Erigor dum corrigor: Ptolemy Corrected or Vindicated?

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This was the keynote lecture of the conference "Ptolemy in Perspective: Use and Criticism of his Work from Antiquity to the Present" at Caltech, May 31–June 2, 2007, which gave rise to the book of (almost) the same title published in 2010. For the portraits of Ptolemy mentioned in the first part of the talk I am especially indebted to Robert Derome's website, "Iconography of Ptolemy's Portrait/Iconographie du portrait de Ptolémée", https://rd.uqam.ca/Ptolemy/.

Though perhaps the shaggiest of the many posthumous imaginings of Ptolemy's appearance, the portrait of disputed authorship that adorns the poster for this year's Bacon Conference, one of a series of 28 illustrious men commissioned by Federico Montefeltro, duke of Urbino, is typical of the species. Richly robed and crowned, King Ptolemy contemplates an armillary sphere that he seems on the point of setting aspin on its axis. If he is not meditating upon a toy armillary or another scientific prop, or reading a book, Ptolemy is generally shown at work, observing the heavens with a quadrant or inscribing a star globe or map.

Fig. 1. Juste de Gand and Pedro Berruguete (attrib.), "Ptolemy," Louvre MI 657.


Figs. 2–4. Portrayals of Ptolemy by Jörg Syrlin the Elder (Ulm Cathedral, c. 1470), and from Gregor Reisch, *Margarita Philosophica* (1504) and Regiomontanus, *Epitoma in Almagestum Ptolemaei* (1496).

In contrast to these images of authority, competence, and business as usual, our earliest Ptolemy, on an early Byzantine silver plate, is apparently engaged in a disputation with Hermes. Each is backed by a female figure, the one behind Ptolemy labelled *skepsis*, sensory inquiry; so here, Ptolemy is no regal authority but an advocate in a debate on the right route to knowledge, the senses versus revelation, and all this conducted at the feet of an enthroned judge whose head is missing but whose pose is oddly suggestive of Christ Pantokrator. From the poses we cannot tell whether Ptolemy is having the best of the argument. He will not be seen again in the disputatious mode until the 1632 frontispiece of Galileo's *Dialogue Concerning the Two Chief World Systems*, where he seems for the nonce to be holding his own comfortably against a decrepit Aristotle and a slouching and generously bearded Copernicus.

Fig. 5. Silver plate with Ptolemy and Hermes (6th century), Getty Villa 83.AM.342.

Fig. 6. Frontispiece of Galileo, *Dialogo* (1632).

But a collapse of Ptolemy's fortunes was imminent, and not quite twenty years later we behold in the famous frontispiece of Riccioli's *Almagestum Novum* a Ptolemy, no longer crowned or turbaned, slumped feebly on the ground, with his world system lying cast off at his feet, as Astraea the starry judge weighs in her scales the Copernican system against a modified Tychonic cosmology. Cheered to see the system that retains vestiges of his own geocentrism preponderate, Ptolemy utters the words *Erigor dum corrigor*, "I am set on my feet even as I am amended."

Fig. 7. Frontispiece of Riccioli, *Almagestum Novum* (1561).

These words could well stand, though perhaps best followed by a question mark, as the motto of the conference that is about to begin, an exploration of other people's responses to what one may call Ptolemy's cosmic sciences: astronomy, astrology, and geography. Beyond the general theme of use and criticism of Ptolemy's works, no effort has been exerted to manipulate the individual contributions into a continuous history of Ptolemy's afterlife; the episodic character of the program directly reflects the spread of problems that the invited speakers judged to be fresh and interesting. Hence, for

example, we may not hear much about what came of Ptolemy's *Planetary Hypotheses* or *Handy Tables*, an omission that would be scandalous if we were aiming to say the first and last word on Ptolemy's impact on later astronomy. But we can expect flashes of intense illumination on pretty well every stage in the generation and corruption of Ptolemy the scientific author—beginning indeed at a time when he was still a young and unknown man making astronomical observations, and before anyone could have turned to his works for theory or tables, and concluding about a century from our own time, when his thought had long ceased to be an agent of current scientific change and the carcase of his writings was being worked over by the maggots of historical scholarship, such as are we.

Since we are going to spend the next two days looking at reflected and refracted images of Ptolemy, it may be appropriate to begin by attempting to inspect him by direct vision—even if, as he himself teaches us in the *Optics*, our mental constructions of the objects we look at are prone to error in proportion as our souls interpret and supplement the reliable but restricted information that we receive from outside. Ptolemy's literal meaning is that our vision apprehends only certain kinds of information objectively: colours, angles between lines of sight that ultimately reach objects, and the lengths of these lines of sight, information that, even if it was exact (which it is not) would leave a great deal unsaid about the objects themselves. Figuratively, Ptolemy's surviving writings stand to us in relation to our perception of Ptolemy the scientist somewhat as he would say that the visually apprehended colours, angles, and distances stand in relation to our perception of a pomegranate.

We can take the analogy a step further. Anyone who has spent a long time with Ptolemy has met someone who questioned whether all the works attributed to him can really be by the same author; this seems to happen most often with people who come to the *Tetrabiblos* after discovering the *Almagest*, but in my experience almost any work in the corpus can give an intelligent person the impression, at least in parts, of representing a manner of thought so unlike that of the *Almagest*—which as it were *defines* Ptolemy—that it must be a false attribution. Now whatever one might make of the content of the various works, there are very strong stylistic arguments for their common authorship. In particular, practically all the works that survive in Greek under Ptolemy's name, including the *Criterion*, the *Harmonics*, the *Tetrabiblos*, the *Geography*, the *Almagest*, Book 1 of the *Planetary Hypotheses*, the *Phaseis*, and the *Analemma* are linked to each other by a surprising number of habitual words and phrases that have nothing to do with their subject matter and that are found nowhere else in Greek literature, or only again in late antiquity, usually in authors such as Theon and Hephaestion who manifestly picked these tics up from immersing themselves in Ptolemy. So unless Ptolemy himself had a profoundly fragmented mind, our perceptions of him based on one or another individual work must be behaving like Ptolemy's account in the *Criterion* of our sensory perceptions of complex and composite objects, which sometimes confound and obstruct one another at the time of immediate impression but are sometimes—only sometimes—sorted out into an accurate and consistent account through subsequent mental processes of combination and reconsideration. Such cases, he admonishes us, admit of doubt and false description.

With this warning in mind, I make no pretense of offering a portrait of Ptolemy in the round, but I do think it is possible to gain a deeper and more internally coherent understanding of the workings and motivations of his individual writings by relating them to certain aspects of his thought that are not dwelt on comprehensively in any one work but that cut across them. There is an obvious element of risk in this kind of intellectual reconstruction: it presumes that statements in separate texts arose out of a consistent and well-thought-out point of view about how the world works. At some level of detail this assumption certainly loses validity. His changes of mind can be documented among the precise numerical parameters and model structures of his astronomy as we move from the *Canobic Inscription* to the *Almagest* and on to the *Handy Tables* and *Planetary Hypotheses*. Concerning certain larger

issues of astronomy Ptolemy appears at the least to have arrived in his later works at a more focused view than he held at, say, the time of writing the *Almagest*. This is true also in his other sciences. His geographical conceptions gained in both breadth and detail between the *Almagest* and the *Geography*; the *Optics* disagrees with certain pronouncements on visual matters in his other works; and we can perhaps detect clues that the astrological applications he envisions for certain kinds of computed data in the *Almagest* are not entirely compatible with the *Tetrabiblos*. But the most fundamental assumptions seem to have remained constant from the earliest to the latest works. It is no surprise that they are in great part utterly foreign to our own ways of explaining the world; but in important respects they also appear to have been less typical of Ptolemy's own time than one might have supposed.

Let me begin with what everyone knows about Ptolemy's cosmology, that he presumes a basically Aristotelian spherical geocentric cosmos partitioned into an outer shell of aetherial matter, the heavens, and an enclosed sphere composed of the four elements earth, water, air, and fire. As so often with Ptolemy, we are easily lulled by the subsequent enduring grip of this conception into assuming that Ptolemy was sticking to a conventional "ancient" notion, but this is not so. While the four Empedoclean elements were the common stuff of non-Atomistic philosophies from the fourth century B.C. on, Aristotle's celestial fifth element, with its property of eternal incorruptibility and its distinctively circular natural motion, was received sceptically or downright rejected even by Aristotle's Peripatetic associates. Stoics and Epicureans agreed in holding that the heavens consist of the same kinds of materials as the world immediately around us, and that the same physical processes are at work there as here. When it comes to Hipparchus and other Hellenistic astronomers, our evidence for their views on the physical nature of the heavenly bodies is negligible.

The earliest hint that someone was again taking Aristotle's fifth element seriously is that the first century B.C. Peripatetic Xenarchus wrote a work attacking the concept and Aristotle's arguments for it. There are in fact many indications that this was the time when the Corpus Aristotelicum, the body of Aristotle's writings roughly corresponding to the ones that we have through the medieval tradition, was for the first time widely read and studied. It remains a matter of dispute whether the reported "edition" of Aristotle's works by Andronicus of Rhodes helped to initiate this new interest in the texts. But it is clear that from this time right through to late antiquity the people who paid most attention to the Corpus Aristotelicum were Platonists. Aristotle was regarded as an accessible avenue to the understanding of Plato's philosophy, with particular value for his presentation of the physical and perceptible world; the way to the *Timaeus* was by way of the *Physics* and *De Caelo*. It is in this intellectual context that we should situate the regeneration of Aristotle's aetherial fifth element.

The Middle Platonists did not merely dust off the cosmology of *De Caelo*, for they were aware that the astronomical models of Eudoxus and Callippus that supplied Aristotle with the geometrical structure of his cosmology were no longer considered tenable. The Platonist response is known to us through an undervalued document, Theon of Smyrna's *Mathematics Useful for Reading Plato*. Theon was a decidedly third-rank philosopher datable by the cut of his hair and beard on his fortuitously surviving portrait bust to the generation before Ptolemy; but for his astronomy Theon drew slavishly on a lost but influential *Timaeus* commentary by Adrastus of Aphrodisias, which can thus hardly have been written much after A.D. 100. Adrastus was acquainted with Hipparchus' solar model in both its eccentric and epicyclic forms, and he understood that, in terms of the sun's resulting geometrical path in space and time, the two model types were equivalent, though his efforts to prove this are rather shaky, at least as Theon retails them. Adrastus further tells us—and I see no reason to disbelieve him—that Hipparchus expressed a special liking for the epicyclic model on the grounds that it was plausible that the heavenly bodies should be alike in a condition of equilibrium and should "hang together" in the same way.

But, he continues, Hipparchus eschewed physical interpretations and thus missed the true philosophical reason for preferring the epicyclic model over the eccentric one. Up to this point Adrastus has exposed the models using diagrams representing epicycles and eccentres as bare circles in the plane of the ecliptic, very much like those in *Almagest* Book 3. But now he explains that an epicycle is really to be conceived as a spinning solid sphere of the fifth element lodged inside a spherical shell, made of the same divine stuff, that is concentric with the earth and revolves around it. This physical model, he believes, is concordant with Aristotle's insistence that the natural motions of the fifth element in the heavens must be concentric with the centre of the cosmos, whereas no acceptable physical instantiation of an eccentric model would be conceivable.

This remarkable passage invites several remarks. First, we should note his explicit testimony that Hipparchus did *not* explain the eccentric or epicyclic models in terms of three-dimensional bodies constituting an aetherial plenum. Secondly, Adrastus is trying, not entirely successfully, to hold on to the *De Caelo* celestial physics—the aether, the plenum, the natural motion in circles concentric with the cosmos—as a fully valid framework for the realities behind a mathematical astronomy that has moved on since Eudoxus. Thirdly, though he has read Hipparchus, he is by no means up to date on planetary theory, for the entirety of which he believes that simple epicyclic models will suffice.

We have no difficulty in recognizing in Adrastus' embedded solid epicycles the same basic conception of the makeup of the heavens that Ptolemy elaborates in Book 2 of the *Planetary Hypotheses*. I think, moreover, that there are several passing remarks in the *Almagest* that indicate that when he wrote that treatise he already took this kind of three- dimensional physical modelling for granted as a general proposition, even if in his models for the planets' latitudes he appears to lapse into thinking of epicycles as flat disks. On the aetherial, eternally unalterable, and divine nature of the celestial matter Ptolemy leaves us in no doubt from the very first pages of the *Almagest*. These are things, moreover, that he expects his reader to accept implicitly.

Ptolemy must have got his fifth-element cosmology from the Platonists. But he was not himself a Platonist in the sense that we speak of Theon of Smyrna, or even of Galen, as a Platonist, and his adoption of the Platonist version of Aristotle's distinction between celestial and mundane physics has to be seen as a deliberate appropriation, not an adherence to the intellectual traditions of a sect. Ptolemy never adduces either the *Timaeus* or *De Caelo* as a touchstone of truth. A remarkable manifestation of his independence is his treatment of the problem of the natural motion of the elements. In the *Almagest* the natural motion of mundane bodies in a straight line towards the centre of the cosmos is presented as an empirically verified fact, to search out the causes of which would be a waste of time, whereas it is taken as axiomatic that the aetherial bodies in the heavens by nature move uniformly in circles, though significantly Ptolemy says not a word about concentricity with the cosmos. By the time that he wrote Book 2 of the *Planetary Hypotheses*, however, Ptolemy had arrived at a more coherent para- Aristotelian theory of natural motion, with which he was sufficiently pleased so that he stated it, according to Simplicius, also in a part of the *Optics* that has not come down to us and again in a lost book *On the Elements*. This theory distinguished the situation when a body composed of an element is outside its "natural place", in which case the body tends to travel in a straight line, evidently towards its natural place, unless it is obstructed; and on the other hand the situation when a body is in its natural place, in which case it either is stationary or revolves. This law applies to all five elements, so that the aetherial bodies in the heavens, being in their natural place, revolve freely whereas the mundane elements strive rectilinearly towards their natural places, which would be either the centre or the periphery of the sublunar cosmos. Thus Ptolemy dispenses with the Aristotelian dichotomy according to which the fifth element's natural motion has to be perpendicular to the natural motion of the mundane elements, and hence concentric with the cosmos itself. And so Adrastus' rationale for

banishing eccentrics from the heavens vanishes. And where did Ptolemy get his law of natural motion? According to Simplicius, it was to be found in the Perepatetic Xenarchus' *attack* on the theory of the fifth element. Again Ptolemy has no concern with the uses, good or bad, that his predecessor made of a principle, so long is it serves to complete his own system.

Let us think through how Ptolemy's law works in practice. The heavy elements earth and water have the centre of the cosmos as their natural place, whereas the natural place of fire and air is the outer surface of the sublunary part of the cosmos. The spherical mass of the earth, thought of as a unitary body, occupies its natural place; so far as Ptolemy's law dictates, the earth has no tendency to motion from place to place, but either stands still or spins. In fact, it is stationary, but we must turn to the partly empirical, partly *a priori* reasonings of *Almagest* Book 1 chapter 8 to establish that. Small independent bodies in our immediate surroundings, close to the surface of the solid terrestrial globe, are remote from their natural places no matter what they are composed of, and thus have a natural tendency to move in straight lines either towards or away from the centre. Up around the boundary between the sublunary and celestial regions is more or less pure spherical shell of fire or fire and air, which, taken as a whole, is in its natural place and thus free either to stand still or to spin. Lastly, the vastly larger part of the cosmos beyond the boundary comprises a large number of exactly contiguous bodies of aether, all in their natural place, and thus free to stand still or, if their geometrical forms allow it, to rotate.

Note that Ptolemy's law does not in any way explain why the aetherial bodies in the heavens do spin or why the earth does not, and in this respect it does less than Aristotle's doctrine of natural motion. For a body to rotate, as Ptolemy conceives it, there must be the absence of certain impediments: obviously there cannot be a physical obstacle to the rotation, so either the body must have axial symmetry or be surrounded by yielding matter; it must not have an overriding tendency to some other natural motion; and it must not have an overriding tendency to stand still either. Ptolemy's law only deals with the second condition. The third condition arises because Ptolemy believes that the denser and heavier elements have a proportionately greater innate resistance to being set or kept in motion, while aether alone is perfectly unresistant; hence the *a priori* absurdity of imagining that the earth makes a daily rotation while the heavens stand still. But even an aetherial body endowed with axial symmetry and situated in its natural place in the superlunary region will not spin just because it is in the right place, has the right shape, and is made of the right stuff.

It may be asked, then, whether Ptolemy's law says *anything* positive about the behaviour of *aether*? Surprisingly, it does, because of another—and perhaps the most audacious—of Ptolemy's divergences from the *De Caelo* cosmology. For Ptolemy does not hold that aether is only to be found in its natural place in the heavens; it is present here on earth. My first item of evidence to justify ascribing this astonishing notion to Ptolemy is, on the face of it, one we might hesitate to rely too much on. The eleventh- century Byzantine scholar Symeon Seth was the author of a book called *Synopsis of Physics*, an eclectic survey of subjects pertaining to cosmology, meteorology, astronomy, natural philosophy, and psychology, drawing on various ancient authorities. He has a series of chapters on the five senses, the first of which, on sight, reviews Aristotle's theory of vision as a transmission of colour from object to eye and the visual ray theory of the "geometers and many of the philosophers," but he judges the demonstrations of the geometers to be "more powerful" because capable of explaining reflection and refraction. And then he closes the chapter with the assertion that "Ptolemy says in the *Optics* that the optic *pneuma* is something aetherial and composed of the fifth element."

No such statement is discernable in the incomplete and sometimes rather distorted surviving twelfth-century Arabo-Latin version of the *Optics* by the Admiral Eugene, but that is not surprising since fundamental questions of the physical basis of vision would have belonged in the lost Book 1.

Now it is by no means out of the question that a copy of the complete *Optics* in Greek could have been around in Byzantium as late as the eleventh century; there are other documented instances of classical Greek texts surviving that late only to disappear soon after, and it is no counterargument that a century later in Sicily Eugene only knew of the *Optics* in its mutilated Arabic form. But Symeon's immediate source may just as well have been a lost second-hand report.

From Galen and other medical authors we know that the optic *pneuma* was a fluid, mobile, and vital material through which the soul imparts the function of vision to the eye, and moreover the *opsis* or visual ray that the eye sends forth to meet external objects in the process of vision is an extension of this *pneuma*. Hence the doctrine that Symeon ascribes to Ptolemy is that a specific part of us, closely connected with our soul, is composed of, or at least contains, *aether*. Now why would Ptolemy make such a strange proposition? I think part of the answer has to do with another thing that Ptolemy is reported to have included in the *Optics* but that cannot be found in the Latin torso, namely a statement of his law concerning the natural motion of the elements. One would have difficulty imagining a context in the *Optics* in which falling or levitating or revolving bodies would be at issue. But if the visual ray is composed of aether, aether moreover that is not at home in its celestial natural place, Ptolemy's law would explain the rectilinear geometry of the ray while the ideal lightness and mobility of the material would explain the imperceptible rapidity of the ray's projection from the eye to even the most astronomically remote object. We know, moreover, that Ptolemy discussed in Book 1 of the *Optics* how the visual ray is closely related in its nature to rays of light, and since the source of light *par excellence* is the sun and the other heavenly bodies, one can even hazard a good guess as to the mechanism by which Ptolemy would have suggested that *aether* comes to be present in the sublunary world.

Lest we should reject all this as a flimsy edifice of reckless speculation founded on the shakiest of testimony, let us look at Ptolemy's most extensive declaration of what the soul is. This account of a strikingly materialistic, physical psychology is to be found in the *Criterion*, and the passage deserves to be quoted at some length: Now we see that even among the simple elements out of which mixtures arise, earth and water are more material and in general passive, while fire and air are more capable of inciting change and are both passive and active, and *aether* is both always in the same condition and solely active. And since among mixtures we apply the name "body" specifically to that which is more material and less energetic, and the name "soul" to that which is capable of inciting change in both the other and itself, it is logical that the body should be organized according to the elements of earth and water, and the soul according to those of fire and air and *aether*. And it is logical that soul is mixed with body to a great degree in proportion as the body has more of heat and moisture, and to a small degree in proportion as it has more of cold and dryness. Hence the soul's power is not present in tendons and bones and all the cold and earthy parts, but it is present in flesh and blood and those parts that have a greater share of moisture and heat. The corollary of these things would be that the very substance of the soul has some differentiation corresponding to the elements that compose it, so that in proportion as it is of air and fire, it is passive and active in the changes that pertain to it, and in proportion as it is of *aether*, it solely activating.

Presently, we learn that this solely activating, predominantly aetherial component of the soul is its intellective, reasoning faculty, and it is concentrated in the head and closely connected with the sensory faculties. This part is preeminently the *hegemonikon*, the faculty of the soul that is "the chief cause of living well." Ptolemy's crowning word on it is that "the intellective part is, potentially and actually, more worthy of honour and more divine, in both the universe and in us, and its place is the highest, namely the heavens for the cosmos, and the head for a human being."

Ptolemy tends to write sentences like this, featuring elevated tone and self- consciously balanced parallels, when he is winding up a programmatic argument, in particular towards the close of an introductory chapter. They are the sublime truths that paradoxically set the reader in the proper frame for study of a technically detailed argument. Perhaps none of these protreptic statements gets so close to the heart of Ptolemy's beliefs about science as this one.

In the first place, it brings us back to a question that we left in suspense earlier on: what is the cause of the celestial revolutions? The first section of Book 2 of the *Planetary Hypotheses* gives Ptolemy's answer, in the context of a working out of a detailed structural cosmology of the heavens that tacitly but unmistakeably frames itself point by point in rivalry with Aristotle's famous chapter on the homocentric spheres in *Metaphysics* Lambda. The mobile spheres, spherical shells, disks, and drums of aether that make up each visible heavenly body's model constitute an animated organism, the soul of which, seated in the visible body, imparts organized motion directly to each component, exactly as the soul of an animal imparts organized motion to its own limbs and organs. These souls are not Aristotelian prime movers, since a prime mover would be an intellect contemplating only itself and unconcerned with the corporeal motions that it unconsciously causes. The analogies that Ptolemy draws between the celestial movements and the actions of mundane living things make sense only if the celestial souls deliberately and rationally choose—though they must choose for all eternity—the complex of individual uniform revolutions performed by the various aetherial bodies corresponding to the eccentres and epicycles of the *Almagest*'s two-dimensional diagrams. Though divine, the souls of the planets are not gods.

To get at the significance of this kinship between human souls and celestial souls, we need to pull back and approach Ptolemy's cosmology from another direction. The opening chapter of the *Almagest* famously endorses Aristotle's partition of theoretical philosophy into theology, mathematics, and physics, and at the same time subverts Aristotle's *a priori* privileging of theology by evaluating the parts of theology according to the criterion of the knowledge attainable by human beings. Ptolemy's definition of mathematics here is interesting because, although it resembles certain of Aristotle's statements about the scope of mathematics, Ptolemy has made the distinction between the objects of mathematics and the objects of physics exactly fit Aristotle's distinction between the primary and secondary objects of sense perception in *De Anima*: the qualities of bodies that the senses directly apprehend, such as white and hot and sweet, being closely tied to the underlying matter, are dealt with by physics, while shape, size, number, and motion, which are perceived secondarily, as it were as patterns in the primary objects of sense, belong to mathematics, and their properties and relations can be deduced rationally through geometrical and number-theoretic argument. Ptolemy makes claims in this chapter that mathematics, and mathematics alone, yields secure knowledge, but these claims ought not to be taken in isolation as an assertion that all the results obtained in the *Almagest* are exact and infallible. I think that in this passage he misrepresents his own more considered views about human knowledge and science in order not to distract with qualifications the distinction he wishes to draw between physics and mathematics. States and processes of bodies with respect to colour and flavour and hardness and so on, he believes, are by nature not regular or subject to exact description. An apple will not rot in just the same way, and a pot of water will not always take the same time to come to a boil, no matter how carefully one recreates the conditions. But the moon's diameter at apogee does always subtend the same angle from the centre of the earth, and the sun does always take the same time to travel from equinox to equinox. We cannot reason exactly and securely about the former kinds of phenomena, because the regularity such reasoning presumes simply does not exist. We *can* reason exactly and securely about the length of the year and the moon's breadth up to a point, subject to the limits of accuracy and discrimination of our senses. Usually Ptolemy qualifies his pronouncements

about the certainty of any knowledge derived from empirical data with phrases like "so far as human faculty extends", *kata ge tên anthrôpinên dynamin*, and indeed the whole immense logical structure of the *Almagest* is built on recursive mathematical methods.

There is an interesting difference in Ptolemy's handling of the issue of exact results in the *Almagest* and the *Harmonics*, which follows partly from the distinction between *geômetria* and *arithmetikê*, geometry and number theory, and partly from the kinds of sensory evidence available for harmonic and astronomical research. The *Harmonics* is a study of the mathematical relations that underly intervals of pitch in music. The validity of the book hinges on an extended argument at the beginning that seeks to demonstrate that musical intervals, that is, the intervals between notes that musicians produce in practice and that auditors accept as esthetically pleasing, are not merely quantitative relations but specifically ratios between whole numbers. Moreover, one's perception that a heard interval is concordant is not a measurement but a kind of irreducible and irrefutable sensory truth; a nearly exactly tuned interval of a fifth, for example, will reliably be sensed as a concord pure and simple, which implies that there is a kind of discreteness about the fact of concordance that can withstand the inevitable modest imprecisions of human production and perception. Consequently, when Ptolemy shows that the fifth is modelled by the ratio of three to two, there is absolutely no question of the exactness of this result. No refinement of experimental technique could ever change that to a ratio of 3.001 to 2.

None of the numerical results stated in the *Almagest* are like that. The mean motions representing the rates of uniform revolution of the celestial bodies are "ugly" numbers written with series of sexagesimal fractional places, the rough equivalent of ten decimal places, and Ptolemy explicitly discusses their status as approximations that could be made better, but never exact, through longer and longer intervals of observation. Spatial dimensions such as epicycle radii and eccentricities are generally rounded to simpler-looking fractions, but Ptolemy appears to do this just for the sake of computational convenience and not because he expects to find simple whole number relations in astronomy. The motions of the heavenly bodies are essentially continuous geometrical and temporal magnitudes, and their orderliness is not expressible in numbers. From a modern point of view this seems like the obvious route to take, but I suspect that for Ptolemy it was a hard-won decision. We know that there were tendencies in Hellenistic and early Roman period astronomy to impose neat numerical relations on astronomical parameters; the Keskintos Inscription with its numerologically derived Great Year comprising whole numbers of planetary cycles is only the most elaborate instance. Ptolemy himself toyed with such relations in the *Canobic Inscription*: the lunar and solar distances expressly set at the sixth powers of two and three respectively, and the eccentricities of Mercury, Venus, and the sun set at 1/24, 1/48, and 1/24 of the deferent radius. It would have been easy for Ptolemy to massage more relations of this kind out of his data, but instead he weeded most of them out between the erection of the inscription and the publication of the *Almagest*.

This is not to forget the mutilated final section of Book 3 of the *Harmonics*, in which Ptolemy maintains that the structural features of his models of musical intervals have analogues in the heavens. I believe that the real point of these chapters, as also of the preceding ones that try to demonstrate such analogies between harmonics and the human soul, is missed if we castigate Ptolemy for undertaking an inherently absurd project and then approaching it by patently hand-waiving arguments. Looking at his works as a whole, we find that Ptolemy employs, even in the exact sciences, many kinds of argument besides strict mathematical reasoning applied to empirical data. For example, the model structures in the *Almagest* are not wholly determined by empirical considerations: simplicity dictates the adoption of an eccentre rather than an epicycle for the sun, and analogy leads to the extension of certain features empirically deduced for one planet's model to those of the others. In Book 1 of the

Harmonics certain key steps of Ptolemy's progressive modelling of musical tuning systems depend on a dialectic appeal to "the good". And Ptolemy also has a counterpart of Dirichlet's Box, or the Pigeonhole Principle: if two sets of entities have been shown to correspond, and one set is shown to exhibit a certain structure, the same structure will be found in the second set. This type of argument, which is particularly fruitful in transferring structure back and forth between the empirical phenomena of musical intervals and the theoretical properties of whole number ratios, is also at work on a larger scale in Book 3 of the *Harmonics*. Chapters 3 and 4 show, through an analysis of science and its objects that parallels that of *Almagest* Book 1 chapter 1 but goes deeper, that *harmonia* is "a form of rational explanation, specifically that which regards commensurability in motions," and moreover that *harmonia* so defined—that is, no longer specifically tied to sounds—must be present to some degree in any being that can initiate movement of its own accord, and especially in rational beings. But in Ptolemy's cosmos there are two kinds of rational beings: people, and heavenly bodies. It follows that whatever we have learned about harmonic structures in sound in the first parts of the *Harmonics* can be expected to apply, *mutatis mutandis*, in human souls and the heavenly bodies, even if, as Ptolemy appears to accept, the harmonic structures in souls and planets do not manifest themselves in ways that our senses can measure quantitatively.

Having defined theology in the narrowest Aristotelian sense in *Almagest* Book 1 chapter 1 as the study of the first cause of the first motion of the universe, Ptolemy dismisses it as guesswork; and though he follows with a concession that astronomy, by studying divine and perceptible beings, might give some hint about imperceptible divinity, his notions of cosmos and science really have no role for god or gods. Nowhere does he suggest that the cosmos as a unit is rationally or deliberately organized. If we take the claims of *Harmonics* Book 3 seriously, rational structures, that is to say mathematical structures, are foreign to inanimate nature, and are only really dominant in connection with reasoning souls, whether mundane or terrestrial. I suspect that in some measure Ptolemy thought of the orderliness and *knowability* of the heavens as something imposed on aetherial matter by the planetary souls, a sort of geometrical music running independently of the harmonic music of the heavens that the *Canobic Inscription* and the *Harmonics* explore. It is a conductorless ensemble performing in perfect counterpoint; and the mathematician is allowed to listen, leaving the physicist outside in a world of senseless noise.