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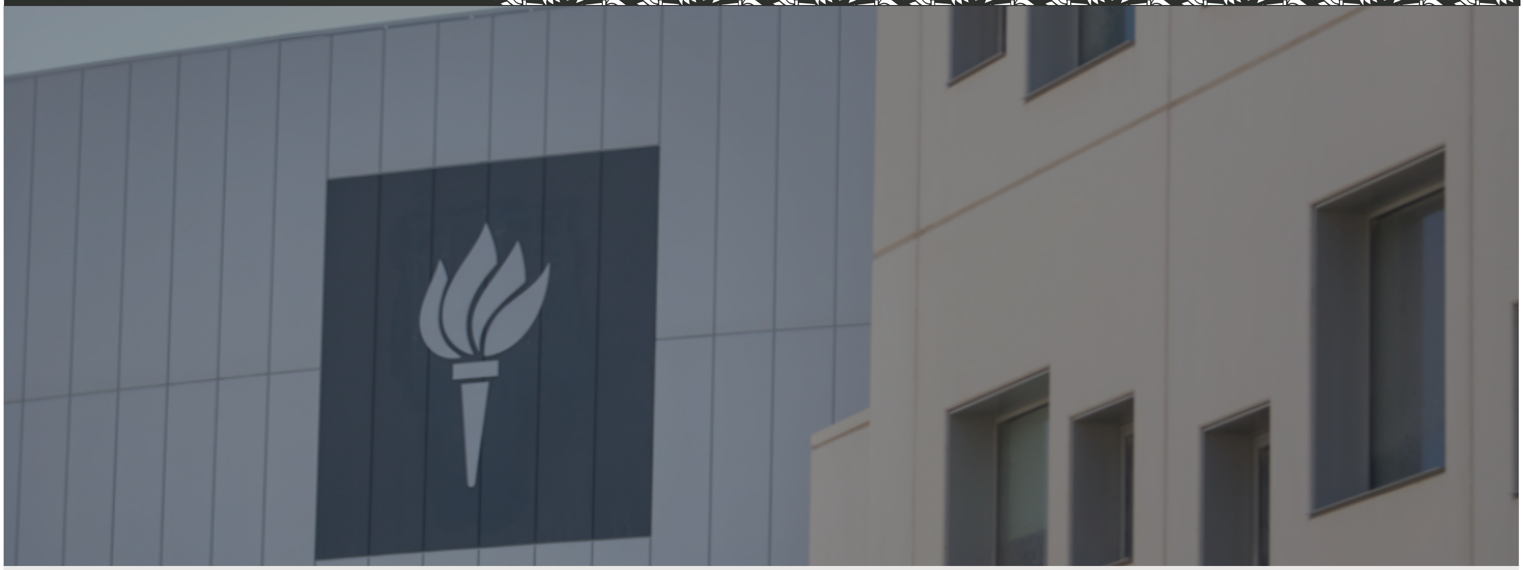

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Working Paper # 0100
May 2024

Division of Social Science Working Paper Series

New York University Abu Dhabi, Saadiyat Island P.O Box 129188, Abu Dhabi, UAE

http://nyuad.nyu.edu/en/academics/academic-divisions/social-science.html

Symmetric Divide-and-Choose

Matt Van Essen[*] John Wooders[†]

May 12, 2024

Abstract

In the United States, a version of divide-and-choose known as the "Texas Shootout" is commonly employed to dissolve partnerships. The Texas Shootout treats symmetric players asymmetrically, with one player designated to be the Divider and the other designated the Chooser. This asymmetry leads to inequitable and inefficient outcomes. This paper introduces a symmetric form of divide-and-choose. Like classical divide-and-choose, the mechanism generates envy-free and proportionally fair outcomes. Unlike classical divide-and-choose, the mechanism treats identical claimants identically and its equilibrium is efficient. Finally, the outcome under maxmin play is closely related to the Shapley value of an associated cooperative game.

JEL Codes: C71, C72, C78

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1 Introduction

The study of fair division begins with divide-and-choose: One person divides, the other person chooses. The idea is simple, practical, and seemingly fair. People have been using divide-and-choose for thousands of years to resolve disputes over contested property.[1] However, despite its widespread use, it is strange that the procedure most identified with fair division is *asymmetric,* treating otherwise identical claimants differently ex-ante as they are assigned different roles – divider or chooser. Further, for the allocation problem studied here, divide-and-choose is inequitable (it favors the chooser) and the equilibrium is inefficient. This paper offers an alternative. We introduce a version of divide-and-choose which treats identical claimants identically, is equitable, is efficient, and which preserves its familiar structure.

The problem we study can be viewed as one of dissolving a partnership: a single indivisible item (the partnership) has to be allocated to one of the two partners, each of whom has an equal claim. In the United States, a version of divide-and-choose known as the "Texas Shootout" is commonly employed to dissolve partnerships. In a shootout, the partner who wants to dissolve the partnership (the "divider") names a price and the other partner (the "chooser") is compelled to either purchase the divider's interest or sell his own interest at the named price. When partners have independent private values, then the equilibrium is inefficient and favors the chooser.

The symmetric divide-and-choose procedure introduced here is a variation of the Texas Shootout, but has the feature that players choose their price before knowing whether they are the divider or the chooser. In the game, each player simultaneously chooses a price and then one player is randomly selected to be the divider. The divider's price is the price at which the partnership is allocated. The chooser's price determines which player buys the other's interest: if the chooser's price exceeds the divider's price, then

[1] While its origin is unknown, written accounts of its use to resolve disputes go back nearly 3000 years. Examples include its description in ancient Greek antidosis laws (see Christ (1990)). Hesiod's *Theogeny* describes a not-so-fair application of divide-and-choose between Zeus and Promethius, called the "Trick at Mecone." Explicit examples of divide-and-choose procedures can be found in the Welsh Code of Venedotion to settle inheritence disputes circa 1000 AD (see Van Essen and Verville (2023)).

the chooser buys the partnership at the divider's price; if the chooser's price is less than the divider's price, then the chooser sells the partnership at the divider's price. In symmetric divide-and-choose, each player chooses a single price which determines his action for both roles.

For symmetric divide-and-choose, we characterize the unique symmetric Bayes Nash equilibrium in increasing bidding strategies when bidders are risk neutral. In contrast to the Texas Shootout, the game is symmetric and its equilibrium is efficient. We also provide necessary and sufficient conditions for a bid function to be a symmetric equilibrium when bidders have general concave utility functions.[2]

A natural starting point when participating in any mechanism is to think about the payoff that one can guarantee for oneself. When dividing a dollar, for example, a sensible divider understands that dividing the dollar into two equal amounts of 50 cents guarantees the divider a payoff of 50 cents and, indeed, this division maximizes the divider's minimum payoff. We show that in symmetric divide-and-choose each player has a unique "maxmin" strategy, i.e., a strategy that maximizes his minimum payoff, which is to bid an amount equal to half his value. If each player follows his maxmin strategy, then the resulting allocation is ex-post efficient and envy free. Furthermore, the allocation that results under maxmin play is related to the Shapley value allocation, which is commonly taken as a benchmark for a fair allocation: If the player with the higher value is the divider, then the "pessimistic" Shapley value allocation obtains, while if the player with the lower value is the divider, then the "optimistic" Shapley value allocation obtains. This result connects maxmin play in symmetric divide-and-choose, a non-cooperative game, to canonically fair solutions dictated by cooperative game theory.

While there are other symmetric mechanisms which generate efficient allocations, an essential and distinguishing feature of the symmetric divide-and-choose mechanism introduced here is that it preserves the structure of a

[2] Both the Texas shoot-out and symmetric divide-and-choose are envy free. In the shoot-out, the allocation is envy free when the divider chooses a price equal to half his value for the partnership. In symmetric divide-and-choose the same price – half the bidder's value – guarantees an envy free allocation.

Texas Shootout, with a divider and a chooser.[3] As de Frutos and Kittsteiner (2008) write "... the buy-sell clause is considered to be such an essential part of the partnership agreements, that a lawyer who fails to recommend to his clients adopting one could be accused of malpractice." Symmetric divide-and-choose is therefore potentially better suited to actual implementation than other mechanisms (e.g., McAfee's Winner's bid auction) that don't preserve the structure of a shootout.

Related Literature

McAfee (1992) studies the Texas Shootout in a Bayesian setting and establishes several key results. First, equilibrium is not ex-post efficient, i.e., the partner who values the partnership the most need not be the one who receives it. Second, there is a payoff disadvantage to being the divider. This second feature has an unfortunate consequence. In practice, the partner who initiates the dissolution is put in the divider role.[4] Since both partners prefer to be the chooser, each prefers that the other initiates dissolution. Thus the partners may engage in a costly war of attrition to determine which partner moves first by naming a price. Brown and Van Essen (2022) model the divider as being chosen by a war of attrition and test the model's prediction with a laboratory experiment. The results support the theory: subjects incur significant waiting costs in order to avoid being the divider.[5]

Khoroshilov (2018) studies partnership dissolution when partners privately know their values, but where the divider, with probability p, knows the chooser's true value for the partnership. It shows that the shootout remains inefficient unless $p = 1$. It also studies, in the same information setting, an auction in which the highest bidder wins the partnership and pays the losing bidder the average of the two bids. It shows that the auction is inefficient unless $p = 0$ or $p = 1$. Hence the presence of an informed partner potentially enhances efficiency in the shootout (i.e., when $p = 1$), but it generally harms

[3]See Cramton, Gibbons, and Klemperer (1987), McAfee (1992), or Van Essen and Wooders (2016), for such mechanisms.

[4]See, for example, Fleischer and Schneider (2012) who provide a legal summary of shoot-out clauses and how they are triggered in practice.

[5]A war of attrition can be avoided by randomly selecting one player to be the divider, but the equilibrium allocation continues to be inefficient, just as in the standard shootout.

efficiency in auctions.

The inefficiency of the Texas Shootout has motivated a search for alternatives. The most relevant to our setting is de Frutos and Kittsteiner (2008) which studies an ascending clock auction in which the first bidder to drop out becomes the divider and receives a payment equal to the price at which he drops. The shootout then commences. The equilibrium of this two-phase mechanism – auction then shootout – is efficient. Here we provide a simple modification of the Texas Shootout that treats players symmetrically and achieves efficiency. In a complete information environment, Nicolò and Velez (2017) obtain interesting results using sequential move fair division games.

The prevalence of the Texas Shootout in practice has led to a growing set of experimental studies of the mechanism in a variety of environments. Kittsteiner, Ockenfels, and Trhal (2013) test the shootout in an independent private values environment. Brown and Velez (2016) test the mechanism in a complete information environment. Brooks, Landeo, and Spier (2010) and Landeo and Spier (2013) study larger dissolution games that incorporate the possibility of shootouts for common value partnerships where only one party is informed. Oechssler and Roomets (2023) examine the shootout in a setting with ambiguity.[6]

In Section 1 we introduce the symmetric divide-and-choose mechanism. In Section 2 we provide the closed-form solution for the risk-neutral Bayes Nash equilibrium. We also provide a necessary condition for equilibrium to be in increasing strategies when bidders are risk averse. In Section 3 we identify the unique maxmin bidding strategy in symmetric divide-and-choose and relate it to maxmin in the Texas Shootout. In Section 4, we connect the outcome under symmetric divide-and-choose to the Shapley value.

[6] Recent studies focus on other aspects of the dissolution process. For example, Hyndman (2021) experimentally studies the role of risk attitudes in a dissolution bargaining game. Fershtman, Szabadi, and Wasser (2023) studies partnership dissolution with potentially unequal claims.

2 The Model

An indivisible item is to be allocated to one of two players, whose private values for the item are independently and identically distributed according to distribution function F, with support $[0, \bar{x}]$ and density $f \equiv F'$, which is continuous and positive. Let x_i denote the value of bidder i. Let x^M denote the median of F, i.e., $F(x^M) = 1/2$. Players have a common vNM utility function u, where $u' > 0$ and $u'' \leq 0$.

In symmetric divide-and-choose, the players simultaneously choose prices p_1 and p_2. Nature then selects one player at random to be the divider and the other to be the chooser. Player i wins the item if $p_i > p_j$. He pays p_i to player j if he is the divider and he pays p_j to player j if he is the chooser. Let $\pi_i(p_1, p_2, x_1, x_2)$ denote the expected utility of player i, when the prices are (p_1, p_2) and the values are (x_1, x_2). For $i, j \in \{1, 2\}$, $i \neq j$, we have

$$\pi_i(p_1, p_2, x_1, x_2) = \begin{cases} \frac{1}{2}u(x_i - p_1) + \frac{1}{2}u(x_i - p_2) & \text{if } p_i > p_j \\ \\ \frac{1}{2}u(p_1) + \frac{1}{2}u(p_2) & \text{if } p_i < p_j. \end{cases}$$

We assume for completeness that if $p_1 = p_2 = p$, then the chooser accepts the divider's price and thus $\pi_i(p_1, p_2, x_1, x_2) = \frac{1}{2}u(x_i - p) + \frac{1}{2}u(p)$. The tie-breaking rule has no impact on any of the results.

In symmetric divide-and-choose, just as in the Texas Shootout, the divider's price determines the price at which the partnership is sold. The chooser is compelled to buy the divider's interest if his price exceeds the divider's price.[7]

3 Equilibrium

We now characterize equilibrium in the symmetric divide-and-choose game. Proposition 1 characterizes the unique symmetric equilibrium of the game when bidders are risk neutral.

[7] This idea of symmetrization is similar to one seen in the early game theory literature in two-person zero-sum games. See, for example, the discussion in Gale (1960, pp. 204-205).

Proposition 1: *Suppose that bidders are risk neutral. The unique symmetric equilibrium in increasing and differentiable bidding strategies of the Symmetric Divide-and-Choose game is given by*

$$\beta(x) = \frac{1}{2}\frac{\int_x^{x^M} q2f(q)\left[\frac{1}{2} - F(q)\right]dq}{\left[\frac{1}{2} - F(x)\right]^2} \text{ for } x \neq x^M/2,$$

and $\beta(x^M) = x^M/2$. *Equilibrium is ex-post efficient.*

Equilibrium in symmetric divide-and-choose is efficient since the highest bidder has the highest value and obtains the partnership, whether selected to be the divider or the chooser.

Proposition 2 gives a necessary and sufficient condition for β to be a symmetric equilibrium in increasing and differentiable bidding strategies when bidders have general concave utility functions.

Proposition 2: *(i) Any symmetric equilibrium β in increasing and differentiable bidding strategies satisfies the differential equation:*

$$u\left(x - \beta(x)\right) - u(\beta(x)) = \beta'(x)\left(\frac{1}{2}u'(x - \beta(x))\frac{F(x)}{f(x)} - \frac{1}{2}u'(\beta(x))\frac{1 - F(x)}{f(x)}\right).$$

(ii) If β is an increasing solution to the differential equation in (i) then it is an equilibrium.

The example below illustrates the efficiency gains of symmetric divide-and-choose over the Texas Shootout, when values are uniformly distributed.

Example 1: If values are distributed $U[0,1]$, then the equilibrium bid function in symmetric divide-and-choose is

$$\beta(x) = \frac{\int_x^{\frac{1}{2}} m(\frac{1}{2} - m)dm}{(\frac{1}{2} - x)^2} = \frac{1}{3}x + \frac{1}{12} \text{ for } x \neq 1/2$$

and $\beta(1/2) = 1/2$. In the Texas Shootout, the equilibrium bid of the Divider with value x_D is

$$\beta_D(x_D) = \frac{1}{4}x_D + \frac{1}{8}.$$

When offered price p, the equilibrium decision for the Chooser with value x_C is

$$\beta_C(x_C, p) = \begin{cases} \text{Buy} & \text{if } p \leq x_C/2 \\ \text{Sell} & \text{if } p > x_C/2, \end{cases}$$

where "Buy" means that the Chooser buys the Divider's interest.

The Texas Shootout is inefficient when either (i) $x_D > x_C$ and $\beta_D(x_D) < x_C/2$, or (ii) $x_D < x_C$ and $\beta_D(x_D) > x_C/2$. The shaded regions in Figure 1 below shows the value profiles (x_D, x_C) for which the equilibrium allocation is ex-post inefficient.

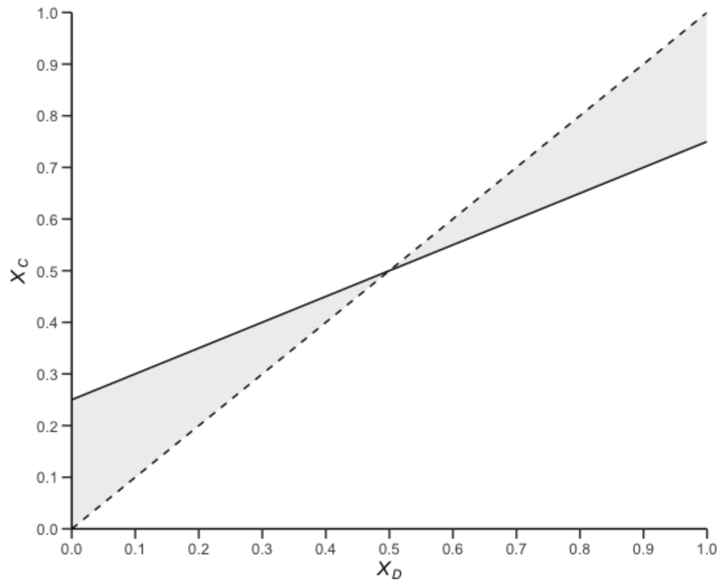

Figure 1

The (expected) surplus lost as a result of the Divider obtaining the partnership when the Chooser has a higher value is

$$\int_0^{\frac{1}{2}} \int_{x_D}^{2\beta_D(x_D)} (x_C - x_D)\, dx_C dx_D = \frac{1}{192},$$

which is also the surplus lost as a result of the Chooser obtaining the partnership when in fact the Divider has a higher value. The expected surplus under symmetric divide-and-choose is 2/3, while the expected surplus if the partnership is not dissolved at all is 1/2. The gains of efficient dissolution are therefore $2/3 - 1/2 = 1/6$. Relative to an efficient mechanism, the share of the surplus forgone as a result of using the Texas Shootout is $(2/192)/(1/6) = 1/16$, i.e., 6.25%. $\triangledown$

4 Maxmin Play

Maxmin play in the Texas Shootout is obvious, which is part of the appeal of the procedure: The Divider with value x for the partnership, by naming a price equal to $x/2$, guarantees himself a payoff of $x/2$. Likewise, the Chooser with value x for the partnership, by choosing their preferred outcome, is also guaranteed a payoff of at least $x/2$. Maxmin play in symmetric divide-and-choose inherits the same structure: A player with value x who bids $x/2$ obtains a payoff of at least $x/2$ regardless of the bid of the other player or whether he is selected to be the divider or the chooser.

Proposition 3: *The unique maxmin strategy in the Symmetric Divide-and-Choose game is* $\gamma(x) = x/2$.

Since maxmin pay guarantees a bidder with value x a payoff of at least $x/2$, the bidder's interim expected equilibrium payoff must also be at least this amount. Hence, each player's interim equilibrium expected payoff is "proportionally fair."[8]

When both players follow their maxmin strategies, the outcome is "envy-free," i.e., neither player prefers the other player's allocation to his own. The divider either obtains the item at price $p = x_D/2$ or receives a payment of $x_D/2$, and is indifferent between these allocations. Since the chooser receives his preferred of these two allocations, he does not envy the divider's allocation.

[8] In an N-player division problem a payoff π is said to be *proportional fair* for a player with value x if $\pi \geq x/N$.

5 Symmetric Divide-and-Choose and the Shapley Value

There is an interesting connection between the allocations achieved by symmetric divide-and-choose and the Shapley value from cooperative game theory. A cooperative game is defined by a set of players and a characteristic function v that gives the value or worth of each possible coalition. In a game with players $\{1, 2\}$ and characteristic function v, Player i's Shapley value ϕ_i is given by

$$\phi_i = \sum_{S \subseteq \{1,2\}} \frac{(|S| - 1)!(2 - |S|)!}{2!} \left[v(S) - v(S \backslash \{i\}) \right].$$

The Shapley value allocation has an interpretation as the fair solution to a cooperative game.

Symmetric divide-and-choose is related to the Shapley value, albeit in a randomized form. Suppose that there is single item to be allocated, for which Ann and Bob have values x_A and x_B, respectively, with $x_A > x_B$. The pessimistic characteristic function, denoted by $\underline{v}$, gives the worst-case worth of a coalition and has

$$\underline{v}(\emptyset) = 0, \underline{v}(A) = 0, \ \underline{v}(B) = 0, \text{ and } \underline{v}(AB) = x_A.$$

In this specification of the characteristic function, a single player can guarantee himself zero, while the coalition of both players has a worth of $\max\{v_A, v_B\} = v_A$. The Shapley values for $\underline{v}$ are $\phi_A = \phi_B = x_A/2$, which equals the maxmin payoffs of symmetric divide-and-choose when Ann is the divider.

The optimistic characteristic function, denoted by $\bar{v}$, gives the best-case worth of a coalition and has

$$\bar{v}(\emptyset) = 0, \bar{v}(A) = x_A, \ \bar{v}(B) = x_B, \text{ and } \bar{v}(AB) = x_A.$$

In this specification, a coalition of a single player has a worth equal to the player's value for the item. The Shapley values for $\bar{v}$ are $\phi_A = x_A - x_B/2$ and $\phi_B = x_B/2$, which are the maxmin payoffs of symmetric divide-and-choose when Bob is the divider.

Hence, under maxmin play, the outcome is always a Shapley value outcome. This result is summarized in Proposition 4.

Proposition 4: *If all players follow their maxmin bidding strategy in the Symmetric Divide-and-Choose game, then the resulting outcome is the Shapley allocation of the pessimistic sharing game when the bidder with the higher value is the divider, and is the Shapley allocation of the optimistic sharing game when the bidder with the higher value is the chooser.*

6 Discussion

Symmetric divide-and-choose retains the familiar structure of the shootout, but induces a game whose equilibrium is ex-post efficient. The timing of the randomization is important. If, for example, the divider and chooser roles are determined randomly but before players choose prices, then the equilibrium is inefficient. Finally, the method of symmetrization used here can be applied to more general divide-and-choose games. This is the subject of ongoing research and should allow more mechanisms to be studied in a Bayesian setting.

7 Appendix

Proof of Proposition 1: Let β be a symmetric equilibrium in strictly increasing and differentiable strategies when bidders are risk neutral. Suppose that Player 2 follows β and Player 1 bids b. If $x_2 \leq \beta^{-1}(b)$, then Player 1 wins the item, he pays b when the divider, and he pays $\beta(x_2)$ when the chooser. If $x_2 > \beta^{-1}(b)$, then receives b when the divider and $\beta(x_2)$ when the chooser. Player 1 rejects Player 2's offer, wins the item, and pays $\beta(x_2)$ to Player 2.

The payoff of Player 1 with value x_1 and bid b, denoted by $\pi_1(b; x_1)$, is

$$\frac{1}{2}\left[\int_0^{\beta^{-1}(b)} (x_1 - b)\, f(x_2)dx_2 + \int_{\beta^{-1}(b)}^{\bar{x}} bf(x_2)dx_2\right]$$
$$+$$
$$\frac{1}{2}\left[\int_0^{\beta^{-1}(b)} (x_1 - \beta(x_2))\, f(x_2)dx_2 + \int_{\beta^{-1}(b)}^{\bar{x}} \beta(x_2)f(x_2)dx_2\right].$$

The first order necessary condition for a maximum is

$$\frac{d}{db}\pi_1(b; x_1) = \frac{1}{\beta'(\beta^{-1}(b))}\left[x_1 - 2b\right] f(\beta^{-1}(b)) + \left[\frac{1}{2} - F(\beta^{-1}(b))\right] = 0.$$

In equilibrium $b = \beta(x_1)$, which yields the differential equation

$$\beta'(x_1)\left[\frac{1}{2} - F(x_1)\right] - 2\beta(x_1)f(x_1) = -x_1 f(x_1).$$

Since $F(x^M) = 1/2$, then $\beta(x^M) = x^M/2$. Multiplying the differential equation on both sides by the integrating factor $\frac{1}{2} - F(x_1)$ yields

$$\frac{d}{dx_1}\left(\beta(x_1)\left[\frac{1}{2} - F(x_1)\right]^2\right) = -x_1 f(x_1)\left[\frac{1}{2} - F(x_1)\right].$$

From the Fundamental Theorem of Calculus we have

$$\beta(x_1)\left[\frac{1}{2} - F(x_1)\right]^2 = -\int_0^{x_1} mf(m)\left[\frac{1}{2} - F(m)\right] dm + C.$$

Since $F(x^M) = 1/2$, at $x_1 = x^M$ the left hand side of the above equation is zero, and hence $C = \int_0^{x^M} mf(m)\left[\frac{1}{2} - F(m)\right] dm$. We have therefore that

$$\beta(x_1) = \frac{\int_{x_1}^{x^M} mf(m)\left[\frac{1}{2} - F(m)\right] dm}{\left[\frac{1}{2} - F(x_1)\right]^2} = \frac{1}{2}\frac{\int_{x_1}^{x^M} m2f(m)\left[\frac{1}{2} - F(m)\right] dm}{\left[\frac{1}{2} - F(x_1)\right]^2}.$$

Proposition 2 establishes this bid function is an equilibrium. □

Proof of Proposition 2: (i) Let β be a symmetric equilibrium in strictly increasing and differentiable strategies when bidders have *vNM* utility function u. Suppose that player 2 follows β and player 1 bids b. The payoff of player 1 with value x_1 and bid b, denoted by $\pi_1^u(b;x_1)$, is

$$\begin{aligned}
&\frac{1}{2}\left[\int_0^{\beta^{-1}(b)} u\left(x_1-b\right) f(x_2)dx_2 + \int_{\beta^{-1}(b)}^{\bar{x}} u(b)f(x_2)dx_2\right] \\
&+ \\
&\frac{1}{2}\left[\int_0^{\beta^{-1}(b)} u\left(x_1-\beta(x_2)\right) f(x_2)dx_2 + \int_{\beta^{-1}(b)}^{\bar{x}} u(\beta(x_2))f(x_2)dx_2\right].
\end{aligned}$$

Differentiating $\pi_1^u(b;x_1)$ with respect to b and substituting $b=\beta(x_1)$ in equilibrium yields the necessary condition

$$u\left(x_1-\beta(x_1)\right)-u(\beta(x_1)) = \beta'(x_1)\left(\frac{1}{2}u'(x_1-\beta(x_1))\frac{F(x_1)}{f(x_1)}-\frac{1}{2}u'(\beta(x_1))\frac{1-F(x_1)}{f(x_1)}\right)=0.$$

This establishes part (i).

(ii) Let $v(x_1,y)$ denote the payoff to a bidder with value x_1 who reports value y, when his rival follows a solution β to the differential equation in part (i) and $\beta'\geq 0$. Then $v(x_1,y)$ is

$$\begin{aligned}
&\frac{1}{2}\left[\int_0^{y} u\left(x_1-\beta(y)\right) f(x_2)dx_2 + \int_{y}^{\bar{x}} u(\beta(y))f(x_2)dx_2\right] \\
&+ \\
&\frac{1}{2}\left[\int_0^{y} u\left(x_1-\beta(x_2)\right) f(x_2)dx_2 + \int_{y}^{\bar{x}} u(\beta(x_2))f(x_2)dx_2\right].
\end{aligned}$$

Part (i) established that

$$\left.\frac{\partial v(x_1,y)}{\partial y}\right|_{y=x_1}=0.$$

Furthermore,

$$\frac{\partial v(x_1,y)}{\partial x_1}=\frac{1}{2}\left[\int_0^{y} u'\left(x_1-\beta(y)\right) f(x_2)dx_2\right]+\frac{1}{2}\left[\int_0^{y} u'\left(x_1-\beta(x_2)\right) f(x_2)dx_2\right],$$

and

$$\frac{\partial^2 v(x_1,y)}{\partial x_1 \partial y}=\frac{1}{2}\left[2u'\left(x_1-\beta(y)\right)f(y)-\int_0^{y} u''(x_1-\beta(y))\beta'(y)f(x_2)dx_2\right]\geq 0,$$

where the inequality follows since $\beta'>0$. Sufficiency of the first order condition follows from McAfee (1994) Lemma 0. □

Proof of Proposition 3: Without loss of generality, consider Player 1 when her value is x_1. We first show that Player 1 guarantees herself a payoff of $x_1/2$ choosing $p_1 = x_1/2$. Suppose $p_1 = x_1/2$. If Player 1 is the divider, then she obtains a payoff of $x_1 - p_1 = x_1/2$ if $p_1 \geq p_2$ and she obtains p_1 if $p_1 < p_2$.[9] If Player 1 is the chooser, then she obtains $x_1 - p_2 > x_1/2$ if $p_1 > p_2$, and she obtains p_2 if $p_1 \leq p_2$.

Next we show that if $p_1 \neq x_1/2$, then Player 1's payoff is in some contingencies less than $x_1/2$. Suppose $p_1 > x_1/2$. If Player 1 is the divider and $p_1 > p_2$, then player's payoff is $x_1 - p_1 < x_1/2$. Suppose $p_1 < x_1/2$. If Player 1 is the divider and $p_2 \in (p_1, x_1/2)$, then Player 1's payoff is p_1. Hence the strategy $\gamma(x) = x/2$ is the unique maxmin strategy. □

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