *Elements* and *Almagest*: he will streamline the texts, eliminate repetition, and make them clearer. He undoubtedly does so in the Middle Books as well, as samples of his proofs in this chapter have shown.

But further, al-Ṭūsī evidently does not see the logic of the proof as immutable either. His edition does not limit itself to presenting the same received proofs in new words – there are cases where instead the editor restructures the logic of the proof as well. His edition transmits mathematical arguments, but he finds it permissible to intervene in these too where he deems it appropriate, acting as a participant in these texts' transmissions who is qualified to dialogue with, correct, and improve on his predecessors. Many of al-Ṭūsī's original contributions are separated out as comments, but these restructured proofs are contributions in their own way. At the same time, al-Ṭūsī occasionally encountered a proof which he must have considered used often enough, or attached to significant enough authorities, that he instead chose to report on it and its argument even where he saw that argument as unsatisfactory. The example of the comment to his *Data* 64 stands out.

There are features, however, which the editor takes a conscious approach to maintaining. As noted, he is very aware that the Middle Books has a long tradition, not just one of being copied and passed down, but of being studied and commented on and referenced. Scholars were accustomed, he says in the preface to his *Almagest*, to discussing and citing particular propositions and diagrams. The referential scholia we have seen throughout the transmissions of these curricula show that they, too, received frequent reference, at the very least in the course of an individual's work with the Middle Books. These kinds of scholarly practices plausibly motivated al-Ṭūsī to maintain the overall structure he received for these texts. Where his sources disagreed on proposition counts, or on whether something should be one proposition or two, he indicates this. His editions were clearly intended to be useful in the context of an ongoing tradition of teaching or studying astronomy with the Middle Books.

CONCLUSION

This dissertation has traced the paths of the Little Astronomy and Middle Books through many long centuries. It is helpful to synthesize the results which have emerged in these chapters into a more continuous whole. We will first note the important conclusions to be taken from the historical chapters – that is, chapters 1, 3, and 5-8. With this long history of transmission and use set out, we will then discuss how the findings in the philological chapters – that is, chapters 2, 4, and 9 – accord with this picture and offer further insights into it.

Chapter 1 considered the question of the so-called Little Astronomy, a group of Greek mathematical and astronomical texts generally accepted by modern scholars to have served some extent of a didactic purpose in late antiquity, the details of which however have been contested. An aim of the chapter was to disentangle what could be said about the Greek Little Astronomy from claims that have been retrojected onto it from reports about the Arabic Middle Books. But the key result of chapter 1's investigation is that some kind of astronomical curriculum comprising many of the texts in question (and this number likely varied over time) did indeed exist, that it had its origins perhaps as far back as the second century CE, and that this was an ordered grouping that proceeded from more general treatises to more particular ones along Aristotelian-inspired lines. Proposition-based texts in Greek geometry were naturally structured so that later propositions could make use of results demonstrated in earlier ones. Chapter 1 found evidence from the orders in manuscripts, from reports by contemporary scholars, and from the referential scholia to show that the curriculum of the Little Astronomy was one which proceeded through its works in a similar manner. The Little Astronomy's prior propositions were being used to understand the arguments of subsequent ones – and this despite the fact that the component texts were originally produced in different contexts and a different order.

So Greek late antiquity saw the circulation of an ordered grouping of texts used since at least the second century CE to study astronomy. The ninth century saw their translation into Arabic – and some made their way into Syriac as well, and perhaps earlier, but what survives of this evidence is limited. Chapter 3 set out how the texts were translated by several different translators, some of them several different times, and how many of them went through further corrections shortly afterwards as well. But a crucial takeaway of that chapter is how very rapidly, already in the lifetimes of their translators, there emerges something called the Middle Books, explicitly described as what was necessary to read before the *Almagest*. Later sources make it clear that the component works of the Little Astronomy comprised the Middle Books.

It is an obvious statement to make, but the *Almagest* was written in Greek, it was produced in a context where the way in which one did mathematics was largely using the methods of Greek geometry. Ptolemy himself notes in his introductory chapters the expectation that his reader will not be unfamiliar with astronomical studies.[1] Studying Ptolemy outside of this Greek context presents an immediate challenge. In light of this, it is not so surprising that in Arabic the refrain "necessary to read before the *Almagest*" starts becoming attached to the *Elements* and to the group of texts which were indeed used to study astronomy in Greek. The report from Galen showed that a curriculum which was or evolved into the Little Astronomy probably already existed before Ptolemy wrote his *Almagest*. The Greek curriculum did not have its origins in preparing a student for the *Almagest*, but the Arabic Middle Books took the extant Little Astronomy and leveraged it to address the need for a preparatory arrangement of Greek geometry and astronomy that would make the *Almagest* more accessible.

Part III of the dissertation – chapters 5, 6, and 7 – addresses what happened to these curricula after the third / ninth century and leading up to the seventh / thirteenth century when the Middle Books

[1] Heiberg (1898) 8.

would receive an influential new edition. This study has demonstrated that in the interval between, the Arabic tradition speaks to a continuing and varied engagement with the Middle Books, and the Greek tradition speaks to what seems to be a stark absence of engagement with the Little Astronomy. The component texts did not fully cease to be copied in Greek, but that is about as much as can be said for the Little Astronomy. The interval between also saw translations into Latin and Hebrew, and these endeavors similarly speak to the ongoing use of the curriculum in Arabic and the comparative lack thereof of the curriculum in Greek: nearly all of these translations were produced from Arabic, and what did enter Latin from Greek were rather the texts which saw circulation outside the Little Astronomy. In the Greek-speaking world, it is not until the end of the thirteenth or beginning of the fourteenth century that we find an individual who speaks of the preparations he undertook before setting out to write an epitome of the *Almagest* as involving the study of many texts which we can recognize as members of the Little Astronomy / Middle Books. This individual, Metochites, claims his own teacher Bryennios had learned astronomy from a man who had been to Persia. It is quite possible that the renewed use of these curricular texts seen here was influenced less by a continuing use of the Little Astronomy in the Byzantine world (for which we find little evidence) and more by the very widespread use of the Middle Books in the Islamicate world. Metochites's study of these texts to support work with the *Almagest*, and, moreover, his interest in producing his own epitome of the *Almagest*, have interesting parallels with ongoing activities in the seventh / thirteenth century Islamicate world.

The entangled study and editing and teaching of the *Elements*, the Middle Books, and the *Almagest* in Arabic was set out by chapter 8. The seventh / thirteenth century was a period of significant political change in the Islamicate world, as the campaigns of Hülagü Khan amassed multiple different political entities under the new reign of the Ilkhanate. But the astronomical scholarly activities under discussion persisted, even as two of their notable actors, Naṣīr al-Dīn al-Ṭūsī and Muḥyī al-Dīn al-Maghribī, were at or caught in the center of the khan's campaigns. Since the astral sciences found a strong supporter in Hülagü Khan, the rise of the Ilkhanate in fact led to the creation of a new astronomical center in Maragha with the founding of the Maragha Observatory. Chapter 8 demonstrated that Maragha was not the impetus for all the many new editions of Greek mathematical and astronomical works in this period – al-Ṭūsī's editing projects and perhaps at least one of al-Maghribī's were begun before, in the Nizari Isma'ili state and (probably) Ayyubid Damascus, respectively. The choices al-Ṭūsī reports making in his own edition speak to the ways in which his contemporaries and scholars before him were already accustomed to working with the *Elements*, the Middle Books, and the *Almagest*. But Maragha concentrated them and astronomers like them in one setting, which then became the destination for many students, one example of which was the famed Syriac scholar Bar Hebraeus.

The Little Astronomy and the Middle Books were undoubtedly related but should not be taken as identical, or as created to serve identical goals. Nor should either curriculum be seen as static. The Little Astronomy's core works appear to have been the *Sphaerica*, *On the Moving Sphere*, the *Phaenomena*, *On Habitations*, *On Days and Nights*, *On Sizes and Distances*, *On Risings and Settings*, and the *Anaphoricus*, but the grouping accumulated other works over time. The *Optics* was perhaps incorporated later, as preliminary material for the *Phaenomena*, whose preface (also perhaps a later addition) references it. The *Optics* is likely responsible for sometimes drawing its related text the *Catoptrics* into manuscripts of the Little Astronomy with it, but the latter did not become a true part of the curriculum. By the ninth century the *Data* seems to have been drawn into the grouping as well, used as supplementary material that supported geometrical studies in general.

Meanwhile, the Middle Books seem to have comprised the same core nine works early on, and the *Data* is very quickly added to the head of this collection. But to this core grouping are added other works, both originally Greek and originally Arabic, and some more consistently than others. The *Spherics* of Menelaus appears frequently, as do works like Thābit's which grapple with the sector theorem – a fundamental theorem of spherical geometry and one which features in the *Almagest*. Other works by Thābit and his patrons the Banū Mūsā see occasional inclusion, as do works attributed to Archimedes and occasional works by later Arabic authors. In the seventh / thirteenth century al-Ṭūsī's edition of the Middle Books comprised the same core ten works plus Menelaus's *Spherics*; the Archimedean *Lemmata*, *On the Sphere and Cylinder*, and *On the Measurement of the Circle*; Thābit's *Assumptions*; and the Banū Mūsā's *Book of Knowledge*. But it is clear from these chapters that this was just one instance of many slightly varied Middle Books seen over the centuries, all of which shared a certain core but which fluctuated in other inclusions.

The philological chapters of this study offer further details on how various scholars throughout history, named and unnamed, interacted with these treatises and with the curricula overall.

The Greek story is largely one of addition, especially of preliminary material. We find evidence of content being incorporated as the "clearer" proof – these are deliberate alterations that would make sense in a didactic context. We see our unknown editors are not beholden to some sort of static received text, but do intentionally contribute to it in ways which they perceive make it more useful or address its gaps. Further, within the Little Astronomy is preserved a version of the *Optics*, recension B, which seems to be older than the version which circulated outside the curriculum (recension A). It is the outside recension which shows more active engagement and alteration, such as alternate proofs, suggesting that the version which became attached to the Little Astronomy partially fossilized – it was perceived as a useful inclusion, but the more active editorial and mathematical interventions were geared towards the treatises on spherical geometry, the actual subject of the Little Astronomy.

We can try in our study of alterations to the Little Astronomy to focus on material introduced before the ninth century, but the fact that most of our extant manuscripts date from the thirteenth century makes it difficult to fully avoid later variants. Comparing the Arabic evidence with the Greek shows that a notable amount of material is absent from the Arabic, and the immediate argument this fact suggests is the argument that the Arabic translated older or otherwise different versions of these texts than the ones which survive today in Greek.

But the Arabic does also show additions of material, or combinations from different versions or from scholia, which seems to show an active attempt to reckon with what varied material they were finding in their sources. Propositions do not appear to have been actively excised. The ways in which alternate proofs are introduced between the Greek and the Arabic offers an interesting comparison. In the Greek, frequently we simply see "alternatively" (ἄλλως) which has a parallel in the Arabic "in another way" (على جهة أخرى). But the Greek also sometimes explicitly identifies the alternate proof as "clearer" (σαφεστέρα), implying a particular reasoning behind the choice to include it. Meanwhile in the Arabic we sometimes see "in another copy" (فى نسخة أخرى), which suggests more of a collation effort. Arabic scholars were working with a tradition received out of Greek which already circulated in several versions, and which saw several versions through the subsequent efforts of different correctors. It is perhaps not so surprising to see more of an active collation effort in this period of the Arabic versus in Greek late antiquity.

One of the other ways in which Arabic scholars worked with these texts, conversely, seems to have been aimed at presenting the texts more logically and comprehensively. So a proposition is introduced to serve as the converse of another, so definitions are added and rearranged to agree with the order in which they are encountered in the propositions.

These various editorial activities, however, are not necessarily connected with the Middle Books as a unit; they could just as easily have been applied to the individual texts more generally. Historically the Middle Books seem to have developed and seen use early on, but the deliberate alterations at this stage cannot be clearly linked to this. While a pattern of intertextual citation in the form of the referential scholia may have been introduced with the translations from the Greek, such a practice of referring to particular propositions by particular numbers apparently did not disincentivize alterations which resulted in proposition numbers changing. The Middle Books were, to differing degrees, in a more unstable structural state in the period after their translation into Arabic.

Fast-forward to the seventh / thirteenth century. The Middle Books have seen circulation and use across the Islamicate world – with this wide circulation of hand-copied texts, it is unsurprising that the manuscripts present many of them in slightly different versions, or with slightly different proposition counts. But in some cases there appear to have been efforts to maintain something more consistent, as suggested by Bodl. Thurston 11's list of Middle Books together with the count of their propositions, or the occasional practice by copyists to indicate in the colophons how many propositions their texts contained.

Chapter 8 introduced some of the statements al-Ṭūsī made on his editions of the *Elements* and *Almagest*, which proved similarly relevant in his edition of the Middle Books, examined in chapter 9. The kinds of alterations that appear in his edition of the Middle Books and some of the comments he makes on these texts show that he was very consciously working within a long and continuing tradition. He structured his editions in ways so that they might be entered into the ongoing practices of studying these texts, of having discussions on particular points within them, of commenting on them or otherwise referencing them in one's own writing. It seems he takes care to introduce proofs that circulated in connection with these texts, even if only to comment on how the proof in question was not satisfactory. Al-Ṭūsī was very much inserting his editions into an ongoing conversation, and himself as an active and competent contributor to the tradition.

Nor did the editor's involvement with these texts cease after he put his editions into circulation. The hints of improving drafts of the *Data* along with al-Ṭūsī's stated intention to improve the edition of the *Phaenomena* should he find a better text show that his editions continued to be a work in progress. And though most of his editions were first produced during his time in the Nizari Isma'ili state, they likely saw continuing work and study and copying during his time in Maragha, which as the contemporary astronomical center probably played some role in how widely copied the editions of al-Ṭūsī subsequently came to be.

The story of the Middle Books hardly ends with al-Ṭūsī. These works continue to circulate and see further transmission and use. Al-Ṭūsī's edition of the *Sphaerica* comes to be translated into Persian.[2] As noted, the practice of studying astronomy via the Middle Books and the *Almagest* in the Islamicate world may have had some influence on astronomical endeavors in the Byzantine world at the end of the thirteenth century. And translations were being produced into Hebrew, also at the end of the thirteenth century. But it is fitting to end this study with the one known historical figure whose name is unquestionably attached to an edition of the whole of the astronomical curriculum.

[2] As do his editions of the *Elements* and *Almagest*. See Rosenfeld and Ihsanoğlu (2003) 212-213, 215.

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