CHAPTER 4

Greco-Roman Sundials: Precision and Displacement

Alexander Jones

In the Greco-Roman world, among the various means known to have existed for determining times of day with any degree of precision, stationary sundials (i.e., sundials installed in a fixed location and with fixed orientation) were by far the most common. The Topoi Excellence Cluster project on Ancient Sundials (henceforth BSDP) has estimated that between 550 and 600 Greco-Roman stationary and portable sundials are extant.[1] Of these, fewer than thirty are portable sundials, while the remains of water clocks—whether of the clepsydra variety that showed the hour as the level of water against a column of marks spaced according to the seasonal variation of day or night, or the more mechanically sophisticated variety that employed a display dial—are comparably scarce.[2] Admittedly, these figures in part reflect differences in the survival rates of ancient artifacts determined by their composition. Most portable sundials, and some clepsydras and components of mechanical water clocks, were made of metal (typically bronze), which was generally melted down and reused when the original object was no longer wanted. Stationary sundials were sculpted from blocks of stone, except for their metal gnomons (which are almost invariably missing now), and even in subsequent repurposing—e.g., as building blocks or filler—they often preserved at least part of their dial surfaces. Nevertheless, we may be confident that when a Greek or Roman of the Hellenistic or Roman period wanted to know the time of day, he or she would most probably have consulted a stationary sundial.

This chapter considers two distinct but related questions concerning the performance of Greco-Roman stationary sundials. The first question is, can the rarity in Greco-Roman textual sources of time specifications to a precision finer than whole hours be attributed to limitations in the ability of sundials to

1 Graßhoff et al. 2016. For the purposes of this paper, Egyptian time-keeping devices that represent pre-Ptolemaic traditions are excluded even if dating from the Greco-Roman period.

2 Schaldach 2016, 91 (inventory of portable sundials), 64–65 (clepsydras), and 81–83 (mechanical water clocks).

display more refined times? In all the varieties of Greco-Roman time-keeping devices, the indicator of time's passage (shadow point, spot of sunlight, water level, moving pointer or revolving dial) had a continuous motion, in the course of which it successively crossed a series of marks or lines corresponding to the moments demarcating the beginnings and endings of seasonal hours. Intermediate marks do not occur. The time read off the device could be thought of as a discrete entity, either a block of time whose identity and character are defined primarily by its position in a sequence of such blocks ("during the *n*th hour"), or a moment subject to a certain tolerance ("at the *n*th hour" or "at the end of the *n*th hour"). However, the Greco-Roman definition of seasonal hours was based on a somewhat simplified model, according to which time was continuous and measurable by the uniform circular revolution of the Sun in the sky through the course of each day. This definition might have encouraged people to think of the output of a time-keeping device as a measurement of elapsed time, such that the hour marks or lines function as a scale of units along a continuum that also allows for intermediate time specifications expressed in hours and fractions of hours. Since we do not see this happening outside of scientific contexts, it is worth investigating what role sundial design might have had in preserving the whole seasonal hour as the smallest quantitative unit of time specification.

My second question is actually twofold. First, how often were sundials installed in localities for which they were not designed? Second, how would such displacements have been apparent to a user, and how would they have affected the accuracy of times displayed on them?

The consequences of geographical displacement of sundials has been treated before in the technical literature on sundials, but still are subject to misconceptions. The danger here is of supposing that, since the lengths of seasonal hours vary significantly with terrestrial latitude as well as with the stage of the year, a displaced sundial will uniformly stretch or diminish the displayed time intervals. What actually happens is more complicated, as some visual illustrations will make clear.

1 Precision of Time Specifications in Ancient Scientific Contexts

In the context of observational astronomy, refined time specifications can be found as early as the 7th century BCE in Babylonia.[3] These observed times, presumed to have been made using water clocks, are expressed in units called

3 See, e.g., Steele's contribution to the present volume.

UŠ, equivalent to 1/360 of a night and day or 4 minutes in our time metrology, and *bēru*, equivalent to 30 UŠ. Thus Babylonian records of lunar and solar eclipses typically included measurements of the time since sunset or sunrise of the beginning of the eclipse, as well as the time intervals taken for the eclipse, either as a whole or divided into stages such as onset, totality, and clearing. The earlier records, up to about the first quarter of the 6th century, typically gave a precision of 5 UŠ (i.e., 20 minutes), later refined to single UŠ, but typical errors remained quite large through the more than six centuries of Babylonian eclipse records, even for short time intervals.[4] Besides eclipse timings, time measurements in UŠ were recorded for a variety of astronomical timings, especially for intervals separating sunset or sunrise from the setting or rising of the Moon and planets when near conjunction or opposition.

Although the Babylonian unit UŠ was carried over into Greek astronomy under the name χρόνος (literally "time period," but usually translated "time degree"), the preserved Greek observational records that specify times with any precision at all employ either seasonal or equinoctial hours. The reports from before Ptolemy's time in Ptolemy's *Almagest* use seasonal hours, sometimes accompanied by a qualitative refinement such as "at the beginning," "at the end," "at the middle," or simply "during." A few give fractions in addition to the whole number of hours since sunset or sunrise: 1/2 (Timocharis, 283 BCE, ed. Heiberg 2.29), 1/3 (anonymous, 200 BCE, ed. Heiberg 1.345, and Hipparchus, 127 BCE, ed. Heiberg 1.374), 2/3 (anonymous, 200 BCE, ed. Heiberg 1.346, and Hipparchus, 128 BCE, ed. Heiberg 1.363). As transmitted by Ptolemy, the reports give no indication of how the times were determined.

Ptolemy's own observation reports (including one unattributed eclipse observation that is likely to be his) use equinoctial hours relative to noon or midnight. Ptolemy's times are usually whole numbers of equinoctial hours, but we also have some fractions, including 1/2 (139 CE, ed. Heiberg 2.283), 1/4 (139 CE, ed. Heiberg 1.362), 3/4 (133 CE, ed. Heiberg 1.314, and 138 CE, ed. Heiberg 2.306), 3/5 (the anonymous eclipse, 125 CE, ed. Heiberg 1.329), and 5/6 (135 CE, ed. Heiberg 1.408). Most of these more precise times, if not all, were ostensibly determined by calculation from the culminating degree of the equator on Ptolemy's observational armillary instrument. The fact that Ptolemy consistently reports this figure to a precision of single degrees—effectively 1 UŠ—even though the times always come out as whole equinoctial hours or hours with simple fractions, possibly indicates that Ptolemy, in actuality, calculated the culminating degrees from the times in hours.

4 Steele 2000, 57–66.

We have few Greek observation reports from the time following Ptolemy—still fewer with precise recorded times—and not all of these turn out to be genuine evidence of observational time precision. In his commentary on Book 6 of the *Almagest*, Theon of Alexandria provides worked examples of how to calculate the circumstances of a lunar eclipse using the *Almagest*'s tables, and those of a solar eclipse using both the *Almagest* and the Handy Tables, choosing for each kind an eclipse that he claims to have observed ἀσφαλέστατα ("most securely"), namely the total lunar eclipse of 364 CE, November 25/26, and the partial solar eclipse of 364 CE, June 16.[5] In both examples, the calculations are preceded by a set of times for the stages of the eclipse. In the case of the solar eclipse, these stages are unambiguously said to have been observed, whereas for the lunar eclipse, Theon gives conflicting indications about whether they were supposed to have been observed or calculated.[6] The times given for the beginning, middle, and end of the solar eclipse are respectively 2 5/6, 3 4/5, and 4 1/2 seasonal hours past noon, whereas the times for the beginning, beginning of totality, end of totality, and end of the lunar eclipse are respectively 14 9/10, 16 17/30, 17 4/15, and 18 3/5 equinoctial hours past noon. Unfortunately, whatever Theon meant his readers to assume about his observational activity, the exact agreement of these times with those that he obtains from the *Almagest* calculations has only one plausible explanation, that they were adjusted or fabricated to produce that very agreement.[7]

5 For the lunar eclipse, the text according to the earliest and most reliable manuscript, Laur. plut. 28.18, f. 231r, is ποιησάμεθα δὲ τὴν ψηφοφορίαν ἐπὶ τῆς ἀσφαλέστατα ἡμῖν τετηρημένης ἐνταῦθα ἐν Ἀλεχανδρείᾳ τῇ πρὸς Αἴγυπτον. ("We have made the computation for the [eclipse] that was observed most securely by us here in Alexandria in Egypt.") The 1538 Basel edition, p. 319, lacks ἡμῖν so that the observation is not expressly attributed to Theon.

6 For the solar eclipse times, Theon writes (Laur. plut. 28.18, f. 242r, 1538 ed. p. 332) ἀσφαλέστατα ἐτηρήσαμεν ("we observed most securely"). His lunar eclipse times (Laur. plut. 28.18, f. 231r, 1538 ed. p. 319), however, are governed by the verb ἐπιλογισάμεθα [*sic* for ἐπελογισάμεθα], "calculated." But at the end of his calculations from the *Almagest*, Theon states that the computed times obtained from the *Almagest* are ἀκολούθως τοῖς κατὰ τὴν τήρησιν γεγενημένοις ἡμῖν τῶν τοιούτων χρόνων ἐπιλογισμοῖς ("in agreement with the calculations made by us of such times on the occasion of the observation"), which must mean times computed *from observational data*.

7 Delambre took the agreement of the ostensibly observed times with the calculations as an indication that the reports were fabricated: "*cette conformité si singulière, qu'on n'obtiendrait aujourd'hui même que par le plus grand des hasards, pourrait faire soupçonner que cette prétendue éclipse est arrangée pour les Tables*." (Delambre 1817, vol. 2, 591 and 594). Theon's solar eclipse has been much discussed in modern scholarship, mostly without awareness that the observation report is suspect; an exception is Mercier (2011, 407), who however mistakenly claims that Theon "never reported his observations as such."

Lastly, among the observation reports believed to have been compiled by the Neoplatonist Heliodorus is a report of an observation of the Moon occulting Saturn on February 21, 503 CE, made by the writer and ὁ φίλτατος ἀδελφός ("my dearest brother," presumably Ammonius), in which the time of the end of occultation is recorded as 5 3/4 seasonal hours (scil. past sunset) obtained ἀπὸ ἀστρολάβου ("from an astrolabe"), which probably means an armillary like Ptolemy's rather than a plane astrolabe, since the plane astrolabe would not function in any straightforward way as an instrument of time measurement at night.[8]

Thus we have good evidence for Greek astronomers making determinations of times of day and night to a precision of a fraction of an hour, though never approaching the 4-minute precision claimed in the Babylonian observation reports. Only Hipparchus's observations of the Moon's elongation from the Sun, which of course were diurnal, could in principle have been made using a sundial. Sundials are more plausible as a source of the birth times in diurnal horoscopes, and—notwithstanding the professions in the astrological literature of extreme time sensitivity, distinguishing horoscopes of people born even a fraction of an hour apart—the birth times in horoscopic documents are invariably given as whole numbers of seasonal hours, at most modified by qualitative expressions such as "at the beginning," but probably never with a quantitative fraction or subunit.[9] Time specifications in civil and administrative contexts too were always by whole numbers of seasonal hours.[10]

2 Reading Fractional Hours on Stationary Sundials

Given the empirical fact that the limit of time precision for non-specialists in the Greco-Roman world was effectively the whole seasonal hour, the question I wish to address now is the speculative one of whether the prevalent sundial technology would have allowed more refined time specifications if people had

8 Jones 2005, 80–83. Since the armillary yields time in time degrees, a time in seasonal hours would have had to be the result of conversion, not direct measurement on the instrument.

9 A possible exception is *P.Oxy.* XII 1476, which gives the time of birth as "10th hour of day completed" followed by "2 degrees": ἡμέρας ὥρ(α) ι/ πεπληρωμένη (*vac.*) μοι() β/. Neugebauer and van Hoesen (1959, 60–61) interpret this as meaning 2 time degrees (i.e., 8 minutes) past the end of the tenth seasonal hour. But such a mixture of time units—moreover, one seasonal, the other constant—makes no sense, and I suspect that the "2 degrees" is a misplaced addition or correction to the astronomical data of the horoscope, not part of the birth time.

10 Remijsen 2007.

wanted them. We will be concerned here not with scientific instrumentation or hypothetical high-precision sundials graduated for fractional hours, but with the kinds of sundial represented by surviving examples, though we can reasonably limit consideration to those that were more carefully executed.

Greco-Roman sundial design originated in a definition of the seasonal hour that was intended to ensure that the duration of all seasonal hours within a single day (or night) was equal. This presumption, that the twelve seasonal hours making up a day or night should be equal in duration, is what made the seasonal hour a unit of time (albeit a seasonally variable unit) and not merely an ordinally-counted partition of a larger interval, and the status of a seasonal hour as a metrical unit is in turn a prerequisite for such expressions as "a third of an hour" to be meaningful.

The definition in question was, naturally, in terms of astronomy. Between sunrise and sunset the Sun was assumed to traverse, at uniform speed, the arc above the horizon of a declination circle on the celestial sphere, so that the seasonal hours of day correspond to equal twelfths of that arc. In terms of the conventional geocentric cosmology, this was a simplification. The proper cosmological assumption was that the fundamental uniform revolution was the one performed by the sphere of the fixed stars, while the Sun superimposes on this uniform revolution its own slower, non-uniform, and oblique revolution around the poles of the ecliptic—in the language of Plato's *Timaeus*, the two revolutions are the motions of the "same" and the "different."[11] In sundial design, however, the Sun is treated as if, during the course of a single day, it moves at a constant rate of right ascension relative to the (uniformly revolving) celestial sphere, with unchanging declination, with a discrete change of declination happening between one day and the next; thus, the small effects of the obliquity of the ecliptic and of solar anomaly on the diurnal change in the Sun's right ascension and declination are disregarded.

We can think of the sky itself, the half of the celestial sphere that is above the horizon, as the prototype of a Greco-Roman sundial. That is, the declination arc that the Sun approximately traces from sunrise to sunset functions as a day curve, indicating the stage of the year, while the fraction of that arc traced by the Sun from sunrise to the present moment indicates the seasonal hour. The day curves inscribed on any mundane sundial should be the projections of the declination arcs, through a gnomon point, upon the sundial's surface, while the hour curve for hour *n* should be the locus of the projections of the points on all declination arcs corresponding to the end of the *n*th seasonal

11 See Sattler's contribution to this volume.

hour—or, in other words, the projection of the locus of points on all declination arcs that lie $n/12$ of the total arc from the eastern horizon.

Thus, the geometry of a sundial's grid of day and hour curves was determined by the kind of sundial surface and, for nonplanar surfaces, the location of the gnomon point. The more common types were the following:

(a) concave spherical surface; gnomon point at the sphere's center
(b) concave surface of a right cone with axis perpendicular to the equator; gnomon point along the axis
(c) planar surface parallel to the equator
(d) horizontal planar surface
(e) vertical plane (oriented in a cardinal or intermediate direction)
(f) downward-facing concave spherical surface; gnomon point (in this case, an eyehole allowing a ray of sunlight to fall on the shaded surface) on the sphere's surface

In types (a), (b), and (c) the declination arcs and their subdivisions project as similar and similarly subdivided circular arcs. The day curves in (d) and (e) are hyperbolas, except for the curve corresponding to the equinoxes (declination 0°), which is a straight line. In (f) the day curves are more complex teardrop or cardioidal curves generated as the intersections of the spherical surface with a cone, except that the curve corresponding to the equinoxes is a circle. All these curves were within the scope of Greek geometry as definable mathematical objects, so that a sundial designer who wished to do so could have constructed them on a stone surface according to accurate mathematical principles.[12]

If Greco-Roman timekeeping had employed the constant equinoctial hour (1/24 of a mean solar day) as the unit of civil time, the points on the celestial sphere corresponding to the boundaries of the equinoctial hours would all have lain along equally spaced great circles passing through the celestial north and south poles. However, even on the celestial sphere itself, the hour curves bounding seasonal hours, as defined above, would not have been tractable to a Greek geometer except as pointwise-constructed loci. Fortunately these loci turn out to diverge only very slightly from great circle arcs on the celestial sphere, so that sundial designers were able to approximate them on concave spherical sundials (type a) by great circle arcs and on planar sundials

12 Interest in the geometrical properties of conic sections is abundantly attested; for possible connections with sundial theory, see Neugebauer 1948 and Rinner 2017. Jones (2017) discusses how a Greek geometer might have treated the day curves of type (f), the so-called roofed spherical sundials.

(types c, d, and e) by straight lines.[13] Most sundial grids comprised just the three day curves corresponding to the summer solstice, the winter solstice, and both equinoxes, and the hour curves corresponding to the ends of the first through the eleventh seasonal hours. To complete such a grid, one usually had to determine the thirty-three points corresponding to the Sun's projected location at the end of each hour on either solstice and the equinoxes (or at least the twenty-two points on the solstitial curves), and then the day and hour curves could be inscribed as passing through the points lying on them. Some grids, however, had day curves not only for the solstices and equinoxes, but also for the other dates when the Sun entered each of the zodiacal signs.[14] For these denser grids, the designer had the option of determining all eleven hour-boundary points on each additional day curve, or merely inscribing the additional day curves across a set of hour curves that were determined just from the solstitial and equinoctial hour-boundary points.

As an illustration of the foregoing, Figure 4.1 shows a complete set of points at which intersect the hour curves and day curves of an ideally accurate horizontal planar sundial intended for latitude 41°, suitable for, say, Pompeii (actual latitude 40° 45′), with day curves for all dates of the Sun's entry into the zodiacal signs. The points were all individually determined by trigonometrical calculation equivalent to the kind of nomographic procedure employed in antiquity for this purpose and known as an *analemma*.[15] The central vertical straight line in the diagram corresponds to the meridian (with north at the top), and its intersection with the horizontal (east-west) straight line would be directly below the gnomon point. The hour curves, as delineated by sets of seven points fanning out from south to north, are visually indistinguishable from straight lines, while the day curves lie precisely on hyperbolic arcs.[16] A horizontal sundial obviously had to be mounted lower than eye-height, hence

13 For a mathematical analysis of the hour curves on the celestial sphere and their deviations from great circle arcs, see Drecker (1925, 12–20). The maximum deviation in hour-angle applying to a locality with latitude 40° is less than half a minute of arc.

14 In Greco-Roman sundial design it was assumed that the solstices and equinoxes coincided with the Sun's entry into Cancer, Libra, Capricorn, and Aries.

15 For a horizontal sundial, the direction and length of the shadow are determined respectively by the Sun's azimuth and altitude, or equivalently, the angles called *horizontalis* and *descensivus* in William of Moerbeke's Latin translation of Ptolemy's *Analemma*. See Drecker 1925, 4–11, and Luckey 1927. Ptolemy's work also provides methods of determining the angles numerically by trigonometry.

16 The straight lines representing the hour curves do not all intersect at a single point, reflecting the fact that the great circles approximating hour curves on the celestial sphere do not all pass through a common point.

there would not have been a great distance between viewer and sundial that might have impeded an accurate reading of the indicated time and season.

In Figure 4.2, the points from Figure 4.1 are superimposed on the grid of a horizontal sundial from Pompeii (Museo Archeologico Nazionale, Naples, inv. 2476). This grid is atypical in having hour curves composed of straight lines joining the corresponding hour points on each of the seven day curves, and these hour points show curious and apparently systematic deviations. The origin of the deviations is not clear, but they might reflect inaccuracies in an underlying analemma construction. On the other hand, the day curves are quite accurately and smoothly drawn.

Figure 4.3 superimposes the points from Figure 4.1 on the grid of another Pompeian horizontal sundial (Soprintendenza Speciale per i Beni Archeologici di Napoli e Pompei inv. 49725) that more conventionally has day curves only for the solstices and equinoxes, showing that this was a reasonably well-executed sundial, except for some inaccuracy in the extremities of the winter solstitial day curve. (For both sundials, the estimated centers of the gnomon's base, marked by a small circle in the diagrams, are displaced from the point vertically below the gnomon point, so the gnomons apparently were sloping.)

Neither of this pair of horizontal sundials is an instance of spectacularly high accuracy, but nevertheless the elements that matter for telling the time of day—namely, the hour curves—are close to their correct theoretical paths, with deviations that are only a small fraction of the spaces between the curves. In other words, at any time of year, when the tip of the gnomon's shadow fell upon an hour curve on either sundial, the true local time in seasonal hours would have been within a few minutes of the indicated time. Moreover, if two people separately consulted one or the other of the sundials, they could coordinate a meeting time with, again, an uncertainty of just a few minutes. Notwithstanding Seneca's gibe that agreement is easier to find among philosophers than among sundials (*Apoc.* 2.2), synchronization to a precision of single seasonal hours by means of decently-executed and properly-oriented sundials was entirely attainable.

The potential for reading off fractional hours on these sundials, however, is more problematic. The spaces between hour curves for the seasonal hours closest to noon are small, perhaps too small for a viewer to identify a displayed time as, say, on the half hour with much confidence. Further from noon, the spaces widen so rapidly—becoming infinite for the first and last hours of the day, when the Sun is in the plane of the horizon—that it becomes a matter of guesswork what point along a day curve would correspond to a time halfway or a third or two-thirds of the way between whole hours.

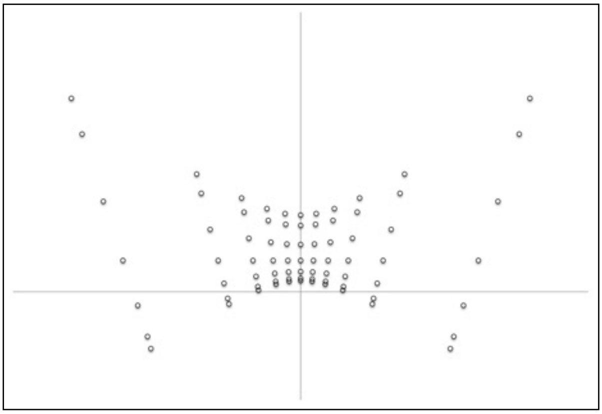

FIGURE 4.1
Seasonal hour-boundary points of a horizontal sundial for latitude 41°, computed for all solar declinations corresponding to the Sun's entry into a zodiacal sign. North is at the top. The uppermost set of points pertain to the winter solstice, and the lowermost to the summer solstice; seasonal hours run from left to right, starting with the end of the first hour of day.

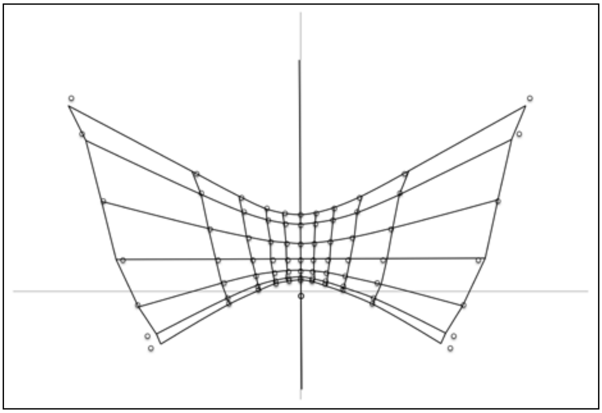

FIGURE 4.2
The hour-boundary points of Figure 4.1 superimposed on the grid of a horizontal sundial from Pompeii
MUSEO ARCHEOLOGICO NAZIONALE, NAPLES, INV. 2476

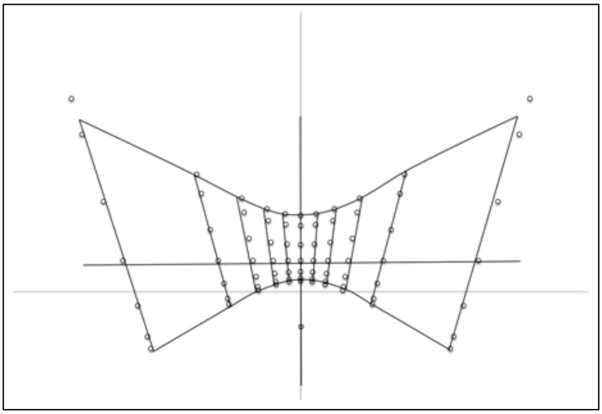

FIGURE 4.3
The hour-boundary points of Figure 4.1 superimposed on the grid of a horizontal sundial from Pompeii
SOPRINTENDENZA SPECIALE PER I BENI ARCHEOLOGICI DI NAPOLI E POMPEI INV. 49725

The same problem of visually interpolating fractional hours arises with all orientations of vertical planar sundials. For example, Figure 4.4 reproduces the grid of the east-facing sundial of the Tower of the Winds in Athens, which of course would have displayed times only before noon. Here, the closely-spaced hour curves are those closest to sunrise, near the top of the grid, whereas noon (with the Sun in the meridian plane) is at infinity, and the non-uniform subdivision of the seasonal hours immediately preceding noon would be difficult for a viewer to estimate correctly. Most viewers, if asked where the shadow indicates the time halfway through an hour on a planar sundial, would probably have pointed to the geometrical midpoint between the points for the whole hours, which is grossly incorrect for the majority of the displayed seasonal hours.

On sundials of type (f), known as “roofed spherical” sundials, the mathematics behind the grid of day and hour curves is rather complex, but an accurately-executed grid results in intervals between the hour-boundary points along the day curves that are comparatively uniform in size, as illustrated in Figure 4.5. In this case, treating each hour interval along a day curve as if the spot of sunlight traversed it with constant speed would result in a subdivision of the hours into fractions that would be entirely satisfactory for everyday purposes.

Lastly, we come to types (a), (b), and (c): the spherical, conical, and equatorial sundials. These types have in common the property (possessed, in fact, by any surface of revolution around the axis of the celestial sphere, when the gnomon point lies on the axis) that any arc of a declination circle on the celestial sphere projects through the gnomon point as a geometrically similar arc on the sundial. Consequently, fractional divisions of the hour intervals on these sundials are exactly proportional to the corresponding fractions of the seasonal hours. Presuming an accurately-drawn grid and that the sundial was mounted where one could see it reasonably well (e.g., not on top of a tall column), a viewer could easily have estimated halves, thirds, and quarters of hours within a few minutes’ precision.

To sum up, four of the six major varieties of Greco-Roman sundial, if executed and aligned accurately and mounted where they could be seen sufficiently clearly, were capable of being read by a layperson to a precision of major fractions of a seasonal hour with sufficient accuracy for any plausible social purposes. Among these, the spherical and conical varieties by themselves account for more than half the known ancient sundials. The fact that Greco-Roman society never went beyond integer hour precision was not determined by limitations of easily available technology.

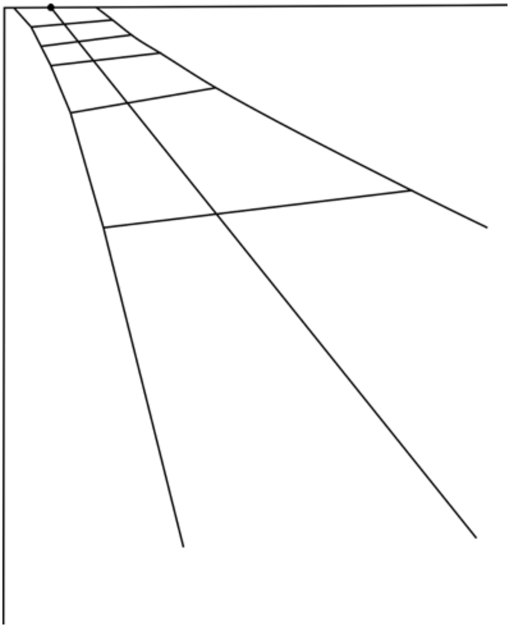

FIGURE 4.4
The grid of the east-facing sundial on the Tower of the Winds, Athens. The gnomon tip would have lain on a line perpendicular to the sundial face and passing through the point marked by a small circle. North is to the right, and the day curves from left to right are respectively for the summer solstice, the equinoxes, and the winter solstice. The hour curves correspond, from top to bottom, to the ends of the first through the fifth seasonal hour.

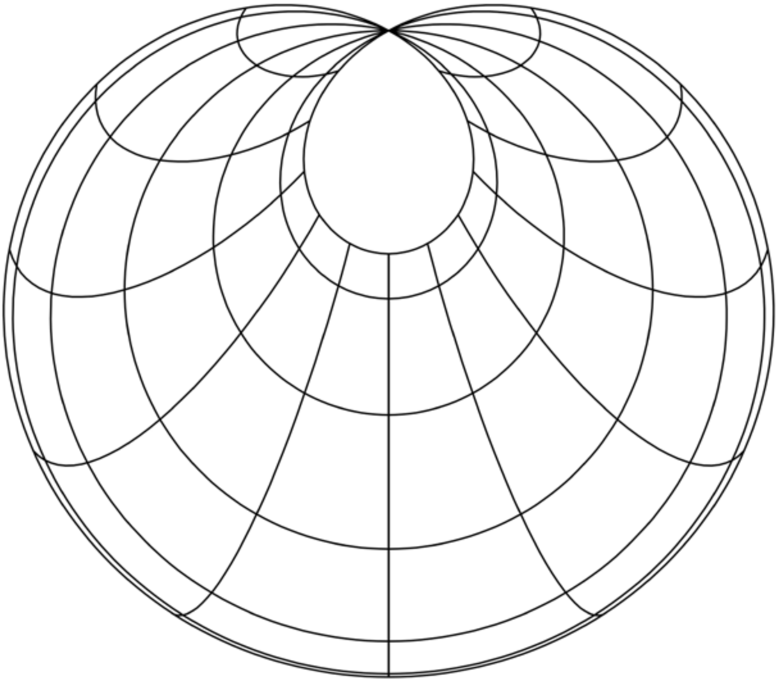

FIGURE 4.5 Grid for a roofed spherical sundial for latitude 48°, projected into the equatorial plane

3 Displaced Sundials

Discussions of geographically displaced sundials in antiquity almost always make reference to Pliny, *HN* 7.214:[17]

> M. Varro primum statutum in publico secundum Rostra in columna tradit bello Punico primo a M'. Valerio Messala cos. Catina capta in Sicilia, deportatum inde post XXX annos quam de Papiriano horologio traditur, anno urbis CCCCLXXXXI. nec congruebant ad horas eius lineae, paruerunt tamen ei annis undecentum, donec Q. Marcius Philippus, qui cum L. Paulo fuit censor, diligentius ordinatum iuxta posuit; idque munus inter censoria opera gratissime acceptum est.

> Marcus Varro reports that the first (sundial in Rome) was set up in public, on a column by the Rostra, during the first Punic War by the consul Manlius Valerius Messala after Catania in Sicily was taken, and that it was brought there 30 years later than is reported for the sundial associated with Papirius, in the year of the City 491 (i.e., 263 BCE).[18] Its lines did not agree with the hours, but nevertheless they obeyed (the sundial) for ninety-nine years, until Quintus Marcius Philippus, who was censor together with Lucius Paullus, installed next to it (another one that was) more carefully designed; this benefaction was received most favorably among the censor's accomplishments.

The passage comes within the context of an account (7.210–215) of three institutions that had come to be adopted universally and without explicit discussion (*gentium consensus tacitus*): use of the Ionian alphabet, shaving the beard, and observance of the division of days into hours (*in horarum observatione*). With respect to the hours, referring back to Book 2 (presumably 2.187) for their original "discovery" in Greece, Pliny focuses on their later adoption by the Romans, for which he cites two reported starting points: an assertion by Fabius Vestalis that the first sundial in Rome was erected by Lucius Papirius Cursor eleven years before the war with Pyrrhus (i.e., around 292 BCE), and Varro's possibly conflicting and more detailed report quoted above.[19]

---

17 A less detailed parallel account is Cens. 23.

18 The year number is corrupt in the manuscript tradition. I adopt Salmasius's emendation rather than Pighi's 490 which appears in some editions of Pliny.

19 It is not clear from his phrasing whether Pliny considers Varro's claim to have been that the sundial from Catania was the first one erected in Rome *at all*, or the first one erected in a public place. In the former case it is given as an alternate story to that of Vestalis, which Pliny complains lacked crucial details; in the latter case, the point would be that

What can we infer from Pliny's narrative? First, the sundial from Catania was placed on top of a column and was used to tell time for nearly a century. This tells us that the sundial was of a type that could be read from ground level when mounted high up—in other words, probably a spherical or conical sundial of type (a) or (b). The sundial's inscribed lines did not show the correct time, but we are not told how this error came to be known or when. Nor are we told that the error was a consequence of the sundial's having been made for a different locality; the remark that the sundial that supplanted it a century later was *diligentius ordinatum* ("more carefully designed") would seem to attribute the inaccuracies to sloppy execution rather than to geographical displacement. In 2.182 Pliny does state that the same time-keeping devices (*vasa horoscopa*) cannot be used everywhere because meridian-shadow-to-gnomon ratios vary from one locality to another, but it is by no means obvious that this was what he had in mind when he wrote the Book 7 passage.[20] Even in the Book 2 passage he fails to state that it is only the north-south component of a displacement that matters.

A naive expectation might be that a sundial made for use at Catania but installed at Rome would show the seasonal hours as if Rome lay on the same parallel of latitude as Catania. In fact, this could be accomplished, but only if the sundial was mounted in a special manner. Catania, at latitude 37° 30′, is situated about 4 1/2 ° south of Rome, latitude 41° 54′ (Figure 4.6).[21] Let us imagine that the sundial from Catania was, in the first instance, accurately constructed for Catania's latitude, and that it was remounted in Rome in an accurate north-south orientation, but tilted 4 1/2 ° from horizontal toward due south. In absolute terms, the sundial would be oriented exactly parallel to the orientation a sundial should have in a location on Catania's parallel, but due south of Rome, a place (actually in the sea) slightly south of Marsala that we may for

Papirius had dedicated his sundial in a sacred precinct, the temple of Quirinus. For a contrasting interpretation of this passage see Wolkenhauer 2011, 67–93.

20 The curious expression *vasa horoscopa* might mean all devices for determining times in seasonal hours, rather than specifically sundials; the supposition that it refers specifically to portable sundials is unjustified. Pliny's incompetence in matters of time-reckoning is glaring in 2.181, where he contends that a long-distance runner could travel the same distance east-to-west in significantly fewer hours than west-to-east because *cum sole iter erat* ("he travels with the Sun"), and moreover that a ship sailing westward can go further during even a winter day than during the nighttime because during the day it is *solem ipsum comitantes* ("travelling with the Sun").

21 The latitudes according to Ptolemy's *Geography*, respectively 37° 45′ and 41° 40′, may be taken as an indication of geographical data that might have been available in Pliny's time, if not in the third century BCE.

FIGURE 4.6 The central Mediterranean, with Rome, Catania, and the fictitious Para-Catania
HTTP://D-MAPS.COM/M/MEDITERRANEAN/MEDITMIN/MEDITMIN03.SVG

convenience name Para-Catania.[22] The result would be that the sundial would always show the correct local time for Para-Catania, as well as the correct stage of the year, so long as a ray of sunlight was able to fall upon the gnomon's tip and cast a shadow on the sundial's inscribed surface. In short, the shadow point always falls on exactly the same spot on the sundial as it would if the sundial were at Para-Catania. But except on an equinox, the sundial would show the correct time for Rome only at noon, whereas on the equinox it would show the correct Roman time all day.

During the half-year from the vernal equinox to the autumnal equinox, the interval of daytime from sunrise to sunset in Para-Catania, like in Catania itself, is longer than daytime in Rome, so that for the first and last bits of the

22 Para-Catania experiences the same variation in the lengths of days and nights through the year as Catania, but Para-Catania's local noon occurs about ten minutes later than Catania's because of their separation in longitude.

Para-Catanian day the sundial would show no time because the Sun would be below Rome's horizon. At the moment of sunrise, the sundial would show a time a fraction of an hour into the first hour of day, and again at sunset the sundial would show a time a fraction of an hour before the end of the twelfth hour. During the other half of the year, from autumnal to vernal equinox, the sundial would indicate no time until some interval after sunrise, because no shadow would fall upon the dial grid until the moment corresponding to sunrise at Para-Catania; and conversely there would be no time-indicating shadow point between sunset at Para-Catania and sunset at Rome.

Incidentally, giving our imaginary transplanted Catanian sundial a small eastward tilt (about 2 1/2 °) in addition to the southward one would put it in the same orientation as it would have had in Catania itself, so that it would display the correct local time at Catania for all moments at which the Sun was up in both Catania and Rome simultaneously. Did any imaginative gnomonist's shop offer a display of variously skewed sundials showing the local time in Rome, Athens, Alexandria, Babylon, etc., like the façade of Tourneau's store in Manhattan?

The behavior of a sundial that has undergone a displacement with a north-south component and that has been mounted in the conventional way, lined up with the local horizon and meridian of the place to which it has been moved, has been discussed previously, but it will bear revisiting.[23] In this situation, it is no longer the case that the shadow point falls on the same spot on the grid as it would have fallen in its original locality, nor is it ever in the same place relative to the day and hour curves as the shadow point on a sundial correctly constructed and mounted for the new location. Since our main concern is with how well it functions as a substitute for a sundial accurately calibrated for its place of exile, we can limit ourselves now to just this comparison—comparing, so to speak, an idealized version of the Catanian sundial installed near the Rostra by Valerius Messala to an idealized version of the sundial installed next to it by Marcius Philippus a century later.

The meridian hour curve marking the end of the sixth hour (i.e., local noon), is the projection of the celestial meridian, which is unaffected by north-south displacement on the Earth. On most sundial types, the meridian curve is also the line of symmetry of the grid.[24] Regardless of displacement, the shadow point will fall upon this line at noon throughout the year. Again, sunrise and

23 Gibbs 1976, 96; Hüttig 2000; Savoie 2001, 317–336; Hannah 2009, 134–136.

24 The principal exceptions are vertical sundials facing directions other than south or north. East-facing and west-facing vertical sundials have no meridian line; sundials facing intermediate directions ("declining" dials) have a meridian line but lack chiral symmetry.

sunset (i.e., the beginning of the first hour and the end of the twelfth) are moments when the Sun lies on the horizon and thus the shadow point lies on the projection of the horizon, whether or not the sundial is displaced. Thus, a displaced sundial of any type automatically displays the correct seasonal hour at these three moments on any day of the year.

To understand why these three times are always correctly displayed on the displaced sundial, as well as what happens with respect to the intermediate times during the morning and afternoon, it may be helpful to recall that the grid of any sundial accurately constructed and mounted for its intended location is a projection of a grid of day and hour curves on the celestial sphere. If we consider this celestial grid in the frame of reference of its poles and equator, the day curves, being the declination circles approximately traced diurnally by the Sun, are the same for all localities, but each locality on the terrestrial globe imposes its own system of hour curves, which include the local horizon (for the beginning of hour 1 and the end of hour 12) and the local meridian (for the end of hour 6). If we displace the sundial to another locality, we are effectively translating the associated celestial grid to a new position on the celestial sphere and pretending that in this position it is the appropriate grid for the sundial's new location.

The translation can be broken down into an east-west and a north-south component. A purely east-west translation is a rotation around the celestial poles. The celestial day curves slide along their declination circles, and the translated grid exactly coincides with the correct grid for the new position. Hence the same sundial will show correct seasonal hours and stages of the year in all localities having the same latitude. A purely north-south translation, on the other hand, is a rotation around a pair of poles lying on the equator, namely the rising and setting points of the Sun on the equinoxes. The translated day curves cease to be arcs of declination circles. The hour curves that were arcs of the local meridian and horizon are translated into arcs of the new local meridian and horizon, but the other translated hour curves will not coincide with the hour curves for the new locality.

Although the sundials of Pliny's anecdote were probably spherical or conical, the phenomena of transplantation can be illustrated more effectively by the grids of horizontal sundials, since using this type removes the awkwardness of representing a three-dimensional surface on the page, while adequately displaying details of sundial behavior through the entire day. Figure 4.7 shows a grid of a horizontal sundial computed for the latitude of Catania (fine black lines) superimposed on a grid computed for Rome's latitude (thick gray lines), on the assumption that the gnomon is identical in position and length for both grids. The grid for Rome represents the parts of the sundial surface that the

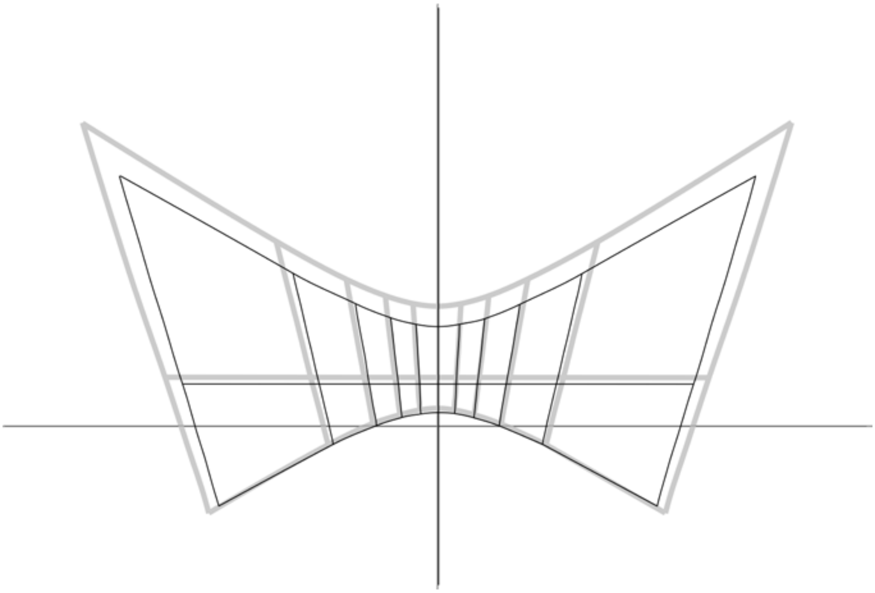

FIGURE 4.7 Grid of a horizontal sundial computed for the latitude of Catania (fine black lines) superimposed on a grid computed for Rome's latitude (thick gray lines).

shadow point can fall upon between the end of the first and the end of the eleventh hour, at any season of the year. Since the hour curves of the two grids are, for most of their length, nearly coincident, the Catanian sundial, if transplanted to Rome, will display seasonal hours that are in error by only a small fraction of an hour.[25] Around the winter solstice, the shadow point will trace paths close to the uppermost gray day curve, that is, noticeably outside the grid inscribed on the Catanian dial, though the point where the gnomon's shaft crosses the uppermost inscribed curve will give a reasonable approximation to the time. Around the summer solstice, the shadow point will never go beyond the lowermost gray day curve, which is slightly above the lowermost inscribed curve, though the horizontal sundial's projection compresses the summer day curves so much that the shortfall would scarcely be noticeable.

The seasonal errors of a displaced sundial could have been noticed simply by observing the disjunction between the inscribed day curves and the actual north-south range covered by the shadow through the year. Errors in time of day, by contrast, could only have been detected empirically if the displaced sundial was confronted with a more trusted sundial or water clock. Hannah

25 Gibbs (1976, 96 n. 25) calculates the greatest error as about 0.07 seasonal hours of the summer.

has suggested that when Pliny writes of the sundial from Catania *nec congruebant ad horas eius liniae*, he employs *hora* (as he does in a few other passages of the *Naturalis Historia*) in the sense of "time of year," thus "its lines did not agree with the seasons" (2009, 135). But the context of 7.214 is all about the Roman adoption of hours as subdivisions of the day, and *paruerunt … ei* only makes sense as stating that the Romans trusted the sundial for marking the hours—they are unlikely to have adopted it as a calendar regulator. One would like to know what motivated Marcius Philippus to erect a new sundial a century later. Did people come to suspect that the sundial showed false times because it manifestly showed false seasons? Or did someone argue on theoretical, mathematical grounds that a sundial constructed for a latitude so far south of Rome must show false times?[26]

In any case, the ancient purchaser of an expensive, high-quality sundial could have taken comfort in the knowledge that, if for any reason he or she had to move to a distant place, the sundial would still work reasonably well as a time-keeping device, if not as a calendar.

4 Determining Intended Latitude and Evaluating Accuracy through Digital 3D Models

How common was it in antiquity for a sundial to be installed in a location whose latitude was significantly different from the one for which it was designed? This question cannot yet be answered satisfactorily on a broad base of data from throughout the Greco-Roman world. Ideally, this base ought to consist of sundials (1) whose findspots are known at least roughly (precise archeological context is not essential), (2) that were designed and executed with care and accuracy with respect to the geometry of their surfaces, day curves, and hour curves, and (3) that are sufficiently well preserved to allow a satisfactory analysis of their geometry.

In her *Greek and Roman Sundials*, Sharon Gibbs not only inventoried the 256 ancient sundials known to her but, for a large fraction of this corpus, reported estimates of the latitudes to which their grids correspond, based in large part on her own measurements.[27] Although the number of known Greco-Roman sundials has more than doubled since 1976—an expansion only partly

26 Wolkenhauer's contribution to this volume explores further motivations for erecting this second sundial alongside the first.

27 Gibbs 1976. One cannot help being astonished by the thoroughness with which Gibbs made personal inspection of the majority of the sundials reported in her monograph.

reflected in Eva Winter's recent survey of ancient time-reckoning devices[28]—with respect to measured data, Gibbs's book has not yet been systematically superseded, though it has been supplemented and corrected in parts by more recent studies of individual sundials and regional corpora. Moreover, the kinds of measurements from which she estimated the intended latitudes of sundials reflected what was practicable using surface measurements on objects that often were very far from completely preserved, and it is not always easy to tell how significant the results are.

The category known as roofed spherical sundials (type [f] above) illustrates the difficulties Gibbs faced. As noted above, this type projects a narrow beam of the Sun's rays upon a shaded spherical surface through an eyehole situated at the zenith of the sphere. The only geometrically simple curves on a roofed spherical sundial's grid are the equinoctial day curve, which is a complete circle passing through the eyehole and parallel to the equator, and the meridian hour curve, which is an arc of a great circle passing through the eyehole and bisecting the equinoctial circle. If the radius of the spherical surface is R, the radius of the equinoctial circle r, and the intended latitude of the sundial is φ, then:

(1) $\cos \varphi = r/R$

However, Gibbs could not measure R and r directly. To estimate R, she measured the arc of the meridian hour curve between the equinoctial circle and either of the solstitial day curves, from which she could derive the circumference and radius of the spherical surface by assuming a plausible value for the obliquity of the ecliptic. For r she measured the arc of the equinoctial circle between any pair of hour curves, since the geometry of the projection requires that these arcs should be 15° times the corresponding number of seasonal hours. Thus, each determination of φ could be affected by inaccuracies in the execution of the spherical surface, the solstitial day curves, and the hour curves, as well as some uncertainty about the assumed value for the obliquity of the ecliptic.

We may take as an example Pompeii Granario inv. 52789, a fragment of a roofed spherical sundial found at Pompeii (latitude 40° 45′, precise findspot unknown). When intact, the sundial took the form of an oversized drinking cup or *skyphos*. Somewhat more than half the grid survives, including much of the equinoctial and solstitial day curves and the hour curves for the ends of the fourth through the tenth hour. Gibbs gives two estimates of φ to 1° precision, derived from measurements (to 1 mm precision) of the meridian arcs between

28 Winter 2013. The technical information in this work is largely derived from earlier publications; because of a rather high rate of inconsistencies and errors, the user needs to check details systematically (see Schaldach 2015).

the equinoctial circle and the winter and summer solstitial curves, and one measurement of the equinoctial arc between the hour curves for the ends of the sixth and seventh hours: 50° using the winter meridian arc, and 53° using the summer meridian arc. These latitudes are not only much higher than that of Pompeii (and appropriate, say, for southern Great Britain), but also significantly different from each other. Similar, and even more extreme, scattering of latitude values occurs for several other roofed spherical sundials that Gibbs examined. One might suspect that this is due partly to imperfections in the mathematically complex solstitial and hour curves, whose errors become magnified in the calculation of r and R, and perhaps also to measurement errors.

Digital 3D models of ancient sundials, derived from either scanning or photogrammetry, offer the possibility of making—and repeating—measurements that would be difficult or impossible through the means that were available to Gibbs. The major repository of digital models is the BSDP, which has up to now made models of a substantial fraction of the known corpus of Greco-Roman sundials available for research. Most of these Berlin Project models are from laser scans, though some are photogrammetric. In the great majority of cases the models are sufficiently detailed and reliable to allow measurements from which the essential parameters of the sundials can be estimated. Of the thirty roofed spherical sundials that I am aware of, the BSDP currently has usable models of eight, and in addition, I have made photogrammetric models of three sundials of this type for which the BSDP currently lacks satisfactory models.

Digital models can be analyzed using specialized software that directly accesses the vertex coordinates of the mesh. However, it is possible to extract key parameters with more than satisfactory precision by applying fairly simple and easily accessible visual tools to the models. In the following analysis of the roofed spherical sundials, the principal software employed comprised MeshLab (open source software for editing 3D mesh models), Inkscape (open source vector graphics software, used here for fitting circles and straight lines to images and measuring lengths and angles), and Microsoft Excel (for calculating ideal sundial grids).

We can illustrate several tests for assessing the quality and estimating the intended latitude of roofed spherical sundials with the Pompeii fragment discussed above. Test 1 is of the quality of the spherical dial surface. After removing most of the surface mesh other than the dial surface (Figure 4.8), we select an arbitrary orientation of the surface and remove a series of thin, parallel planar slices (Figure 4.9). The perimeters of the remaining slices compare well with a set of concentric reference circles superimposed on the image (Figure 4.10), confirming that this is an accurate surface of revolution. Repeating this procedure using a different orientation for the slicing suffices to establish that the surface is accurately spherical.

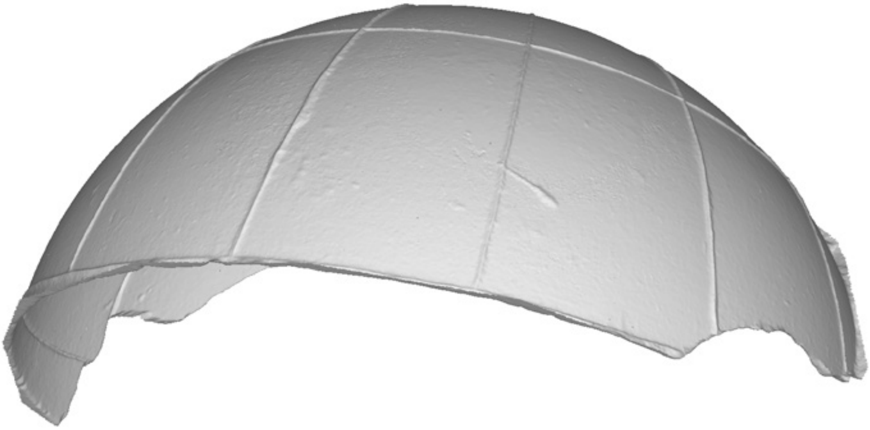

FIGURE 4.8 Orthographic image of the BSDP digital model of the dial surface of Pompeii Granario inv. 52789

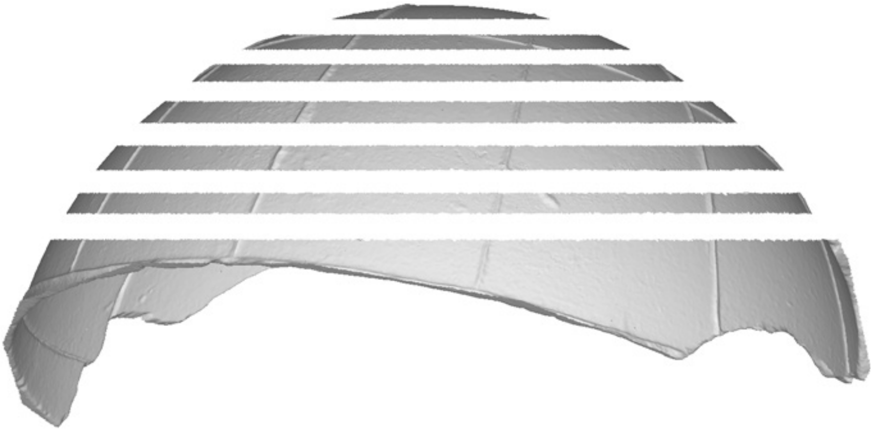

FIGURE 4.9 Digital model of the dial surface with planar slices removed

FIGURE 4.10
Orthogonal image of the sliced model in a plane parallel to the slices, with concentric reference circles in black

The remaining tests are methods of estimating the intended latitude of the sundial. For test 2, we cut away half the surface model of the complete sundial fragment along its meridian plane, and generate an orthographic image in the meridian plane (Figure 4.11). Fortunately the sundial's base survives, taking the form of a triangular projection behind the dial face. The angle between the base line and the plane of the equinoctial circle (which projects as expected as an almost exact straight line) is 52.5° to the nearest half degree (Figure 4.12). If the sundial was supposed to be mounted on a horizontal surface, this angle would be the local elevation of the equatorial plane, so that the intended latitude would be 37.5°, significantly south of Pompeii's actual latitude, 40° 45′.

If, on the other hand, the sundial was supposed to be mounted at a sufficient forward tilt of about 3° so that the equinoctial circle was parallel to the equatorial plane for Pompeii, there ought to be a corresponding deviation of the position of the eyehole from the point of the spherical surface that would be the zenith point if the sundial was installed horizontally. To check this, we fit a circle to the outline of the meridian arc and produce the projection of the equinoctial circle to meet it at the eyehole point. The angle subtended at the sphere's center between the eyehole point and the zenith point relative to the baseline is approximately 4°, near enough to the expected value to confirm that a forward tilt of the sundial was intended. We thus report the latitude resulting from Test 2 as 37.5° + 4°.

Test 3 uses the same image with the fitted meridian circle and projection of the equinoctial circle. Assuming that the eyehole was at the zenith of the bowl (relative to the local horizon), then from equation (1) we find φ is 40.5° to the nearest half degree.

Lastly, in Test 4 we generate an orthographic image of the dial surface in the plane of the equinoctial circle (that is, the equatorial plane) (Figure 4.13), upon which we superimpose equatorial projections of accurately-computed grids for different latitudes at 1° intervals, employing 23° 40′ as the obliquity of the ecliptic and assuming that the eyehole was at the zenith relative to the local horizon. The match for 41° (Figure 4.14) is quite impressive, with only very small deviations apparent along the winter solstitial curve and the hour curves, most noticeably where the hour curves approach the winter equinoctial curve. This was evidently a very well constructed sundial after all, and made for the latitude of Pompeii.[29] In this instance the principal cause of error in Gibbs's

29 It is true that the sundial would have functioned correctly if installed horizontally at a latitude about 37° 30′. But if it was made for that latitude and later displaced to Pompeii, one would have to explain why the eyehole was placed at just such an angle behind the zenith point so that the grid ended up being the normal one for the latitude of Pompeii.

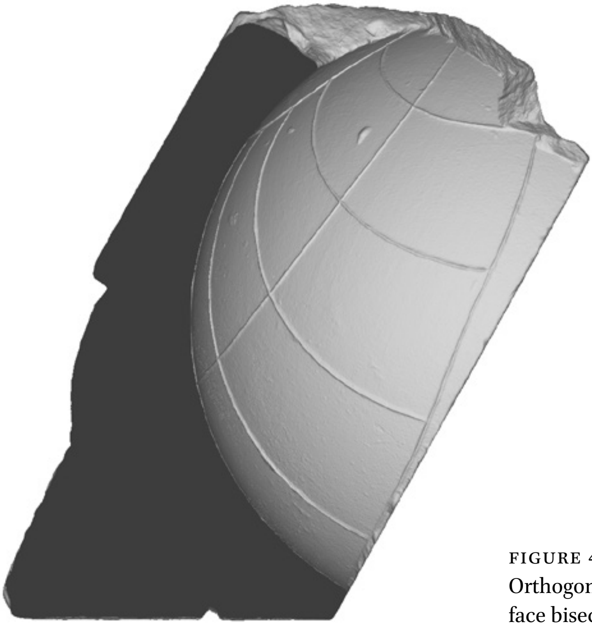

FIGURE 4.11
Orthogonal image of the sundial surface bisected along the meridian plane

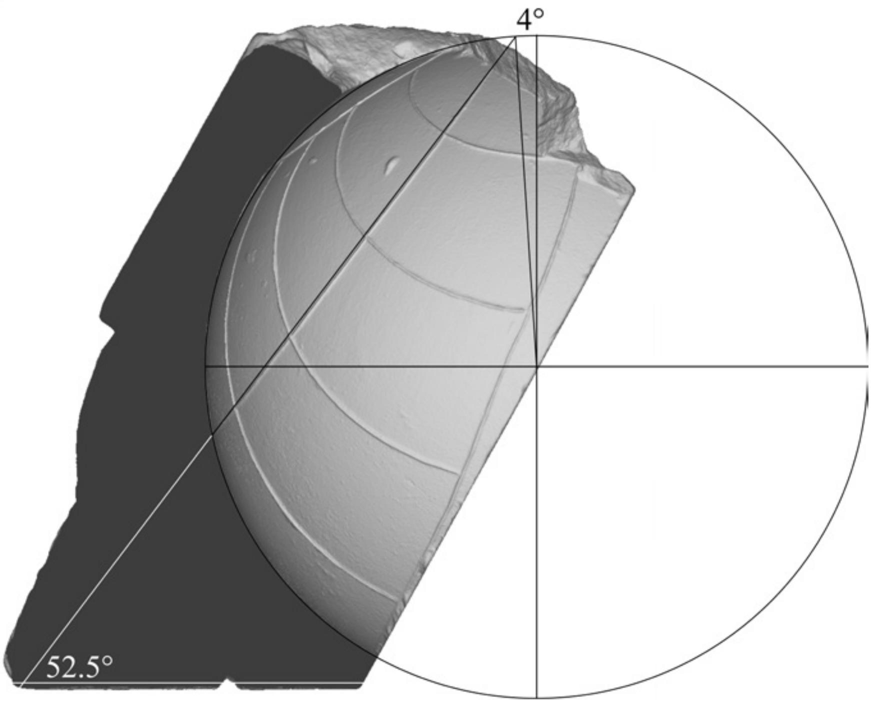

FIGURE 4.12 The orthogonal image from Figure 4.11 with lines traced for Tests 2 and 3

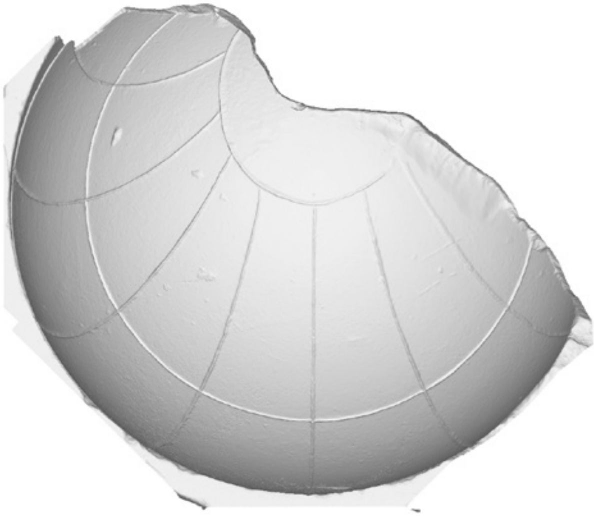

FIGURE 4.13 Orthogonal image of the dial surface in the equatorial plane

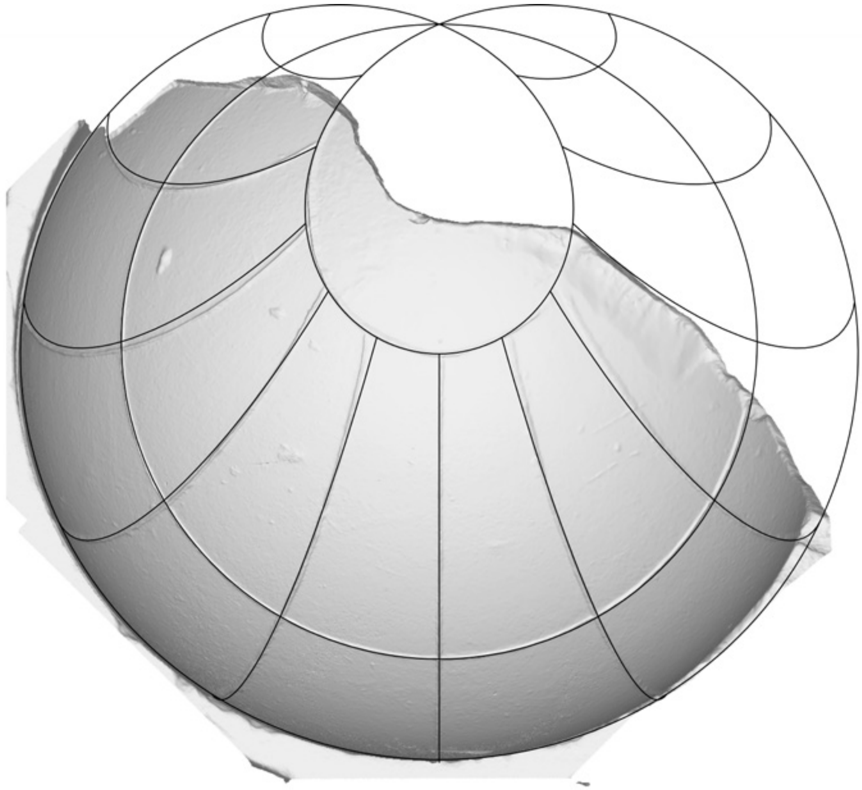

FIGURE 4.14 The dial surface as in Figure 4.13, with ideal grid for latitude 41° superimposed

TABLE 4.1 Analyses of eleven roofed spherical sundials

Gibbs	Winter	BSDP dialface	Present location	Findspot	Latitude of findspot
2001G	Altinum 3	1	Altino, Veneto	Altino	45° 35′
2018G	Fundort unbekannt 10	94	Bologna	unknown	44° 30′? (Bologna)
2019G	Pompeji 15	95	Pompeii	Pompeii	40° 45′
2020	Baelo Claudia 1	96	Madrid	Baelo Claudia	36° 5′
2021G	—	97	Vatican	unknown	41° 54′? (Rome)
2023G	—	98	Berlin	Rome? (Acquired there)	41° 54′?
7001G	Tenos 1	280	Tinos	Tinos	37° 33′
—	Karthago 1	366	Paris	Carthage? (Reported)	36° 51′?
—	Ariminum 1	437	Verucchio	Verucchio?	43° 59′
—	Concordia Sagittaria 1	480	Concordia Sagittaria	Concordia Sagittaria	45° 45′
—	Ravenna 1	521	Ravenna	unknown	44° 25′? (Ravenna)

results does not seem to be inaccuracies in the sundial's day and hour curves but some fault in her measurements.

In Table 4.1 I summarize the results of similar analyses of eleven roofed spherical sundials. This is followed by notes on the individual sundials.

1. Museo Archeologico Nazionale di Altino, Altino, Italy, inv. AL 10. Found near Quarto d'Altino. The sundial bowl, partly broken away at the top, is sculpted from the upper part of a block of marble, the lower part of which is a plinth provided with a decoration in the form of a boat. The plinth has several ostensibly horizontal edges, and the estimate of latitude from Test 2 is with respect to edges running from front to back of the plinth. The estimates from Tests 3 and 4 are close enough to the findspot's latitude to confirm that the sundial was made for this location; however, to function correctly it would have had to be installed at a forward tilt of about 12°. In fact the extrapolated

Digital model	Test 1 (quality of spherical surface)	Test 2	Test 3	Test 4	Notes
BSDP photogrammetry	fair	34° + 10.5° = 44.5°	44.5°	44°	1
Jones photogrammetry	poor	N/A	44.5°	45°	2
BSDP laser	good	37.5° + 4° = 41.5°	40.5°	41°	3
Jones photogrammetry	good	48° – 11.5° = 36.5°	37°	36°	4
BSDP laser	good	43° + 0° = 43°	42°	N/A	5
BSDP laser	fair	50° – 8° = 42°	41.5°	42°	6
Jones photogrammetry	good	37.5° + 1° = 38.5°	39°	37°	7
BSDP photogrammetry	good	44.5° – 2° = 42.5°	42°	42°	8
BSDP photogrammetry	fair	46° + 3° = 49°	46°	45°	9
BSDP photogrammetry	good	44° – 5° = 39°	41.5°	41°	10
BSDP laser	fair	N/A	49	50°	11

highest point of the equinoctial day circle, which would have been where the eyehole was located, is about this angle to the rear of the extrapolated zenith of the spherical surface, assuming that the sundial was installed with no tilt.

Gibbs gives four rather scattered estimates of φ to 1° precision, derived from measurements (to 1 mm precision) of the meridian arcs between the equinoctial circle and the winter and summer solstitial curves, and measurements of the equinoctial arcs between the hour curves for the ends of the sixth, seventh, and eighth hours:

winter solstice, hours 6 and 7 $\varphi = 46°$
winter solstice, hours 7 and 8 $\varphi = 43°$
summer solstice, hours 6 and 7 $\varphi = 44°$
summer solstice, hours 7 and 8 $\varphi = 41°$

2. Museo Civico Archeologico, Bologna, without inventory number. The findspot is unknown. Parts of the sundial bowl are broken away along the top and sides. The digital model, which I reconstructed from BSDP photographs since the grid curves were not clear on the BSDP model, lacks any satisfactory indication of the intact sundial's vertical or horizontal directions. Despite the rather poor geometry of the bowl's surface, the latitude estimates are in good agreement with the latitude of Bologna. Using the meridian arc between the equinoctial circle and the summer solstitial curve with the arcs of the equinoctial circle between hours 1, 2, 3, and 4, Gibbs obtained somewhat scattered latitude estimates of 45°, 44° 13′, and 47° 30′.

3. Pompeii, Granario inv. 52789. (Analysis discussed in detail above.) The exact findspot is unknown but definitely Pompeii. The bowl is broken away along the top, sides, and part of the bottom edge.

4. Museo Arqueológico Nacional, Madrid, inv. 33.185. Found in the kitchen of a house at Baelo Claudia; it must have been moved from an outdoor location, perhaps for use as a table.[30] The sundial is essentially complete and in excellent preservation, though missing the metal plate that would have been perforated by the eyehole. The original publication of the sundial by Paris et al. reported the intended latitude as 41° 30′, measuring the elevation from horizontal of a line from the zenith of the bowl to the intersection of the meridian and equinoctial circles on the grid, on the assumption that the sundial would have been erected horizontally.[31] Raya Román, realizing that the eyehole must have been forward of the ostensible zenith point, applied the first step of our Test 2 and obtained 47° 30′, without considering the possibility that the sundial was meant to be installed at a tilt.[32]

5. Musei Vaticani inv. 53875. Findspot unknown. The majority of the bowl is broken away; what remains is most of the part between the equinoctial and summer solstitial day circles. Tests 2 and 3 are close enough to Rome's latitude to suggest that the sundial could have been made for Rome. Using the arc on the equinoctial circle between hours 6 and 7 and the meridian arc between the equinoctial circle and the summer solstitial curve, Gibbs estimated the latitude as 41° 13′.

6. Staatliche Museen zu Berlin inv. SK1049. Findspot unknown, but reportedly acquired in Rome.[33] The sundial is complete except for some breakage

30 Paris et al. 1923, 167.
31 Paris et al. 1923, 167 fig. 65.
32 Raya Román 1984, 111.
33 Schaldach n.d.

around where the lost eyehole plate would have been. Tests 2 (assuming an 8° backward tilt), 3, and 4 are all in good agreement with Rome's latitude. This sundial has received several previous analyses. Drecker found the intended latitude to be 41° 40′ from the ratio of directly-measured radii of the equinoctial circle and the sundial bowl.[34] Using the arc of the equinoctial circle between the 5th and 6th hours together with the (approximately equal) meridian arcs between the two solstitial curves and the equinoctial circle, Gibbs found 39°; this is the outlier among the estimates. Following Gibbs's method for the bowl's radius, but with direct measurement of the radius of the equinoctial circle, Schaldach gives 41.5° and, after more recent measurements, 41.67°.[35]

7. **Archaeological Museum of Tinos inv. A139.** Found near the altar of Poseidon at Kionia, Tinos. In addition to the roofed spherical bowl on the south side, the sundial has east-facing and west-facing vertical planar sundials, and a spherical bowl (type [a]) on its north side. The upper part of the roofed spherical bowl is broken away, except for some fragments that have been embedded in a (not particularly accurate) restoration. The latitudes from Tests 2, 3, and 4 are close enough to that of Tinos for the sundial to have been designed for that location. Using the arc of the equatorial circle between hours 4 and 5 together with the meridian arcs between the two solstitial curves and the equinoctial circle, Gibbs obtained latitude estimates of 37° (summer curve) and 20° (winter curve). Her measurement of the winter arc is suspect.

8. **Musée du Louvre inv. MNE 1178, *nº usuel* Ma 5074.** The sundial, in the form of an elaborately decorated *skyphos*, is complete and in excellent condition. Acquired by purchase, the findspot was reportedly Carthage.[36] The latitudes resulting from Tests 2 (assuming a 2° backwards tilt), 3, and 4 are consistent with each other and incompatible with the reported findspot, Carthage. Savoie and Lehoucq likewise estimated 41° 6′ from the ratio of radii of the equinoctial circle and the spherical bowl, with confirmation from measurements involving the other inscribed curves.[37] They conclude that the sundial was designed for the latitude of Rome but displaced to Carthage, remarking that the errors in the seasonal hours resulting from the displacement would have

34 Drecker 1925, 31.

35 Schaldach 2001, 107 (with a valuable discussion of the complications involved in estimating the latitude); Schaldach n.d.

36 According to Gagnaire (1998, 179), it was found in excavations of a Roman Villa at Carthage before the Second World War. Its earliest publicly accessible documentation seems to be its inclusion in an exhibition at The Hague in 1990; see Turner 1990, 60–61. The Louvre purchased it in 2000.

37 Savoie and Lehoucq 2001, 30.

been negligible. However, since the summer solstitial curve grazes the lower rim of the bowl (as was normal for roofed spherical sundials), at Carthage the spot of sunlight would have fallen not only outside the grid but entirely off the bowl's surface for an interval around the summer solstice. Rather, one may suspect that the findspot has not been accurately reported.

9. Museo Civico Archeologico, Verucchio. Findspot unknown, but presumably near Verucchio. The top and sides of the bowl are broken away. Because of the bowl's deviations from sphericity, its center cannot be determined with precision, so that the tilt component of Test 2 is not trustworthy. In any case Tests 2, 3, and 4 all suggest that the sundial was designed for a latitude slightly farther north than Verucchio. I am not aware of any previous estimate of the intended latitude.

10. Soprintendenza Archeologica del Veneto, Concordia Sagittaria. Found at Concordia Sagittaria. The bowl is completely preserved except for some breakage around the eyehole. Tests 2 (assuming tilt), 3, and 4 clearly indicate that the sundial was designed for a latitude several degrees south of Concordia Sagittaria. I am not aware of any previous estimate of the intended latitude of this sundial.

11. Museo Nazionale di Ravenna. Findspot unknown. The bowl is broken away at the top and sides; the model provides no satisfactory horizontal or vertical element for Test 1. Tests 2 and 3 give a latitude about 5° north of Ravenna (subject to modest uncertainty since the equinoctial circle is rather coarsely executed), so this may have been a displaced sundial if it was a local find, as seems likely. The same reservations concerning its usability in the summer would apply as for the "Carthage" sundial now in the Louvre. Mario Arnaldi, on the other hand, gives 45° as the latitude for which the sundial was made, without indicating how he obtained this figure.[38]

Of the eleven sundials examined above, five have securely-known findspots, and among these, four (Altino, Pompeii, Baelo Claudia, and Tinos) were constructed, with some care, for the latitudes of the localities where they were found. The sundial from Concordia Sagittaria, on the other hand, appears to have been made for somewhere significantly further south—say, between Rome and Naples. There are also four for which the default assumption would be that the sundial was found near its present location: those now at Bologna and the Vatican fit the presumed findspots, whereas those now at Verucchio and Ravenna seem to be displaced. Of the two with reported places of origin, the one now in Berlin may well have been made for Rome, where it was purchased if not actually found. The sundial now in Paris, however, though said

38 Arnaldi 1996, 14 n. 3.

to have been excavated at Carthage, clearly was made for somewhere around Rome's latitude—but, given the impossibility of confirming the claimed findspot or tracing the object's history during the half century following its supposed discovery, we should be circumspect about identifying this as a displaced sundial. Even so, three out of eleven is a surprisingly high rate of certain or probable displaced sundials. It would be interesting to find out whether extending the examination to a larger sample would yield similar results.

5 Conclusion

One way of describing the distinctive character of Greco-Roman sundials is that they were an embodiment of a fairly sophisticated mathematical theory in objects fashioned by craftspeople representing a wide range of skill, for practical use (as well as ornament) by lay people who were mostly ignorant of the underlying theory. The foregoing article highlights two strengths of the mathematical theory of sundials: the potential of certain popular types of sundial to facilitate reasonably accurate reading of the time of day to a precision significantly finer than whole seasonal hours, and the robustness of a well-executed system of hour curves with respect to even fairly large disparities between the latitude for which the sundial was designed and the latitude where it was installed. The former strength seems not to have been appreciated, at least outside the scientific community; the unit of the seasonal hour remained through antiquity the limit to which the capabilities of technology drew society in the direction of more refined time-management.

The frequency of demonstrable or probable instances of latitudinal displacement found in our—admittedly very small—sampling of roofed spherical sundials might suggest that the owners of these sundials were informed of how little effect terrestrial latitude has on the hour curves in the lateral (east-west) dimension. I suspect, however, that these displacements were not really symptomatic of mathematical sophistication on the part of the sundials' owners. In particular, the special characteristic of the roofed spherical type, that the time was indicated not by the shadow of a stick-like gnomon but by a small spot of sunlight, combined with the fact that the summer solstitial day curve in this type normally ran very close to the lip of the bowl for the middle part of the day, would have made this type useless for a large fraction of the summer days if displaced far south of its intended location: the narrow beam of light would miss the dial surface entirely. Probably the owners of the displaced sundials valued them more as prestige objects than as instruments for regulating daily life.

References

Arnaldi, Mario. 1996. "Due frammenti di orologio solare romano al museo nazionale di Ravenna." *Ravenna: Studi e ricerche* 3: 14–28.

Delambre, Jean B.J. 1817. *Histoire de l'astronomie Ancienne*. 2 vols. Paris: Courcier.

Drecker, Joseph. 1925. *Die Theorie der Sonnenuhren. Die Geschichte der Zeitmessung und der Uhren I E*. Berlin: de Gruyter.

Gagnaire, Paul. 1998. "Le Scaphé de Carthage." *L'astronomie* 112 (Juin–Juillet): 179–82.

Gibbs, Sharon L. 1976. *Greek and Roman Sundials*. New Haven, CT: Yale University Press.

Graßhoff, Gerd, Elisabeth Rinner, Karlheinz Schaldach, Bernhard Fritsch, and Liba Taub. 2016. *Ancient Sundials*. [Online database] accessed June 5, 2019, https://doi.org/10.17171/1-1.

Hannah, Robert. 2009. *Time in Antiquity*. New York: Routledge.

Heiberg, Johan L., ed. 1898. *Claudii Ptolemaei opera quae exstant omnia*. Vol. I. Leipzig: Teubner.

Hüttig, Manfred. 2000. "The Conical Sundial from Thyrrheion: Reconstruction and Error Analysis of a Displaced Antique Sundial." *Archive for History of Exact Sciences* 55 (2): 163–76.

Jones, Alexander. 2005. "Ptolemy's Canobic Inscription and Heliodorus' Observation Reports." *SCIAMVS* 6: 53–97.

Jones, Alexander. 2017. "The Roofed Spherical Sundial and the Greek Geometry of Curves." In *Studies in the Ancient Exact Sciences in Honour of Lis Brack-Bernsen*, edited by John Steele and Mathieu Ossendrijver, 183–203. Berlin: Edition Topoi.

Luckey, Paul. 1927. "Das Analemma des Ptolemäus." *Astronomische Nachrichten* 230 (5498): 18–46.

Mercier, Raymond. 2011. "Book Review, Ptolemy in Perspective, Edited by Alexander Jones." *Journal for the History of Astronomy* 42 (3): 405–8.

Neugebauer, Otto. 1948. "The Astronomical Origin of the Theory of Conic Sections." *Proceedings of the American Philosophical Society* 92 (3): 136–38.

Neugebauer, Otto, and Henry B. Van Hoesen. 1959. *Greek Horoscopes*. Philadelphia: American Philosophical Society.

Paris, Pierre, Jorge Eduardo Bonsor, Alfred Laumonier, Robert Ricard, and Cayetano de Mergelina. 1923. *Fouilles de Belo* (*Bolonia, Province de Cadix*) (*1917–1921*). *I. La ville et ses dépendances. Bibliothèque de l'école des hautes études hispaniques, Fascicule VI*. Bordeaux: Féret.

Raya Román, José María. 1984. "Reloj solar de Belo." *Boletin del Museo Arqueológico Nacional* 2 (1): 103–15.

Remijsen, Sofia. 2007. "The Postal Service and the Hour as a Unit of Time in Antiquity." *Historia* 56 (2): 127–40.

Rinner, Elizabeth. 2017. "Ancient Greek Sundials and the Theory of Conic Sections Revisited." In *Studies in the Ancient Exact Sciences in Honour of Lis Brack-Bernsen*, edited by John Steele and Mathieu Ossendrijver, 165–82. Berlin: Edition Topoi.

Savoie, Denis. 2001. *La Gnomonique. Nouvelle édition revue et augmentée*. Paris: Les Belles Lettres.

Savoie, Denis, and Roland Lehoucq. 2001. "Étude gnomonique d'un cadran solaire découvert à Carthage." *Revue d'archéométrie* 21: 25–34.

Schaldach, Karlheinz. n.d. "107898. Sonnenuhr. Berlin, Staatliche Museen, Antikensammlung Berlin." *Central Object database of the German Archaeological Institute (DAI) and the Archaeological Institute of the University of Cologne. Arachne*. [Online database entry] accessed June 5, 2019, http://arachne.uni-koeln.de/item/objekt/107898.

Schaldach, Karlheinz. 2001. *Römische Sonnenuhren. Eine Einführung in die antike Gnomonik*. 3rd ed. Frankfurt am Main: Harri Deutsch.

Schaldach, Karlheinz. 2015. "Book Review of Eva Winter, Zeitzeichen." *Gnomon* 87 (2): 144–49.

Schaldach, Karlheinz. 2016. "Measuring the Hours: Sundials, Water Clocks, and Portable Sundials." In *Time and Cosmos in Greco-Roman Antiquity*, edited by Alexander Jones, 63–93. Princeton: Princeton University Press.

Steele, John. 2000. *Observations and Predictions of Eclipse Times by Early Astronomers*. Archimedes: New Studies in the History and Philosophy of Science and Technology 4. Dordrecht: Kluwer.

Turner, Anthony J. 1990. *Time*. The Hague: Tijd voor Tijd Foundation.

Winter, Eva. 2013. *Zeitzeichen: Zur Entwicklung und Verwendung antiker Zeitmesser*. 2 vols. Berlin: de Gruyter.

Wolkenhauer, A. 2011. *Sonne und Mond, Kalender und Uhr: Studien zur Darstellung und poetischen Reflexion der Zeitordnung in der römischen Literatur.* Berlin: de Gruyter.